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THE NUMERICAL SOLUTION OF A SECOND-ORDER DIFFERENTIAL EQUATION WITH NEUMANN BOUNDARY CONDITIONS VIA DIFFERENTIAL TRANSFORMATION METHOD

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ABSTRACT

In this study, the differential transformation method is used for finding the numerical solution of a second-order Neumann problem. Numerical examples are included to demonstrate the efficiency and the accuracy of this method for the studied problem and a comparison is made with the existing results. The present method is easy to implement and yields very accurate results.

Keywords: Differential transformation method, Neumann problem, Numerical solution.

NEUMANN SINIR ŞARTLI İKİNCİ MERTEBEDEN BİR DİFERENSİYEL DENKLEMİN DİFERENSİYEL DÖNÜŞÜM METODU İLE YAKLAŞIK ÇÖZÜMÜ

ÖZ

Bu çalışmada, ikinci mertebeden bir Neumann probleminin yaklaşık çözümü için diferensiyel dönüşüm metodu kullanıldı. Çalışılan problem için bu metodun etkinliğini ve duyarlılığını gösteren sayısal örnekler verildi ve mevcut sonuçlar ile bir karşılaştırma yapıldı. Mevcut metodun uygulanması kolaydır ve çok duyarlı sonuçlar verir.

Anahtar Kelimeler: Diferensiyel dönüşüm metodu, Neumann problemi, Yaklaşık çözüm.

1. INTRODUCTION

In this study, we consider the second-order Neumann boundary value problem of the form

$$-y''(x) = f(x, y(x)), \quad x \in [0,1], \tag{1.1}$$

with the boundary conditions

$$y'(0) = \alpha, \quad y'(1) = \beta. \tag{1.2}$$

In (Khan, 2005), the author studied the existence of a solution to Eq. (1.1), including the approximation of solutions via the quasi-linearization method. An approach that is based on semi-orthogonal B-spline wavelets is suggested in (Lakestani and Dehghan, 2006) for solving problem (1.1) and (1.2). The aim of the current study is to approximate the solution of the above problem by means of the differential transformation method.

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The differential transformation method is based on the Taylor's series expansion, and provides an effective numerical means of solving linear and non-linear initial value problems. The differential transformation method may be employed to solve both ordinary and partial differential equations. For example, In (Ertürk and Momani, 2007), the authors successfully applied the one-dimensional differential transformation method to the solution of a general fourth order boundary problem. The authors of (Kurnaz et. al, 2005) presented the generalization of the differential transformation method to n -dimensional case in order to solve partial differential equations.In (Jang et al, 2001), the authors applied the two dimensional differential transformation method to solve partial differential equations, too. Finally, the author of (Hassan, 2002) adopted the differential transformation method to solve some eigenvalue problems.

In this paper, the differential transformation technique is applied to solve problem (1.1) and (1.2). The method can be used to evaluate the approximating solution by the finite Taylor series and by an iteration procedure described by the transformed equations obtained from the original equation using the operations of differential transformation.

The sections of this paper are organized as follows. In the next section we describe the differential transformation method. In Section 3, numerical examples have been presented to illustrate the effectiveness of the present method and a comparison is made with the existing results. Section 4 ends this paper with a brief conclusion. Note that we have computed the numerical results by Matematica programming.

2. DIFFERENTIAL TRANSFORMATION METHOD

Let $y(x)$ be an analytic function in a domain D and let $x = x_i$ represent any point in D. The function $y(x)$ is then represented by a power series whose center is located at x_i. The Taylor series expansion function of $y(x)$ is expressed as:

$$y(x) = \sum_{k=0}^{\infty} \frac{(x-x_i)^k}{k!} \left[\frac{d^k y(x)}{dx^k} \right]_{x=x_i} \quad \text{for} \quad \forall x \in D. \tag{2.1}$$

The particular case of Eq. (2.1) when $x_i = 0$ is referred to as the Maclaurin series of $y(x)$, and is given by:

$$y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} \left[\frac{d^k y(x)}{dx^k} \right]_{x=0} \quad \text{for} \quad \forall x \in D. \tag{2.2}$$

