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Hedging Petroleum Futures with Multivariate GARCH Models

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ABSTRACT: This paper examined the petroleum futures volatility comovements and spillovers for crude oil, gasoline, heat oil and natural gas. The results of volatility analysis were used to calculate the optimal two-petroleum portfolio weights and hedging ratios. The data used in this study was the daily data from 2009 to 2014. The three Multivariate GARCH models, namely the VAR (1)-diagonal VECH, the VAR (1)-diagonal BEKK and the VAR (1)-CCC, were employed. The empirical results overall showed that the estimates of the multivariate GARCH parameters were statistically significant in almost all cases except in the case of RGASOLINE with RNG. This indicates that the short run persistence of shocks on the dynamic conditional correlations was greatest for RCRUDE with RHEATOIL, while the largest long run persistence of shocks to the conditional correlations for RCRUDE with RGASOLINE. Finally, the results from these optimal portfolio weights base on the VAR (1)-diagonal VECH estimates suggested that investors should had more heat oil than crude oil and other petroleum in their portfolio to minimize risk without lowering the expected return.

Keywords: The petroleum futures volatility; comovements and spillovers; multivariate GARCH models; optimal portfolio weights; hedging ratios
JEL Classifications: C13; C32; G13

1. Introduction

All countries consume petroleum. Both producers and consumers are highly concerned about petroleum prices. The petroleum prices are being directly affected by several economic, political, geopolitical, technological factors, and also oil reserves, available stocks and weather conditions, among others. On other hand the petroleum price fluctuations influence directly the world economy. Compared to financial assets petroleum prices have had an elevated volatility in recent years. Therefore, studies of petroleum price movements and co-movements are highly complex. Therefore the academics and practitioners are developing many studies about themes related with petroleum prices. Economic agents indirectly involved in petroleum negotiations, such as firm or government planners, are looking for related petroleum price forecasting models, elaborating studies, while the agents directly involved are looking for the hedge strategies studies as well. The hedge strategies allow negotiators that have short and long positions in the market protection against price fluctuations.

The motivation of this work is the relevance of petroleum international market growth, the biggest market among the commodity markets. This led to a development of derivative markets of this commodity, in particular, future contract markets, or simply future markets. This development brought sophisticated strategies. Among these strategies there are many for risk reduction of physical positions, investments in petroleum or others related to this commodity movements.

Futures contracts are firm commitments to make or accept delivery of a specified quantity and quality of a commodity during a specific month in the future at a price agreed upon at the time the commitment is made. The buyer, known as the long, agrees to take delivery of the underlying commodity. The seller, known as the short, agrees to make delivery. Only a small number of contracts traded each year result in delivery of the underlying commodity. Instead, traders generally offset (a buyer will liquidate by selling the contract, the seller will liquidate by buying back the contract) their futures positions before their contracts mature. The difference between the initial purchase or sale price and the price of the offsetting transaction represents the realized profit or loss.

Futures contracts trade in standardized units in a highly visible, extremely competitive, continuous open auction. In this way, futures lend themselves to widely diverse participation and efficient price discovery, giving an accurate picture of the market.

To do this effectively, the underlying market must meet three broad criteria: The prices of the underlying commodities must be volatile, there must be a diverse, large number of buyers and sellers, and the underlying physical products must be fungible, that is, products are interchangeable for purposes of shipment or storage. All market participants must work with a common denominator. Each understands that futures prices are quoted for products with precise specifications delivered to a specified point during a specified period of time.

In this research, we are interested in petroleum four types include crude oil, gasoline, heat oil and natural gas. These are of interest to investors. The investment is more interesting for petroleum futures. We can explain more in the next section, which is related to the literature reviews, research methodology and empirical results.

The purpose is to analyze the petroleum future volatility comovements and spillovers among major petroleum including crude oil (WTI market), gasoline, heat oil and natural gas by using multivariate GARCH, namely the diagonal VECH, the diagonal BEKK and CCC model and choose the best way for such analysis. In addition, continue to manage in hedging strategies.

2. Literature Review

The previous studies on petroleum markets centers mainly on the issues such as price discovery and market interrelationships. For example, many studies investigate the issue of price discovery, efficiency and causal relationship between oil spot and futures prices; Crowder and Hamed (1993), Moosa and Al-Loughani (1995), Peroni and McNown (1998) and Silvapulle and Moosa (1999). A number of studies also investigate linkages both in conditional return and variances between spot and futures of crude oil markets in different geographical locations; Ewing and Harter (2000), Lin and Tamvakis (2001), Lanza et al. (2006), Manera et al. (2006), Chang et al. (2009a, 2009b).

It has not been widely acknowledged in the literature that risk in the oil market can be minimized through futures hedging. Knill et al. (2006) suggest that if an oil and gas company uses futures contracts to hedge risk, they hedge only the downside risk. On the while, Daniel (2001) shows that hedging strategies can substantially reduce oil price volatility without significantly reducing returns, and with the added benefit of greater predictability and certainty. Haigh and Holt (2002) specify the time-varying hedge ratio of BEKK model of Engle and Kroner (1995) for crude oil (WTI), heating oil and unleaded gasoline futures contracts to examine volatility spillovers. Using the VECM and BEKK models, Alizadeh et al. (2004) examine the effectiveness of hedging marine bunker price fluctuations in Rotterdam, Singapore and Houston using different crude oil and petroleum futures contracts traded on the New York Mercantile Exchange (NYMEX) and the International Petroleum Exchange (IPE) in London. Jalali-Naini and Kazemi-Manesh (2006) find that the OHRs are time varying for all contracts, and higher duration contracts had higher perceived risk, a higher OHR mean, and standard deviations using weekly spot prices of WTI and futures prices of crude oil contracts one month to four months on NYMEX.

In order to estimate time varying optimal hedge ratios, two distinct approaches have been developing. One approach is basically to follow a Markov regime-switching model, which is firstly used for estimating optimal hedge ratios by Alizadeh and Nomikos (2004). Lee et al. (2006) and Lee and Yoder (2007a,b) propose various forms of Markov regime-switching models with allowing the hedge ratio to be both time varying and state-dependent, and find that all of these models outperform state-independent GARCH models.

The other approach is to estimate time varying optimal hedge ratios by using mixed normal GARCH models. In fact, finite mixing two or more conditionally normal and heteroskedastic components exhibit quite complex dynamics, as often observed in financial markets. For example, there may be components provided by nonstationary dynamics, another is not, but the overall mixing process might be a covariance stationary. This implies that markets are stable most of the time, but, occasionally, subject to severe and temporal fluctuations. In this regards, Alexander and Lazar (2004, 2005, 2006), and Haas et al. (2002, 2004) recently proposed family of univariate mixed normal GARCH processes, which has been shown to be particularly well suited for analyzing and forecasting financial volatility. However, estimating time varying optimal hedge ratios is inherently multivariate and Haas et al. (2006) and Bauween et al. (2006) thus generalize the univariate mixed normal GARCH model to the multivariate specification. As a consequence, the hedge ratios estimated from mixed normal GARCH models are both time varying and asymmetric.

Finally, Moschini and Myers (2002) develop a different bivariate GARCH parameterization for cash and futures markets, with a flexible functional form for time-varying volatility that is suitable for testing whether the optimal hedge ratio is constant, and whether the time variations in the optimal hedge ratios are due solely to deterministic seasonality and time-to-maturity effects. Statistical tests reject both null hypotheses.

