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Sniper Rifle Selection Using Evidential Fuzzy Multi-Criteria Decision Making

Kanıtsal Bulanık Çok Kriterli Karar Vermeyi Kullanarak Keskin Nişancı Tüfeği Seçimi

Galip Cihan YALÇIN

Kırıkkale University, Faculty of Science, Department of Mathematics, Kırıkkale, Türkiye.

Kırıkkale Üniversitesi, Fen Fakültesi, Matematik Bölümü, Kırıkkale, Türkiye.

Güvenç ARSLAN

Kırıkkale University, Faculty of Engineering and Natural Sciences, Department of Statistical, Kırıkkale, Türkiye.

Kırıkkale Üniversitesi, Mühendislik ve Doğa Bilimleri Fakültesi, İstatistik Bölümü, Kırıkkale, Türkiye.

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Sorumlu Yazar/Corresponding author:
Galip Cihan Yalçın
E-mail: pgcy2014@gmail.com

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ABSTRACT

Some of the main reasons for the uncertainties that are effective in the decision-making processes are imprecision, randomness, and ambiguity. One of the methods to deal with these uncertainties is the DST method. DST stands out in applications, especially with its ability to cope with both random and incomplete information and inconsistency. The main purpose of this study is to compare the results obtained in a previous sniper rifle selection problem with the results obtained by the DST method using EFMCDM and to evaluate whether the results obtained by the two methods are compatible with each other. In this study 4 sniper rifles were evaluated with respect to 6 criteria. From the research findings it was concluded that the DST method provides similar results to the outranking based fuzzy decision-making method for the sniper rifle selection problem. In addition, the results show that the security forces can use the DST method for this type of selection problem. In conclusion, it has been demonstrated that the EFMCDM method based on the belief entropy method can be used in many similar selection problems.

Keywords*:* Fuzzy logic, evidential fuzzy multi-criteria decision making, Dempster-Shafer theory, sniper gun.

JEL Codes: F40, E20

ÖZ

Karar verme süreçlerinde etkili olan belirsizliklerin temel nedenlerinden bazıları kesin olmama, rastgelelik ve muğlaklıktır. Bu belirsizliklerle başa çıkma yöntemlerinden biri de DST yöntemidir. DST, uygulamalarda özellikle hem rastgele ve eksik bilgi hem de tutarsızlık ile baş edebilme yeteneği ile öne çıkmaktadır. Bu çalışmanın temel amacı daha önce yapılmış bir keskin nişancı tüfeği seçim probleminde elde edilen sonuçlarla, EFMCDM kullanılarak DST yöntemiyle elde edilen sonuçları karşılaştırmak ve iki yöntemle elde edilen sonuçların birbiriyle uyumlu olup olmadığını değerlendirmektir. Çalışmada 4 keskin nişancı tüfeği 6 kritere göre değerlendirilmiştir. Araştırma bulgularından, DST yönteminin keskin nişancı tüfeği seçim problemi için üstünlük esaslı bulanık çok ölçütlü karar verme yöntemine benzer sonuçlar verdiği sonucuna varılmıştır. Ayrıca sonuçlar, güvenlik güçlerinin bu tür bir seçim problemi için DST yöntemini kullanabileceğini göstermektedir. Sonuç olarak, inanç entropisi yöntemine dayalı EFMCDM yönteminin benzer birçok seçim probleminde kullanılabileceği ortaya konmuştur.

Anahtar Kelimeler*:* Bulanık mantık, kanıtsal bulanık çok kriterli karar verme, Dempster-Shafer teorisi, keskin nişancı silahı.

JEL Kodları: F40, E20

Intruduction

The main purpose of MCDM methods is to determine the most suitable alternative by comparing the alternatives according to the determined criteria. The alternatives are evaluated with respect to these criteria, which are expressed with numerical measurements or evaluations of users/experts. On the other hand, in most real applications numerical expressions are insufficient due to lack of precise information. Therefore, it may be necessary to use alternative approaches such as fuzzy logic (Lin & Hung, 2011). In addition, in many applications, expert knowledge can be expressed verbally more comfortably and easily. For this reason, more reliable results can be obtained by using methods such as fuzzy logic in many decision-making methods (Chatterjee & Namin, 2021).

Many methods have been developed to deal with situations involving incompleteness, imprecision, uncertainty, and inconsistency that negatively affect the decision-making process. Some of them are Probability Theory, Fuzzy Set Theory and DST (Zhu et al., 2021). Among these theories, Fuzzy Set Theory deals with ambiguity, while Probability Theory is concerned mainly with randomness (Chatterjee & Namin, 2021). DST is more effective than most of the other theories for dealing with inconsistency, incomplete information, and randomness (Xiao, 2020). In addition, DST introduced the concept of conflict arising from the assumption that available information may be inconsistent. One of the most important advantages of DST is that it considers with uncertainty and uses it in the decision-making process (Turhan, 2014; Sun et al., 2020; Chatterjee & Namin, 2021). Another important advantage is that there is no obligation to give a clear value to the variables as in Bayesian inference (Danaee et al., 2017). This shows how flexible the theory is.

DST has been successfully used in many different applications ranging from artificial intelligence to medical diagnosis, from statistical classification to data fusion, from face recognition to risk assessment, and from target identification to multi-criteria decision analysis (Seçkin, 2015).

Seçkin (2015) used the Dempster-Shafer/Analytical Hierarchy Process (DS/AHP) method in the supplier selection problem. In the study of Çavdur (2005), search engines used the DST method to detect subject changes in user login. Aygün and Adalı (2010) used the DST method to combine classification algorithms in their study. In his study, Beynon et al. Beynon et al., (2000) talked about the lessons to be taken about the DS/AHP method. Turhan (2014) conducted a study on decision making using the DST method in target tracking applications. Danaee et al. (2017) used the DS/AHP method while determining the optimum criteria list in his study. Sun et al. (2020) and Xiao (2020) discussed the supplier selection problem using the DST method in their studies. Fei et al. (2019) have handled the supplier selection problem by using the ELECTRE method based on the DST method. Chinnasamy et al. (2022) developed a MCDM system on deletion and selective translation reality with the fuzzy ELECTRE method based on the DST method. Liu and Gao (2019) discussed the air conditioning system selection problem by using the intuitionistic fuzzy power Bonferroni mean operator in the DST method. Wu and Tang (2020) proposed an improved failure mode and effect analysis method based on the uncertainty measure in the steel sheet production process with the DST method. Fei and Feng (2021) dealt with the air conditioner selection problem in the DST method with Intuitionistic fuzzy numbers in their studies. Zhong et al. (2023) discussed the smart device selection problem in the DST method with Power Muirhead Mean Operators of Interval-Valued Intuitionistic Fuzzy Values. Dymova et al. (2021) discussed the supplier selection problem in the DST method with intuitionistic fuzzy TOPSIS in their study. Qin et al. (2020), in their study, discussed an example of the selection of the best enterprise resource planning system in the DST method with picture fuzzy values. In the study of Si et al. (2023), a drug selection that can be more efficient in the treatment process of a COVID-19 patient is discussed by using picture fuzzy set (PFS) and DS evidence theory methods. In the study of Mokarram and Sathyamoorthy (2023), geographic information system (GIS) based fuzzy AHP and DST methods were used to identify suitable locations for gas power plant construction. Wu et al. (2023) evaluated the risk analyses of aircraft turbine rotor blade failure modes using the DST method based on belief entropy. In their study, Fei and Ma (2023) used a method based on DST method that takes into account the intuitionistic fuzzy environment for emergency alternative selection in a case of flood disaster in China. Zhang et al. (2023) used the DST-based PROMETHEE method to identify a public safety training base in China. In their study, Ngo et al. (2023) addressed the service innovation evaluation problem for three banks in Vietnam with the DST method. In their study, Qin et al. (2023) used the interval-valued intuitionistic fuzzy DST method to determine the best alternative and compared it with other methods. Rashki and Faes (2023) used the DST method to minimize the probability of error in an efficient reliability analysis. In their study, Dutta and Shome (2023) used the DST method based on belief correlation measure for target recognition fusion of sensors.

One of the main aims of this study is to determine whether EFMCDM (Xiao, 2020) based on the belief entropy is compatible with the outranking based FMCDM method (Aouam et al., 2003). In addition, we also investigate how similar the results are for a sample study, namely the selection of sniper rifles, which was studied in Arslan and Aydın (2009). The main contributions of this paper are as follows: i) Application of the EFMCDM based on belief entropy to the sniper rifle selection problem. ii) EFMCDM method has been compared with an outranking based FMCDM method and the results were found to be consistent. iii) The results demonstrate that some of the uncertainties that are effective in the decision-making process can be overcome with the EFMCDM based on belief entropy.

In Section 1, the basics of fuzzy logic and DST method are explained. In addition, the steps of the EFMCDM based on belief entropy are explained. In Section 2 of the research, an application was made by using the sniper rifle selection problem with data from Arslan and Aydın (2009). We also performed sensitivity analysis in this part. In the conclusion part, a summary of the application is presented.