As shown by (Zhou, 1986), the differential transform of function $y(x)$ is defined as:

$$Y(k) \equiv \frac{H^k}{k!} \left[\frac{d^k y(x)}{dx^k} \right]_{x=0}, \quad k = 0,1,2,\ldots,\infty \tag{2.3}$$

where $Y(k)$ represents the transformed function (commonly referred to as the T-function) and $y(x)$ is the original function. The differential spectrum of $Y(k)$ is confined within the interval $x \in [0, H]$, where H is a constant.

The differential inverse transform of $Y(k)$ is defined as follows:

$$y(x) = \sum_{k=0}^{\infty} \left(\frac{x}{H} \right)^k Y(k). \tag{2.4}$$

From the above, it is clear that the differential transformation technique is based upon the Taylor series expansion. Note that the original functions are denoted by lowercase letters, while their transformed functions (i.e. their T-functions) are indicated by the corresponding uppercase letters.

The values of function $Y(k)$ at specific values of the argument k are referred to as discretes, i.e. $Y(0)$ is known as the zero discrete, $Y(1)$ as the first discrete etc. The greater the number of discretes considered, the more precisely the unknown function can be restored. The function $y(x)$ is expressed in terms of the T-function $Y(k)$, and its value is given by the sum of the T-function using $\left(x/H\right)^k$ as its coefficient.

Table 1 presents some important properties of the differential transformation method derived using the expressions presented in Eqs. (2.3) and (2.4) above.

In real applications, it is found that the number of arguments required to restore the unknown function precisely can be reduced by specifying an appropriate value of the constant H. In other words, the function $y(x)$ can be expressed in terms of a finite series and Eq. (2.3) can be written as

$$y(x)=\sum_{k=0}^{n}\left(\frac{x}{H}\right)^k Y(k). \tag{2.5}$$

Eq. (2.5) implies that the value of $\sum_{k=n+1}^{\infty}\left(x/H\right)^k Y(k)$ is negligible.

Table 1. Specific functions, y(x) , and their corresponding differential transforms, $Y(k)$

Original function	Transformed function $Y(k)$
$y(x)=g(x)\pm h(x)$	$Y(k)=G(k)\pm H(k)$
$y(x)=\alpha\, g(x)$	$Y(k)=\alpha\, G(k)$
$y(x)=\dfrac{d^m g(x)}{dx^m}$	$Y(k)=(k+1)(k+2)\ldots(k+m)\times G(k+m)$
$y(x)=g(x)h(x)$	$Y(k)=\sum_{l=0}^{k}G(l)H(k-l)$
$y(x)=x^m$	$Y(k)=\delta(k-m)=\begin{cases}1, & if\ k=m\\ 0, & if\ k\neq m\end{cases}$

3. NUMERICAL EXAMPLES

To demonstrate the accuracy of the present method, we consider the examples given in (Lakestani and Dehghan, 2006) in this section. Our method differs from the method presented in (Lakestani and Dehghan, 2006) and thus these examples could be used as a basis for comparison. The results obtained by the present method are found to be in good agreement with the results obtained in (Lakestani and Dehghan, 2006).

Example 3.1 Consider the following linear Neumann problem (Lakestani and Dehghan, 2006)

$$-y''(x)=(2-4x^2)y(x), \quad x\in[0,1] \tag{3.1}$$

subject to the boundary conditions

$$y'(0)=0, \quad y'(1)=-2/e. \tag{3.2}$$

The exact solution of this problem is

$$y(x) = e^{-x^2}. \tag{3.3}$$

Taking the differential transform of Eq.(3.1) with respect to time x gives:

$$Y(k+2) = \frac{1}{(k+1)(k+2)}\left(\sum_{l=0}^{k}[4\delta(l-2) - 2\delta(l)]Y(k-l)\right), \tag{3.4}$$

where $Y(k)$ is the differential transformation of function $y(x)$.

