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Analytical solution of the (2+1)-dimensional Zoomeron equation by rational sine-Gordon Method

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Abstract
The current study is about the solution of the Zoomeron equation, one of the important models of mathematics and physics. In this study, the rational Sine-Gordon expansion method (RSGEM) is used to obtain various analytical solutions of the model. Compared to other methods, this method is quite effective and the desired results were obtained. Although there are many analytical solutions to the model used in the literature, we present rational type solutions for the first time with this method. We obtained rational hyperbolic function solutions, and also classified all soliton solutions (kink-like, kink, singular kink, anti-kink, dark, bright). In addition, geometric representations of the solutions in two-, and three-dimensional space and contour shape are made with the Mathematica software program.

Keywords: *RSGEM; Zoomeron equation; Analytical method*

Rasyonel sine-Gordon metodu ile (2+1) boyutlu Zoomeron denkleminin analitik çözümü

Öz

Mevcut çalışma, matematiğin ve fiziğin önemli modellerinden biri olan Zoomeron denkleminin çözümü ile ilgilidir. Bu çalışmada, modelin çeşitli analitik çözümlerini elde etmek için rasyonel Sinüs-Gordon açılım yöntemi (RSGEM) kullanılmıştır. Kullanılan modelin literatürde birçok analitik çözümü bulunmasına rağmen, bu yöntemle ilk kez rasyonel tip çözümler sunuyoruz. Rasyonel hiperbolik fonksiyon çözümleri elde ettik ve ayrıca tüm soliton çözümlerini (kıvrımlı benzeri, kıvrımlı, tekil kıvrımlı, ters kıvrımlı,

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koyu, parlak) sınıflandırdık. Ayrıca, Mathematica yazılım programı ile çözümlerin iki ve üç boyutlu uzayda ve kontur şeklinde geometrik gösterimleri yapılmıştır.

Anahtar kelimeler: *Rasyonel Sine-Gordon açılım metodu, Zoomeron denklemi, Analitik metot*

1. Introduction

Differential equations are important mathematical models because they are used to represent problems in various fields such as science and engineering. The capacity of differential equations to model intricate systems is one of their most important advantages. It is frequently impossible to comprehend a system's behavior without the use of differential equations. Differential equations describe natural life problems; for instance, they are employed in fluid dynamics to simulate the movement of fluids through pipes and other structures. And they are also used to simulate how gases and other materials behave under various conditions.

Non-linear partial differential equations have wave solutions such as soliton, compaction etc. and these solitons make it easier to create physical interpretations of problems.

Solitons are strengthening their wave bundles that keep their structure as they travel at a constant velocity in various environments. Solitons appear when nonlinear and dispersive effects in the medium are removed. Dispersive effects are characteristics of some systems in which the frequency of a wave affects its speed. Solitons are also physical system solutions to several nonlinear dispersive PDEs. They have been observed in water waves, the study of solid-state, plasma physics, particle physics, biological systems, nonlinear optics, and etc. [2,3,9,27].

There are techniques for solving partial differential equations in addition to ordinary differential equations, although these techniques may not work for all equations. Finding an approach to solving the provided partial differential equation is crucial in this situation. This is particularly valid for non-linear equations. The numerical method can be applied in certain situations in which the analytical method is inapplicable. This method solves mathematical problems on a computer-aided level [4,5,11,12,14,16,17,20,22-31,34-36].

In this study, we use the RSGEM, which is an analytical method. The method considers the solutions of the nonlinear partial differential equation (NPDE) as polynomials of trigonometric functions. To facilitate the search for these polynomials in the next step, we use a transformation with the help of the sine-Gordon (SG) equation. The literature shows that this method is quite effective and leads to the desired results [8,15,19,32,33].

2. A brief introduction to the RSGEM method

In order to proceed to the main part of this study, we must present some conclusions about the characteristics of the method we will use. These features have also been reported in the literature [8,15,19,32,33].

Considering the SG equation as

$$\varphi_{xx} - \varphi_{tt} = m^2 \sin(\varphi), \tag{2.1}$$

where $\varphi = \varphi(x,t)$, m is a constant number. By applying $\varphi = \varphi(x,t) = \Phi(\xi)$, $\xi = \mu\left(x - ct\right)$ transform to (2.1), we have

$$\Phi'' = \frac{m^2}{\mu^2\left(1-c^2\right)}\sin\left(\Phi\right), \tag{2.2}$$

where $\Phi = \Phi(\xi)$, μ and c are physical parameters. Then, we can see easily

$$\left[\left(\frac{\Phi}{2}\right)'\right]^2 = \frac{m^2}{\mu^2\left(1-c^2\right)}\sin^2\left(\frac{\Phi}{2}\right) + C, \tag{2.3}$$

here C is an integration constant different from above c. By replacing $C = 0, \varpi\left(\xi\right) = \frac{\Phi}{2}$ and $a^2 = \frac{m^2}{\mu^2\left(1-c^2\right)}$ in (2.3) gives,

$$\varpi' = a\sin\left(\varpi\right), \tag{2.4}$$

taking $a = 1$ in Eq.(2.4), we have easily

$$\varpi' = \sin\left(\varpi\right). \tag{2.5}$$

If we apply the separation of variables, a classical method in the theory of differential equations, to (2.5), we easily obtain the following equations.

