ANADOLU ÜNİVERSİTESİ

Bilim ve Teknoloji Dergisi B-Teorik Bilimler
Cilt: 2 Sayı: 2 2013
Sayfa:127-142

ARAŞTIRMA MAKALESİ / **RESEARCH ARTICLE**

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MARKOV MODELS IN CALCULATING CLV

ABSTRACT

The paper presents a method of calculating customer lifetime value and finding optimal remarketing strategy basing on Markov model with short-term memory of client's activity. Furthermore, sensitivity analysis of optimal strategy is conducted for two types of retention rate functional form defining transition probabilities.

Keywords: Customer lifetime value, Retention rate, Marketing strategy, Markov chain

MÜŞTERİ ÖMÜR DEĞERİNİN HESAPLANMASINDA MARKOV MODELLERİNİN KULLANILMASI

ÖZ

Bu çalışmada, müşteri ömür değerinin hesaplanması ve optimal pazarlama stratejisinin bulunmasına yönelik kısa sureli müşteri faaliyetini esas alan bir Markov modeli öne sürülmüştür. Ayrıca optimal stratejinin duyarlılık analizi, geçiş olasılıkları tanımlanan fonksiyonel formdaki iki tür müşteri elde tutma oranı için gerçekleştirilmiştir.

Anahtar Kelimeler: Müşteri ömür değeri, Müşteri elde tutma oranı, Pazarlama stratejileri, Markov zinciri

JEL: C23, C69, M31

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Recieved: 28 November 2012 **Revised:** 24 April 2013 **Accepted:** 18 June 2013

1. INTRODUCTION

Huge marketing databases containing not only demographic, descriptive and address information about company's clients but also the whole history of client – company relations are a powerful device for statistical data mining enabling to estimate mathematical models used to support relationship oriented marketing decisions. The approach to marketing focused on effective use of information about individual customers is called database marketing. According to various definitions database marketing is:

- "an interactive approach to marketing, which uses the individually addressable marketing media and channels (such as mail, telephone and the sales force): to extend help to a company's target audience; to stimulate their demand; and to stay close to them by recording and keeping an electronic database memory of the customer, prospect and all commercial contacts, to help improve all future contacts and to ensure more realistic of all marketing" (Shaw and Stone, 1988);

- "managing and computing relational database, in real time, of comprehensive, up-to-date, relevant data on customers, inquiries, prospects and suspects, to identify our most responsive customers for the purpose of developing a high quality, long-standing relationship of repeat business by developing predictive models which enable us to send desired messages at the right time in the right form to the right people – all with the result of pleasing our customers, increasing our response rate per marketing dollar, lowering our cost per order, building our businesses, and increasing our profits" (National Center for Database Marketing).

What makes database marketing a distinguished discipline of marketing is concentration on data mining in order to model customers behaviour. Its development is connected with new digital technologies of collecting and analyzing information and interactive communication with customer. With the beginning in 80s of XX century database marketing is becoming more and more important due to extended abilities to process huge amounts of data and increasing popularity of internet commerce. Econometric and statistical models used in database marketing are logit, tobit, hazard models, RFM analysis, cluster analysis, decision trees, neural networks, Markov models and many others.

Markov models are stochastic processes with wide applications in marketing researches such as modelling customer behaviour and loyalty, brand switching, identification of purchasing patterns or customer relationship management. Hereby the attention is focused on applying Markov models in calculating customer lifetime value. The paper refers to the approach suggested by Pfeifer and Carraway2 basing on a Markov chain with transition probabilities resulting from RFM analysis. A modification of this approach consisting in including additional information (such as remarketing expenditure) is presented, followed by a method of finding optimal remarketing strategy.

2. CUSTOMER LIFETIME VALUE

One of the crucial concepts in customer relationship management is CLV (customer lifetime value) defined as discounted value of revenue ever generated by a client. Proper calculation of CLV enables to quantify company long-term profitability and support decisions related to the level of expenditure on customer acquiring and retention, planning promotions, choosing distributions channels. Calculating CLV of individual customer is particularly important in direct marketing.

[2] Ph.E. Pfeifer, R.L. Carraway (2000).