By using (2.3) and (2.2), the following transformed boundary conditions at $x = 0$ can be obtained:

$$Y(1) = 0, \ \sum_{k=0}^{n} kY(k) = -\frac{2}{e}. \tag{3.5}$$

Utilizing the recurrence relation in (3.4) and the first one of the transformed boundary conditions in (3.5), the following solution up to $O(x^{24})$ is obtained:

$$\begin{aligned} y(x) = {} & a - ax^2 + \frac{a}{2}x^4 - \frac{a}{6}x^6 + \frac{a}{24}x^8 - \frac{a}{120}x^{10} + \frac{a}{720}x^{12} - \frac{a}{5040}x^{14} + \frac{a}{40320}x^{16} \\ & - \frac{a}{362880}x^{18} + \frac{a}{3628800}x^{20} - \frac{a}{39916800}x^{22} + O(x^{24}), \end{aligned} \tag{3.6}$$

where, according to Eq.(2.3),

$$a = y(0) = Y(0) \tag{3.7}$$

The constant a is evaluated from the second one of the transformed boundary conditions in Eq. (3.5) as follows:

$$a = 1. \tag{3.8}$$

Substituting a into (3.6), we get the following series solution

$$\begin{aligned} y(x) = {} & 1 - x^2 + 0.5x^4 - 0.166667x^6 + 0.0416667x^8 \\ & - 0.00833333x^{10} + 0.00138889x^{12} - 0.000198413x^{14} \\ & + 0.0000248016x^{16} - 2.75573\times10^{-6}x^{18} \\ & + 2.75573\times10^{-7}x^{20} - 2.50521\times10^{-8}x^{22} + O(x^{24}). \end{aligned} \tag{3.9}$$

In Table 2, we report the absolute value of the errors of the differential transform method for $n = 22$ together with the results given in (Lakestani and Dehghan, 2006) and the exact solutions.

Table 2. Exact solution and absolute errors for $y(x)$ for Example 3.1

x	Exact solution	Method of (Lakestani and Dehghan, 2006)	Present method
0.0	1.00000000	1.3×10^{-7}	6.0×10^{-6}
0.1	0.99004983	5.9×10^{-6}	0.0
0.2	0.96078944	5.6×10^{-6}	0.0
0.3	0.91393119	5.2×10^{-6}	1.0×10^{-7}
0.4	0.85214379	2.2×10^{-6}	1.0×10^{-7}
0.5	0.77880078	4.4×10^{-7}	5.0×10^{-8}
0.6	0.69767633	4.0×10^{-7}	5.0×10^{-8}
0.7	0.61262639	1.8×10^{-6}	0.0
0.8	0.52729242	1.1×10^{-6}	0.0
0.9	0.44485807	4.0×10^{-6}	1.0×10^{-7}
1.0	0.36787944	2.3×10^{-6}	0.0

Example 3.2 Consider the following nonlinear Neumann problem (Lakestani and Dehghan, 2006) with the boundary conditions by Eqs. (3.11) and (3.12):

$$-y''(x) = -2y^3(x), \quad x \in [0,1], \tag{3.10}$$

$$y'(0) = -1, \tag{3.11}$$

$$y'(1) = -1/4. \tag{3.12}$$

The exact solution of this problem is

$$y(x) = \frac{1}{1+x}. \tag{3.13}$$

Taking the differential transform of both sides of Eq. (3.10), we obtain the following recurrence relation:

$$Y(k+2) = \frac{2}{(k+1)(k+2)} \sum_{k_2=0}^{k} \sum_{k_1=0}^{k_2} Y(k_1) Y(k_2 - k_1) Y(k - k_2). \tag{3.14}$$

