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Some Functions via δ-Semiopen Sets

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Abstract: In this paper, it is introduced and studied new classes of generalizations of some non-continuous functions concerning the concepts of weak forms of δ-semiopen sets in topological spaces. And also it is given some of their properties.

Keywords: α-open set, Semiopen set, Preopen set, β-open set.

δ-Yarıaçık Kümelerle Bazı Fonksiyonlar

Öz Bu makalede, topolojik uzaylarda δ-yarıaçık kümelerin zayıf kavramlarıyla bazı sürekli olmayan fonksiyonların yeni genelleştirmeleri ortaya konuldu ve çalışıldı. Aynı zamanda, bu fonksiyonların sağladığı bazı özellikler verildi.

Anahtar kelimeler: α-açık küme, Yarıaçık küme, Önaçık küme, β-açık küme.

1. Introduction

Recall the concepts of α-open (Njåstad, 1965) (resp. semiopen (Levine, 1963), preopen (Mashhour et al., 1982), β-open (Abd El-Monsef et al., 1983), g-closed (Levine, 1970), rg-closed (Palaniappan and Rao, 1993), αlc-set (Al-Nashef, 2002)) sets in topological spaces.

The purpose of this paper is to define and investigate the notions of new classes of functions, namely δαlc-semi-continuous, δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous functions, and to obtain some properties of these functions in topological spaces.

2. Preliminaries

Throughout this paper, spaces always mean topological spaces and f:X→Y denotes a single valued function of a space (X,τ) into a space (Y,υ). Let S be a subset of a space (X,τ). The closure and the interior of S are denoted by Cl(S) and Int(S), respectively.

Here we recall the following known definitions and properties.

Definition 2.1. A subset S of a space (X,τ) is said to be α-open (Njåstad, 1965) (resp. semiopen (Levine 1963), preopen (Mashhour et al., 1982), β-open (Abd El-Monsef et al., 1983)) if $S \subset Int(Cl(Int(S)))$ (resp. $S \subset Cl(Int(S))$, $S \subset Int(Cl(S))$, $S \subset Cl(Int(Cl(S)))$).

The family of all α-open (resp. semiopen, preopen, β-open) sets in a space (X,τ) is denoted by $\alpha(X)$ (resp. $SO(X)$, $PO(X)$, $\beta O(X)$). It is shown in (Njåstad, 1965) that $\alpha(X)$ is a topology for X. Moreover, $\tau \subset \alpha(X) = PO(X) \cap SO(X) \subset PO(X) \cup SO(X) \subset \beta O(X)$.

Definition 2.2. A subset A of a space (X,τ) is called

(1) a generalized closed (briefly, g-closed) set (Levine, 1970) if $Cl(A) \subset U$ whenever $A \subset U$ and U is open.

(2) a regular generalized closed (briefly, rg-closed) set (Palaniappan and Rao, 1993) if $Cl(A) \subset U$ whenever $A \subset U$ and U is regular open.

(3) an αlc-set (Al-Nashef, 2002) if $A=S\cap F$, where S is α-open and F is closed.

(4) an slc-set (Beceren et al., 2006) if $A=S\cap F$, where S is semi open and F is closed.

(5) a plc-set (Beceren et al., 2006) if $A=S\cap F$, where S is preopen and F is closed.

(6) a βlc-set (Beceren et al., 2006) if $A=S\cap F$, where S is β-open and F is closed.

(7) an αglc-set (Beceren et al., 2006) if $A=S\cap F$, where S is α-open and F is g-closed.

(8) an sglc-set (Beceren et al., 2006) if $A=S\cap F$, where S is semi open and F is g-closed.

(9) a pglc-set (Beceren et al., 2006) if $A=S\cap F$, where S is preopen and F is g-closed.

(10) a βglc-set (Beceren et al., 2006) if $A=S\cap F$, where S is β-open and F is g-closed.

(11) an αrglc-set (Beceren et al., 2006) if $A=S\cap F$, where S is α-open and F is rg-closed.