The concept of CLV in relation to direct marketing[3] has been popularized by Dwyer[4] who provided examples of its application in situations called *retention* and *migration*. Mathematical models of CLV in these two situations have been given by Berger and Nasr[5]. Blattberg and Deighton[6] have constructed CLV model oriented on finding balance between acquisition and retention expenditures.

Retention situation may be characterized as follows:

- customer who has break in relations (for instance no purchase in previous time period) is considered lost for good, in case of another purchase in the future the history of previous relations is not taken into account,
- is adequate in contract relations with observed churning.

Migration models refer to situation when:

- breaks in relationship are allowed,
- migration consists in ability of moving between distinguished states, for that reason Markov chains are quite often applied,
- states are defined basing on RFM (*recency*, *frequency*, *monetary*) analysis, used since years 50s of XX century in direct marketing as a method of segmentation. The marketers noticed that most information about customer's behaviour is concentrated in three variables: recency (time elapsed from the previous purchase), frequency of purchases and their monetary value.

A basis formula[7] of customer lifetime value is following:

$$CLV = \sum_{t=1}^{\infty} \frac{\mathrm{E}(V_t)}{(1+d)^t} \tag{1}$$

with V_t representing revenue in period *t* generated by a customer, *d* – one period interest rate. Extended formula of CLV[8] contains decomposition of V_t on various cost and income categories,

$$CLV = -AC + \sum_{t=1}^{T} \left[r^t \frac{(AR_t + UR_t + CR_t + RV_t) - (SC_t + RC_t)}{(1+d)^t} - r^{t-1}(1 - r)\frac{TC}{(1+d)^t} \right], \tag{2}$$

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[3] History of direct marketing has the beginnings in 1872 in the USA when a cloths selling company published a list of items which could be ordered and purchased by mail. The term "direct marketing" was introduced in 1967 and it refers to direct mail, telemarketing and online techniques. According to Direct Marketing Association, founded in 1917 to support businesses and non-profit organization using direct marketing tools and techniques, in 2009 direct marketing accounted for 8.3% of total US GDP and spending on it was over 54% of all advertising expenditure.

[4] F.R. Dwyer (1989).

[5] P.D. Berger, N.J. Nasr (1998).

[6] R.C. Blattberg, J. Deighton (1996).

[7] R.C. Blattberg, B.D. Kim, S.A. Neslin (2008).

[8] H.H. Bauer, M. Hammerschmidt (2005).

AC – acquisition cost,
r – retention rate,
AR_t – autonomous revenue in period *t*,
UR_t – up selling revenue in period *t*,
CR_t – cross selling revenue in period *t*,
RV_t – gross contribute from reference activity of customer in period *t*,
SC_t – cost of serving in period *t*,
RC_t – cost of retaining customer in period *t*,
TC – cost of terminating the relationship,
D – interest rate,
T – time horizon, expected time of relationship.

Recently definition of CLV has been expended by including the value of influences made by a customer during his network activity. Weinberg and Berger[9] propose CSMV (Customer Social Media Value) to represent the value of customer's influence on other consumers by his interactions with other social media users.

An important element of formula (2) is retention rate, determining the percentage of customers who remain active in next period. Models of retention are presented in next section. Another important issue is expected time of relationship. Kumar[10] underlines that in non-contractual relationships it is usually hard to identify moment of beginning and ending. Besides, allowing longer time horizon may result in problems with interest rates stability and precision of predicting monetary value of incomes and costs. In practice CLV is usually calculated for 3-years period (except from motor industry and insurance).

3. RETENTION RATE MODELS

The simplest model of retention assumes constant retention rate. The formula for CLV is then[11]

$$CLV = \sum_{t=1}^{T} r^{t-1}\left(R_t - C_t\right)\left(\tfrac{1}{1+d}\right)^{t-1},$$

with following notation: R_t - revenue in period *t*, C_t - costs in period *t*, *r* – retention rate, *d* – interest rate. With additional assumption of constant costs and revenues its limit value is.

$$CLV^{\infty} = \sum_{t=1}^{\infty} r^{t-1}\left(R - C\right)\left(\tfrac{1}{1+d}\right)^{t-1} = (R - C)\,\tfrac{1+d}{1+d-r}$$

Blattberg and Deighton[12] suggested a model with retention rate depending exponentially on remarketing expenditure *M* incurred to maintain relationship with customer.

$$r = a(1 - \exp(-bM)),\ a, b > 0. \qquad (3)$$

[9] B.D. Weinberg, P.D. Berger (2011).