Material Method

Fuzzy Logic

Fuzzy logic was introduced with the article named “Fuzzy Sets” published in (Zadeh, 1965). Due to the uncertain environment in our daily life, better decisions can be made with the help of some theories such as decision theory and probability theory. Therefore, the concepts of uncertainty and randomness are a naturel part of modern decision making. Randomness is used to describe the uncertainty of occurrence for a member of a set, whereas fuzziness is used to describe situations where various degrees of membership are possible for members of a set. In other words, the decision maker is not forced to use only 2-valued logic, that is there are generalizations to statements such as "true/false" or "yes/no" in fuzzy logic. Intermediate values such as "I strongly disagree, I disagree, I am undecided, I agree, I completely agree" can be used (Taban, 2019). It should be noted also that linguistic expressions used to explain perception or reasoning are always subjective and ambiguous.

Scientifically, fuzziness is defined as uncertainty. Uncertainty is a situation that cannot be fully known, contains subjective data, and expresses the different opinions of decision makers. This uncertainty has led to the development of fuzzy logic. The fuzziness is expressed as uncertainty or not definite in expressions defining a purpose and a system. In other words, perception differences in human thought, subjective behaviors and uncertainties in their goals can be explained by fuzziness.

In fuzzy set theory, a flexible and gradual transition is allowed from objects that are fully members of the set to objects that are not completely members of the set (Taban, 2019). The main difference between traditional sets and fuzzy sets is their membership functions. The membership functions of fuzzy sets represent linguistically uncertain concepts in a meaningful way. Thus, the uncertainty of managers in decision making is reduced.

Sets in which an object takes the values 0 and 1 depending on whether it is a member of the set or not are called classical sets. This idea is based on Aristotle's 1 and 0 logic. Full membership status is 1 and non-member status is 0. That is, an element is either a member of the set or not. In fuzzy sets, on the other hand, each of the objects is assigned a value consisting of numbers between 0 and 1, representing their degree of membership in the relevant cluster (meaning belonging, not belonging to that cluster, or how much they belong to the cluster). This value (membership degree) can have an infinite number of values in the continuous range of [0,1].

Fuzzy numbers can be considered as special subsets of fuzzy sets. There are many types of fuzzy numbers, and the most commonly used ones are trapezoidal and triangular fuzzy numbers. In this study, we use triangular fuzzy numbers.

Triangular fuzzy numbers will be denoted by (l, m, u), where:

l→The smallest possible value

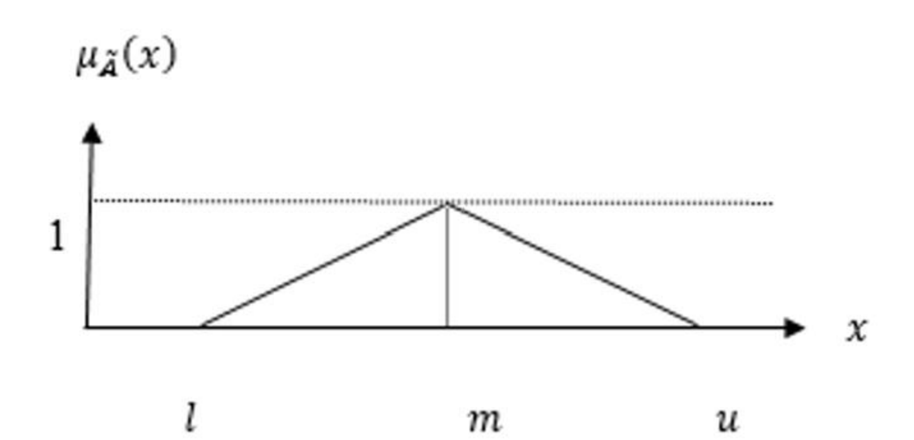
m→Most expected value

u →The greatest possible value (Taban, 2019).

The membership function of a triangular fuzzy number (l, m, u) is shown in Figure 1.

Figure 1

The triangular fuzzy numbers



DST Method

The decision-making process can be briefly defined as the process of selecting the best alternative by evaluating alternatives according to one or more criteria. The decision maker should combine the available data while evaluating the alternatives according to the criteria. There may be uncertainties in many of these data to be combined. Some common causes of uncertainty are; imprecision, randomness and ambiguity. Inconsistency is expressed as errors arising from the measurements made. Ambiguity refers to the uncertainties arising from linguistic expressions such as "good", "bad" during decision making.

DST emerged with A.P. Dempster's work in 1967 on determining the lower and upper limits of probability. It was later developed with some additions in G. Shafer's book, The Mathematical Theory of Proof, in 1976. That's why the theory was named DST, named after both fathers of ideas. This theory is a numerical method and has been developed to combat incomplete and uncertain information (Seçkin, 2015).

DST finds the total probability of the event affected by these events by combining information about the probabilities of more than one event. It is also an information aggregation rule that aims to increase the confidence level of information from different sources of evidence (Tang et al., 2021). DST is an effective tool for combining aggregated basic probability assignment (bpa) values and demonstrates how robust it is to make decisions with simple calculations in an uncertain and imprecise environment (Turhan, 2014).

DST can be considered as a development of Bayesian probability theory in a sense. It has the advantage of showing "uncertainty" by distributing probability to multiple event hypotheses rather than a single event (Zhu & Xiao, 2021). In Bayesian probability theory, the weights assigned to the states are called "probability", while in DST they are called "mass". The most important distinguishing feature is; in DST, there is no obligation to give a clear value to the variables as in Bayesian probability theory (Danaee et al., 2017).

DST has the ability to represent information, even if there is a lack of prior knowledge of all possible outcomes. It also provides a simple computational method for combining evidence from multiple sources, resulting in a numerical score for each view and belief (Chatterjee & Namin, 2021).

If a variable can take Θ different values in DST, that variable has 2^{Θ} subsets and each subset is assigned a mass value. And the sum of the mass values of these subsets is equal to 1 (Danaee et al., 2017).

Belief function (bel) and bpa function, which are one-to-one functions, express the information in DST. DST defines the state space in probability theory as the frame of discernment (Danaee et al., 2017). As the number of elements in the frame of discernment increases, the computational complexity also increases. Also, when the evidence is highly contradictory, the consolidation result may not be normal. It can easily be seen that all of these problems are related to the bpa values, thus determining the bpa values is the first step that directly affects the calculation of the DST (Zhu et al., 2021).

In this study we are interested in application of DST to MCDM problems. Therefore, in this setting the frame of discernment represents the set of possible actions or selections, which will be denoted by $\Theta = \{A_1, A_2, \dots, A_i, \dots, A_N\}$. The power set of Θ denoted by 2^{Θ} contains all subsets of Θ, that is $2^{\Theta} = \{\emptyset, \{A_1\}, \{A_2\}, \dots, \{A_N\}, \{A_1, A_2\}, \dots, \{A_1, A_2, \dots, A_i\}, \dots, \Theta\}$. If the following conditions given in equations (2) and (3) are met for a function $m: 2^{\Theta} \to [0,1]$, then this function is a bpa function (Büyükyazıcı & Sucu, 2009):

$m(\emptyset) = 0$ (1)

$\sum_{x \in 2^{\Theta}} m(x) = 1$ (2)

The values of the bpa function here are defined as the probabilities of each element in the frame of discernment, Θ. If each subset element here contains only one element, then in this case, DST overlaps with Bayesian deduction and bpa function is considered as probability density function. Therefore, DST is considered as a generalized form of Bayesian inference. The m function is called the bpa function, and $m(x)$ is called the basic probability of x (Xiong et al., 2021).

The sum of the mass values of the subsets is equal to 1. It can also attain 0 because there is no assignment to the empty set. When x is considered as any subset of Θ and the bpa value is different from 0, x becomes a focal element (Xiong et al., 2021). The value taken for any focal element belonging to the bel function is called the degree of belief (Büyükyazıcı & Sucu, 2009).

There are two confidence criteria: bel function and plausibility (pls) function. As the first confidence criterion, the one-to-one bel function is defined as $bel: 2^{\Theta} \to [0,1]$ and this value is obtained from the sums of the propositions as Equation (3) (Beynon et al., 2000).

$bel\ (A) = \sum_{B \subseteq A} m(B),\ \forall A \subseteq \Theta$ (3)

The $bel(A)$ value here indicates the level of confidence in any subset of A or A itself. That is, it is the definitive support for A.

Another confidence criteria, the one-to-one function pls (Equation 4), is defined as $pls: 2^{\Theta} \to [0,1]$.

$pls\ (A) = 1 - bel(\bar{B}) = \sum_{B \cap A \neq \emptyset} m(B),\ \forall A \subseteq \Theta$ (5)

The $pls(A)$ value here also indicates the failure level in case of distrust for A or any subset where A is present. So, it is more plausible to support A.

$bel(A)$ and $pls(A)$ represent the lower and upper limits of the A function. The lower bound indicates the hypothesis's degree of support and the upper bound the maximum degree of support expected to be assigned to the hypothesis. From here, it is also understood that $pls(A) \geq bel(A)$ (Xiao, 2020).