(12) an srglc-set (Beceren et al., 2006) if $A=S\cap F$, where S is semi open and F is rg-closed.

(13) a prglc-set (Beceren et al., 2006) if $A=S\cap F$, where S is preopen and F is rg-closed.

(14) a βrglc-set (Beceren et al., 2006) if $A=S\cap F$, where S is β-open and F is rg-closed.

The family of all αlc-sets (resp. plc-sets, slc-sets, βlc-sets, αglc-sets, pglc-sets, sglc-sets, βglc-sets, αrglc-sets, prglc-sets, srglc-sets, βrglc-sets) in a space (X,τ) is denoted by $\alpha LC(X)$ (resp. $PLC(X)$, $SLC(X)$, $\beta LC(X)$, $\alpha GLC(X)$, $PGLC(X)$, $SGLC(X)$, $\beta GLC(X)$, $\alpha RGLC(X)$, $PRGLC(X)$, $SRGLC(X)$, $\beta RGLC(X)$). Moreover, $\alpha(X) \subset \alpha LC(X) \subset PLC(X) \subset \beta LC(X)$ and $PO(X) \subset PLC(X)$ (Beceren et al., 2006).

Remark 2.1 (Noiri, 1996). It is known that closed ⇒ g-closed ⇒ rg-closed. In general, none of the implications is reversible.

Lemma 2.1 (Beceren and Noiri, 2008). Let (X,τ) be a topological space. Then we have

(1) $\alpha LC(X) \subset \alpha GLC(X) \subset \alpha RGLC(X)$.

(2) $PLC(X) \subset PGLC(X) \subset PRGLC(X)$.

(3) $SLC(X) \subset SGLC(X) \subset SRGLC(X)$.

(4) $\beta LC(X) \subset \beta GLC(X) \subset \beta RGLC(X)$.

A topological space (X,τ) is called a $T_{1/2}$-space (Levine, 1970) (resp. T_{rg}-space (Rani and Balachandran, 1997)) iff every g-closed (resp. rg-closed) subset of X is closed (resp. g-closed).

Lemma 2.2 (Beceren et al., 2006). Let (X,τ) be a $T_{1/2}$-space. Then we have

(1) $\alpha GLC(X) = \alpha LC(X)$.

(2) $PGLC(X) = PLC(X)$.

(3) $SGLC(X) = SLC(X)$.

(4) $\beta GLC(X) = \beta LC(X)$.

Lemma 2.3 (Beceren et al., 2006). Let (X,τ) be a T_{rg}-space. Then we hold

(1) $\alpha RGLC(X) = \alpha GLC(X)$.

(2) $PRGLC(X) = PGLC(X)$.

(3) $SRGLC(X) = SGLC(X)$.

(4) $\beta RGLC(X) = \beta GLC(X)$.

Lemma 2.4 (Al-Nashef, 2002). Let (X,τ) be a topological space. Then $SO(X)=\beta O(X)\cap\alpha LC(X)$.

Let A be a subset of a space X. A point $x\in X$ is called the δ-cluster point of A if $A\cap Int(Cl(U))\neq\emptyset$ for every open set U of X containing x. The set of all δ-cluster points of A is called the δ-closure of A, denoted by $Cl_\delta(A)$. A subset A of X is called δ-closed if $A=Cl_\delta(A)$. The complement of a δ-closed set is called δ-open (Veličko, 1968).

A subset A of a space X is said to be a δ-semiopen set if there exists a δ-open set U of X such that $U\subset A\subset Cl(U)$. The complement of a δ-semiopen set is called δ-semiclosed (Park et al., 1997).

A point $x\in X$ is called the δ-semicluster point of A if $A\cap U\neq\emptyset$ for every δ-semiopen set U of X containing x. The set of all δ-semicluster points of A is called the δ-semiclosure of A, denoted by $\delta Cl_s(A)$ (Caldas et al., 2009).