[10] V. Kumar (2010).

[11] R.C. Blattberg, B.D. Kim, S.A. Neslin, op.cit., pp. 111.

[12] R.C. Blattberg, J. Deighton , op.cit.

Another approach[13] consists in applying logistic function.

$$r = \frac{a}{1+\exp(-bM)},\ a,b>0\,. \tag{4}$$

According to formula (3), retention rate is an increasing function of remarketing expenditure and no expenditure results in zero retention rate. In case of logistic function (4) even null expenditure results in positive value of retention rate. This can be easily corrected by including an intercept in exponent and obtaining

$$r = a(1-\exp(-c-bM)),\ a,b,c>0\,. \tag{3a}$$

$$r = \frac{a}{1+\exp(-c-bM)},\ a,b,c>0\,. \tag{4a}$$

In all cases parameter $a<1$ refers to limit retention rate achieved with unconstrained budget. An alternative to functions (3) or (4) could be applying logistic regression.

4. RETENTION RATE WITH LAST PURCHASE MEMORY

Ma, Li and Chen[14] suggested a model in which retention rate depends on customer's history. They applied a stochastic process $\{Y_t\}$,

$$Y_t = \begin{cases} 1, \text{if customer is active in period } t, \\ 0, \text{if customer is not active,} \end{cases}$$

satisfying Markov property

$$\mathrm{P}(Y_t = j \mid Y_0, Y_1, \ldots, Y_{t-1} = i) = \mathrm{P}(Y_t = j \mid Y_{t-1} = i) = p_{ij}(M)\,. \tag{5}$$

Therefore they obtained a two-state Markov chain with transition probabilities (which are equivalent to probabilities of being active in the next period) depending on customer's activity in actual period and remarketing expenditure *M*. By introducing a variable describing revenues,

$$R_t = \begin{cases} R \text{ if } Y_t = 1, \\ 0 \text{ if } Y_t = 0, \end{cases}$$

and denoting time of relationships by T_δ, they obtained customer lifetime value formula

$$CLV_\delta = R + \sum_{t=1}^{T_\delta} \frac{R_t - M}{(1+d)^t}\,. \tag{6}$$

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[13] M. Ma, Z. Li, J. Chen (2008).

[14] Ibidem.

Parameter δ denotes time of inactivity after which customer is considered lost for good and no more remarketing expenditure are taken. In order to obtain expected values of R_t, T_δ and CLV_δ Ma, Li and Chen used an auxiliary Markov chain $\{X_t\}$ with state space $\{0,1,...,2^\delta - 1\}$. Transient states refer to binary sequences of last δ realizations of original chain $\{Y_t\}$,

$$X_n = Y_n + 2 \cdot Y_{n-1} + 2^2 \cdot Y_{n-2} + ... + 2^{\delta-1} \cdot Y_{n-(\delta-1)},$$

and state 0 is an absorbing one and refers to a sequence of δ visits in non-purchasing state.

The method of deriving CLV with analogous assumptions and making use of the concept of Pfeifer and Carraway[15] is now presented. The situation considered is following:

- company makes effort to acquire and maintain a customer, if succeeds it obtains revenue R for each purchase made,
- breaks in customer-company relationship are admitted,
- company suffers remarketing expenditure M each period when customer is considered active,
- probability of purchase in the next period depends of customer's recency (time from the previous purchase).

This is a migration model as customers "migrate" among states defined in terms of how recently they have purchased from the company. In opposition to retention model customer might skip one or more periods and still be considered an active one.