The mass values, the relations of the observed variables with each other and the relations of the values obtained from the previous observations are obtained using Dempster Rule of Combination (DRC). This rule is used both to combine previously known data belonging to a single decision maker and observed data, and to combine data belonging to more than one decision maker (Çavdur, 2005). Information obtained from different information sources can be easily combined with DRC (Xiong et al., 2021).

When the independent evidence sources m_1 and m_2 are combined with Equation (5), the toa function $m = m_1 \oplus m_2: 2^{\Theta} \to [0,1]$ is obtained [4]. The $\oplus$ operator has commutative and associativity features (Tong et al., 2021).

$$m(A) = \begin{cases} 0 & , \quad A = \emptyset \\ \frac{\sum_{B,C \in 2^{\Theta} | B \cap C = A} m_1(B) m_2(C)}{1-K} & , \quad A \neq \emptyset \end{cases} \quad (5)$$

Here K is expressed as the conflict coefficient between m_1 and m_2 and is calculated with Equation (6):

$K = \sum_{B,C \in 2^{\Theta} | B \cap C = \emptyset} m_1(B) m_2(C)$ (6)

For DRC to be useful; the K coefficient should be less than 1 (Xiao, 2020). If $K=0$, there is no contradiction between the two proofs, that is, each focus set m_1 intersects all focus sets of m_2. If $K=1$, the two sets of evidence are logically contradictory and as a result they cannot be combined. The mass function $m_1 \oplus m_2$ is called the orthogonal sum of m_1 and m_2 (Denoeux et al., 2020). K is the mass related to opposing beliefs and is determined as the result of the mass of each empty intersection (Chatterjee & Namin, 2021).

EFMCDM

In this subsection we give an outline of the steps for applying EFMCDM introduced by Xiao (2020) to MCDM problems.

When applying the DST method to MCDM problems, the frame of discernment is the set of alternatives, which is defined as $\Theta = \{A_1, A_2, \ldots., A_m\}$ and the set of criteria as $C = \{C_1, C_2, \ldots., C_n\}$. The i^th^ alternative is shown as $A_i (i = 1,2, \ldots, m)$ and the j^th^ criterion as $C_j (j = 1,2, \ldots, n)$. Evaluation of the i^th^ alternative according to the j^th^ criterion is denoted by $\{\tilde{x}_{ij}, i = 1,2, \ldots, m\ ve\ j = 1,2, \ldots, n\}$, whereas the weighting of the criteria is denoted as $\{\widetilde{w}_j, j = 1,2, \ldots, n\}$. Since these are fuzzy numbers, we have $\tilde{x}_{ij} = (\tilde{x}_{ij1}, \tilde{x}_{ij2}, \tilde{x}_{ij3})$ and $\widetilde{w}_j = (\widetilde{w}_{j1}, \widetilde{w}_{j2}, \widetilde{w}_{j3})$ (Xiao, 2020).

The linguistic expressions and fuzzy number values to be used while evaluating the alternatives are shown in Table 1 (Arslan & Aydın, 2009).

The steps to be performed in the application are shown below step by step:

Step-1: The weights of the criteria are determined in Equation (7) (j=1, 2, …, n).

Table 1

Linguistic Expressions for Alternatives and Corresponding Fuzzy Numbers

Linguistic expressions	**Abbreviations**	**Fuzzy number values**
Very High	VH	(0.8, 1.0, 1.0)
High	H	(0.6, 0.8, 1.0)
Medium	M	(0.3, 0.5, 0.7)
Low	L	(0.0, 0.2, 0.4)
Very Low	VL	(0.0, 0.0, 0.2)

$$\widetilde{W} = [\widetilde{w}_1 \ldots \widetilde{w}_j \ldots \widetilde{w}_n] \quad (7)$$

Step-2: The decision matrix (Equation 8) containing the values of the i^th^ alternative according to the j^th^ criterion is arranged (i=1, 2, …, m).

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1j} & \cdots & \tilde{x}_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \tilde{x}_{i1} & \cdots & \tilde{x}_{ij} & \ldots & \tilde{x}_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mj} & \ldots & \tilde{x}_{mn} \end{bmatrix} (8)$$

Step-3: Elements of the weighted decision matrix $\left(\tilde{x}_{ij}^{w}\right)$ are found with Equation (9). Values in this equation, Equation (10) is found with.

$$\tilde{x}_{ij}^{w} = \tilde{x}_{ij} * \widetilde{w}_j = (x_{ij1}^{w}, x_{ij2}^{w}, x_{ij3}^{w})$$

$$\tilde{x}_{ij1}^{w} = \tilde{x}_{ij1} * \widetilde{w}_{j1} \quad (9)$$

$$\tilde{x}_{ij2}^{w} = \tilde{x}_{ij2} * \widetilde{w}_{j2}$$

$$\tilde{x}_{ij3}^{w} = \tilde{x}_{ij3} * \widetilde{w}_{j3} \quad (10)$$

The weighted decision matrix $(\widetilde{D})$ is created as Equation (11):

$$\widetilde{D}=\begin{bmatrix} \tilde{x}_{11}^{w} & \cdots & \tilde{x}_{1j}^{w} & \cdots & \tilde{x}_{1n}^{w} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \tilde{x}_{i1}^{w} & \cdots & \tilde{x}_{ij}^{w} & \dots & \tilde{x}_{in}^{w} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1}^{w} & \cdots & \tilde{x}_{mj}^{w} & \dots & \tilde{x}_{mn}^{w} \end{bmatrix} \quad (11)$$

Step-4: For the benefit (Equation 12) and cost criteria (Equation 13), the fuzzy values combined separately with Equation (14) are calculated.

$$\tilde{x}_{ij3}^{w+} = \max_{i}\{\tilde{x}_{ij3}^{w}\}, \text{ for benefit criteria } (BC_j) \quad (12)$$

$$\tilde{x}_{ij1}^{w-} = \min_{i}\{\tilde{x}_{ij1}^{w}\}, \text{ for cost criteria } (CC_j) \quad (13)$$

$$\bar{\tilde{x}}_{ij}^{w} = \begin{cases} \left(\frac{\tilde{x}_{ij1}^{w}}{\tilde{x}_{ij3}^{w+}}, \frac{\tilde{x}_{ij2}^{w}}{\tilde{x}_{ij3}^{w+}}, \frac{\tilde{x}_{ij3}^{w}}{\tilde{x}_{ij3}^{w+}}\right), for\ BC_j \\ \left(\frac{\tilde{x}_{ij1}^{w}}{\tilde{x}_{ij1}^{w-}}, \frac{\tilde{x}_{ij2}^{w}}{\tilde{x}_{ij1}^{w-}}, \frac{\tilde{x}_{ij3}^{w}}{\tilde{x}_{ij1}^{w-}}\right), \ for\ CC_j \end{cases} \quad (14)$$

Thus, the elements needed to normalize the $\widetilde{D}$ matrix are found. The normalized version of the combined weighted decision matrix $(\bar{\widetilde{D}})$ is shown as Equation (15):

$$\bar{\widetilde{D}}=\begin{bmatrix} \bar{\tilde{x}}_{11}^{w} & \cdots & \bar{\tilde{x}}_{1j}^{w} & \cdots & \bar{\tilde{x}}_{1n}^{w} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \bar{\tilde{x}}_{i1}^{w} & \cdots & \bar{\tilde{x}}_{ij}^{w} & \dots & \bar{\tilde{x}}_{in}^{w} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \bar{\tilde{x}}_{m1}^{w} & \cdots & \bar{\tilde{x}}_{mj}^{w} & \dots & \bar{\tilde{x}}_{mn}^{w} \end{bmatrix} \quad (15)$$

Values here are displayed in $\bar{\tilde{x}}_{ij}^{w} = (\bar{x}_{ij1}^{w}, \bar{x}_{ij2}^{w}, \bar{x}_{ij3}^{w})$ format.

Step-5: The defuzzification process is performed by applying Equation (16) to the normalized combined decision matrix elements. The defuzzification process is defined as the conversion of a fuzzy number to a definite number (Ross, 2005):

$$Def\left(\bar{\tilde{x}}_{ij}^{w}\right) = \frac{\int \mu(x)xdx}{\int \mu(x)dx} = \frac{-\bar{x}_{ij1}^{w}\bar{x}_{ij2}^{w}+\bar{x}_{ij2}^{w}\bar{x}_{ij3}^{w}+\frac{1}{3}\left(\bar{x}_{ij3}^{w}-\bar{x}_{ij2}^{w}\right)^{2}-\frac{1}{3}\left(\bar{x}_{ij2}^{w}-\bar{x}_{ij1}^{w}\right)^{2}}{-\bar{x}_{ij1}^{w}+\bar{x}_{ij3}^{w}} \quad (16)$$

The defuzzied matrix $\left(Def\left(\bar{\widetilde{D}}\right)\right)$ with the resulting elements is shown as Equation (17):

$$Def\left(\bar{\widetilde{D}}\right)=\begin{bmatrix} Def(\bar{\tilde{x}}_{11}^{w}) & \cdots & Def(\bar{\tilde{x}}_{1j}^{w}) & \cdots & Def(\bar{\tilde{x}}_{1n}^{w}) \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ Def(\bar{\tilde{x}}_{i1}^{w}) & \cdots & Def(\bar{\tilde{x}}_{ij}^{w}) & \dots & Def(\bar{\tilde{x}}_{in}^{w}) \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ Def(\bar{\tilde{x}}_{m1}^{w}) & \cdots & Def(\bar{\tilde{x}}_{mj}^{w}) & \dots & Def(\bar{\tilde{x}}_{mn}^{w}) \end{bmatrix} \quad (17)$$