A subset S of a space (X,τ) is δ-semiopen (resp. δ-semiclosed) if $S\subset Cl(Int_\delta(S))$ (resp. $Int(Cl_\delta(S))\subset S$) (Park et al., 1997).

Remark 2.2 (Park et al., 1997). It is known that every δ-semiopen set is semiopen but the converse is not true in general.

Lemma 2.5 (Park et al., 1997). The intersection (resp. union) of an arbitrary collection of δ-semiclosed (resp. δ-semiopen) sets in (X,τ) is δ-semiclosed (δ-semiopen). And $A \subset X$ is δ-semiclosed if and only if $A=\delta Cl_s(A)$.

Lemma 2.6 (Caldas et al., 2009). Let A and B be subsets of a space (X,τ). Then we have

(1) If A is δ-semiopen in X and B is δ-open in X, then $A \cap B$ is δ-semiopen in B.

(2) If A is δ-semiopen in B and B is δ-open in X, then A is δ-semiopen in X.

A function $f : (X,\tau) \to (Y,\upsilon)$ is said to be δ-semi-continuous (Caldas et al., 2003) if $f^{-1}(V)$ is δ-semiopen in X for every δ-semiopen set V in Y.

3. Generalizations of Some Types of Functions

Definition 3.1. A function $f : (X,\tau) \to (Y,\upsilon)$ is said to be δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous) if $f^{-1}(V)$ is δ-semiopen in X for every αlc-set (resp. plc-set, slc-set, βlc-set, αglc-set, pglc-set, sglc-set, βglc-set, αrglc-set, prglc-set, srglc-set, βrglc-set) V in Y.

The proofs of the other parts of the following theorems follow by a similar way and are thus omitted.

Theorem 3.1. If $f:(X,\tau)\to(Y,\upsilon)$ is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous) and A is a δ-open subset of X, then the restriction $f_{/A}: A \to Y$ is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous).

Proof. Let V be any αlc-set (resp. plc-set, slc-set, βlc-set, αglc-set, pglc-set, sglc-set, βglc-set, αrglc-set, prglc-set, srglc-set, βrglc-set) of Y. Since f is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous), then $f^{-1}(V)$ is a δ-semiopen set in X. Since A is δ-open in X, $(f_{/A})^{-1}(V) = A \cap f^{-1}(V)$ is δ-semiopen in A by Lemma 2.6. Hence $f_{/A}$ is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous).

Theorem 3.2. Let f:(X,τ)→(Y,υ) be a function and $\{A_\lambda: \lambda\in\Lambda\}$ be a cover of X by δ-open sets of (X,τ). Then f is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous) if $f_{/A_\lambda} : A_\lambda \to Y$ is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous) for each $\lambda\in\Lambda$.

Proof. Let V be any αlc-set (resp. plc-set, slc-set, βlc-set, αglc-set, pglc-set, sglc-set, βglc-set, αrglc-set, prglc-set, srglc-set, βrglc-set) of Y. Since $f_{/A_\lambda}$ is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous), $(f_{/A_\lambda})^{-1}(V)=f^{-1}(V)\cap A_\lambda$ is δ-semiopen in A_λ. Since A_λ is δ-open in X, then $(f_{/A_\lambda})^{-1}(V)$ is δ-semiopen in X for each $\lambda\in\Lambda$ by Lemma 2.6. Therefore, $f^{-1}(V) = X\cap f^{-1}(V) = \cup\{A_\lambda\cap f^{-1}(V): \lambda\in\Lambda\} = \cup\{(f_{/A_\lambda})^{-1}(V): \lambda\in\Lambda\}$ is δ-semiopen in X by Lemma 2.5. Hence f is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous).

Theorem 3.3. Let f:X→Y be a δ-semi-continuous function and g:Y→Z be a function. If g is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous), then the composition gof:X→Z is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous).