A mathematical model of a situation described is δ+1-states Markov chain with rewards. First δ states are defined as the number of periods elapsed form the previous purchase and the last state is an absorbing one and refers to terminating the relation. It is equivalent to stating that company does not believe in coming back of a customer who has been inactive for last δ periods and considers such customer to be lost for good. Transition matrix of the Markov chain involved is following,

$$\mathbf{P} = \begin{bmatrix} p_1 & 1-p_1 & 0 & \dots & 0 \\ p_2 & 0 & 1-p_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_\delta & 0 & 0 & \dots & 1-p_\delta \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}, \qquad (7)$$

with p_i for $i = 1,2,...,\delta$ denoting probability of purchasing after i periods of being inactive. Vector of rewards is equal

$$\mathbf{R} = [R - M \quad -M \quad \dots \quad -M \quad 0]^T.$$

[15] Ph. E. Pfeifer, R.L. Carraway, op.cit.

Following the theory of Markov chains with rewards the elements of vector

$$\mathbf{CLV}^T = \sum_{t=0}^{T} \left(\tfrac{1}{1+d}\mathbf{P}\right)^t \mathbf{R}$$

equal the value of customer starting in states $1,2,...,\delta$ with time horizon T. For infinite time horizon vector of customer lifetime value takes form

$$\mathbf{CLV} = \sum_{t=0}^{\infty} \left(\tfrac{1}{1+d}\mathbf{P}\right)^t \mathbf{R} = \left[\mathbf{I} - \tfrac{1}{1+d}\mathbf{P}\right]^{-1} \mathbf{R} .$$

Specific form of transition matrix $\mathbf{P}$ enables to present elements of the first row of matrix $\mathbf{A} = \left[\mathbf{I} - \tfrac{1}{1+d}\mathbf{P}\right]^{-1}$ as

$$a_{11} = \left[1 - \sum_{k=1}^{\delta}\left(\left(\tfrac{1}{1+d}\right)^k p_k \prod_{l=1}^{k-1}(1-p_l)\right)\right]^{-1},$$

$$a_{12} = a_{11}\tfrac{1}{1+d}(1-p_1),$$

$$a_{1\delta} = a_{1,\delta-1}\tfrac{1}{1+d}(1-p_{\delta-1}),$$

$$a_{1,\delta+1} = a_{1\delta}\tfrac{1}{d}(1-p_{\delta}),$$

and CLV for a customer in state 1, that is the most "recent" customer is then

$$CLV = a_{11}(R-M) - M(a_{12}+a_{13}+...+a_{1\delta}) =$$

$$= a_{11}R - a_{11}M\left(1 + \sum_{k=1}^{\delta-1}\left(\tfrac{1}{1+d}\right)^k \prod_{l=1}^{k}(1-p_l)\right). \tag{8}$$

The second row of the matrix $\mathbf{A} = \left[\mathbf{I} - \frac{1}{1+d}\mathbf{P}\right]^{-1}$ takes form:

$$a_{21} = \frac{1 - a_{22}}{\frac{1}{1+d}\left(p_1 - 1\right)},$$

$$a_{22} = \left[1 - \sum_{k=2}^{\delta}\left(\left(\frac{1}{1+d}\right)^k p_k \prod_{l=1}^{k-1}\left(1 - p_l\right)\right)\right]^{-1},$$

$$a_{23} = a_{22}\,\frac{1}{1+d}\left(1 - p_1\right),$$

$$a_{2\delta} = a_{2,\delta-1}\,\frac{1}{1+d}\left(1 - p_{\delta-1}\right),$$

$$a_{2,\delta+1} = a_{2\delta}\,\frac{1}{d}\left(1 - p_{\delta}\right)$$

and enables to calculate directly CLV for a customer in state 2, that is customer whose last purchase was made two periods ago. Analogous derivation is possible for customer in each state.