However, this study we use the popular multivariate GARCH include the diagonal VECH, the diagonal BEKK and the CCC model as detailed below.

3. Research Methodology

Multivariate GARCH Models

The basic idea to extend univariate GARCH models to multivariate GARCH models is that it is significant to predict the dependence in the comovement of the petroleum future returns in a portfolio. To recognize this feature through a multivariate model would generate a more reliable model than separate univariate models.

In the first place, one should consider what specification of a multivariate GARCH model should be imposed. On the one hand, it should be flexible enough to state the dynamics of the conditional variances and covariances. On the other hand, as the number of parameters in a multivariate GARCH model increase rapidly along with the dimension of the model, the specification should be parsimonious to simplify the model estimation and also reach the purpose of easy interpretation of the model parameters. However, parsimony may reduce the number of parameters, in which situation the relevant dynamics in the covariance matrix cannot be captured. So it is important to get balance between the parsimony and the flexibility when designing the multivariate GARCH model specification. Another feature that multivariate GARCH models must satisfy is that the covariance matrix should be positive definite.

Several different multivariate GARCH model formulations have been proposed in the literature, and the most popular of these are the diagonal VECH, the diagonal BEKK and CCC models. Each of these is discussed briefly in turn below; for a more detailed discussion, see Kroner and Ng (1998).

The diagonal VECH model

The first multivariate GARCH model was introduced by Bollerslev, Engle and Wooldridge in 1988, which is called VECH model. It is much general compared to the subsequent formulations. In the VECH model, every conditional variance and covariance is a function of all lagged conditional variances and covariances, as well as lagged squared returns and cross-products of returns. The model can be expressed below:

$$VECH(H_t) = c + \sum_{j=1}^{q} A_j VECH(\varepsilon_{t-j}\varepsilon'_{t-j}) + \sum_{j=1}^{p} B_j VECH(H_{t-j}), \qquad (1)$$

where $VECH(H_t)$ is an operator that stacks the columns of the lower triangular part of its argument square matrix, H_t is the covariance matrix of the residuals, N presents the number of variables, t is the index of the t^{th} observation, c is an $\frac{N(N+1)}{2} \times 1$ vector, A_j and B_j are $\frac{N(N+1)}{2} \times \frac{N(N+1)}{2}$ parameter matrices and ε is an $N \times 1$ vector.

The condition for H_t is to be positive definite for all t is not restrictive. In addition, the number of parameters equals to $(p+q) \times \left(\frac{N(N+1)}{2}\right)^2 + \frac{N(N+1)}{2}$, which is large. Furthermore, it demands a large quantity of computation.

The diagonal VECH model, the restricted version of VECH, was also proposed by Bollerslev, et al (1988). It assumes the A_j and B_j in equation (1) are diagonal matrices, which makes it possible for H_t to be positive definite for all t. Also, the estimation process proceeds much smoothly compared to the complete VECH model. However, the diagonal VECH model with $(p+q+1)\times N\times\frac{(N+1)}{2}$ parameters is too restrictive since it does not take into account the interaction between different conditional variances and covariances.

The diagonal BEKK model

To ensure positive definiteness, a new parameterization of the conditional variance matrix H_t was defined by Baba, Engle, Kraft and Kroner (1990) and became known as the BEKK model, which is viewed as another restricted version of the VECH model. It achieves the positive definiteness of the conditional variance by formulating the model in a way that is property is implied by model structure.

The form of the BEKK model is as follows

$$H_t = CC' + \sum_{j=1}^{q}\sum_{k=1}^{K} A'_{kj}\varepsilon_{t-j}\varepsilon'_{t-j}A_{kj} + \sum_{j=1}^{p}\sum_{k=1}^{K} B'_{kj}H_{t-j}B_{kj} \tag{2}$$

where A_{kj}, B_{kj} and C are $N\times N$ parameter matrices, and C is a lower triangular matrix. The purpose of decomposing the constant term into a product of two triangle matrices is to guarantee the positive semi-definiteness of H_t. Whenever $K>1$, an identification problem would be generated for the reason that there are not only single parameterizations that can obtain the same representation of the model.

The first order BEKK model is

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \tag{3}$$

The BEKK model also has its diagonal form by assuming A_{kj}, B_{kj} matrices are diagonal. It is a restricted version of the diagonal VECH model. The most restricted version of the diagonal BEKK model is the scalar BEKK one with $A = aI$ and $B = bI$ where a and b are scalars.

Estimation of a BEKK model still bears large computations due to several matrix transpositions. The number of parameters of the complete BEKK model is $(p+q)KN^2 + \frac{N(N+1)}{2}$. Even in the diagonal one, the number of parameters soon reduces to $(p+q)KN + \frac{N(N+1)}{2}$, but it is still large. The BEKK form is not linear in parameters, which makes the convergence of the model difficult. However, the strong point lies in that the model structure automatically guarantees the positive definiteness of H_t. Under the overall consideration, it is typically assumed that $p=q=K=1$ in BEKK form's application.

The Constant Conditional Correlations (CCC) model

The CCC model was introduced by Bollerslev in 1990 to primarily model the condition covariance matrix indirectly by estimating the conditional correlation matrix. The conditional correlation is assumed to be constant while the conditional variances are varying.

Consider the CCC model of Bollerslev (1990):

$$y_t = E\langle y_t \mid F_{t-1}\rangle + \varepsilon_t \quad , \quad \varepsilon_t = D_t\eta_t \tag{4}$$

$$\operatorname{var}\langle \varepsilon_t \mid F_{t-1}\rangle = D_t\Gamma D_t$$

where $y_t = (y_{1t}, ..., y_{mt})'$, $\eta_t = (\eta_{1t}, ..., \eta_{mt})'$ is a sequence of independently and identically distributed (i.i.d) random vectors, F_t is the past information available at time t, $D_t = diag\left(h_t^{1/2},...,h_m^{1/2}\right)$, m is the number of returns, and $t = 1,\ldots,n,$. As $\Gamma = E\langle \eta_t \eta_t' | F_{t-1} \rangle = E(\eta_t \eta_t')$, where $\Gamma = \{\rho_{ij}\}$ for $i, j = 1,\ldots m$, the constant conditional correlation matrix of the unconditional shocks, η_t, is equivalent to the constant conditional covariance matrix of the conditional shocks, ε_t, from (4), $\varepsilon_t \varepsilon_t' = D_t \eta_t \eta_{t-1}' D_t$, $D_t = (diag\ Q_t)^{1/2}$, and $E\langle \varepsilon_t \varepsilon_{t-1}' | F_{t-1} \rangle = Q_t = D_t \Gamma D_t$, where Q_t is the conditional covariance matrix.

The CCC model assumes that the conditional variance for each return $h_{it}, i = 1,\ldots,m$, follows a univariate GARCH process, that is

$$h_t = \omega_t + \sum_{j=1}^{r} \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{j=1}^{s} \beta_{ij} h_{i,t-j} \quad , \tag{5}$$

where α_{ij} represents the ARCH effect, or short run persistence of shocks to return i, β_{ij} represents the GARCH effect, and $\sum_{j=1}^{r} \alpha_{ij} + \sum_{j=1}^{s} \beta_{ij}$ denotes the long run persistence.

