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USING A NEW METHOD BASED ON FINSLER GEOMETRY FOR WIND SPEED MODELLING

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Abstract

Accurately modelling of wind speed is very important for the assessment of wind energy potential of a certain region. Before the installation of a wind energy conversion system in a region, the wind speed potential of that region needs to be determined and modelled. For this reason, different distribution functions such as two-parameter Weibull, Gamma, Lognormal, Rayleigh etc. are proposed for accurately modeling wind speed in the literature. In this paper, new probability and cumulative probability density functions based on Finsler geometry are proposed for wind speed modelling. Two-dimensional Finsler space metric function is obtained for Weibull distribution. Monthly analysis for Yalova, Turkey is realized using a new method based on Finsler geometry and two-parameter Weibull distribution. Wind data, consisting of hourly wind speed records between October 2015-September 2016 were obtained from the Yalova station of Turkish State Meteorological Service. The performances of the models are given comparatively by using root mean square error (RMSE).

Key words: *Finsler Geometry, Wind Speed, Modelling, Weibull Distribution, Renewable Energy*

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1. Introduction

Energy is an essential part of our daily lives. Clean form of energy production in parallel with the needs of today's growing demand for power is of great importance. For this reason, the use of renewable energy resources are increasing rapidly. Consequently, it is foreseen that the share of renewable energy in the electricity generation should reach 20% by the year 2020 and the main part of this renewable energy was assigned to wind energy that should be 12% [1].

The detailed knowledge of wind characteristics should be investigated while determining the wind energy potential for the selected region [2]. Several distribution functions were proposed for wind speed modeling in literature. The most commonly used distribution functions for modelling are the log-normal distribution, the inverse Gaussian distribution, the wake by, three-parameter log normal, the gamma distribution, two-parameter gamma distribution, hybrid distributions, the three parameter generalized gamma distribution etc. [3-17]. It can be seen that the wind speed characteristics for lots of regions are fitted to the two-parameter Weibull distribution.

In this paper, new probability and cumulative probability density functions based on Finsler geometry are proposed for wind speed modelling. Finsler geometry allows accurate modelling and describing ability for asymmetric structures. First the two-dimensional Finsler space metric function is obtained for Weibull distribution and then the metric definition for two-parameter Weibull probability density function which has shape (k) and scale (c) parameters in two-dimensional Finsler space is realized using a different approach by Finsler geometry. Based on all these, new probability and cumulative probability density functions based on Finsler geometry are proposed for wind speed modelling. The use of the new model based on Finsler geometry is analysis comparatively for a sample region that is Yalova, Turkey. Two-parameter Weibull distribution function structure is presented in Section 2. Finsler metrics that have two-parameter Weibull distribution function family of curve and their geodesics are given for nonnegative different n numbers, comparatively in Sect. 3. Finally, conclusions are given in Sect. 4.

2. Two-Parameter Weibull Distribution

Two-parameter Weibull distribution is widely used for wind speed modeling. In this section, the structure of the two-parameter Weibull distribution function will be discussed on the real world problem such as the wind speed distribution which has a nonlinear structure in the asymmetric platform.

Two-parameter Weibull distribution is one of the widely used statistical methods in the modeling of wind speed data. The Weibull distribution function is given by Equation (1) [18-24]: Materials, equipments, devices, mathematical formulas etc. used in study, and methodology can present at this section.

$$f(\mathrm{v}) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^{k}}, \tag{1}$$

where f(v) is the frequency or probability of occurrence of wind speed v, c is the Weibull scale parameter with unit equals to the wind speed unit (m/s) and, k is the unitless Weibull shape parameter. The higher value of c indicates that the wind speed is higher, while the value of k shows the wind stability [25]. The cumulative Weibull distribution function F(v) gives the probability of the wind speed exceeding the value v. It is expressed by Equation (2) [26-27]:

$$\mathrm{F(v)} = 1 - e^{-\left(\frac{v}{c}\right)^{k}}. \tag{2}$$

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3. Finsler Geometry

To construct the geometry in non-flat spaces in order to understand nature has great importance in terms of applied science. Finsler geometry allows accurate modelling and describing ability for asymmetric structures in this application area.