Elements of fundamental matrix $\mathbf{N} = \left(\mathbf{I} - \mathbf{S}\right)^{-1}$, with S denoting matrix of transition probabilities between transient states $1,2,\ldots,\delta$ of a Markov chain with transition matrix (7), take form:

$$\text{for } i = 1,2,\ldots,\delta - 1 \qquad n_{ij} = \begin{cases} \dfrac{1 - \prod_{l=i}^{\delta}\left(1 - p_l\right)}{\prod_{k=j}^{\delta}\left(1 - p_k\right)} & \text{for } j < i \\ \dfrac{1}{\prod_{k=j}^{\delta}\left(1 - p_k\right)} & \text{for } j \geq i \end{cases},$$

$$\text{and for } i = \delta \qquad n_{\delta j} = \frac{p_\delta}{\prod_{k=j}^{\delta}\left(1 - p_k\right)}.$$

The expected absorbing time, which refers to expected time of customer-company relationship, is derived by summing elements in each row of fundamental matrix. Particularly, for customer in state 1 (customer whose last purchase was made a period ago) the expected time of relationship equals.

$$t^{(1)} = \sum_{j=1}^{\delta} \frac{1}{\prod_{k=j}^{\delta}\left(1 - p_k\right)}. \qquad (9)$$

Next step consists in defining relations between probabilities p_i of transition matrix (7) and retention rates from the Markov chain $\{Y_t\}$ (see formula (5)):

$$p_1 = \mathrm{P}(Y_t = 1 \mid Y_{t-1} = 1) = p_{11}(M),$$

$$p_2 = \mathrm{P}(Y_t = 1, Y_{t-1} = 0 \mid Y_{t-2} = 1) = p_{10}(M) \cdot p_{01}(M),$$

$$\ldots,$$

$$p_\delta = p_{10}(M)\left[p_{00}(M)\right]^{\delta-2} p_{01}(M).$$

These relations enable to apply directly formulae (8) and (9) to calculate customer lifetime value and expected time of his relationship with company.

5. OPTIMAL STRATEGY

This section illustrates the method of calculating CLV described above. Retention rates take form of logistic function (4) and are put into a transition matrix of a Markov chain $\{Y_t\}$:

$$\mathbf{P}(M) = \begin{bmatrix} 1 - \frac{a_0}{1+\exp(-bM)} & \frac{a_0}{1+\exp(-bM)} \\ 1 - \frac{a_1}{1+\exp(-bM)} & \frac{a_1}{1+\exp(-bM)} \end{bmatrix}. \tag{10}$$

Parameters $a_0, a_1 \in (0,1)$ refer to retention ceiling, the limit retention rates for both states. With zero remarketing expenditure M retention rates equal half retention ceiling. Parameter b refers to the speed of convergence to retention ceiling (see Figure 1).

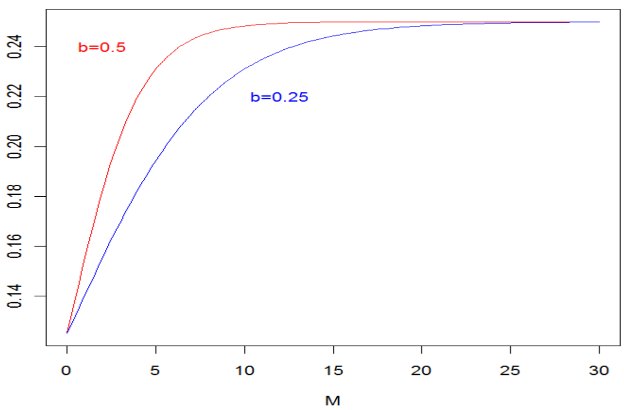

Figure 1. Retention rate (4a), a=0.25

Source: Author's computation

All the parameters of the transition matrix (10) are positive so the Markov chain is ergodic and its limit distribution takes form

$$\mathbf{e} = \left[\frac{1-a_1+\exp(-bM)}{1+a_0-a_1+\exp(-bM)} \quad \frac{a_0}{1+a_0-a_1+\exp(-bM)}\right]. \tag{11}$$

Transition matrix (10) enables to calculate expected time of relationship for various strategies of customer "disactivation" δ and his CLV. Table 1 contains the results obtained for exemplary set of parameters: $a_0 = 0{,}25$, $a_1 = 0{,}75$, $b = 0{,}5$, $\frac{1}{1+d} = 0{,}9$, R=100. Figure 2 illustrates the curve of CLV for various strategies depending on remarketing expenditure M. Figure 3 shows relation between CLV and frequency of purchases. Expected time of relationship for a set of parameters chosen is presented on Figure 4.