Model estimation for multivariate GARCH

Under the assumption of conditional normality, the parameters of the multivariate GARCH models of any of the above specifications can be estimated by maximizing the log-likelihood function.

$$\ell(\theta) = -\frac{TN}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^{T} (\log|H_t| + \varepsilon_t' H_t^{-1} \varepsilon_t) \tag{6}$$

where θ denotes all the unknown parameters to be estimated, N is the number of the petroleum future prices and T is the number of observations and all other notation is as above. The maximum-likelihood estimates for θ is asymptotically normal, and thus traditional procedures for statistical inference are applicable.

4. Data

The data used in this study is the daily data from 4 November 2009 to 29 October 2014. We will get 1251 observations. The data is derived from www.quandl.com and trade in CME (Chicago Mercantile Exchange) market. Moreover, data analysis can be carried out using EVIEWS 8. The four petroleum future return is defined as:

$$R_t = \log\left(\frac{FP_t}{FP_{t-1}}\right) \tag{7}$$

where FP_t is the petroleum future price at time t and FP_{t-1} is the petroleum future price at time $t-1$. The R_t of equation (7) will be used in observing the volatility of the petroleum between the selected petroleum over the period 2009 to 2014. We can create the variables of the return on the petroleum futures as follows:

The returns of crude oil future = RCRUDE, the returns of gasoline future = RGASOLINE, the returns of heat oil future = RHEATOIL and the returns of natural gas future = RNG

In addition, we can show the movement of the daily four petroleum future prices and returns according to Figure 1 and Figure 2.

Figure 1. the daily four petroleum future prices

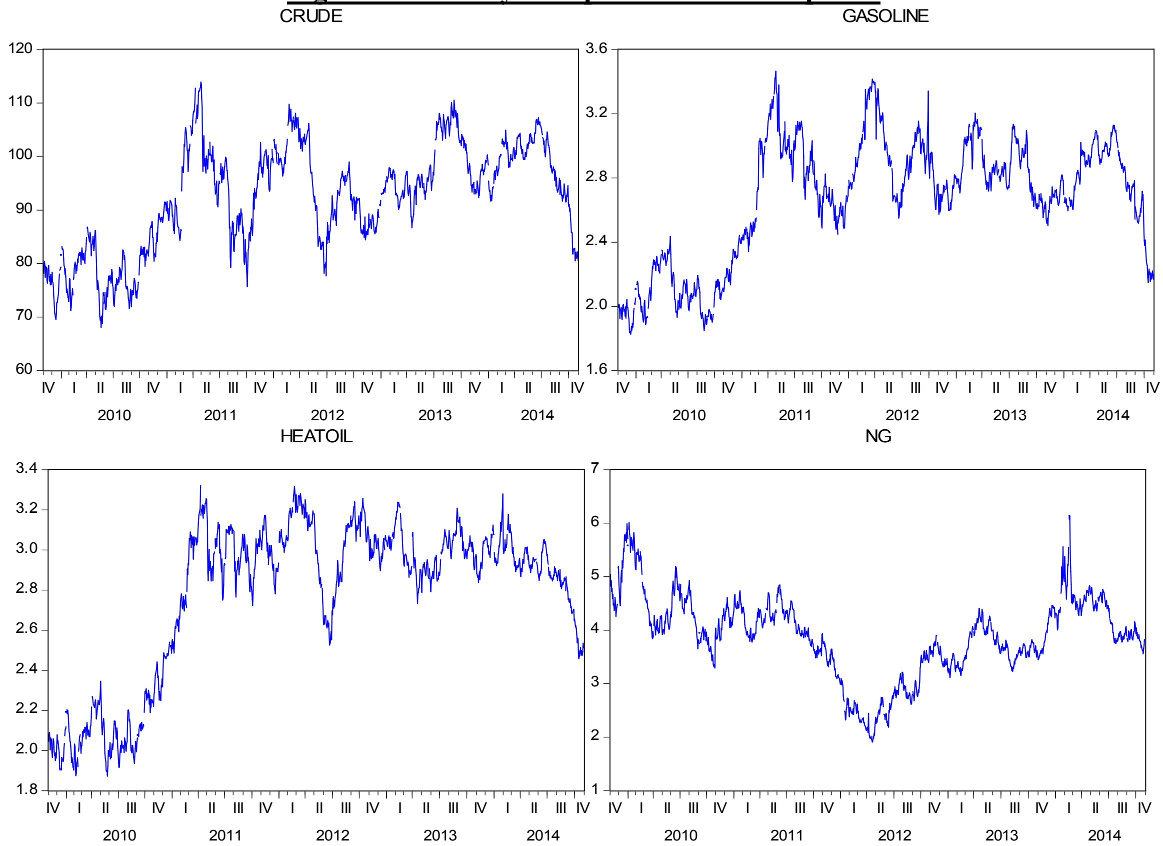
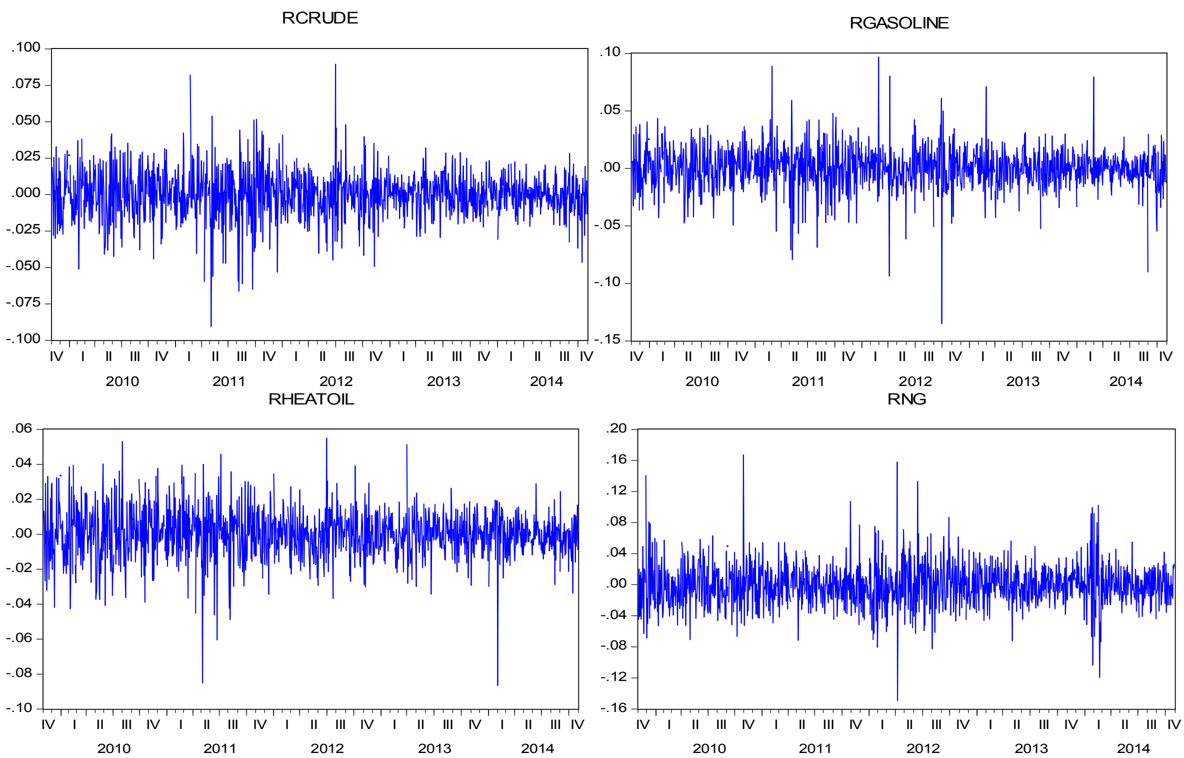


Figure 2. the daily four petroleum future returns



The descriptive statistics are given in Table 1. The daily future returns of natural gas (RNG) display the greatest variability with the mean of -0.000221%, a maximum of 0.1699%, and a minimum of -0.1491%. Furthermore, the skewness, the kurtosis and the Jarque-Bera Lagrange multiplier statistics of all petroleum future returns are statistically significant, thereby implying that the distribution is not normal. Besides, the return series will be used to construct the conditional mean and the conditional variances in next.

