

Determining Which Date Is The Day Of The Week

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Abstract

This study is about determining which day of the week a day falls in history using mathematical equations. Various computer programs or some calendar tables have been prepared on this subject. This study aims to study mathematical equations covering all centuries after Christ. The Gregorian calendar used today was first used as the Julian calendar, and in 1582, it was determined that there was a small error in this Julian calendar. This error was corrected and the Gregorian calendar was created. The aim of this study is to find the equations that correspond to the days of the Julian and Gregorian calendars.

Keywords: “Historical days, days of the week, calculation of days.”

1. Introduction

The Gregorian calendar used today is Hz. Based on the birth of Jesus, A.D. It started to be used as a Julian calendar in 325. A retrospective period of 325 years was established. According to this calendar, Friday, January 1, 325, was considered the first day. This calendar was created by accepting 365 days and 6 hours. The 6-hour excess is called a leap year by taking February as 29 days instead of 28 days every 4 years. But in 1582, it was determined that the earth revolves around the sun in 365 days, 5 hours, 48 minutes and 46 seconds. For this reason, it turned out that there was a difference of 11 minutes and 14 seconds. This shows that it has advanced by approximately 3 days in 400 years. The day after 04 October 1582, 11 days were added and became 15 October 1582, thus correcting the 11-day error of the Julian calendar. This new calendar system was called the Gregorian calendar. In addition, the calendar has been determined so that years that are multiples of 400 are leap years, but years that are multiples of 100 are not leap years. That is, the year 1600 was a leap year, but the years 1700, 1800 and 1900 were not leap years. This calendar started to be used in our country on January 1, 1926.

2. Findings and Determinations

The 1st day of the week is Monday, the 2nd day is Tuesday, the 3rd day is Wednesday, the 4th day is Thursday, the 5th day is Friday, the 6th day is Saturday, and the 7th day is Sunday (day 7=day 0).

1. Definition: The largest integer that is not greater than x , $x \in \mathbb{R}$, is called the exact value of x and is denoted by the symbol $\llbracket x \rrbracket$. For example, $\llbracket 3,25 \rrbracket = 3$, $\llbracket -2,8 \rrbracket = -3$. [1], [6]

2. Definition: Representing each month with a number is called the month's representation number, denoted by T .

1. Theorem (Julian Calendar - Before 04 October 1582 [7]): The x value in the

$$YYYY + \left\llbracket \frac{YYYY}{4} \right\rrbracket + T + GG \equiv x \pmod{7} \quad (1)$$

equation, represented by the symbols day GG , month AA , year $YYYY$, gives the Julian calendar, that is, the day of the week on dates before 04 October 1582. Here, January 4, February 0, March 0, April 3, May 5, June 1, July 3, August 6, September 2, October 4, November 0, December 2 are represented by the numbers.

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Proof: Since there are 365 days in a year and 7 days in a week, K happens, which means that a certain day coincides with the next day in the following year.

After YYYYY years,

$$365 \cdot YYYYY \equiv YYYYY \pmod{7} \quad (2)$$

is obtained. But there is a leap year situation. Since February will have 29 days every 4 years, there will be an excess of $\left\lfloor \frac{YYYYY}{4} \right\rfloor$.

For January 01, 0325 to be on Friday, the month of January should be represented by the number 4. Because

$$0325 + \left\lfloor \frac{0325}{4} \right\rfloor + 4 + 1 = 411 \equiv 5 \pmod{7} \quad (3)$$

is done. Therefore, the month of January is represented by 4. Since January has 31 days, it is

$$31 + 0 \equiv 3 \pmod{7} \quad (4)$$

and February is represented by the number 0. Since February has 28 days (a leap year will be given as a result), it is

$$28 + 0 \equiv 0 \pmod{7} \quad (5)$$

and March is represented by the number 0. Since March has 31 days,

$$31 + 0 \equiv 3 \pmod{7} \quad (6)$$

is represented by the number 3 in April. Since April has 30 days, it is

$$30 + 3 \equiv 5 \pmod{7} \quad (7)$$

and May is represented by 5. Similarly, other months are also found. In this case, the representative numbers of the month T are formed. Accordingly, the date GG/AA/YYYY is found as

$$YYYY + \left\lfloor \frac{YYYY}{4} \right\rfloor + T + GG \equiv x \pmod{7} \quad (8)$$

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Example: Let's research the date of birth of Prophet Muhammad, June 30, 570. The month of April is represented by 3. In equation (1),

$$0570 + \left\lfloor \frac{0570}{4} \right\rfloor + 1 + 30 = 743 \equiv 1 \pmod{7} \quad (9)$$

$x = 1$ and 30 June 570 is Monday. [2]

Example: Let's investigate the date of the Battle of Manzikert, August 26, 1071, which is accepted as the date of the Turks' entry into Anatolia [3]. The month of August is represented by 6. In equation (1),

$$1071 + \left\lfloor \frac{1071}{4} \right\rfloor + 6 + 26 = 1370 \equiv 5 \pmod{7} \quad (10)$$

$x = 5$ and 26 August 1071 is Friday.