Step-6: The defuzzied decision matrix $\left(\overline{Def}\left(\bar{\tilde{x}}_{ij}^{w}\right)\right)$ elements are normalized by applying Equation (18):

$$\overline{Def}\left(\bar{\tilde{x}}_{ij}^{w}\right) = \frac{Def\left(\bar{\tilde{x}}_{ij}^{w}\right)}{\sum_{s=1}^{m} Def\left(\bar{\tilde{x}}_{sj}^{w}\right)}, \ j = 1, 2, \dots, n \quad (18)$$

The normalized defuzzied matrix $\left(\overline{Def}\left(\bar{\widetilde{D}}\right)\right)$ is shown as Equation (19):

$$\overline{Def}\left(\bar{\widetilde{D}}\right)=\begin{bmatrix} \overline{Def}(\bar{\tilde{x}}_{11}^{w}) & \cdots & \overline{Def}(\bar{\tilde{x}}_{1j}^{w}) & \cdots & \overline{Def}(\bar{\tilde{x}}_{1n}^{w}) \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \overline{Def}(\bar{\tilde{x}}_{i1}^{w}) & \cdots & \overline{Def}(\bar{\tilde{x}}_{ij}^{w}) & \dots & \overline{Def}(\bar{\tilde{x}}_{in}^{w}) \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \overline{Def}(\bar{\tilde{x}}_{m1}^{w}) & \cdots & \overline{Def}(\bar{\tilde{x}}_{mj}^{w}) & \dots & \overline{Def}(\bar{\tilde{x}}_{mn}^{w}) \end{bmatrix} \quad (19)$$

Step-7: The belief entropy $\left(E_d(C_j)\right)$ of the C_j criterion is calculated by Equation (20) (Kang & Deng, 2019):

$$E_d(C_j) = -\sum_{i=1}^{m} \overline{Def}\left(\tilde{x}_{ij}\right) log \frac{\overline{Def}\left(\tilde{x}_{ij}\right)}{2^{|A_i|}-1} \quad (20)$$

As a result of applying Equation (21) to the calculated belief entropy, the degree of uncertainty of the C_j criterion is found:

$$U(C_j) = e^{E_d(C_j)} = e^{-\sum_{i=1}^{m} \overline{Def}(\tilde{x}_{ij}) log \frac{\overline{Def}(\tilde{x}_{ij})}{2^{|A_i|}-1}} \quad (21)$$

Step-8: The degree of uncertainty of the $C_j$criterion is normalized by Equation (22):

$$\bar{U}(C_j) = \frac{U(C_j)}{\sum_{h=1}^{n} U_h}, \; j = 1, 2, \dots, n \quad (22)$$

Step-9: The bpa value of the A_i alternative in terms of the C_j criterion is calculated by Equation (23), Equation (24) and Equation (25) (Sun et al., 2020):

$$m_{C_j}(\emptyset) = 0 \quad (23)$$

$$m_{C_j}(A_i) = \overline{Def}(\tilde{x}_{ij}^{w}) * (1 - \bar{U}_j) \quad (24)$$

$$m_{C_j}(\Theta) = 1 - \sum_{i=1}^{m} m_{C_j}(A_i), i = 1, 2, \dots, m; \; j = 1, 2, \dots, n \quad (25)$$

Thus, it is concluded that the bpa value of the frame of discernment is $\sum_{B \subseteq \Theta} m_{C_j}(\mathrm{B}) = 1$.

Step-10: Multiple proofs are combined with Equation (26) as follows:

$$m_C = \left(\left(m_{C_1} \oplus m_{C_2} \right)_1 \oplus \dots \oplus m_{C_n} \right)_{(n-1)} \quad (26)$$

Thus, in terms of the C_j criterion, the final bpa value of the $A_i (i = 1, 2, \dots, m)$ alternative is found, and this value gives the belief values of the alternatives (Equation 27):

$$Bel(A_i) = m_C(A_i) \quad (27)$$

Step-11: A_i alternatives are ranked according to their belief values and the best alternative is decided by Equation (28):

$$\propto = \underset{1 \le i \le m}{argmax} \{Bel(A_i)\} \quad (28)$$

$A_\propto$ would be the best alternative.

The Research Findings and Discussion

Application

The sniper rifle selection data in Arslan and Aydın (2009) were used in this study. They used an outranking based FMCDM method that was proposed by Auouam et al. (2003). Since 4 sniper rifles were considered, there are 4 alternatives and 6 criteria. The criteria were determined as minute of angle (MOA), weight of sniper gun (Weight), effective range (Range), binocular equipment (Binocular), ergonomic and upgradeability, respectively. Alternatives are Dragunov (Russia), SR25 (USA), Acc.L96A1 (UK) and Steyr SSG-69 (Austria). Detailed information about the alternatives and criteria is available in Arslan and Aydın (2009). Although the methodology is similar to Xiao (2020) study, Xiao (2020) used trapezoidal fuzzy numbers in her study. In this study, triangular fuzzy numbers, which are a special case of trapezoidal fuzzy numbers, are used since triangular fuzzy numbers are used in Arslan and Aydın (2009).

The set of alternatives is defined as $\Theta = \{\mathrm{Dragunov\ (A1), SR25\ (A2), Acc.L96A1\ (A3), Steyr\ SSG-69\ (A4)}\}$ and the set of criteria as $C =${MOA, Weight, Range, Binocular, Ergonomic and Upgradeability}.

The steps in the study to be done are as follows:

Step-1: As a result of the study conducted by Arslan and Aydın (2009), the criteria importance weights in Table 2 were used.

Table 2

Criterion Importance Weights

MOA (C_1)	Weight (C_2)	Range (C_3)	Binocular (C_4)	Ergonomic (C_5)	Upgradeability (C_6)
0.3	0.1	0.2	0.2	0.1	0.1

Step-2: The linguistic values given to the alternatives in the fuzzy MCDM based on an outranking relation study conducted by Bozkaya and Arslan (2008) and Arslan and Aydın (2009) are given in Table 3 and the fuzzy number values are given in Table 4.

Table 3

Rating of Alternatives by Linguistic Values According to the Criteria Evaluated by the Decision Maker

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
Dragunov	VL	H	L	M	L	M
SR25	H	M	VH	H	VH	VH
Acc.L96A1	VH	VL	VH	VH	H	VH
Steyr SSG-69	VH	M	VH	M	L	M

Table 4

Rating of Alternatives by Linguistic Values According to the Criteria Evaluated by the Decision Maker

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
Dragunov	(0.00;0.00;0.20)	(0.60;0.80;1.00)	(0.00;0.20;0.40)	(0.30;0.50;0.70)	(0.00;0.20;0.40)	(0.30;0.50;0.70)
SR25	(0.60;0.8.0;1;00)	(0.30;0.50;0.70)	(0.80;1.00;1.00)	(0.60;0.80;1.00)	(0.80;1.00;1.00)	(0.80;1.00;1.00)
Acc.L96A1	(0.80;1.00;1.00)	(0.00;0.00;0.20)	(0.80;1.00;1.00)	(0.80;1.00;1.00)	(0.60;0.80;1.00)	(0.80;1.00;1.00)
Steyr SSG-69	(0.80;1.00;1.00)	(0.30;0.50;0.70)	(0.80;1.00;1.00)	(0.30;0.50;0.70)	(0.00;0.20;0.40)	(0.30;0.50;0.70)

Step-3: Using Equation (9) and (10), the weighted decision matrix $(\widetilde{\mathrm{D}})$ is obtained in Table 5.

Table 5

Weighted Decision Matrix

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
Dragunov	(0.00;0.00;0.06)	(0.06;0.08;0.10)	(0.00;0.04;0.08)	(0.06;0.10;0.14)	(0.00;0.02;0.04)	(0.03;0.05;0.07)
SR25	(0.18;0.24;0.30)	(0.03;0.05;0.07)	(0.16;0.20;0.20)	(0.12;0.16;0.20)	(0.08;0.10;0.10)	(0.08;0.10;0.10)
Acc.L96A1	(0.24;0.30;0.30)	(0.00;0.00;0.02)	(0.16;0.20;0.20)	(0.16;0.20;0.20)	(0.06;0.08;0.10)	(0.08;0.10;0.10)
Steyr SSG-69	(0.24;0.30;0.30)	(0.03;0.05;0.07)	(0.16;0.20;0.20)	(0.06;0.10;0.14)	(0.00;0.02;0.04)	(0.03;0.05;0.07)

Step-4: Using Equation (12), (13) and (14), the normalized weighted decision matrix $(\overline{\overline{D}})$ is obtained in Table 6. Here, the x_{ij1}^{w-} value was determined as 0.18 for the MOA criterion, which was determined as the cost criterion, and the x_{ij1}^{w-} value for the weapon weight criterion was determined as 0.03.