Table 1. Descriptive Statistics

Returns	RCRUDE	RGASOLINE	RHEATOIL	RNG
Mean	4.06E-05	7.85E-05	0.000171	-0.000221
Median	0.000299	7.54E-05	0.000230	-0.000760
Maximum	0.0894	0.0968	0.0549	0.1699
Minimum	-0.0903	-0.1349	-0.0865	-0.1491
Std. Dev.	0.0164	0.0182	0.0142	0.0279
Skewness	-0.1510	-0.4351	-0.3430	0.4722
Kurtosis	5.6766	8.3840	5.7790	6.6720
Jarque-Bera	378.5033	1551.6920	427.4340	749.9592

5. Empirical Results

5.1 Unit Root Tests

Standard econometric practice in the analysis of financial time series data begins with an examination of unit roots. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are used to test for all the petroleum future returns under the null hypothesis of a unit root against the alternative hypothesis of stationarity. The results from unit root tests are presented in Table 2. The tests yield negative values in all cases for levels, such that the individual returns series reject the null hypothesis at the 1% significance level, so that all returns are stationary.

Table 2. Unit Root Tests

Returns	Augmented Dickey-Fuller Test				Phillips-Perron Test			
	Constant		Constant and Trend		Constant		Constant and Trend	
	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)
RCRUDE	-35.712***	-17.304***	-35.717***	-17.297***	-35.712***	-811.866***	-35.717***	-810.068***
RGASOLINE	-36.319***	-19.835***	-36.353***	-19.827***	-36.343***	-549.900***	-36.457***	-550.837***
RHEATOIL	-34.294***	-15.271***	-34.323***	-15.264***	-34.288***	-316.475***	-34.309***	-316.093***
RNG	-38.190***	-18.018***	-38.181***	-18.011***	-38.198***	-309.877***	-38.235***	-309.762***

*** denote significance at the 1% level

5.2. Vector Autoregression Model

An important task is to model the conditional mean and conditional variances of the return series. Therefore, the appropriate multivariate conditional volatility model given as VAR (1)-diagonal VECH, VAR (1)-diagonal BEKK and VAR (1)-CCC models is estimated. The conditional mean comes from VAR (Vector Autoregression Model) which can display the source as follows:

Let $Y_t = (Y_{1t}, Y_{2t}, ..., Y_{nt})'$ denote a $k \times 1$ vector of petroleum future return series variables. The basic vector autoregressive model of order p, VAR (p), is

$$Y_t = c + \Pi_t Y_{t-1} + \Pi_2 Y_{t-2} + ... + \Pi_p Y_{t-p} + \mu_t, \qquad t = 1,...T, \tag{8}$$

where Π_t are $k \times k$ matrices of coefficients, c is a $k \times 1$ vector of constants and μ_t is an $k \times 1$ unobservable zero mean white noise vector process with covariance matrix $\sum$.

As in the univariate case with AR processes, we can use the lag operator to represent VAR (p) $\Pi(L)Y_t = c + \mu_t$, where $\Pi(L) = I_n - \Pi_1 L - ... - \Pi_p L^p$

If we impose stationarity on Y_t in (8), the unconditional expected value is given by

$$\mu = (I_n - \Pi_1 - ... - \Pi_p)^{-1} c.$$

Lag Length Selection**:** a reasonable strategy how to determine the lag length of the VAR model is to fit VAR (p) models with different orders $p = 0,...,p_{\max}$ and choose the value of p which minimizes some model selection criteria. Model selection criteria for VAR (p) could be base on Akaike (AIC), Schewarz-Bayesian (BIC) and Hannan-Quinn (HQ) information criteria.

Before we construct the conditional mean, the first thing to do is to find the right lag of VAR model as shown in the table 3. From the various criterions are found to be selected lag that 1 and 3. Most of them will choose lag 1. We therefore conclude that lag 1 should be suitable for the conditional mean.

After all multivariate conditional volatility models in this paper are already estimated. The next step, we will have to explain that the results of each model and select the best model.

The VAR (1)-diagonal VECH estimates of the conditional correlation between the volatilities of the four petroleum future returns base on estimating the univariate GARCH (1,1) model for each the petroleum are given in Table 4. The estimates of the VAR (1) - diagonal VECH parameters that θ_1 and θ_2 are statistically significant in almost all cases except in the case of $\rho_{RGA.\_RNG.}$ (gasoline with natural gas). This indicates that the short run persistence of shocks on the dynamic conditional correlations is greatest for RCRUDE with RHEATOIL at 0.075 (θ_1), while the largest long run persistence of shocks to the conditional correlations is 0.990 ($\theta_1 + \theta_2$) for RCRUDE with RGASOLINE.

Table 3. Lag order selection

Lag	LR	FPE	AIC	SC	HQ
0	NA	4.50e-15	-21.682	-21.666	-21.676
1	298.686	3.63e-15*	-21.897*	-21.815*	-21.866*
2	23.257	3.66e-15	-21.890	-21.742	-21.835
3	35.799*	3.64e-15	-21.894	-21.680	-21.813
4	12.914	3.70e-15	-21.879	-21.598	-21.773
5	13.980	3.75e-15	-21.864	-21.518	-21.734
6	23.184	3.78e-15	-21.858	-21.446	-21.703
7	11.364	3.84e-15	-21.841	-21.363	-21.662
8	10.107	3.91e-15	-21.824	-21.280	-21.619

Note: * indicates lag order selected: LR= Sequential modified LR test statistic, FPE=Final prediction error, AIC=Akaike information criterion, SC=Schwarz information criterion, HQ=Hannan-Quinn information criterion

The VAR (1)-diagonal BEKK estimates of the conditional correlation between the volatilities of the four petroleum future returns are given in Table 5. The estimates of the diagonal BEKK parameters that θ_1 and θ_2 are statistically significant in all cases. This indicates that the short run persistence of shocks on the dynamic conditional correlations is greatest at 0.059 for RCRUDE with RHEATOIL, while the largest long run persistence of shocks to the conditional correlations is 0.991 ($\theta_1 + \theta_2$) for RCRUDE with RGASOLINE, RCRUDE with RHEATOIL and RGASOLINE with RHEATOIL.