$$\sin(\varpi) = \sin\left(\varpi\left(\xi\right)\right) = \left.\frac{2pe^{\xi}}{p^2 e^{2\xi}+1}\right|_{p=1} = \sec h(\xi), \tag{2.6}$$

$$\cos(\varpi) = \cos\left(\varpi\left(\xi\right)\right) = \left.\frac{p^2 e^{2\xi}-1}{p^2 e^{2\xi}+1}\right|_{p=1} = \tan h(\xi). \tag{2.7}$$

Now, we will consider a NPDE in general form as

$$P\left(\varphi, \varphi_x, \varphi_t, \varphi_{xx}, \varphi_{tt}, \varphi_{xt}, \varphi_{xxx}, \varphi_{xxt}, \ldots\right) = 0, \tag{2.8}$$

$$N\left(\Phi, \frac{d\Phi}{d\xi}, \frac{d^2\Phi}{d\xi^2}, \ldots\right) = 0. \tag{2.9}$$

The solution form for Equation (2.8) in SGEM is given below.

$$\Phi\left(\xi\right) = \sum_{i=1}^{N} \tanh^{i-1}\left(\xi\right)\left[b_i \sec h\left(\xi\right) + a_i \tanh\left(\xi\right)\right] + a_0. \tag{2.10}$$

If we rewrite (2.9) with (2.6) and (2.7), we have

$$\Phi\left(\varpi\right) = \sum_{i=1}^{N} \cos^{i-1}\left(\varpi\right)\left[b_i \sin\left(\varpi\right) + a_i \cos\left(\varpi\right)\right] + a_0. \tag{2.11}$$

It would be more appropriate to look for solutions rationally because mathematically rational functions are more general than polynomial functions. Consequently, the wave solutions we are searching for will have the shape that we want. Let us consider the following solution form [8,15,19,32,33],

$$\Phi(\xi)=\frac{\sum_{i=1}^{N}\tanh^{i-1}(\xi)\left[a_i \sec h(\xi)+c_i\tanh(\xi)\right]+a_0}{\sum_{j=1}^{M}\tanh^{j-1}(\xi)\left[b_j \sec h(\xi)+d_j\tanh(\xi)\right]+b_0.}, \tag{2.12}$$

which is also written as

$$\Phi(\varpi)=\frac{\sum_{i=1}^{N}\cos^{i-1}(\varpi)\left[a_i \sin(\varpi)+c_i\cos(\varpi)\right]+a_0}{\sum_{j=1}^{M}\cos^{j-1}(\varpi)\left[b_j \sin(\varpi)+d_j\cos(\varpi)\right]+b_0}. \tag{2.13}$$

$a_i, b_i, c_i, d_i, a_0, b_0$ are constants which will be obtained in next steps. It appears that not all constants are zero simultaneously. At this stage, we consider the balance principle between the nonlinear term with the highest power and the highest derivative in the NPDE. Then M and N are specified. The next step consists of a set of algebraic equations for $a_i, b_i, c_i, d_i, a_0, b_0$. We solve this system and find the values using the software. Finally, we put them into (2.13) and get the new travelling wave solutions for (2.8).

3. Application of RSGEM method

The aim of this section is to present the solution of the Zoomeron equation, which has an important place in the class of NPDE, using the RSGEM method. Let us consider Zoomeron equation

$$\left(\frac{U_{xy}}{U}\right)_{tt}-\left(\frac{U_{xy}}{U}\right)_{xx}+2\left(U^2\right)_{xt}=0. \tag{3.1}$$

A well-liked model for illustrating the new phenomena associated with boomerons and trappons, the Zoomeron equation is also frequently employed to explain the development of a single scalar field [1,10,13,18,21]. Historically, this equation has always attracted the attention of people working in the field of applied mathematics. The reason for this is that it provides the desired results by some methods and also represents a physical event as a model. However, in recent times, this type of equations has been defined and the desired solutions have been obtained [1,10,13,18,21]. Let us look briefly at the literature on this equation. In [1], author gave explicit traveling wave solutions the Zoomeron equation, by using the another method, in [18.32], it was shown that the solution of the Zoomeron equation was related to the dromion solutions of Davey-Stewartson-III equation.
Assume that the wave transformation is in the form