Table 1. Optimal expenditure and CLV

δ	CLV_{opt}	M_{opt}	$t^{(1)}$	Retention rate for state 0*	Retention rate for state 1*	Mean interval between purchases**
3	281,36	8,087	6,45	98,0%	98,3%	2,070
4	282,14	7,790	7,71	98,0%	98,0%	2,081
5	281,87	7,555	8,93	97,8%	97,8%	2,092
* as percentage of retention ceiling ** minimal interval for unconstrained budget equals 2						

Source: Author's computation

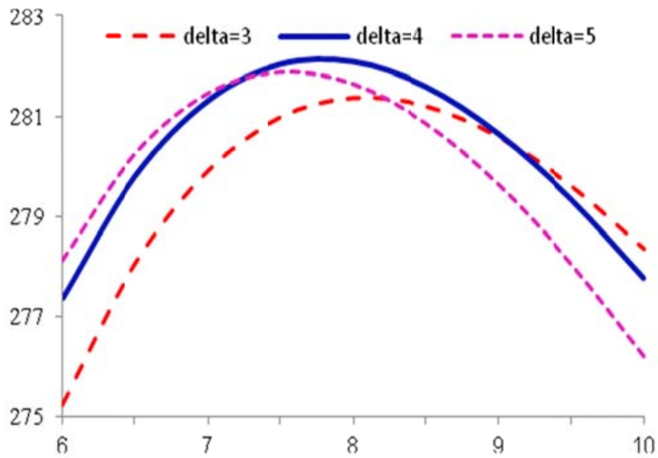

Figure 2. CLV curve for various strategies

Source: Author's computation

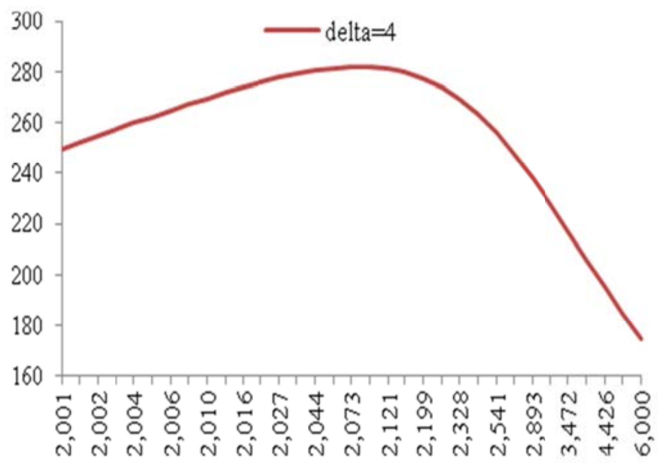

Figure 3. CLV and mean interval between purchases

Source: Author's computation

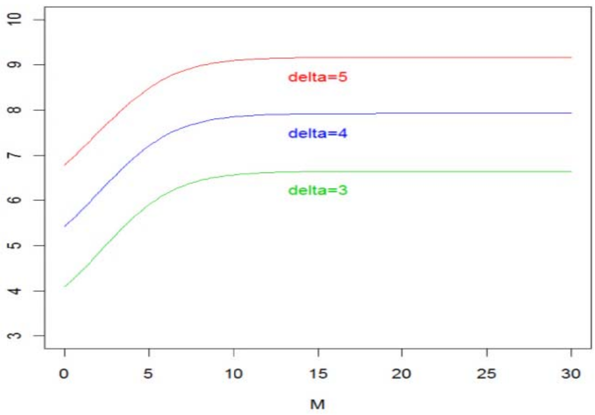

Figure 4. Expected time of relationship

Source: Author's computation

Table 2 shows optimal remarketing expenditure M and CLV for various values of a_0 and strategy $\delta = 3$. Increasing value of parameter a_0 (retention ceiling for state 1) is followed by increasing value of optimal CLV and decreasing value of remarketing expenditure and mean interval between purchases. Optimal solution is always obtained for retention probability oscillating around 98% of its ceiling. Table 3 sums up the reaction of optimal remarketing expenditure and CLV on parameter's changes by 0.01 for different strategies. Sensitivity of parameters a_0 and a_1 is not significant due to a fact that each time the optimal values of M and CLV are achieved for retention rate close to its ceiling. A stronger sensitivity is observed for parameter b responsible for convergence rate to retention ceiling. Figures 5 and 6 show how optimal CLV and remarketing expenditure react on increasing retention ceiling a_0 with other parameters being unchanged.