The boundary conditions given in Eqs.(3.11) and (3.12) can be transformed at $x_0 = 1$ as

$$Y(1) = -\frac{1}{4}, \ \sum_{k=1}^{n} kY(k)(-1)^{k-1} = -1. \tag{3.15}$$

Utilizing the recurrence relation in (3.14) and the first one of the transformed boundary conditions in (3.15), the following solution up to $O(x^{21})$ is obtained:

$$y(x) = a + \frac{1-x}{4} + a^3(x-1)^2 - \frac{1}{4}a^2(x-1)^3 + \left(\frac{a}{32} + \frac{a^5}{2}\right)(x-1)^4 + \left(-\frac{1}{640} - \frac{9a^4}{40}\right)(x-1)^5$$
$$+\left(\frac{7a^3}{160} + \frac{3a^7}{10}\right)(x-1)^6 + \left(-\frac{3a^2}{640} - \frac{7a^6}{40}\right)(x-1)^7 + \left(\frac{3a}{10240} + \frac{3a^5}{64} + \frac{7a^9}{40}\right)(x-1)^8$$
$$+\left(-\frac{1}{122880} - \frac{29a^4}{3840} - \frac{61a^8}{480}\right)(x-1)^9 + \left(\frac{121a^3}{153600} + \frac{209a^7}{4800} + \frac{61a^{11}}{600}\right)(x-1)^{10}$$
$$+\left(-\frac{11a^2}{204800} - \frac{59a^6}{6400} - \frac{71a^{10}}{800}\right)(x-1)^{11} + \left(\frac{11a}{4915200} + \frac{409a^5}{307200} + \frac{709a^9}{19200} + \frac{71a^{13}}{1200}\right)(x-1)^{12}$$
$$+\left(-\frac{11}{255590400} - \frac{2177a^4}{15974400} - \frac{9653a^8}{998400} - \frac{1253a^{12}}{20800}\right)(x-1)^{13}$$
$$+\left(\frac{211a^3}{21299200} + \frac{2357a^7}{1331200} + \frac{2453a^{11}}{83200} + \frac{179a^{15}}{5200}\right)(x-1)^{14}$$
$$+\left(-\frac{211a^2}{425984000} - \frac{19031a^6}{79872000} - \frac{45839a^{10}}{4992000} - \frac{12497a^{14}}{312000}\right)(x-1)^{15}$$
$$+\left(\frac{211a}{13631488000} + \frac{5127a^5}{212992000} + \frac{499a^9}{245760} + \frac{56249a^{13}}{2496000} + \frac{12497a^{17}}{624000}\right)(x-1)^{16}$$
$$+\left(-\frac{211}{926941184000} - \frac{26479a^4}{14483456000} - \frac{609573a^8}{1810432000} - \frac{35383a^{12}}{4352000} - \frac{370811a^{16}}{14144000}\right)(x-1)^{17}$$
$$+\left(\frac{2137311a^3}{2085617664000} + \frac{1404499a^7}{32587776000} + \frac{951957a^{11}}{452608000} + \frac{2121559a^{15}}{127296000} + \frac{370811a^{19}}{31824000}\right)(x-1)^{18}$$
$$+\left(-\frac{11249a^2}{2780823552000} - \frac{187481a^6}{43450368000} - \frac{2244767a^{10}}{5431296000} - \frac{1159541a^{14}}{16972800} - \frac{719129a^{18}}{42432000}\right)(x-1)^{19}$$
$$+\left(\frac{11249a}{111232942080000} + \frac{783671a^5}{2317352960000} + \frac{13848569a^9}{217251840000} + \frac{27457409a^{13}}{13578240000}\right.$$
$$\left. + \frac{6795551a^{17}}{565760000} + \frac{719129a^{21}}{106080000}\right)(x-1)^{20} + O(x^{21}),$$