Table 6

Normalized Weighted Decision Matrix

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
Dragunov	(0.00;0.00;0.33)	(2.00;2.67;3.33)	(0.00;0.20;0.40)	(0.30;0.50;0.70)	(0.00;0.20;0.40)	(0.30;0.50;0.70)
SR25	(1.00;1.33;1.67)	(1.00;1.67;2.33)	(0.80;1.00;1.00)	(0.60;0.80;1.00)	(0.80;1.00;1.00)	(0.80;1.00;1.00)
Acc.L96A1	(1.33;1.67;1.67)	(0.00;0.00;0.67)	(0.80;1.00;1.00)	(0.80;1.00;1.00)	(0.60;0.80;1.00)	(0.80;1.00;1.00)
Steyr SSG-69	(1.33;1.67;1.67)	(1.00;1.67;2.33)	(0.80;1.00;1.00)	(0.30;0.50;0.70)	(0.00;0.20;0.40)	(0.30;0.50;0.70)

Step-5: Using Equation (16), the defuzzied decision matrix $\left(Def\left(\bar{\bar{D}}\right)\right)$ is obtained in Table 7.

Table 7

Defuzzied Decision Matrix

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
Dragunov	0.11	2.67	0.20	0.50	0.20	0.50
SR25	1.33	1.67	0.93	0.80	0.93	0.93
Acc.L96A1	1.56	0.22	0.93	0.93	0.80	0.93
Steyr SSG-69	1.56	1.67	0.93	0.50	0.20	0.50

Step-6: Using Equation (18), the normalized defuzzied decision matrix $\left(\overline{Def}\left(\bar{\bar{D}}\right)\right)$ is obtained in Table 8.

Table 8

Normalized defuzzied Decision Matrix

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
Dragunov	0.0244	0.4286	0.0667	0.1829	0.0938	0.1744
SR25	0.2927	0.2679	0.3111	0.2927	0.4375	0.3256
Acc.L96A1	0.3415	0.0357	0.3111	0.3415	0.3750	0.3256
Steyr SSG-69	0.3415	0.2679	0.3111	0.1829	0.0938	0.1744

Step-7: Using Equation (20) and (21), the belief entropy values $\left(E_d(C_j)\right)$ and uncertainty degrees of the criteria are obtained in Table 9.

Table 9

Belief Entropy Values and Uncertainty Degrees of Criteria

	C_1	C_2	C_3	C_4	C_5	C_6
$E_d(C_j)$	1.7081	1.7137	1.8327	1.9447	1.6927	1.9330
$U(C_j)$	5.5187	5.5493	6.2505	6.9917	5.4344	6.9104

Step-8: Using Equation (22), the uncertainty degrees of the C_j criterion are normalized. Table 10 includes the normalized version of the uncertainty degrees of the criteria.

Table 10

Normalized Uncertainty Degrees of Criteria

$\bar{U}(C_1)$	$\bar{U}(C_2)$	$\bar{U}(C_3)$	$\bar{U}(C_4)$	$\bar{U}(C_5)$	$\bar{U}(C_6)$
0.1506	0.1514	0.1705	0.1907	0.1483	0.1885

Step-9: The bpa values of the alternatives are calculated using Equation (23), (24) and (25), are shown in Table 11.

Step-10: Belief values of alternatives obtained using Equation (26) and (27), are shown in Table 12.

Table 11

BPA Values of Alternatives

	C_1	C_2	C_3	C_4	C_5	C_6
m(A_1)	0.0207	0.3637	0.0553	0.1480	0.0799	0.1415
m(A_2)	0.2486	0.2273	0.2581	0.2369	0.3726	0.2642
m(A_3)	0.2901	0.0303	0.2581	0.2763	0.3194	0.2642
m(A_4)	0.2901	0.2273	0.2581	0.1480	0.0799	0.1415
m(Θ)	0.1506	0.1514	0.1705	0.1907	0.1483	0.1885

Table 12

Belief Values of Alternatives

$Bel(A_1)$	$Bel(A_2)$	$Bel(A_3)$	$Bel(A_4)$
0.0400	0.5340	0.2765	0.1479

Step-11: The best alternative is determined by using Equation (28). As a result of the operations, the alternative order is determined as $A_2 > A_3 > A_4 > A_1$.

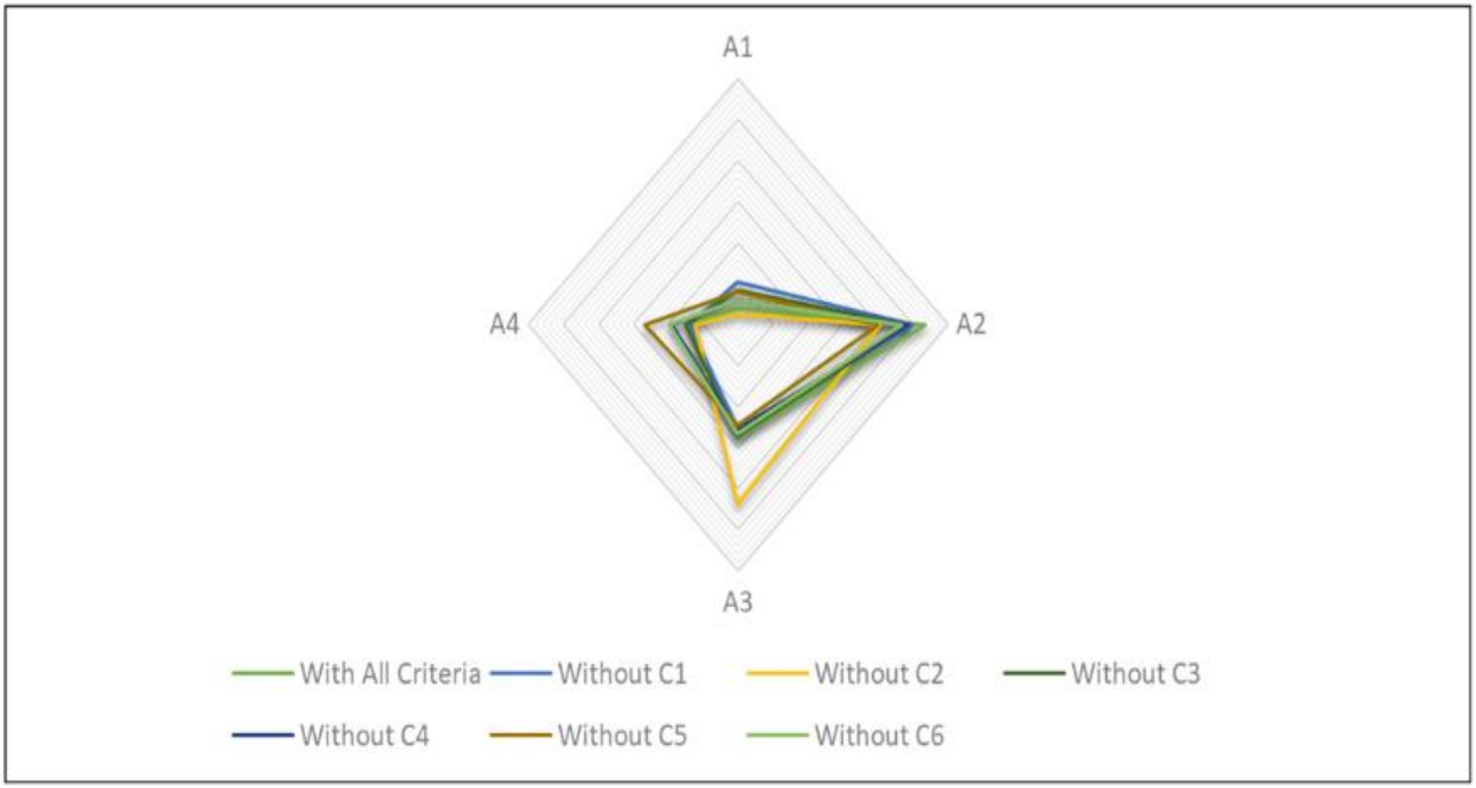
Sensitivity Analysis

The values obtained for the ranking of the alternatives are respectively; 0.0400 for A1 alternative; 0.5340 for A2 alternative; 0.2765 for the A3 alternative; and 0.1479 for the A4 alternative. Thus, the order for the alternatives is obtained as A2>A3>A4>A1.