Proof. Let W be any αlc-set (resp. plc-set, slc-set, βlc-set, αglc-set, pglc-set, sglc-set, βglc-set, αrglc-set, prglc-set, srglc-set, βrglc-set) of Z. Since g is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous), $g^{-1}(W)$ is δ-semiopen in Y. Since f is δ-semi-continuous, then $(gof)^{-1}(W) = f^{-1}(g^{-1}(W))$ is δ-semiopen in X and hence gof is δαlc-semi-continuous (resp. δplc-semi-continuous, δslc-semi-continuous, δβlc-semi-continuous, δαglc-semi-continuous, δpglc-semi-continuous, δsglc-semi-continuous, δβglc-semi-continuous, δαrglc-semi-continuous, δprglc-semi-continuous, δsrglc-semi-continuous, δβrglc-semi-continuous).

Theorem 3.4. Let (Y,υ) be a $T_{1/2}$-space and let f:(X,τ)→(Y,υ) be a function. Then we have

(1) δαlc-semi-continuity ⇔ δαglc-semi-continuity,

(2) δplc-semi-continuity ⇔ δpglc-semi-continuity,

(3) δslc-semi-continuity ⇔ δsglc-semi-continuity,

(4) δβlc-semi-continuity ⇔ δβglc-semi-continuity.

Proof. This follows immediately from Lemma 2.2.

Theorem 3.5. Let (Y,υ) be a T_{rg}-space. For a function f:(X,τ)→(Y,υ), we hold

(1) δαglc-semi-continuity ⇔ δαrglc-semi-continuity,

(2) δpglc-semi-continuity ⇔ δprglc-semi-continuity,

(3) δsglc-semi-continuity ⇔ δsrglc-semi-continuity,

(4) δβglc-semi-continuity ⇔ δβrglc-semi-continuity.

Proof. It is obvious from Lemma 2.3.

Corollary 3.1. Let (Y,υ) be a $T_{1/2}$-space and T_{rg}-space. For a function f : (X,τ) → (Y,υ), we hold

(1) δαlc-semi-continuity ⇔ δαglc-semi-continuity ⇔ δαrglc-semi-continuity,

(2) δplc-semi-continuity ⇔ δpglc-semi-continuity ⇔ δprglc-semi-continuity,

(3) δslc-semi-continuity ⇔ δsglc-semi-continuity ⇔ δsrglc-semi-continuity,

(4) δβlc-semi-continuity ⇔ δβglc-semi-continuity ⇔ δβrglc-semi-continuity.

Proof. This is an immediate consequence of Theorems 3.4 and 3.5.

We recall that a space (X,τ) is said to be submaximal (Bourbaki, 1966) if every dense subset of X is open in X and extremally disconnected (Njåstad, 1965) if the closure of each open subset of X is open in X. The following theorem follows from the fact that if (X,τ) is a submaximal and extremally disconnected space, then τ=α(X)=SO(X)=PO(X)=βO(X) ((Janković, 1983), (Nasef and Noiri, 1998)).

Theorem 3.6. Let (Y,υ) be a submaximal and extremally disconnected space and let f:(X,τ)→(Y,υ) be a function. Then we have

(1) δαlc-semi-continuity ⇔ δplc-semi-continuity ⇔ δslc-semi-continuity ⇔ δβlc-semi-continuity.

(2) δαglc-semi-continuity ⇔ δpglc-semi-continuity ⇔ δsglc-semi-continuity ⇔ δβglc-semi-continuity.

(3) δαrglc-semi-continuity ⇔ δprglc-semi-continuity ⇔ δsrglc-semi-continuity ⇔ δβrglc-semi-continuity.

From the definitions, Lemma 2.4 and Remark 2.2, the following implication is hold for a function f:(X,τ)→(Y,υ):

δαlc-semi-continuity⇒δ-semi-continuity.

4. Conclusion

The area of mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Then, the generalizations of continuity are one of the most important subjects in general topology. Hence, it is obtained some of their properties and some non-continuous functions in topology.

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