In a two-dimensional space, a continuous function $F: TM \to [0,\infty)$ is called a Finsler metric on a C^∞ manifold M if it satisfies the following conditions:

i. $F(\mathrm{x},\mathrm{y};\dot{\mathrm{x}},\dot{\mathrm{y}})$ is C^∞ on $TM\setminus\{0\}$.

ii. $F(\mathrm{x},\mathrm{y};\lambda\dot{x},\lambda\,\dot{\mathrm{y}}) = \lambda\,\mathrm{F}(\mathrm{x},\mathrm{y};\dot{x},\dot{\mathrm{y}}), \lambda > 0.$

iii. $g_{ij}(\mathrm{x},\mathrm{y};\dot{x},\dot{\mathrm{y}})$, the fundamental metric tensor is positive define. Where (x,y) denotes the coordinates of $p \in M$ and $(\mathrm{x},\mathrm{y};\dot{x},\dot{\mathrm{y}})$ denotes the local coordinates of $(\dot{x},\dot{\mathrm{y}}) \in T_pM$.

On the purpose of determination of Finsler metrics and their geodesics in two-dimensional Finsler space belonging to two-parameter Weibull distribution that has scale (k) and shape (c) parameters are made calculations as following based on Matsumoto article [30].
Fundamental metric function is given by [30]:

$$\mathrm{L}(\mathrm{x},\mathrm{y},\dot{x},\dot{\mathrm{y}}) = \dot{x}\int_0^z (\mathrm{z}-\mathrm{t})\,\mathrm{H}(\mathrm{t},\mathrm{y}-\mathrm{tx})\,\mathrm{dt} + \dot{x}E_x + \dot{y}\,\mathrm{E}_y. \tag{3}$$

Different Finsler metrics for two-parameter Weibull distribution function was obtained for n arbitrarily nonnegative real number by Dokur et al. [31] choice of $\mathrm{H}(\alpha,\beta) = \beta^n$ in Equation (3). Hence, with the selection of $\mathrm{H}(\mathrm{z},\mathrm{y}-\mathrm{zx}) = \beta^n = (y-zx)^n$ and E=constant in Equation (3), metric function that has Weibull distribution is obtained in the form of

$$L(\mathrm{x},\mathrm{y},\dot{\mathrm{x}},\dot{\mathrm{y}}) = \frac{y^{(\mathrm{n}+2)}\dot{x}}{\mathrm{x}^2}\sum_{k=0}^{n}\binom{n+2}{k+2}\left(-\frac{x\dot{y}}{y\dot{x}}\right)^{(k+2)}. \tag{4}$$

It can be easily seen that the obtained function provides the Finsler metric conditions.
L_n and G_n^i, respectively, metric function defined in n value and spray coefficients, for integer selection of n=2 are calculated by [31] as

$$L_2(\mathrm{x},\mathrm{y},\mathrm{p},\mathrm{q}) = \frac{q^2x^2\left(q^2x^2 - 4pqxy + 6p^2y^2\right)}{x^2p^3} \tag{5}$$

Spray coefficients for Finsler metrics related to these equations are found by [31]

$$G_2^{\,1} = \frac{p^2q\left(-qx+4py\right)}{q^2x^2 - 4pqxy + 6p^2y^2}$$
$$G_2^{\,2} = \frac{pq^2\left(-qx+4py\right)}{q^2x^2 - 4pqxy + 6p^2y^2} \tag{6}$$

Same calculation steps are repeated for arbitrary positive rational numbers, n = 1 / 2,11 / 12 , we get

$$L_{1/2}(\mathrm{x},\mathrm{y},\mathrm{p},\mathrm{q}) = \frac{15q^2x^2\sqrt{y}}{8x^2p}$$
$$L_{11/12}(\mathrm{x},\mathrm{y},\mathrm{p},\mathrm{q}) = \frac{805q^2x^2y^{11/12}}{288x^2p}. \tag{7}$$

Spray coefficients are found

$$G_{1/2}{}^{1} = G_{11/12}{}^{1} = 0$$
$$G_{1/2}{}^{2} = \frac{q^2}{8y} \tag{8}$$
$$G_{11/12}{}^{2} = \frac{11q^2}{48y}.$$

Substituting spray coefficients in Equation (8) to Equation (7), we obtain second order differential equation of y with respect to x.