$$U(x,y,t)=u(\xi),\ \xi=x+\eta y-wt \tag{3.2}$$

where η, w are constants and we obtain value of them later.
By using (3.2), we obtain the partial derivative of U respect to *x, y* and *t* and accordingly inserting them in (3.1), we have an ordinary differential equation

$$\eta(w^2-1)\left(\frac{u''}{u}\right)''-2w(u^2)''=0 \tag{3.3}$$

Integrating (3.3) two times with respect to η and getting the integration constant to zero, then we obtain

$$\eta(w^2-1)u''-ku-2wu^3=0 \tag{3.4}$$

Regarding the balance between the nonlinear term u^3 and the highest order term u'' in (3.3), we obtain easily that $M=1$ and $N=1$. Therefore, we can write the solutions of this equation as

$$\Phi(\xi)=\frac{a_1\sec h(\xi)+c_1\tanh(\xi)+a_0}{b_1\sec h(\xi)+d_1\tanh(\xi)+b_0},$$

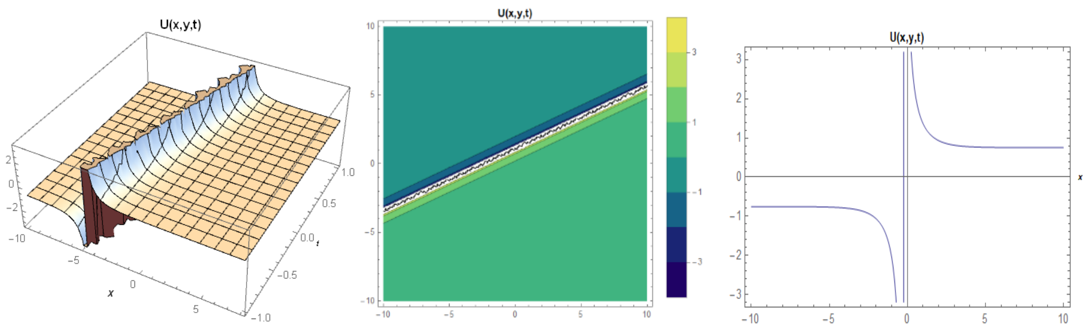
In below, we get diagrams of the wave solutions for different cases in 2D and 3D. These graphs are very important to better understand the physical meaning of both the solution and the equation. It is possible to draw graphs when different constants are considered.

CASE 1: Regarding the coefficients' values,

$$a_0=\frac{\sqrt{-1+w^2}\sqrt{\eta}d_1}{2\sqrt{w}},a_1=\frac{\sqrt{-1+w^2}\sqrt{\eta b_1^2+d_1^2)}}{2\sqrt{w}},c_1=0,b_0=0,r=-\frac{1}{2}(-1+w^2)\eta\,.$$

We determine the solution and the accompanying contour, 2-D, and 3-D graphs.

$$u(x,y,t)=\frac{\sqrt{-1+w^2}\sqrt{\eta}Cosh(tw-x-y\eta)d_1+\sqrt{b_1^2+d_1^2}}{2\sqrt{w}\left[b_1-Sinh(tw-x-y\eta)d_1)\right]}$$



a **b**

Figure.1. 3-D,2-D and contour graphs for a; η=1.3, w=2.2,b1=0.7,d1=0.5 and for b; y=1, t=1.