Finally, in Table 6 presents the estimates for the VAR (1)-CCC model, with $p = q = r = s = 1$. The ARCH and GARCH estimates of the conditional variance between the four petroleum future returns are statistically significant in all cases. The ARCH (α) estimates are generally small (less than 0.2), and the GARCH (β) estimates are generally high (more than 0.8) and close to one. Therefore, the long run persistence ($\alpha + \beta$), is generally to one, indicating a near long memory process. This indicates a near long memory process. In addition, since $\alpha + \beta < 1$, all petroleum satisfies the second moment and log-moment condition, which is a sufficient condition for the QMLE (quasi-maximum likelihood) to be consistent and asymptotically normal. VAR (1)-CCC estimates of the constant conditional correlation between RCRUDE and RHEATOIL with the highest in 0.725. This indicates that the standardized shock on the constant conditional correlation for RCRUDE with RHEATOIL is 0.725.

Table 4. VAR (1) - diagonal VECH model estimates

VAR (1)	**RCR.**	**RGA.**	**RHE.**	**RNG.**	ρ RCR._RGA.	ρ RCR._RHE.	ρ RCR._RNG.	ρ RGA._RHE.	ρ RGA._RNG.	ρ RHE._RNG.
RCR.(-1)	-0.051 (0.034)	0.082** (0.039)	0.012 (0.026)	0.034 (0.075)	-	-	-	-	-	-
RGA.(-1)	0.386*** (0.015)	0.118*** (0.026)	0.363*** (0.012)	0.040 (0.040)						
RHE.(-1)	-0.088** (0.037)	-0.024 (0.046)	-0.101*** (0.034)	0.085 (0.086)						
RNG.(-1)	0.012 (0.011)	-0.0002 (0.0130)	0.016* (0.009)	-0.060* (0.033)						
ω (Constant)	2.73E-06** (7.04E-07)	.76E-05*** (2.06E-06)	2.19E-06*** (4.64E-07)	1.31E-05*** (4.50E-06)	-	-	-	-	-	-
α	0.074*** (0.007)	0.077*** (0.006)	0.074*** (0.006)	0.045*** (0.008)	-	-	-	-	-	-
β	0.915*** (0.007)	0.875*** (0.008)	0.914*** (0.006)	0.936*** (0.010)	-	-	-	-	-	-
$\alpha+\beta$	0.989	0.952	0.980	0.981	-	-	-	-	-	-
θ_0 (Constant)	-	-	-	-	4.67E-07 (4.96E-07)	1.43E-06*** (3.92E-07)	3.04E-06 (2.42E-06)	6.05E-07* (3.60E-07)	1.82E-06 (2.14E-06)	3.99E-06 (3.38E-06)
θ_1	-	-	-	-	0.069*** (0.005)	0.075*** (0.006)	0.020* (0.012)	0.070*** (0.007)	-0.004 (0.010)	0.024** (0.010)
θ_2	-	-	-	-	0.921*** (0.006)	0.914*** (0.006)	0.870*** (0.089)	0.919*** (0.005)	0.934*** (0.077)	0.821*** (0.130)
$\theta_1+\theta_2$	-	-	-	-	0.990	0.989	0.890	0.989	0.930	0.981
	Log-likelihood=14172.68				AIC=-22.578		SIC=-22.373		HQ=-22.501	

Note: standard error in parenthesis, *** denote significance at the 1% level, **denote significance at the 5% level and * denote significance at the 10% level, RCR. =the returns of crude oil, RGA.=the returns of gasoline, RHE.=the returns of heat oil and RNG.=the returns of natural gas

Furthermore, we will choose the best model next by considering the value of log-likelihood, AIC, SIC and HQ. From the Table 4, 5 and 6, we found that the VAR (1)-diagonal VECH model is highest log-likelihood equal 14172.68. AIC, SIC and HQ are lowest, equal -22.578, -22.373 and -22.501, respectively. Thus, it can be concluded that we should choose the VAR (1)-diagonal VECH model in volatility analysis of the petroleum future returns and the results of this model are used to calculate the optimal two-petroleum portfolio weights and hedging ratios.

However, we can show the movement of the conditional covariance and the conditional correlation of the four petroleum future returns in each model according to Figure 3, 4, 5, 6 and 7, respectively.

5.3. Multivariate GARCH diagnostic tests

The multivariate GARCH models consist of the VAR (1)-diagonal VECH, the VAR (1)-diagonal BEKK and the VAR (1)-CCC model. We can diagnostic check on the system residuals to determine efficiency of estimator according to the Table 7. We found that system residuals have no autocorrelations up to lag 6 and are not normally distributed. Therefore, it can be concluded that the estimators of multivariate GARCH model are efficient.

Table 5. VAR (1) - diagonal BEKK model estimates

VAR (1)	RCR.	RGA.	RHE.	RNG.	ρ RCR._RGA.	ρ RCR._RHE.	ρ RCR._RNG.	ρ RGA._RHE.	ρ RGA._RNG.	ρ RHE._RNG.
RCR.(-1)	-0.041 (0.033)	0.105** (0.042)	0.022 (0.027)	0.025 (0.074)	-	-	-	-	-	-
RGA.(-1)	0.386*** (0.015)	0.136*** (0.025)	0.364*** (0.012)	0.051 (0.039)						
RHE.(-1)	-0.100*** (0.037)	-0.050 (0.048)	-0.110*** (0.034)	0.078 (0.083)						
RNG.(-1)	0.010 (0.011)	-0.0004 (0.013)	0.013 (0.009)	-0.057** (0.033)						
ω (Constant)	1.92E-06*** (5.08E-07)	3.57E-06*** (7.01E-07)	1.66E-06*** (3.61E-07)	2.09E-05*** (5.75E-06)	-	-	-	-	-	-
α^2	0.059***	0.046***	0.060***	0.034***	-	-	-	-	-	-
β^2	0.933***	0.945***	0.931***	0.938***	-	-	-	-	-	-
$\alpha^2+\beta^2$	0.992	0.991	0.991	0.972	-	-	-	-	-	-
θ_0 (Constant)	-	-	-	-	7.70E-07** (3.00E-07)	1.07E-6*** (2.85E-07)	6.87E-07 (5.78E-07)	8.72E-7*** (2.22E-07)	7.77E-07 (5.87E-07)	6.84E-07 (4.86E-07)
θ_1	-	-	-	-	0.052***	0.059***	0.045***	0.053***	0.040***	0.045***
θ_2	-	-	-	-	0.939***	0.932***	0.935***	0.938***	0.941***	0.934***
$\theta_1+\theta_2$	-	-	-	-	0.991	0.991	0.980	0.991	0.981	0.979
	Log-likelihood=14124.700				AIC=-22.520		SIC=-22.364		HQ=-22.462	

Note: standard error in parenthesis, *** denote significance at the 1% level, **denote significance at the 5% level and * denote significance at the 10% level, RCR. =the returns of crude oil, RGA.=the returns of gasoline, RHE.=the returns of heat oil and RNG.=the returns of natural gas