$$y'' = K\frac{y'^2}{y} \tag{9}$$

where K is a coefficient dependent of n . It is apparent that K are -1/4,-11/24 for $n = 1/2, 11/12$, respectively. In this case, it can be seen relation between n and K is $K = -\frac{1}{2}n$. For all non-negative rational numbers, when the differential equation in Equation (9) is solved,

$$y = \left(C_2 x + \frac{2}{n+2}C_1\right)^{\frac{2}{n+2}} \tag{10}$$

is found. Where C_1 and C_2 are the integration constants. After applying some mathematical calculations, new two-parameter cumulative function is given as follow [31]:

$$F_{new}(\mathrm{v};\mathrm{C}_1,\mathrm{C}_2) = 1 - e^{-\frac{2}{n+2}v^{C_2}e^{\frac{2}{n+2}C_1}} \tag{11}$$

setting $a = \frac{2}{n+2}$, it is rewritten in the form

$$F_{new}(\mathrm{v};\mathrm{C}_1,\mathrm{C}_2) = 1 - e^{-av^{C_2}e^{aC_1}}. \tag{12}$$

Probability density function is calculated by $f_{new} = \frac{dF_{new}}{dv}$

$$f_{new}(\mathrm{v};\mathrm{C}_1,\mathrm{C}_2) = aC_2 e^{a(\mathrm{C}_1 - \mathrm{V}^{C_2}\mathrm{e}^{aC_1})} v^{C_2 - 1}. \tag{13}$$

While the solution of the differential equation that is obtained by using the arbitrary values of non-negative integer n gives the same geodesics as the two-parameter Weibull function, new function that is defined non-negative rational numbers of n is derived for two-dimensional family of curve.

4. Wind Speed Modelling

Wind speed data, consisting of hourly wind speed records between October 2015-September 2016 were obtained from the Yalova Meteorological Stations. The geographical data and wind speed period for these region are given in Table 1.

Table 1. The geographical data and wind data period of sample region

Station	Latitude (ºN)	Longitude (ºE)	Altitude (m)	Wind Data Period
Yalova	40º 39'	31º 29'	30	October 2015-September 2016

Analysis of wind speed modeling is realized using the new probability density function based on Finsler geometry for all months of Yalova region. The parameters of new function is calculated for different cases of n by boundary value problem. When n is chosen as the natural number, the model gives same results with Weibull distribution (WD). All monthly analysis results is given in Table II. n=11/12 is better results than the other choice of n, according to the performance criteria.

Probability and cumulative probability density of wind speed models are given comparatively for January in Figure 1. As seen in Fig.1, new probability and cumulative probability density functions based on Finsler geometry are better performance for choosing n=11/12 than two-parameter Weibull distribution functions.

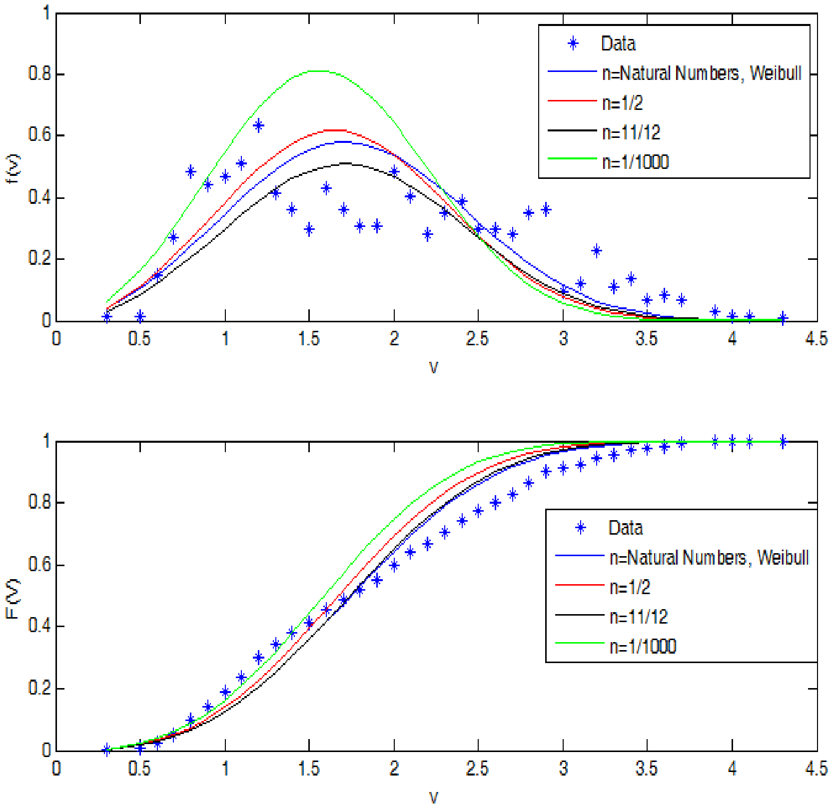

Fig. 1. Probability density and cumulative probability density function for sample month using proposed function based on Finsler geometry.