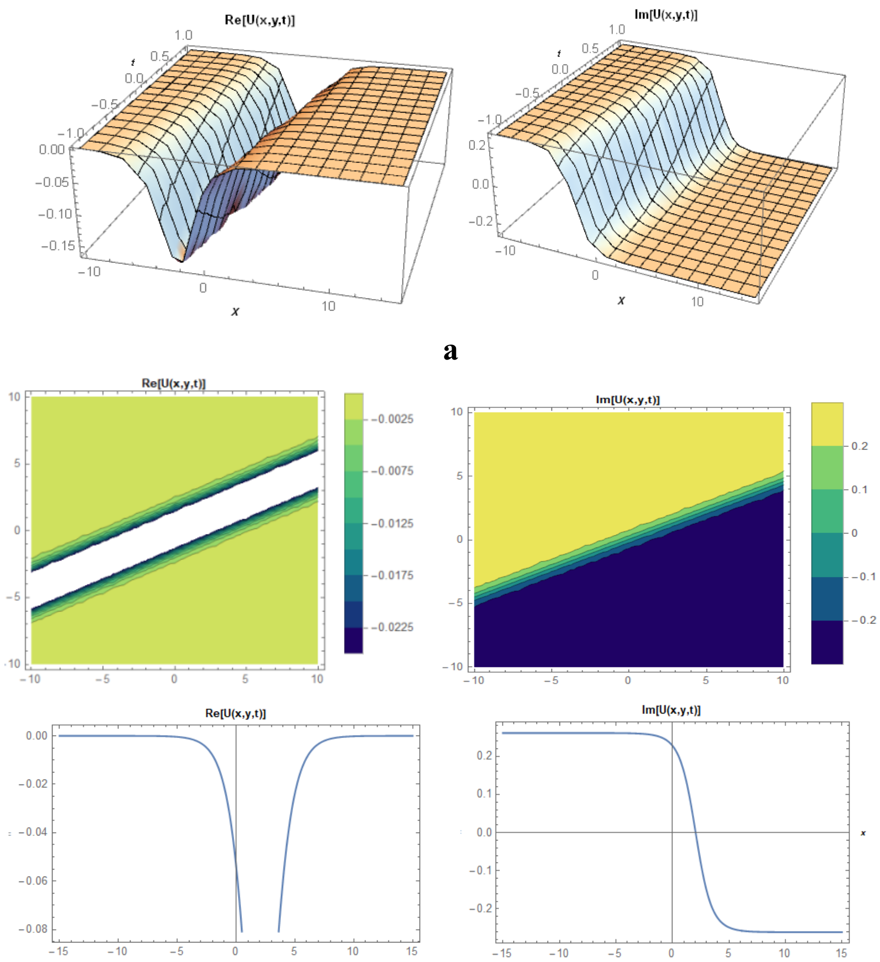
CASE 2

Regarding the coefficients' values

$$a_0=\frac{i\sqrt{r}d_1}{2\sqrt{w}},a_1=\frac{\sqrt{r}\sqrt{b_0^2-b_1^2-d_1^2}\sqrt{\eta}\left(b_1^2+d_1^2\right)}{\sqrt{2w}},c_1=\frac{i\sqrt{r}b_0}{\sqrt{2w}},\eta=-\frac{2r}{-1+w^2},$$

We determine the solution and the accompanying contour, 2-D, and 3-D graphs.

$$u(x,y,t)=-\frac{\sqrt{r}\left(-i\,\mathrm{Sin}\,h\left(tw-x+\frac{2ry}{-1+w^{2}}\right)b_{0}+i\,\mathrm{Cosh}\left(tw-x+\frac{2ry}{-1+w^{2}}\right)d_{1}+\sqrt{b_{0}^{2}-b_{1}^{2}-d_{1}^{2}}\right)}{\sqrt{2w}\,\mathrm{Cosh}\left(tw-x+\frac{2ry}{-1+w^{2}}\right)b_{0}+b_{1}-\mathrm{Sin}\,h\left(tw-x+\frac{2ry}{-1+w^{2}}\right)d_{1}},$$



b

Fig.2. Graphs for a; r=0.3, w=2.2, b_0=1.7, b_1=0.7, d_1=0.5 and for b; y=1, t=1.

CASE 3

Regarding the coefficients' values as

$$a_{0}=-\frac{\sqrt{\eta\left(-1+w^{2}\right)}d_{1}}{2\sqrt{w}},a_{1}=\frac{\sqrt{\eta\left(-1+w^{2}\right)}\sqrt{-b_{0}^{2}+b_{1}^{2}+d_{1}^{2}}}{2\sqrt{w}},c_{1}=-\frac{\sqrt{\eta\left(-1+w^{2}\right)}b_{0}}{2\sqrt{w}},r=-\frac{1}{2}(-1+w^{2})\eta,$$

We determine the solution and the accompanying contour, 2-D, and 3-D graphs.

$$u(x,y,t)=-\frac{\sqrt{\eta\left(-1+w^{2}\right)}\left[\left(d_{1}-\mathrm{Sech}\left(x+\eta y-wt\right)\right)\sqrt{-b_{0}^{2}+b_{1}^{2}+d_{1}^{2}}+b_{0}\,\mathrm{Tanh}\left(x+\eta y-wt\right)\right]}{2\sqrt{w}\left[b_{0}+\mathrm{Sech}\left(x+\eta y-wt\right)b_{1}+d_{1}\,\mathrm{Tanh}\left(x+\eta y-wt\right)\right]},$$

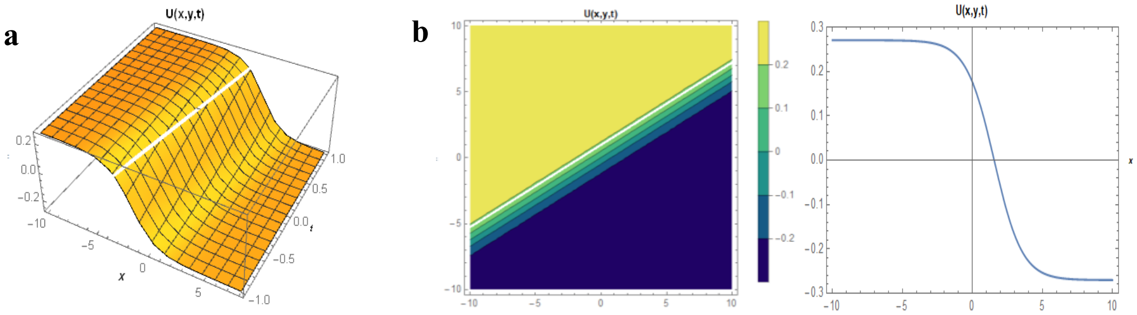


Figure.3. Graphs for a; η=0.3, w=1.6, b_0=0.7, b_1=1.1, d_1=1.5 and for b; y=1, t=1.