Sensitivity analysis (Figure 2) was performed to demonstrate the robustness. For this, each criterion was excluded from the evaluation one by one, and the alternatives were reordered. When the MOA criterion is excluded, the value obtained for the A1 alternative is 0.1021; the value obtained for alternative A2 is 0.5004; the value obtained for the A3 alternative was calculated as 0.2516 and the value obtained for the A4 alternative was calculated as 0.1382. Thus, the alternative order was obtained as A2>A3>A4>A1. When the weight criterion is excluded, the value obtained for the A1 alternative is 0.0258; the value obtained for alternative A2 is 0.4069; the value obtained for the A3 alternative was calculated as 0.4372 and the value obtained for the A4 alternative was calculated as 0.1241. Thus, the alternative order was obtained as A3>A2>A4>A1. When the range criterion is excluded, the value obtained for the A1 alternative is 0.0841; the value obtained for alternative A2 is 0.4862; the value obtained for the A3 alternative was calculated as 0.2715 and the value obtained for the A4 alternative was calculated as 0.1505. Thus, the alternative order was obtained as A2>A3>A4>A1. When the binocular criterion is excluded, the value obtained for the A1 alternative is 0.0595; the value obtained for alternative A2 is 0.4881; the value obtained for the A3 alternative was calculated as 0.2545 and the value obtained for the A4 alternative was calculated as 0.1904. Thus, the alternative order was obtained as A2>A3>A4>A1. When the ergonomic criterion is excluded, the value obtained for the A1 alternative is 0.0805; the value obtained for the A2 alternative is 0.4011; the value obtained for the A3 alternative was calculated as 0.2449 and the value obtained for the A4 alternative was calculated as 0.2656. Thus, the alternative order was obtained as A2>A4>A3>A1. When the upgradeability criterion is excluded, the value obtained for the A1 alternative is 0.0615; the value obtained for the A2 alternative is 0.4693; the value obtained for the A3 alternative was calculated as 0.2646 and the value obtained for the A4 alternative was calculated as 0.1969. Thus, the alternative order was obtained as A2>A3>A4>A1.

Figure 2

Sensitivity Analysis



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Conclusions

In this study we applied the EFMCDM method introduced by Xiao (2020) to the sniper rifle selection problem studied in [30]. The results show that indeed some of the uncertainties that are effective in the decision-making process can be overcome with EFMCDM. In addition, the same rankings for the sniper rifles have been obtained with both MCDM approaches.

In Arslan and Aydın (2009), the MOA criterion (C1) weight was 0.3; weight criterion (C2) weight is 0.1; range criterion (C3) weight 0.2; binocular criterion (C4) weight is 0.2; ergonomic criterion (C5) weight was determined as 0.1 and upgradeability criterion (C6) weight was determined as 0.1.

When the degree of uncertainty of the criteria is evaluated in our study, the criterion with the lowest degree of uncertainty is ergonomic

(C5) criterion. The criterion with the highest degree of uncertainty is binoculars (C4). The degrees of uncertainty of the other criteria are as follows, from low to high: MOA criterion (C1) is second, weight criterion (C2) is third, range criterion (C3) is fourth, and upgradeability criterion (C6) is fifth. A decrease in the level of uncertainty indicates an increase in the importance of the criterion. For this reason, while the gun ergonomics criterion provides the greatest effect in calculating the belief values of the alternatives, the binocular magnification and viewing angle criteria show the lowest effect.

In Arslan and Aydın (2009), the superiority value of the SR25 alternative was 1.23; the superiority value of the Acc.L96A1 alternative was 1.07; the superiority value of the Steyr SSG-69 alternative was calculated as 0.45 and the superiority value of the Dragunov alternative was calculated as -2.75.

As a result of the findings obtained in this study, the belief value of the SR25 alternative was 0.5340; the belief value of the Acc. L96A1 alternative was 0.2765; the belief value of Steyr SSG-69 alternative was calculated as 0.1479 and the belief value of Dragunov alternative was calculated as 0.0400. The same ranking was obtained with the ranking of the alternatives in the study conducted by Arslan and Aydın (2009). Thus, it has been demonstrated that DST, which tries to minimize uncertainty, and outranking based FMCDM method yield results that support each other.

Finally, the best alternative was determined as the second alternative with the criteria accepted within the scope of this study. Technical information about the criteria plays an important role in ranking the alternatives. According to the results obtained based on the evaluations of the experts with the entropy belief approach, the SR25 weapon was determined as the best weapon among the alternatives.

The limitations of the study are as follows: i) In the case of developing and changing the decision matrix of this research, different findings can be encountered. ii) Different results can be obtained with the same data with different MCDM methods. iii) Changes in the number of decision makers/experts can directly affect the result.

The suggestions as a result of the study are as follows: i) The results of the sniper rifle selection problem handled with the EFMCDM method based on belief entropy yielded the same results with an outranking based FMCDM method, and two main research questions developed within the scope of the research were answered. ii) With the study, it has been shown that the security forces can achieve a healthy result in the sniper rifle selection problem by supporting the previous study and obtaining the same results. iii) Researchers can conduct new studies with different criteria that can be obtained as a result of literature review. iv) Studies can be updated by adding newly developed sniper rifles to alternatives. v) Different selection problems can be handled with the EFMCDM method based on belief entropy.

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References

Aouam, T., Chang, S. I., & Lee, E. S. (2003). Fuzzy MADM: An outranking method. *European Journal of Operational Research*, *145*(2), 317-328. https://doi.org/10.1016/S0377-2217(02)00537-4

Arslan, G., & Aydın, Ö. (2009). A new software development for fuzzy multicriteria decision-making. *Technological and Economic Development of Economy*, *15*(2), 197-212. https://doi.org/10.3846/1392-8619.2009.15.197-212

Aygün, H., & Adalı, E. (2006). Dempster-Shafer algoritmasının kullanımı ile sınıflandırma algoritmalarının birleştirilmesi. *İTÜDERGİSİ*, *5(4)*. http://www.itudergi.itu.edu.tr/index.php/itudergisi_d/article/viewFile/462/401

Beynon, M., Curry, B., & Morgan, P. (2000). The Dempster–Shafer theory of evidence: an alternative approach to multicriteria decision modelling. *Omega*, *28*(1), 37-50. https://doi.org/10.1016/S0305-0483(99)00033-X

Bozkaya, N., & Arslan, G. (2008). Üstünlük esaslı bulanık çok ölçütlü karar verme yönteminin keskin nişancı tüfeği seçimi problemine uygulaması. *Savunma Bilimleri Dergisi*, *7*(1), 40-54. https://dergipark.org.tr/en/pub/khosbd/issue/19232/204362

Büyükyazıcı, M., & Sucu, M. (2009). Matematiksel kanıt kuramı'nda uzlaşma üretici yöntemler için bir çerçeve. *İstatistikçiler Dergisi: İstatistik ve Aktüerya*, *2*(1), 19-27. https://dergipark.org.tr/en/pub/jssa/issue/10040/123863

Chatterjee, M., & Namin, A. S. (2021). A fuzzy Dempster–Shafer classifier for detecting web spams. *Journal of Information Security and Applications*, *59*. https://doi.org/10.1016/j.jisa.2021.102793

Chinnasamy, S., Ramachandran, M., & Kurinjimalar Ramu, P. A. (2022). Study on fuzzy ELECTRE method with various methodologies. *REST Journal on Emerging trends in Modelling and Manufacturing*, *7*(4), 108-115. https://doi.org/10.46632/7/4/2

Çavdur, F. (2005). *Arama motorları kullanıcı oturumlarındaki konu değişikliklerinin tespit ve tahmin yöntemleri* (Publication No. 198634)[MsC. Dissertation, Uludağ University]. YÖK National Thesis Center.

Danaee, P., Ghaeini, R., & Hendrix, D. A. (2017). A deep learning approach for cancer detection and relevant gene identification. In *Pacific symposium on biocomputing 2017* (pp. 219-229). https://doi.org/10.1142/9789813207813_0022

Denœux, T., Dubois, D., & Prade, H. (2020). Representations of uncertainty in artificial intelligence: probability and possibility. In Marquis, P., Papini, O., Prade, H. (Eds.), *A Guided Tour of Artificial Intelligence Research* (pp.69-117). Springer. https://doi.org/10.1007/978-3-030-06164-7_3

Dutta, P., & Shome, S. (2023). A new belief entropy measure in the weighted combination rule under DST with faulty diagnosis and real-life medical application. *International Journal of Machine Learning and Cybernetics*, *14*(4), 1179-1203. https://doi.org/10.1007/s13042-022-01693-6

Dymova, L., Kaczmarek, K., Sevastjanov, P., Sułkowski, Ł., & Przybyszewski, K. (2021). An approach to generalization of the intuitionistic fuzzy TOPSIS method in the framework of evidence theory. *Journal of Artificial Intelligence and Soft Computing Research*, *11*(2), 157-175. https://doi.org/10.2478/jaiscr-2021-0010

Fei, L., & Feng, Y. (2021). Intuitionistic fuzzy decision-making in the framework of Dempster–Shafer structures. *International Journal of Intelligent Systems*, *36*(10). https://doi.org/10.1002/int.22517

Fei, L., & Ma, Y. (2023). A hybrid decision-making framework for selecting the emergency alternatives. *International Journal of Fuzzy Systems*, 1-15. https://doi.org/10.1007/s40815-023-01467-4

Fei, L., Xia, J., Feng, Y., & Liu, L. (2019). An ELECTRE-based multiple criteria decision making method for supplier selection using Dempster-Shafer theory. *IEEE Access*, *7*. https://doi.org/10.1109/ACCESS.2019.2924945

Kang, B., & Deng, Y. (2019). The maximum Deng entropy. *IEEE Access*, *7*. https://doi.org/10.1109/ACCESS.2019.2937679