(3.16)

where

$$a = y(0) = Y(0) \tag{3.17}$$

The constant a is evaluated from the second one of the transformed boundary conditions in Eq. (3.15) as follows:

$$a = 0.5000023. \tag{3.18}$$

Substituting a into (3.16), we have series solution as follows:

$$y(x) = 0.500002 + \frac{1-x}{4} + 0.125002(x-1)^2 - 0.0625006(x-1)^3$$
$$+ 0.0312504(x-1)^4 - 0.0156253(x-1)^5 + 0.00781265(x-1)^6$$
$$- 0.00390633(x-1)^7 + 0.00195317(x-1)^8 - 0.000976589(x-1)^9$$
$$+ 0.000488296(x-1)^{10} - 0.000244149(x-1)^{11} + 0.000122075(x-1)^{12} \quad (3.19)$$
$$- 0.0000610375(x-1)^{13} + 0.0000305188(x-1)^{14} - 0.0000152595(x-1)^{15}$$
$$+ 7.62975\times10^{-6}(x-1)^{16} - 3.81488\times10^{-6}(x-1)^{17} + 1.90745\times10^{-6}(x-1)^{18}$$
$$- 9.53726\times10^{-7}(x-1)^{19} + 4.76864\times10^{-7}(x-1)^{20} - O(x^{21}).$$

In Table 3, we report the absolute value of the errors of the differential transform method for $n = 20$ together with the results given in (Lakestani and Dehghan, 2006) and the exact solutions.

Table 3. Exact solution and absolute errors for $y(x)$ for Example 3.2

x	Exact solution	Method of (Lakestani and Dehghan, 2006)	Present method
0.0	1.00000000	5.6×10^{-6}	5.1×10^{-6}
0.1	0.99004983	2.6×10^{-5}	4.0×10^{-6}
0.2	0.96078944	1.7×10^{-5}	4.0×10^{-6}
0.3	0.91393119	1.6×10^{-5}	3.0×10^{-6}
0.4	0.85214379	1.4×10^{-5}	3.0×10^{-6}
0.5	0.77880078	1.2×10^{-5}	2.0×10^{-6}
0.6	0.69767633	1.0×10^{-5}	3.0×10^{-6}
0.7	0.61262639	7.2×10^{-6}	2.4×10^{-6}
0.8	0.52729242	5.3×10^{-6}	2.3×10^{-6}
0.9	0.44485807	5.5×10^{-6}	2.3×10^{-6}
1.0	0.36787944	1.6×10^{-6}	2.3×10^{-6}

Example 3.3. Consider the following linear Neumann problem(Lakestani and Dehghan, 2006)

$$-y''(x) = 4y(x) - 2, \quad x \in [0,1], \quad (3.20)$$

subject to the boundary conditions

$$y'(0) = 0, \quad (3.21)$$

$$y'(1) = \sin(2) \quad (3.22)$$

The exact solution of this problem is

$$y(x) = \sin^2(x). \quad (3.23)$$

Using the transformational operations in Table 1 and by taking differential transform for the both sides of (3.20), we have

$$Y(k+2)=\frac{2\delta(k)-4Y(k)}{(k+1)(k+2)}. \tag{3.24}$$

By using (2.3), the boundary condition (3.21) becomes

$$Y(1)=0. \tag{3.25}$$

By using (2.2), the boundary condition (3.22) becomes

$$\sum_{k=0}^{n} kY(k)=\sin(2). \tag{3.26}$$

Utulizing the recurrence relation in (3.24) and the transformed boundary condition given in Eq. (3.15), the following series solution up to 16-term is obtained:

$$\begin{aligned} y(x) &= a+(1-2a)x^2+\left(\frac{2a-1}{3}\right)x^4+\left(\frac{2-4a}{45}\right)x^6+\left(\frac{2a-1}{315}\right)x^8 \\ &+\left(\frac{2-4a}{14175}\right)x^{10}+\left(\frac{4a-2}{467775}\right)x^{12}+\left(\frac{4-8a}{42567525}\right)x^{14} \\ &+\left(\frac{2a-1}{638512875}\right)x^{16}+O(x^{18}), \end{aligned} \tag{3.27}$$

where, according to Eq.(2.3),

$$a=y(0)=Y(0) \tag{3.28}$$

The constant a is evaluated from the transformed boundary condition given in (3.26) as follows:

$$a=-2.00283\times 10^{-10}. \tag{3.29}$$

Substituting a into (3.27), we get the following solution:

$$\begin{aligned} y(x) &= -2.00283\times 10^{-10}+x^2-0.333333x^4+0.0444444x^6-0.0031746x^8 \\ &+0.00014109x^{10}-4.2755\times 10^{-6}x^{12}+9.39683\times 10^{-8}x^{14} \\ &-1.56614\times 10^{-9}x^{16}+O(x^{18}). \end{aligned} \tag{3.30}$$

In Table 4, we compare the absolute values of the errors of the differential transform method for $n=16$ together with the results given in (Lakestani and Dehghan, 2006) and the exact solutions.