CASE 4

Regarding the coefficients' values as,

$$a_0 = -\frac{c_1 d_1}{b_0}, a_1 = \frac{i c_1 \sqrt{b_0^2 - b_1^2 - d_1^2}}{b_0}, \eta = -\frac{4 w c_1^2}{(-1+w^2) b_0^2}, r = -\frac{2 w c_1^2}{b_0^2}.$$

We determine the solution and the accompanying contour, 2-D, and 3-D graphs.

$$u(x,y,t) = -\frac{c_1 \left(d_1 + i \,\mathrm{Sech}\left(x+\eta y - wt\right)\right)\sqrt{b_0^2 - b_1^2 - d_1^2} + b_0 \,\mathrm{Tanh}\left(x + \eta y - wt\right)}{b_0 \left[b_0 + \mathrm{Sech}\left(x+\eta y - wt\right) b_1 + d_1 \,\mathrm{Tanh}\left(x+\eta y - wt\right)\right]}.$$

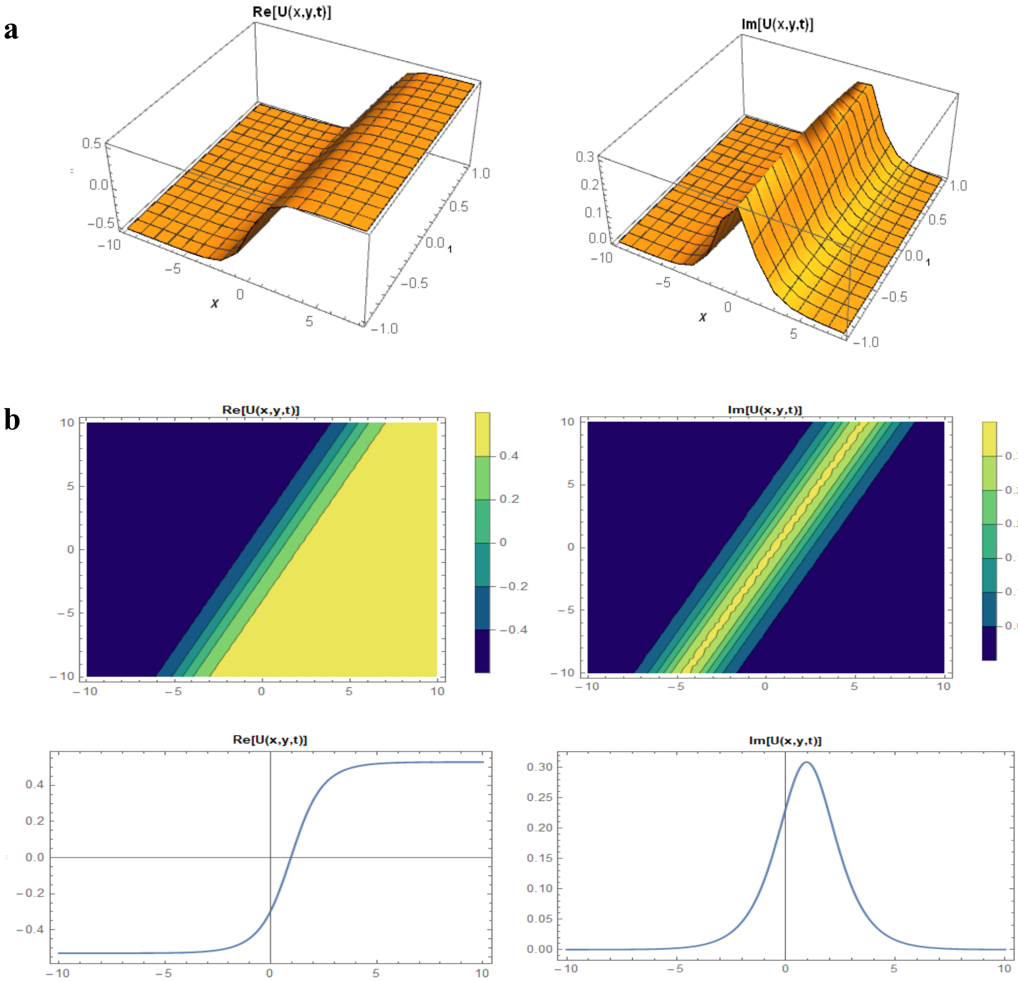


Figure.4. Graphs for a; w=0.5, b_0=1.7, b_1=0.8,d_1=0.5, c_1=0.9 and for b; y=1, t=1 .