Table 6. VAR (1) - CCC model estimates

VAR (1)	RCR.	RGA.	RHE.	RNG.	ρ RCR._RGA.	ρ RCR._RHE.	ρ RCR._RNG.	ρ RGA._RHE.	ρ RGA._RNG.	ρ RHE._RNG.
RCR.(-1)	-0.068* (0.040)	0.082* (0.050)	0.006 (0.034)	0.034 (0.074)	-	-	-	-	-	-
RGA.(-1)	0.386*** (0.017)	-0.008 (0.035)	0.368*** (0.015)	0.051 (0.040)						
RHE.(-1)	-0.084* (0.046)	-0.107* (0.058)	-0.100** (0.043)	0.077 (0.085)						
RNG.(-1)	0.011 (0.014)	0.015 (0.017)	0.011 (0.011)	-0.068** (0.031)						
ω (Constant)	5.59E-06*** (1.40E-06)	0.0001*** (2.29E-05)	8.17E-06*** (1.55E-06)	1.28E-05*** (4.18E-06)	-	-	-	-	-	-
α	0.032*** (0.007)	0.148*** (0.022)	0.055*** (0.006)	0.047*** (0.007)	-	-	-	-	-	-
β	0.939*** (0.012)	0.495*** (0.084)	0.889*** (0.013)	0.936*** (0.008)	-	-	-	-	-	-
$\alpha+\beta$	0.971	0.643	0.944	0.983	-	-	-	-	-	-
Constant Conditional Correlation	-	-	-	-	0.311*** (0.022)	0.725*** (0.011)	0.093*** (0.029)	0.345*** (0.023)	0.043 (0.029)	0.069** (0.027)
	Log-likelihood=13886.940				AIC=-22.140		SIC=-21.984		HQ=-22.082	

Note: standard error in parenthesis, *** denote significance at the 1% level, **denote significance at the 5% level and * denote significance at the 10% level, RCR. =the returns of crude oil, RGA.=the returns of gasoline, RHE.=the returns of heat oil and RNG.=the returns of natural gas

Figure 3. Conditional Covariance (VAR (1) - diagonal VECH estimates)

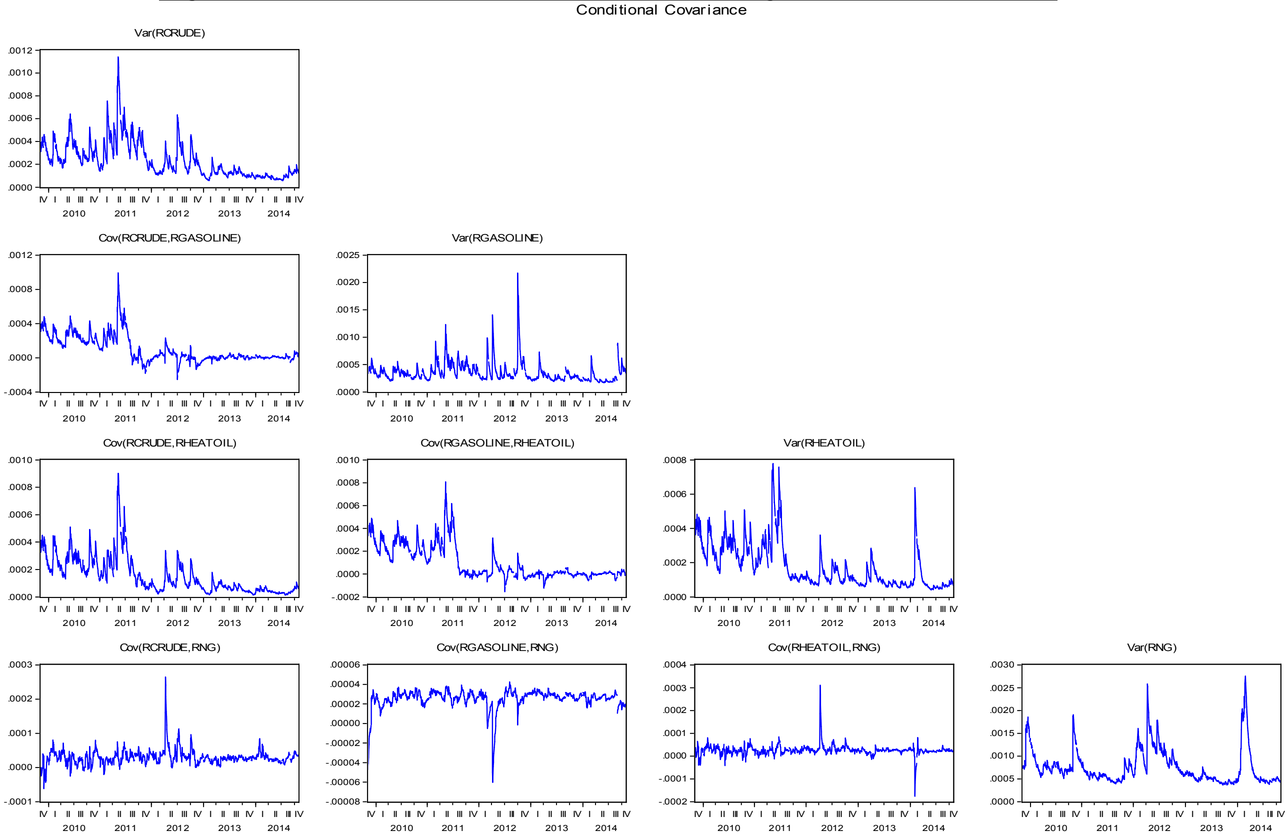


Figure 4. Conditional Covariance (VAR (1) - diagonal BEKK estimates)

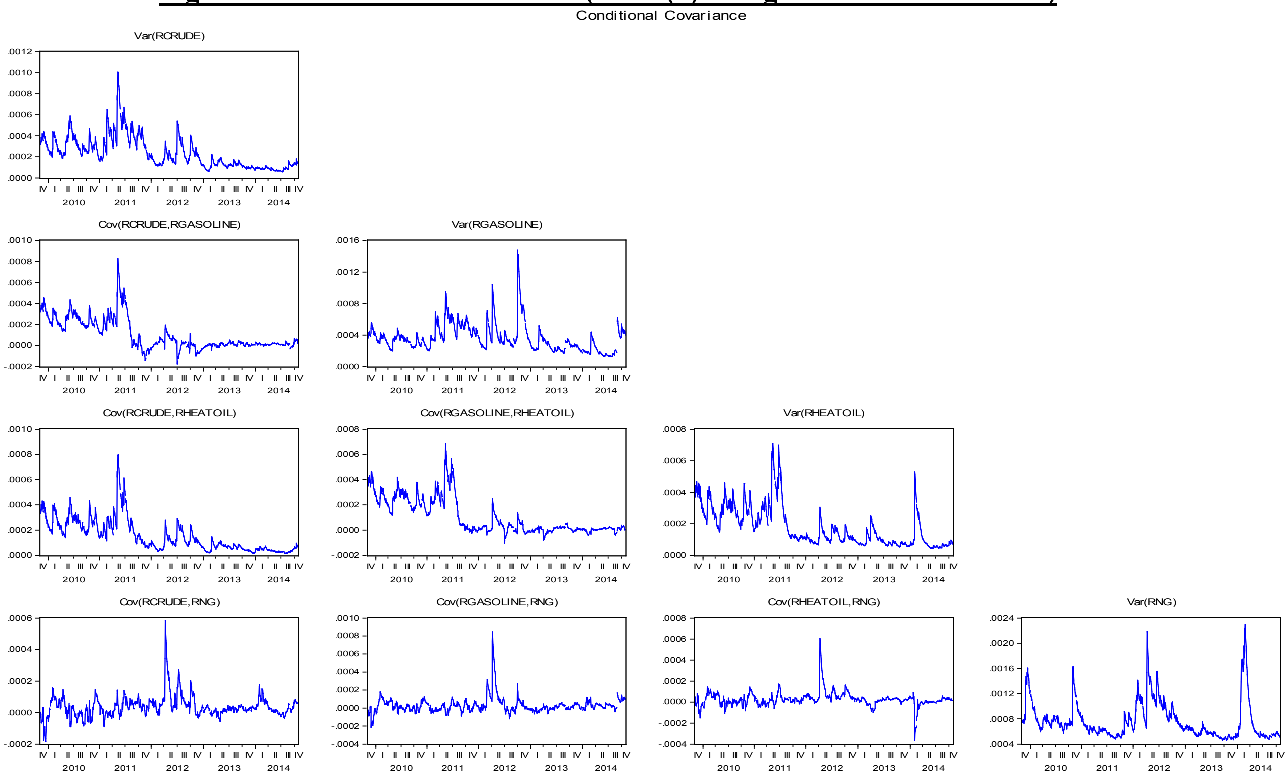


Figure 5. Conditional Covariance (VAR (1) - CCC estimates)