Example: Now let's look at the date of the conquest of Istanbul, which is May 29, 1453 [3]. The month of May is represented by 5. In equation (1),

$$1453 + \left\lfloor \frac{1453}{4} \right\rfloor + 5 + 29 = 1850 \equiv 2 \pmod{7} \quad (11)$$

$x = 1$ and May 29, 1453 is a Tuesday.

1. Result: In the calculation of leap years, in January and February,

$$YYYY + \left\lfloor \frac{YYYY}{4} \right\rfloor + T + GG - 1 \equiv x \pmod{7} \quad (12)$$

equation is valid.

Example: Let's examine the date January 15, 1032. Since the year 1032 is a leap year and the month of January is represented by 4 and in equation (2),

$$1032 + \left\lfloor \frac{1032}{4} \right\rfloor + 4 + 15 - 1 = 1308 \equiv 6 \pmod{7} \quad (13)$$

$x = 6$ and January 15, 1032 is Saturday.

2. Theorem (Gregorian Calendar - After October 15, 1582 [8]): The day is GG, the month is AA, the year is YYYY and the first two digits of the year are shown as YY symbols.

$$YYYY + \left\lfloor \frac{YYYY}{4} \right\rfloor + YY - \left\lfloor \frac{YY}{4} \right\rfloor + T + GG \equiv x \pmod{7} \quad (14)$$

The x value in the equation gives the day of the week for dates after October 15, 1582. (The number of representations of months is as in the 1st theorem.)

Proof: In the 1st theorem, information was given about YYYY, $\left\lfloor \frac{YYYY}{4} \right\rfloor$, T and GG. Information about $YY - \left\lfloor \frac{YY}{4} \right\rfloor$ needs to be given here.

In 1582, the Gregorian calendar was adopted because it was determined that the earth's circulation around the sun was 365 days, 5 hours, 48 minutes and 46 seconds. The calendar has been determined so that years with a 3-day difference, which occurs approximately every 400 years, are leap years that are multiples of 400, but years that are multiples of 100 are not leap years. That is, the year 1600 is a leap year, but the years 1700, 1800 and 1900 are not leap years. For this reason, $YY - \left\lfloor \frac{YY}{4} \right\rfloor$ should be added to the 1st theorem. For example, when examining the year 1583, $15 - \left\lfloor \frac{15}{4} \right\rfloor = 12$ days should be added.

Example: Let's investigate the date of August 26, 1922, the date of the start of the great offensive in the War of Independence [4]. Here $YY = 19$ and August is represented by 6. In equation (3),

$$1922 + \left\lfloor \frac{1922}{4} \right\rfloor + 19 - \left\lfloor \frac{19}{4} \right\rfloor + 6 + 26 = 2449 \equiv 6 \pmod{7} \quad (15)$$

$x = 6$ and August 26, 1922 is Saturday.

Example: 1. Our President and head teacher Mustafa Kemal Atatürk passed away on November 10, 1938 [5]. Let's examine the date of death. Again, here $YY = 19$ and November is represented by 0. In equation (3)

$$1938 + \left\lfloor \frac{1938}{4} \right\rfloor + 19 - \left\lfloor \frac{19}{4} \right\rfloor + 0 + 10 = 2447 \equiv 4 \pmod{7} \quad (16)$$

$x = 4$ and November 10, 1938 is Thursday.

2. Result: In the calculation of leap years after October 15, 1582, in January and February

$$YYYY + \left\lfloor \frac{YYYY}{4} \right\rfloor + YY - \left\lfloor \frac{YY}{4} \right\rfloor + T + GG - 1 \equiv x \pmod{7} \quad (17)$$

The equation is valid.

Example: Let's search for the date February 15, 2024. Since the year 2024 is a leap year and February is represented by 0 and in equation (4),

$$2024 + \left\lfloor \frac{2024}{4} \right\rfloor + 20 - \left\lfloor \frac{20}{4} \right\rfloor + 0 + 15 - 1 = 2559 \equiv 4 \pmod{7} \quad (18)$$

3. Conclusion

By examining the calendar changes in the historical process, this study will be able to obtain clearer results from the data used, especially in historical science. It is hoped that this will shed light on historical studies.

In addition, there is an average of 0.000125 days of small error per year, which corresponds to 10.8 seconds per year. Approximately 8000 years would have to pass for this one-day error to occur in the calendar calculation. The use of this calendar in 8000 should be reviewed.

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