Table 2. Optimal CLV for various values of a_0

a_0	CLV	M	$p_{01}(M)$	$\frac{p_{01}(M)}{p_{01}(Inf)}$	$p_{11}(M)$	$\frac{p_{11}(M)}{p_{11}(Inf)}$	purchase interval
0,2	277,812	8,126	0,196	98,00%	0,737	98,27%	2,335
0,21	278,535	8,118	0,206	98,10%	0,737	98,27%	2,272
0,22	279,251	8,11	0,216	98,18%	0,737	98,27%	2,215
0,23	279,962	8,102	0,226	98,26%	0,737	98,27%	2,162
0,24	280,667	8,095	0,235	97,92%	0,737	98,27%	2,114
0,25	281,367	8,087	0,245	98,00%	0,737	98,27%	2,07
0,26	282,06	8,079	0,255	98,08%	0,737	98,27%	2,029
0,27	282,748	8,072	0,265	98,15%	0,736	98,13%	1,991
0,28	283,43	8,064	0,275	98,21%	0,736	98,13%	1,956
0,29	284,105	8,056	0,284	97,93%	0,736	98,13%	1,923
0,3	284,775	8,049	0,294	98,00%	0,736	98,13%	1,892

Sources: Author's computation

Table 3. Sensitivity of optimal CLV on parameters changes

	δ = 3		**δ = 4**		**δ = 5**	
	CLV_{opt}	**M_{opt}**	**CLV_{opt}**	**M_{opt}**	**CLV_{opt}**	**M_{opt}**
a_0, a_1	↑ 0,2-0,3%	↓ 0,1%	↑ 0,3%	↓ 0,1%	↑ 0,2-0,3%	↓ 0,1%
b	↑ 0,2-0,3%	↓ 1,3-1,5%	↑ 0,2-0,3%	↓ 1,3-1,5%	↑ 0,3-0,4%	↓ 1,3-1,5%