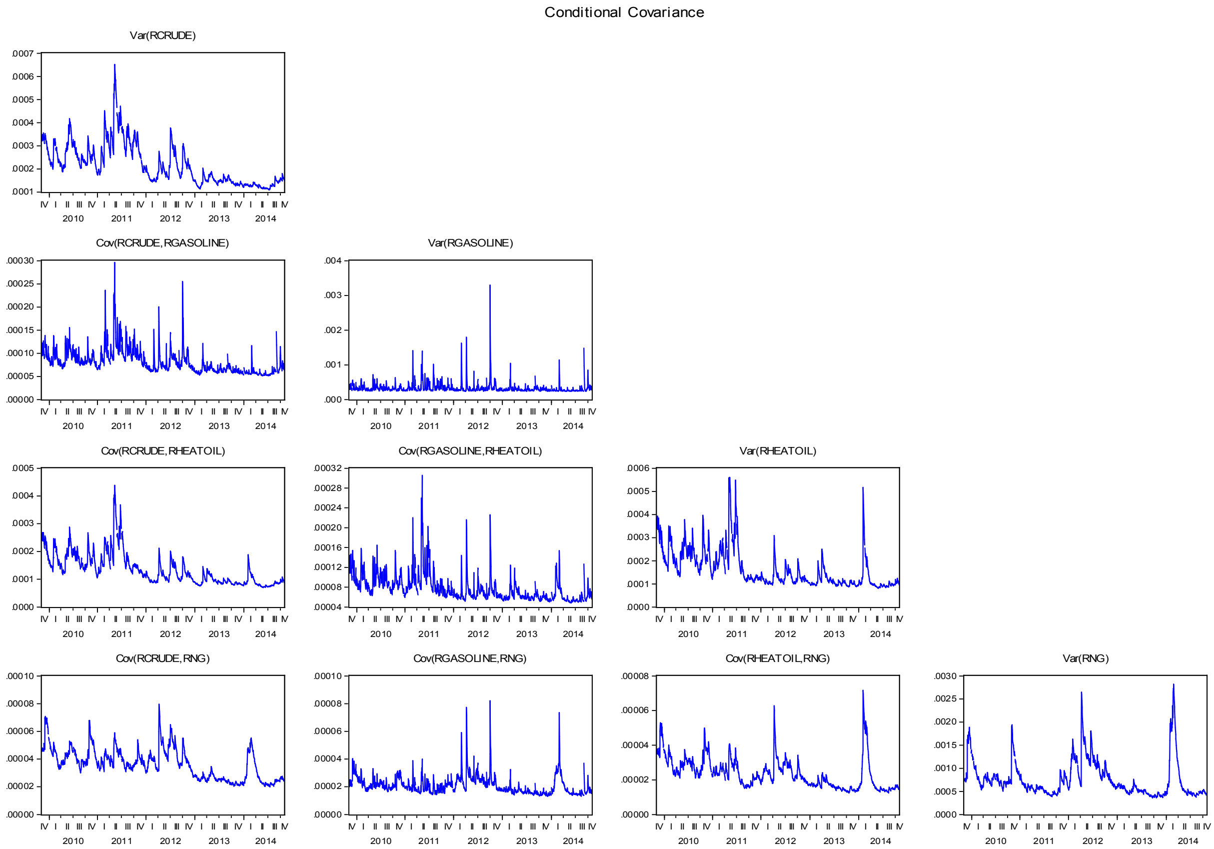


Figure 6. Conditional Correlation (VAR (1) - diagonal VECH estimates)

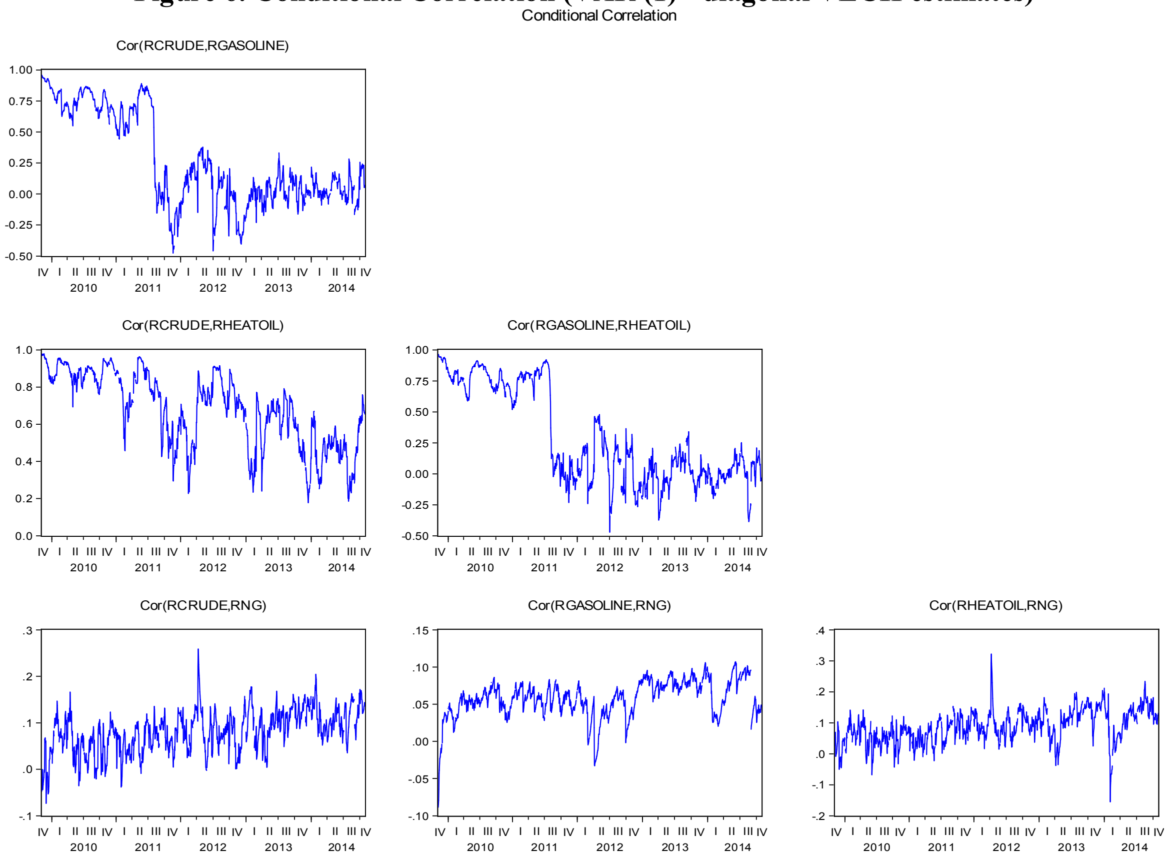


Figure 7. Conditional Correlation (VAR (1) - diagonal BEKK estimates)