CASE 5

Regarding the coefficients' values as

$$a_0 = 0, d_1 = 0, b_0 = -\frac{2\sqrt{w}c_1}{\sqrt{-1+w^2\eta}}, b_1 = \frac{2\sqrt{w\left(a_1^2+c_1^2\right)}}{\sqrt{-1+w^2\eta}}, r = -\frac{1}{2}\left(-1+w^2\right)\eta.$$

We determine the solution and the accompanying contour, 2-D, and 3-D graphs.

$$u(x,y,t) = \frac{\sqrt{\left(-1+w^2\right)\eta}\left(a_1 + c_1 \operatorname{Sin}h\left(x+\eta y - wt\right)\right)}{-2\sqrt{w}\,c_1 \operatorname{Cos}h\left(x+\eta y - wt\right) + 2\sqrt{w\left(a_1^2+c_1^2\right)}}.$$

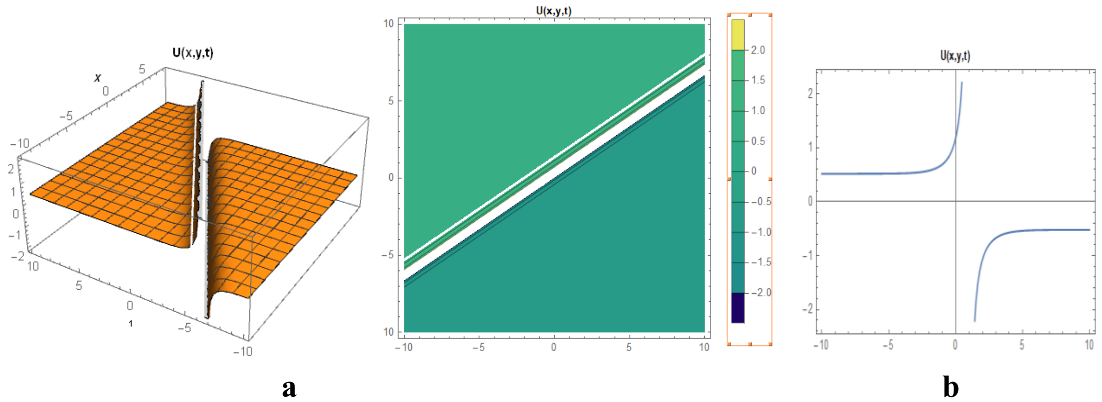


Figure.5. Graphs for a; η=1.3; w=1.5; a_1=1.3; c_1=1.6 and for b; y=1; t=1.

CASE 6

Regarding the coefficients' values as,

$$a_0 = \frac{\sqrt{w\left(a_1^2+c_1^2\right)}}{w}, b_0 = \frac{2\sqrt{w}c_1}{\sqrt{-1+w^2\eta}}, b_1 = 0, d_1 = \frac{2\sqrt{w\left(a_1^2+c_1^2\right)}}{\sqrt{-1+w^2\eta}}, r = -\frac{1}{2}\left(-1+w^2\right)\eta.$$

We determine the solution and the accompanying contour, 2-D, and 3-D graphs

$$u(x,y,t) = \frac{\sqrt{(-1+w^2)\eta}\sqrt{w}\operatorname{Sech}\left(x+\eta y - wt\right)\left(a_1 + c_1 Sinh\left(x+\eta y - wt\right)\right) + \sqrt{w(a_1^2+c_1^2)}}{2wc_1 + 2\sqrt{w^2(a_1^2+c_1^2)}\operatorname{Tanh}\left(x+\eta y - wt\right)}.$$

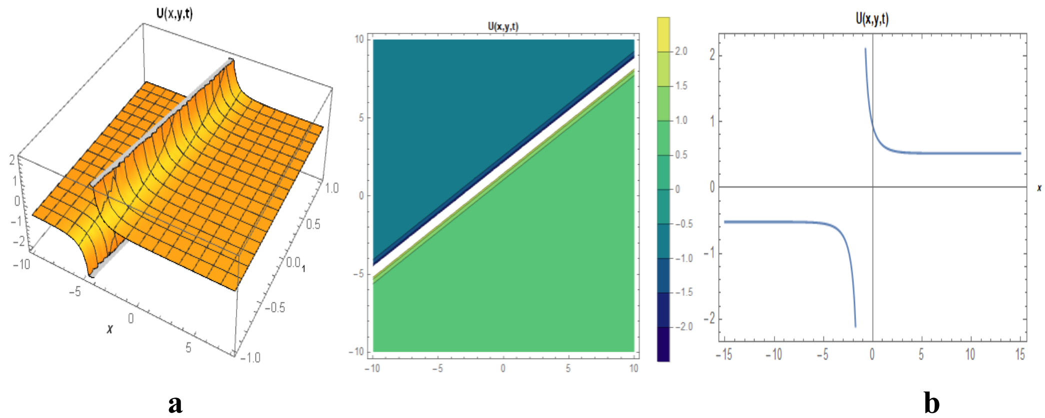


Figure.5. Graphs for a η=1.3, w=1.5,a_1=0.3, c_1=0.6 and for b y=1,t=1.