Lin, K. P., & Hung, K. C. (2011). An efficient fuzzy weighted average algorithm for the military UAV selecting under group decision-making. *Knowledge-Based Systems*, *24*(6), 877-889. https://doi.org/10.1016/j.knosys.2011.04.002

Liu, P., & Gao, H. (2019). Some intuitionistic fuzzy power Bonferroni mean operators in the framework of Dempster–Shafer theory and their application to multicriteria decision making. *Applied Soft Computing*, *85*, 105790. https://doi.org/10.1016/j.asoc.2019.105790

Mokarram, M., & Sathyamoorthy, D. (2023). Determination of suitable locations for the construction of gas power plant using multicriteria decision and Dempster–Shafer model in GIS. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, *45*(1), 2846-2861. https://doi.org/10.1080/15567036.2019.1666189

Ngo, N. D. K., Tansuchat, R., Cu, P. V., Mau, T. N., Kohda, Y., & Huynh, V. N. (2023). A customer-driven evaluation method for service innovation in banking. *IEEE Access*, 23419242. https://doi.org/10.1109/ACCESS.2023.3292123

Qin, Y., Qi, Q., Shi, P., Scott, P. J., & Jiang, X. (2020). Novel operational laws and power Muirhead mean operators of picture fuzzy values in the framework of Dempster-Shafer theory for multiple criteria decision making. *Computers & Industrial Engineering*, *149*. https://doi.org/10.1016/j.cie.2020.106853

Qin, Y., Qi, Q., Shi, P., Scott, P. J., & Jiang, X. (2023). A novel weighted averaging operator of linguistic interval-valued intuitionistic fuzzy numbers for cognitively inspired decision-making. *Cognitive Computation*, 1-19. https://doi.org/10.1007/s12559-023-10167-y

Rashki, M., & Faes, M. G. (2023). No-free-lunch theorems for reliability analysis. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, *9*(3), 04023019. https://doi.org/10.1061/AJRUA6.RUENG-1015

Ross, T. J. (2009). *Fuzzy logic with engineering applications*. John Wiley & Sons.

Seçkin, F. (2015). *A Model development to formulate buyer-supplier integration levels and evalution criteria for sustainable supply chain management* [Unpublished doctoral dissertation]. National Defense University, Turkish Air Force Academy.

Si, A., Das, S., & Kar, S. (2021). Picture fuzzy set-based decision-making approach using Dempster-Shafer theory of evidence and grey relation analysis and its application in COVID-19 medicine selection. *Soft Computing*, 1-15. https://doi.org/10.1007/s00500-021-05909-9

Sun, C., Li, S., & Deng, Y. (2020). Determining weights in multi-criteria decision making based on negation of probability distribution under uncertain environment. *Mathematics*, *8*(2), 191. https://doi.org/10.3390/math8020191

Taban, C. (2019). *UAV hub selection with fuzzy multi criteria decision making techniqoues for ensuring maritime safety* [Unpublished MsC. dissertation]. Sakarya University.

Tang, X., Gu, X., Rao, L., & Lu, J. (2021). A single fault detection method of gearbox based on random forest hybrid classifier and improved Dempster-Shafer information fusion. *Computers & Electrical Engineering*, *92*. https://doi.org/10.1016/j.compeleceng.2021.107101

Tong, Z., Xu, P., & Denoeux, T. (2021). An evidential classifier based on Dempster-Shafer theory and deep learning. *Neurocomputing*, *450*, 275-293. https://doi.org/10.1016/j.neucom.2021.03.066

Turhan, H. İ. (2014). *Decision making in tracking applications by using Dempster-Shafer Theory* (Publication No. 384969) [MsC. Dissertation, Middle East Technical University]. YÖK National Thesis Center. https://open.metu.edu.tr/handle/11511/23984

Wu, D., & Tang, Y. (2020). An improved failure mode and effects analysis method based on uncertainty measure in the evidence theory. *Quality and Reliability Engineering International*, *36*(5), 1786-1807. https://doi.org/10.1002/qre.2660

Wu, L., Tang, Y., Zhang, L., & Huang, Y. (2023). Uncertainty management in assessment of FMEA expert based on negation information and belief entropy. *Entropy*, *25*(5). https://doi.org/10.3390/e25050800

Xiao, F. (2019). EFMCDM: Evidential fuzzy multicriteria decision making based on belief entropy. *IEEE Transactions on Fuzzy Systems*, *28*(7), 1477-1491. https://doi.org/10.1109/TFUZZ.2019.2936368

Xiong, L., Su, X., & Qian, H. (2021). Conflicting evidence combination from the perspective of networks. *Information Sciences*, *580*, 408-418. https://doi.org/10.1016/j.ins.2021.08.088

Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

Zhang, Y., Dai, Y., & Liu, B. (2023). Identifying qualified public safety education venues using the Dempster–Shafer theory-based PROMETHEE method under linguistic environments. *Mathematics*, *11*(4), 1011. https://doi.org/10.3390/math11041011

Zhong, Y., Zhang, H., Cao, L., Li, Y., Qin, Y., & Luo, X. (2023). Power muirhead mean operators of interval-valued intuitionistic fuzzy values in the framework of Dempster–Shafer theory for multiple criteria decision-making. *Soft Computing*, *27*(2), 763-782. https://doi.org/10.1007/s00500-022-07595-7

Zhu, C., Qin, B., Xiao, F., Cao, Z., & Pandey, H. M. (2021). A fuzzy preference-based Dempster-Shafer evidence theory for decision fusion. *Information Sciences*, *570*, 306-322. https://doi.org/10.1016/j.ins.2021.04.059

Zhu, C., & Xiao, F. (2021). A belief Hellinger distance for D–S evidence theory and its application in pattern recognition. *Engineering Applications of Artificial Intelligence*, *106*, 104452. https://doi.org/10.1016/j.engappai.2021.104452

Genişletilmiş Özet

Giriş

Çok kriterli karar verme (ÇKKV) metodolojisinin temel amacı, önceden belirlenmiş ölçütler doğrultusunda alternatifleri kıyaslayarak en elverişli alternatifi tespit etmektir. Alternatifler, nicel ölçümler veya uzmanların değerlendirmeleriyle belirlenen kriterlere göre değerlendirilmektedir. Ancak gerçek hayatta, veri eksikliği nedeniyle nicel ifadelerin yetersiz olduğu durumlarla sıkça karşılaşılmaktadır. Bu gibi durumlarda, bulanık mantık gibi alternatif sıralama yaklaşımlarının kullanılması gerekebilmektedir. Ayrıca, birçok durumda uzman görüşleri sözlü olarak daha açık ve kolay ifade edilebilmektedir. Bu sebeple, bulanık mantık gibi yöntemlerin kullanılmasıyla karar verme süreçlerinde daha güvenilir sonuçlar elde edilebilmektedir.

Yöntem

Karar verme sürecindeki eksiklik, belirsizlik ve tutarsızlık gibi durumlarla başa çıkmak için çeşitli yöntemler geliştirilmiştir. Bu yöntemlerden bazıları Olasılık Teorisi, Bulanık Küme Teorisi ve DST olarak adlandırılmaktadır. Bulanık Küme Teorisi belirsizlikle ilgilenirken Olasılık Teorisi genellikle rastlantısallık üzerine odaklanmaktadır. DST ise tutarsızlık, eksik bilgi ve rastlantı durumlarıyla başa çıkmak için diğer teorilere kıyasla daha etkili olabilmektedir. Bunun yanı sıra DST, mevcut bilgilerin tutarsızlığıyla ilgili çatışma kavramını ortaya atarak belirsizliği ele almakta ve karar verme sürecinde kullanılmaktadır. Ayrıca, DST'nin önemli bir diğer avantajı da değişkenlere kesin bir değer atama zorunluluğunun olmamasıdır. Bu durum da teorinin esnekliğini göstermektedir.

DST, yapay zekâ tabanlı tıbbi teşhislerden istatistiksel sınıflandırmaya, veri birleştirme işlemlerinden yüz tanıma sistemlerine, risk analizinden çok kriterli karar analizine kadar geniş bir yelpazedeki uygulamalarda başarılı bir şekilde kullanılmaktadır.

Bu çalışmanın esas hedeflerinden biri, İnanç Entropisine Dayalı Kanıtsal Bulanık Çok Kriterli Karar Verme (EFMCDM) Yönteminin, Üstünlük Esaslı Bulanık Çok Ölçütlü Karar Verme Yöntemi ile uyumlu olup olmadığını belirlemektir. Ayrıca, Arslan ve Aydın (2009) tarafından incelenen keskin nişancı tüfeği seçimi gibi bir örneğe odaklanarak sonuçların ne derece benzer olduğu da araştırılmıştır. Bu makalenin temel katkıları şunlardır: i) EFMCDM Yönteminin keskin nişancı tüfeği seçimi problemine uygulanması, ii) EFMCDM yönteminin, Üstünlük Esaslı Bulanık Çok Ölçütlü Karar Verme Yöntemi ile karşılaştırılması ve sonuçların tutarlı olup olmadığının gözlemlenmesi, iii) Karar verme sürecinde etkili olan belirsizliklerin EFMCDM yöntemi ile aşılabileceğinin gösterilmesi.