Table 4. Exact solution and absolute errors for $y(x)$ for Example 3.3

x	Exact solution	Method of (Lakestani and Dehghan, 2006)	Present method
0.0	0.000000000	1.2×10^{-8}	0.0
0.2	0.039469503	5.6×10^{-6}	0.0
0.4	0.151646645	3.1×10^{-6}	0.0
0.6	0.318821123	9.2×10^{-7}	0.0
0.8	0.514599761	4.5×10^{-7}	0.0
1.0	0.708073418	1.8×10^{-8}	0.0

Example 3.4 Consider the linear Neumann problem (Lakestani and Dehghan, 2006)

$$-y''(x) = -y(x), \quad x \in [0,1], \tag{3.31}$$

subject to the boundary conditions

$$y'(0) = 0, \tag{3.32}$$

$$y'(1) = \sinh(1). \tag{3.33}$$

The exact solution of this problem is

$$y(x) = \cosh(x). \tag{3.34}$$

The differential transform of Eq. (3.31) yields to

$$Y(k+2) = \frac{Y(k)}{(k+1)(k+2)}. \tag{3.35}$$

The boundary conditions are transformed to be:

$$Y(1) = 0, \tag{3.36}$$

$$\sum_{k=0}^{n} kY(k) = \sinh(1). \tag{3.37}$$

Using Eqs. (3.35) and (3.36), $Y(k)$ is obtained up to $n = 12$ and then using the inverse transformation rule in Eq. (2.4), the following series solution is obtained:

$$\begin{aligned} y(x) = a &+ \frac{1}{2}ax^2 + \frac{1}{24}ax^4 + \frac{1}{720}ax^6 + \frac{1}{40320}ax^8 + \frac{1}{3628800}ax^{10} \\ &+ \frac{1}{479001600}ax^{12} + O(x^{14}), \end{aligned} \tag{3.38}$$

where, according to Eq.(2.3),

$$a = y(0) = Y(0) \tag{3.39}$$

The constant a is evaluated from the transformed boundary condition given in (3.37) as follows:

$$a = 1. \tag{3.40}$$

Substituting a into (3.38), we get the following solution:

$$\begin{aligned} y(x) &= 1 + 0.5x^2 + 0.0416667x^4 + 0.00138889x^6 + 0.0000248016x^8 \\ &+ 2.75573\times10^{-7}x^{10} + 2.08768\times10^{-9}x^{12} + O(x^{14}). \end{aligned} \tag{3.41}$$

Numerical results for $n = 12$ with comparison to (Lakestani and Dehghan, 2006) and the exact solution (3.34) are given in Table 5.

Table 5. Exact solution and absolute errors for $y(x)$ for Example 3.4

x	Exact solution	Method of (Lakestani and Dehghan, 2006)	Present method
0.0	1.0000000000	2.3×10^{-9}	1.0×10^{-10}
0.2	1.0200667556	3.4×10^{-6}	0.0
0.4	1.0810723718	3.8×10^{-6}	0.0
0.6	1.1854652182	4.2×10^{-6}	0.0
0.8	1.3374349463	4.4×10^{-6}	0.0
1.0	1.5430806348	4.6×10^{-9}	2.0×10^{-10}

CONCULUSION

The differential transform method is used to solve a second-order Neumann problem. The present method is computationally attractive and applications are demonstrated through illustrative examples. The results obtained show that this method can solve the problem effectively.

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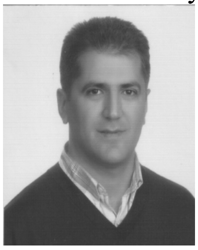

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