4. Conclusion

In this study, we examine a few new soliton solutions to the zoomeron equation. We concluded that certain new solutions to the Zoomeron problem are obtained by applying the efficient RSGEM approach, which uses the Mathematica program. We illustrated 2D, 3D, and contour simulations to more accurately depict the physical characteristics of the derived wave solution. Figure 1 represents kink-like soliton solution. For figure 2, the real part is dark soliton and the imaginary part is singular kink solitons. Figure 3 has an anti- kink wave soliton solution. Figure 4 represents singular kink soliton and bright soliton for the real part and the imaginary part of the solution, respectively. Figure 5 and 6 show solitary wave solutions. We used Wolfram Mathematica-12 to verify each solution. This method is a powerful and easy to apply method that can produce a wide range of different types of solutions to such mathematical models. The RSGEM has unveiled novel soliton solutions with applications in engineering and mathematical physics. The obtained soliton solutions may be useful in providing an expanded comprehension of the nonlinear physical phenomena described by the governing equation. The concept employed in this work can be used to solve further partial differential equations and various fractional models, in mathematical physics. Moreover, multiple solitons, rogue waves, breathers, bifurcation analysis can be studied and explored.

References

[1] Abazari, R.: The solitary wave solutions of Zoomeron equation. **Appl. Math. Sci**. 5(59), 2943–2949 (2011)

[2] Ablowitz, M.J., Clarkson, P.A.: Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform. **Cambridge University Press, Cambridge** (1991)

[3] Arshad, M., Seadawy, A.R., Lu, D.: Bright-dark solitary wave solutions of generalized higher-order nonlinear Schrödinger equation and its applications in optics. **J. Electromagn. Waves Appl**. 31, 1711–1721 (2017)

[4] Ata E. Kıymaz O. New generalized Mellin transform and applications to partial and fractional differential equations, **International Journal of Mathematics and Computer in Engineering**, 1(2023)

[5] Ata, E., & Kıymaz, İ. O. Generalized Gamma, Beta and Hypergeometric Functions Defined by Wright Function and Applications to Fractional Differential Equations. **Cumhuriyet Science Journal**, 43(4), 684-695, (2022)

[6] Baskonus, H.M., Sulaiman, T.A., Bulut, H.: On the new wave behavior to the Klein–Gordon–Zakharov equations in plasma physics. **Indian J. Phys.** 93(3), 393–399 (2019)

[7] Baskonus, H. M., Bulut, H., Atangana, A. On the complex and hyperbolic structures of the longitudinal wave equation in a magneto-electro-elastic circular rod. **Smart Materials and Structures**, 25(3), 035022 (2016)

[8] Baskonus, H.M., Bulut, H., Sulaiman, T.A.: New complex hyperbolic structures to the Lonngren-wave equation by using sine-Gordon expansion method. **Appl. Math. Nonlinear Sci**. 4(1), 141–150 (2019)

[9] Bulut, H., Sulaiman, T.A., Baskonus, H.M.: Dark, bright and other soliton solutions to the Heisenberg ferromagnetic spin chain equation. **Superlattices Microstruct**. 123, 12–19 (2018)

[10] Degasperis, A., Rogers, C., Schief, W.K.: Isothermic surfaces generated via Bäcklund and Moutard Transformations: Boomeron and Zoomeron connections. **Stud. Appl. Math**. 109, 39–65 (2002)

[11] Dokuyucu M. A., Çelik E., Bulut H., Baskonus H. M., Cancer treatment model with the Caputo-Fabrizio fractional derivative, **The European Physical Journal Plus**, 133, 1-6 (2018)

[12] Durur, H., Ilhan, E., Bulut, H. Novel Complex Wave Solutions of the (2+1)-Dimensional Hyperbolic Nonlinear Schrödinger Equation. **Fractal and Fractional,** *4*(3), 41. (2020).

[13] Gao W., Rezazadeh, H. Pinar Z., Baskonus H. M., Sarwar S. and Yel G., (2020) Novel explicit solutions for the nonlinear Zoomeron equation by using newly extended direct algebraic technique, **Optical and Quantum Electronics** 52:52 https://doi.org/10.1007/s11082-019-2162-8

[14] Ilhan OA., Sulaiman TA., Bulut H. and Baskonus HM, On the new wave solutions to a nonlinear model arising in plasma physics, **Eur. Phys. J. Plus** 133: 27 (2018)

[15] Ismael, H. F. Bulut, H., Baskonus, H. M. Optical soliton solutions to the Fokas–Lenells equation via sine-Gordon expansion method and (m+(G'/G))-expansion method. **Pramana,** 94(1), 35 (2020).