Zadeh tarafından 1965 yılında yayınlanan "Fuzzy Sets" başlıklı makale, bulanık mantığı anlatmaktadır. Günlük hayatımızın belirsiz doğası, karar teorisi ve olasılık teorisi gibi çeşitli teorilerle daha iyi kararlar almayı amaçlamaktadır. Bu sebeple, bulanıklık ve rastlantısallık kavramları modern karar verme süreçlerinin önemli bir parçası olmaktadırlar. Rastlantısallık, bir kümenin üyeleri için meydana gelebilecek belirsizliği ifade ederken bulanıklık ise bir kümenin üyeleri için çeşitli üyelik derecelerinin mümkün olduğu durumları tanımlamaktadır. Bu bağlamda, karar verici artık sadece "doğru/yanlış" veya "evet/hayır" gibi ifadelerle sınırlı kalmamakta; bulanık mantık, "kesinlikle katılmıyorum, katılmıyorum, kararsızım, katılıyorum, tamamen katılıyorum" gibi ara değerleri de içerebilmektedir.

Bilimsel olarak, bulanıklık belirsizlik olarak nitelendirilmektedir. Belirsizlik, tam olarak bilinmeyen, öznel verileri içeren ve karar vericilerin farklı görüşlerini içeren bir durumu ifade etmektedir. Belirsizlik, bir amacı ve bir sistemi tanımlayan ifadelerdeki belirsizlik ya da kesin olmama durumu olarak açıklanmaktadır. Yani, insan düşüncesindeki algı farklılıkları, öznel davranışlar ve hedeflerdeki belirsizlikler bulanıklık kavramıyla açıklanabilmektedir.

Bir nesnenin kümenin üyesi olup olmamasına bağlı olarak 0 ve 1 değerlerini aldığı kümelere klasik kümeler denilmektedir. Bu fikir Aristoteles'in 0 ve 1 mantığına dayanmaktadır. Tam üyelik durumu 1, üye olmama durumu ise 0'dır. Yani bir eleman ya kümenin üyesidir ya da değildir. Bulanık kümelerde ise nesnelerin her birine ilgili kümeye üyelik derecelerini temsil eden 0 ile 1 arasındaki sayılardan oluşan bir değer atanmaktadır. Bu değer [0,1] aralığında sonsuz sayıda değere sahip olabilmektedir.

DST, 1967 yılında A.P. Dempster'ın olasılığın alt ve üst sınırlarını belirlemeye yönelik çalışmasıyla ortaya çıkmıştır. Daha sonra 1976 yılında G. Shafer'ın The Mathematical Theory of Proof adlı kitabında bazı eklemelerle geliştirilmiştir. Bu nedenle teoriye, her iki fikir babasının adını taşıyan DST adı verilmiştir. Bu teori sayısal bir yöntem olup eksik ve belirsiz bilgilerle mücadele etmek için geliştirilmiştir.

DST, birden fazla olayın olasılıkları hakkındaki bilgileri birleştirerek bu olaylardan etkilenen olayın toplam olasılığını bulmaktadır. Aynı zamanda farklı kanıt kaynaklarından gelen bilgilerin güven düzeyini artırmayı amaçlayan bir bilgi birleştirme kuralıdır. DST, toplanmış temel olasılık atama (TOA) değerlerini birleştirmek için etkili bir araç olarak kullanılmakta ve belirsiz ve kesin olmayan bir ortamda basit hesaplamalarla karar vermenin ne kadar sağlam olduğunu göstermektedir.

DST bir anlamda Bayes olasılık teorisinin bir genellemesi olarak düşünülmektedir. Olasılığı tek bir olay yerine birden fazla olay hipotezine dağıtarak "belirsizliği" gösterme avantajı bulunmaktadır. Bayes olasılık teorisinde, durumlara atanan ağırlıklar "olasılık" olarak adlandırılırken DST'de "kütle" olarak adlandırılmaktadır. En önemli ayırt edici özelliklerden biri ise DST'de Bayes olasılık teorisinde olduğu gibi değişkenlere net bir değer verme zorunluluğunun olmamasıdır.

Uygulama

Bu çalışma, Arslan ve Aydın'ın (2009) keskin nişancı tüfeği seçim verilerini temel almaktadır. Bu verilerde, Auouam vd., (2003) tarafından önerilen Üstünlük Esaslı Bulanık Çok Ölçütlü Karar Verme Yöntemi kullanılmıştır. Çalışmada 4 farklı keskin nişancı tüfeği ve bunların değerlendirilmesinde kullanılan 6 kriter bulunmaktadır. Bu kriterler sırasıyla doğruluk, silah ağırlığı, etkili menzil, dürbün büyütme ve görüş açısı, silah ergonomisi ve geliştirilme yeteneği olarak belirlenmiştir. Dragunov (Rusya), SR25 (ABD), Acc.L96A1 (İngiltere) ve Steyr SSG-69 (Avusturya) ise incelenen keskin nişancı tüfeği alternatifleridir. Detaylı bilgi, Arslan ve Aydın (2009) çalışmasında yer almaktadır. Bu çalışma, metodoloji açısından Xiao'nun (2020) çalışmasına benzerlik göstermektedir ancak Xiao'nun çalışmasında yamuksal bulanık sayılar kullanılmıştır. Bu çalışmada ise Arslan ve Aydın'ın çalışmasında üçgensel bulanık sayılar kullanıldığı için yamuksal bulanık sayıların özel bir durumu olan üçgensel bulanık sayılar tercih edilmiştir.

Elde ettiğimiz değerlere dayanarak alternatifler için sıralama şu şekildedir: A2 en yüksek değere sahipken onu A3, ardından A4 ve en son A1 takip etmektedir. Dolayısıyla elde edilen sıralama A2 > A3 > A4 > A1 şeklindedir.

Sonuç

Bu çalışmada, Arslan ve Aydın (2009) tarafından incelenen keskin nişancı tüfeği seçim problemine Xiao (2020) tarafından tanıtılan EFMCDM yöntemi uygulanmıştır. Elde edilen sonuçlar, EFMCDM'nin aslında karar verme sürecinde etkili olan bazı belirsizlikleri ele alabildiğini göstermektedir. Ayrıca, her iki ÇKKV yaklaşımıyla da keskin nişancı tüfekleri için aynı sıralamalar elde edilmiştir.

Arslan ve Aydın (2009) çalışmasında, doğruluk kriterinin (C1) ağırlığı 0,3 olarak belirlenmiştir. Silah ağırlığı kriterinin (C2) ağırlığı 0,1; etkili menzil kriterinin (C3) ağırlığı 0,2; dürbün büyütme ve görüş açısı kriterinin (C4) ağırlığı 0,2; silah ergonomisi kriterin (C5) ağırlığı 0,1 ve geliştirilme yeteneği kriterinin (C6) ağırlığı ise 0,1 olarak hesaplanmıştır.

Bu çalışmada, kriterlerin belirsizlik düzeyleri incelendiğinde, belirsizlik derecesi en düşük olan kriter silah ergonomisi kriteri (C5) olarak belirlenmiştir. En yüksek belirsizlik derecesine sahip kriter ise dürbün büyütme ve görüş açısı (C4) olarak belirlenmiştir. Diğer kriterlerin belirsizlik dereceleri sıralı olarak şu şekildedir: doğruluk kriteri (C1) ikinci, silah ağırlığı kriteri (C2) üçüncü, etkili menzil kriteri (C3) dördüncü ve geliştirilme yeteneği kriteri (C6) beşinci sırada yer almaktadır. Belirsizlik düzeyinin azalması, bir kriterin öneminin arttığını göstermektedir. Bu bağlamda, silah ergonomisi kriteri alternatiflerin inanç değerlerinin hesaplanmasında en büyük etkiye sahipken dürbün büyütme ve görüş açısı kriterleri en düşük etkiyi göstermektedir.

Arslan ve Aydın (2009) çalışmasında, SR25 alternatifinin üstünlük değeri 1,23 olarak belirlenmiştir. Acc.L96A1 alternatifinin üstünlük değeri 1,07; Steyr SSG-69 alternatifinin üstünlük değeri 0,45 ve Dragunov alternatifinin üstünlük değeri ise -2,75 olarak hesaplanmıştır.

Bu çalışmada elde edilen bulgular sonucunda SR25 alternatifinin inanç değeri 0,5340; Acc. L96A1 alternatifinin inanç değeri 0,2765; Steyr SSG-69 alternatifinin inanç değeri 0,1479 ve Dragunov alternatifinin inanç değeri 0,0400 olarak hesaplanmıştır. Arslan ve Aydın (2009) tarafından yapılan çalışma sonucunda elde edilen sıralama ile çalışmamız sonucunda elde edilen alternatif sıralamalarının aynı olduğu görülmektedir. Böylece belirsizliği en aza indirmeye çalışan EFMCDM Yöntemi ile Üstünlük Esaslı Bulanık Çok Ölçütlü Karar Verme Yönteminin birbirini destekleyen sonuçlar verdiği ortaya konulmuştur.