Sources: Author's computation

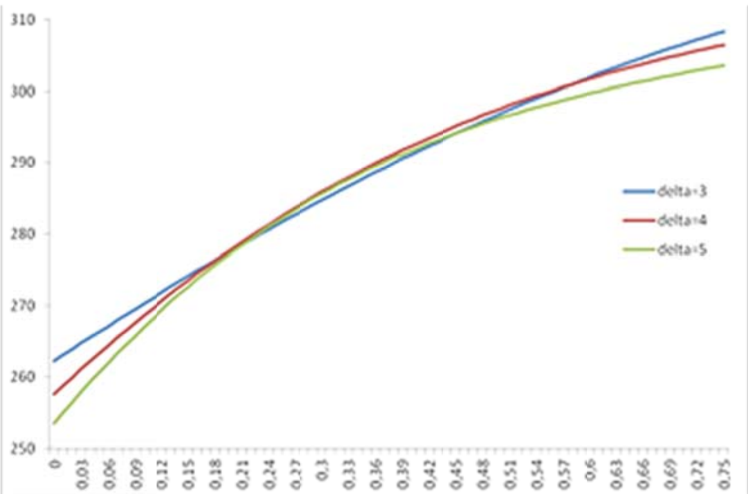

Figure 5. Optimal CLV for increasing retention ceiling

Source: Author's computation

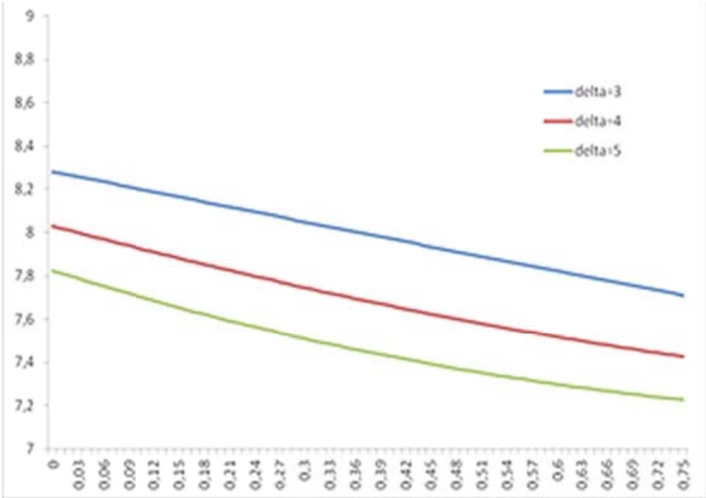

Figure 6. Optimal remarketing expenditure for increasing retention ceiling

Source: Author's computation

Analogous computation have been conducted for retention rate of form (3). This time the transition matrix of the Markov chain involved is

$$\mathbf{P} = \begin{bmatrix} 1 - a_0(1 - \exp(-bM)) & a_0(1 - \exp(-bM)) \\ 1 - a_1(1 - \exp(-bM)) & a_1(1 - \exp(-bM)) \end{bmatrix}, \tag{12}$$

and its limit distribution

$$\mathbf{e} = \begin{bmatrix} \frac{1 - a_1(1 - \exp(-bM))}{1 - (a_1 - a_0)(1 - \exp(-bM))} & \frac{a_0(1 - \exp(-bM))}{1 - (a_1 - a_0)(1 - \exp(-bM))} \end{bmatrix}. \tag{13}$$

Parameters $a_0, a_1 \in (0,1)$ refer to retention ceilings again and parameter $b > 0$ to effectiveness of maintaining customers, the rate of convergence to retention ceiling with increasing expenditures. Table 4 contains the results of computations analogous to those presented in Table 1.

Table 4. Optimal values of expenditure and CLV

δ	CLV_{opt}	M_{opt}	Retention rate for state 0*	Retention rate for state 1*	Mean interval between purchases**
3	281,209	8,158	98,3%	98,3%	2,069
4	281,935	7,872	98,0%	98,0%	2,080
5	281,616	7,648	97,8%	97,8%	2,089
* as percentage of retention ceiling ** minimal interval for unconstrained budget equals 2					

Source: Author's computation

Simulation taken for various values of model parameters lead to following conclusions:

- increasing parameter δ referring to time of considering inactive customer to be lost for good results in longer expected time of relationship but generally does not increase the optimal customer lifetime value,

- optimal level of remarketing expenditure and maximal CLV is obtained for retention rates leading 98% of their ceilings and mean purchases frequency is then slightly lower than frequency accessible with unconstrained budget,

- consequently, optimal values of remarketing expenditure and CLV are not significantly sensitive for parameters changes, particularly for changes of retention ceilings,

- model with last purchase memory seems to be a good basis for proper calculation of CLV and the method of its calculation presented is easy to apply and computationally non demanding.

Those conclusions remain valid for both retention rate in form of exponential function (3) and logistic function (4).

6. MARKOV CHAINS IN MARKETING

Probably the most common application of Markov chains in marketing is in modelling customers loyalty[16] (brand switching). Assuming that customer making brand purchasing decision remembers only the last brand chosen a Markov chain describing customers migration between brands is constructed with transition probabilities referring to retention, loyalty or churn rates. The models enables predicting market structure and in classical version is a homogeneous one however a natural modification would be modelling transition probabilities as functions of marketing efforts, price relations etc.

Identification of purchasing patterns of financial and insurance products has been analyzed by Prinzie and Poel[17] who used a third-order Markov model and associated mixture transition model (MTD). Bozetto[18] et al. examined customers relationship with an insurance company by means of various types of Markov models (homo and nonhomogeneous, first and second ordered, mover-stayer model) to identify purchasing dynamics patterns and predict future number and acquisition structure. Ching[19] et al. applied Markov chain with rewards methods to optimize CLV and derive optimal promotion strategy.

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