[16] Khalique, C.M., Mhlanga, I.E. Travelling waves and conservation laws of a (2+1)-dimensional coupling system with Korteweg-de Vries equation. **Appl. Math. Nonlinear Sci.** 3(1), 241–254 (2018)

[17] Khalique, C.M., Adeyemo, O.D., Simbanefayi, I.: On optimal system, exact solutions and conservation laws of the modifed equal-width equation. **Appl. Math. Nonlinear Sci**. 3(2), 409–418 (2018)

[18] Khan, K., Akbar, A.M.: Traveling wave solutions of the (2+1)-dimensional Zoomeron equation and the Burgers equations via the MSE method and the exp-function method. **Ain Shams Eng. J**. 5, 247–256 (2014)

[19] Kundu,P.R. Fahim Md. R. Islam Md. E. and Akbar, M.A. The sine-Gordon expansion method for higher-dimensional NLEEs and parametric analysis, **Heliyon,** 7(3), e06459 (2021)

[20] Ma, W.X., Huang, T., Zhang, Y.: A multiple exp-function method for nonlinear diferential equations and its application. **Phys. Scr**. 82(065003), 1–10 (2010)

[21] Morris, M.R., Leach, P.G.L.: Symmetry reductions and solutions to the Zoomeron equation. **Phys. Scr.** 90(015202), 1–5 (2014)

[22] Pandey, P.K.: A new computational algorithm for the solution of second order initial value problems in ordinary diferential equations. **Appl. Math. Nonlinear Sci**. 3(1), 167–174 (2018)

[23] Peng, Y. Z., & Shen, M. On exact solutions of the Bogoyavlenskii equation. **Pramana,** 67(3), 449-456. (2006)

[24] Raza, N., Javid, A.: Optical dark and singular solitons to the Biswas–Milovic equation in nonlinear optics with spatio-temporal dispersion. **Optik** 158, 1049–1057 (2018)

[25] Seadawy, A.R.: Exact solutions of a two dimensional nonlinear Schrödinger equation. **Appl. Math. Lett**. 25, 687–691 (2017)

[26] Seadawy, A.R., Lu, D.: Bright and dark solitary wave soliton solutions for the generalized higher order nonlinear Schrödinger equation and its stability. **Results Phys**. 7, 43–48 (2017)

[27] Sulaiman, T.A., Bulut, H., Yel, G., Atas, S.S.: Optical solitons to the fractional perturbed Radhakrishnan– Kundu–Lakshmanan model. **Opt. Quant. Electron**. 50(372), 372–378 (2018b)

[28] Sulaiman, T.A., Bulut, H., Baskonus, H.M.: Optical solitons to the fractional perturbed NLSE in nano-fibers. **Discrete Contin. Dyn. Syst.** S 13(3), 925–936 (2020)

[29] Veeresha, P., Prakasha, DG, Baskonus,HM. New numerical surfaces to the mathematical model of cancer chemotherapy effect in Caputo fractional derivatives, **Chaos: An Interdisciplinary Journal of Nonlinear Science** 29 (1) (2019)

[30] Wazwaz, A.M.: **Partial Diferential Equations: Methods and Applications**. Balkema, Leiden (2002)

[31] Wazwaz, A.M.: **Partial Diferential Equations and Solitary Wave Theory**. Higher Education Press, Beijing and Springer-Verlag, Berlin Heidelberg (2009)

[32] Yamgou, S.B., Deffo G.R. ,and Pelap, F. C., A new rational sine-Gordon expansion method and its application to nonlinear wave equations arising in mathematical physics, **Eur. Phys. J. Plus** 134: 380 (2019)

[33] Yel, G., Baskonus, H.M., Bulut, H.: Novel archetypes of new coupled Konno-Oono equation by using sine– Gordon expansion method. **Opt. Quant. Electron**. 49(285), 1–10 (2017)

[34] Yel, G., Baskonus, H.M., Bulut, H.: Regarding some novel exponential travelling wave solutions to the Wu– Zhang system arising in nonlinear water wave model. **Indian J. Phys**. 93(8), 1031–1039 (2019)

[35] Yel, G., New wave patterns to the doubly dispersive equation in nonlinear dynamic elasticity, **Pramana – J. Phys**. 94(1):79 (2020)

[36] Zhao, Z., Han, B.: Lump solutions of a (3+1)-dimensional B-type KP equation and its dimensionally reduced equations. **Anal. Math. Phys**. 9(1), 119–130 (2019)