As seen in Table 2 and Fig. 1, the proposed new distribution function based on Finsler geometry gives more accurate results than Weibull distribution which is commonly used wind speed modelling. Especially, it is foreseen that more accurate models for different regions can be realized by choosing of n and estimation of parameters.

As a result, it is aimed to propose more accurate models by using this novel approach than the models which have two-parameter Weibull probability density function, especially used for determination of wind energy potential of a region.

5. Conclusion

To construct the geometry in non-flat spaces in order to understand nature has great importance in terms of applied science. Finsler geometry allows accurate modelling and describing ability for asymmetric structures in this application area. With the help of Finsler geometry's modelling ability of physical phenomena that are genuinely asymmetric and/or non-isotropic more accurate modeling can be achieved.

In this paper, a sample application is given for the proposed new probability and cumulative probability functions based on Finsler geometry. According to analysis results, new probability and cumulative probability density functions based on Finsler geometry are better performance for choosing n=11/12. It is expected that the proposed Finsler metric based function can be applied many different regions for wind speed modeling. It is foreseen that the performed analysis using this function will bring a new approach to the literature.

Table 2. The results of comparative monthly analysis of using Finsler geometry.

Months	Finsler Geometry Method	Yalova		
		C_1	C_2	RMSE
January	n=Natural Number, WD	2.9184	1.9795	0.1300
	n=0.5	-2.0957	2.9776	0.1418
	n=11/12	-2.2370	2.9771	0.1253
	n=1/1000	-1.8916	2.9704	0.2088
February	n=Natural Number, WD	3.1464	2.3995	0.2366
	n=0.5	-3.0911	3.2338	0.2323
	n=11/12	-2.2840	4.3197	0.2257
	n=1/1000	-2.7565	3.1466	0.2457
March	n=Natural Number, WD	3.5536	2.0998	0.2344
	n=0.5	-2.9033	3.6659	0.2306
	n=11/12	-3.0750	3.6219	0.2257
	n=1/1000	-2.5388	3.6603	0.2727
April	n=Natural Number, WD	3.3804	2.1255	0.3229
	n=0.5	-2.8032	3.4792	0.3183
	n=11/12	-2.9942	3.4542	0.3153
	n=1/1000	-2.4672	3.4718	0.3440
May	n=Natural Number, WD	3.8679	1.8359	0.3398
	n=0.5	-2.5048	4.0153	0.3424
	n=11/12	-2.6198	3.9636	0.3224
	n=1/1000	-2.2203	4.0091	0.4022
June	n=Natural Number, WD	3.3339	2.0970	0.3063
	n=0.5	-2.6889	3.4753	0.2992
	n=11/12	-2.8948	3.4666	0.3025
	n=1/1000	-2.3747	3.4751	0.3275
July	n=Natural Number, WD	3.9369	2.0775	0.2799
	n=0.5	-3.2017	4.0814	0.2873
	n=11/12	-3.5337	4.0938	0.2699
	n=1/1000	-2.7559	4.0584	0.3655
August	n=Natural Number, WD	3.7333	2.2730	0.2733
	n=0.5	-3.4655	3.8547	0.2677
	n=11/12	-3.8425	3.8665	0.2749
	n=1/1000	-2.9701	3.8280	0.3059
September	n=Natural Number, WD	3.2977	2.3663	0.3399
	n=0.5	-3.1978	3.3758	0.3344
	n=11/12	-3.5081	3.2712	0.3464
	n=1/1000	-2.2408	3.9710	0.3680
October	n=Natural Number, WD	3.9863	2.0373	0.3287
	n=0.5	-3.1434	4.1359	0.3315
	n=11/12	-3.4656	4.1486	0.3189
	n=1/1000	-2.7094	4.1125	0.3866
November	n=Natural Number, WD	3.5409	2.0948	0.3865
	n=0.5	-2.8807	3.6527	0.3827
	n=11/12	-3.0537	3.6112	0.3755
	n=1/1000	-2.5214	3.6474	0.4072
December	n=Natural Number, WD	3.3205	2.0985	0.3333
	n=0.5	-2.6938	3.4172	0.3299
	n=11/12	-2.8822	3.3998	0.3270
	n=1/1000	-3.4108	3.6474	0.8945

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