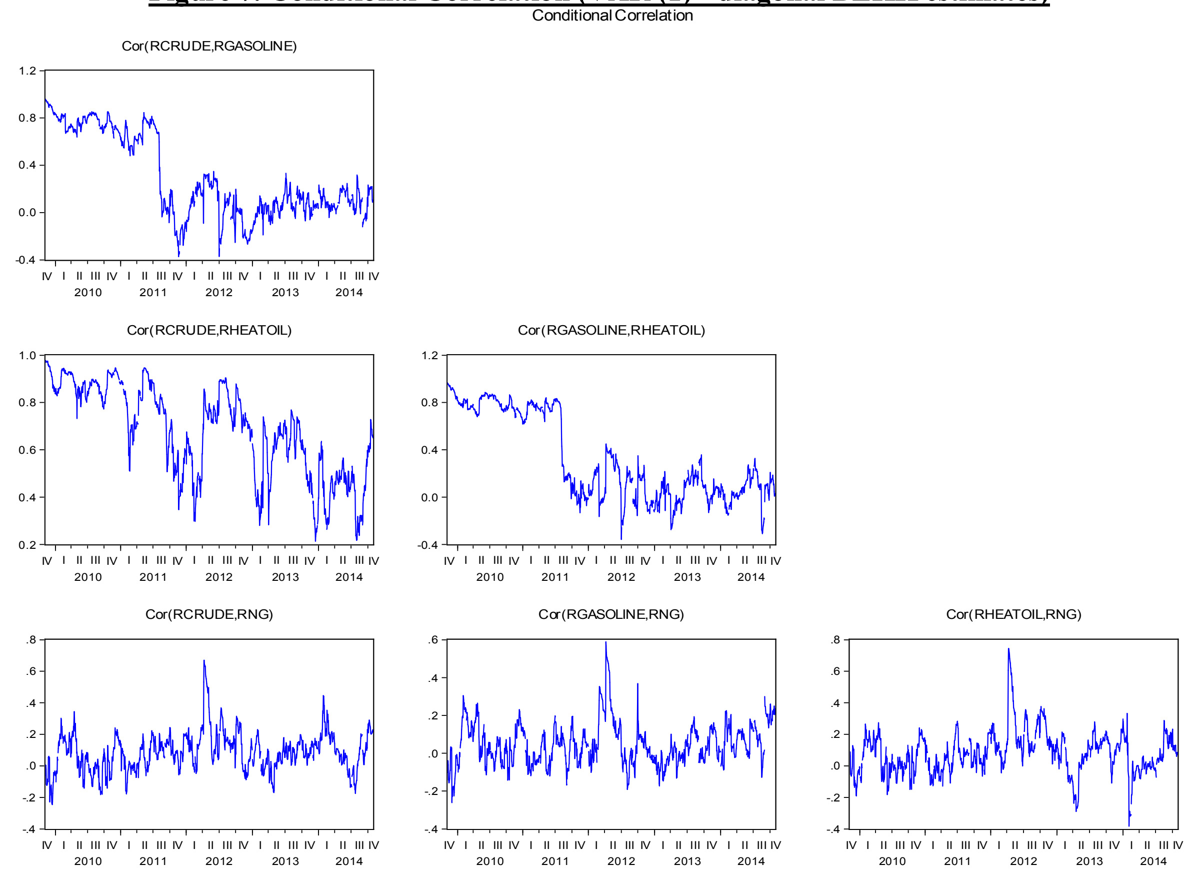


Table 7. Multivariate GARCH diagnostic tests

Test	VAR(1)-diagonal VECH					
	Lags	Value	Probability	Test	Value	Probability
System Residual Tests for Autocorrelations H_0=no residual autocorrelation (Q-Stat)	1	17.472	0.355	System Residual Normality Tests H_0=Multivariate normal		
	2	41.002	0.132			
	3	60.203	0.111			
	4	69.748	0.290			
	5	84.451	0.345	-Skewness (Chi-sq)	83.522	0.000
	6	104.294	0.264	-Kurtosis (Chi-sq)	3125.908	0.000
				-Jarque-Bera	3209.431`	0.000
Test	VAR (1) – diagonal BEKK					
	Lags	Value	Probability	Test		
System Residual Tests for Autocorrelations H_0=no residual autocorrelation (Q-Stat)	1	22.052	0.141	System Residual Normality Tests H_0=Multivariate normal		
	2	41.683	0.117			
	3	60.093	0.113			
	4	69.896	0.286			
	5	83.382	0.375	-Skewness (Chi-sq)	51.080	0.000
	6	102.976	0.294	-Kurtosis (Chi-sq)	3990.282	0.000
				-Jarque-Bera	4041.363	0.000
Test	VAR (1) - CCC					
	Lags	Value	Probability	Test		
System Residual Tests for Autocorrelations H_0=no residual autocorrelation (Q-Stat)	1	3.251	0.999	System Residual Normality Tests H_0=Multivariate normal		
	2	28.200	0.659			
	3	50.570	0.372			
	4	60.661	0.595			
	5	73.413	0.685	-Skewness (Chi-sq)	111.783	0.000
	6	92.505	0.582	-Kurtosis (Chi-sq)	5480.235	0.000
				-Jarque-Bera	5592.018	0.000

6. Implications for Portfolio Designs and Hedging Strategies

We provide two examples for constructing optimal portfolio designs and hedging strategies using our best estimates of model VAR (1)-diagonal VECH for the petroleum.

The first example follows Kroner and Ng (1998) by considering a portfolio that minimize risk without lowering expected returns. If we assume the expected returns to be zero, the optimal portfolio weight of one petroleum (or asset) to the other in a two petroleum (asset) portfolio is given by:

$$w_{12,t} = \frac{h_{22,t} - h_{12,t}}{h_{11,t} - 2h_{12,t} + h_{22,t}} \tag{9}$$

and

$$w_{12,t} = \begin{cases} 0, & if \quad w_{12,t} < 0 \\ w_{12,t}, & if \quad 0 \leq w_{12,t} \geq 1 \\ 1, & if \quad w_{12,t} > 1 \end{cases} \tag{10}$$

where $w_{12,t}$ is the weight of the first petroleum in one dollar portfolio of two petroleum at time t, $h_{12,t}$ is the conditional covariance between petroleum 1 and 2 and $h_{22,t}$ is the conditional variance of the second petroleum in the one dollar portfolio is $1 - w_{12,t}$.

The average values of $w_{12,t}$ base on VAR (1)-diagonal VECH estimates are reported in the first column of Table 8. For instance, the average value of $w_{12,t}$ of a portfolio comprising crude oil and heat oil is 0.23. This suggests that the optimal holding of crude oil in one dollar of crude oil/heat oil portfolio be 23 cents and 77 cents for heat oil. These optimal portfolio weights suggest that investors should have more heat oil than crude oil and other petroleum in their portfolio to minimize risk without lowering the expected return. The petroleum between crude oil and gasoline, investors should have more crude oil than gasoline (64% to 36%) in their portfolios. When it comes to the petroleum between heat oil and natural gas, the optimal portfolio should be 82% to 18% and investors should have more heat oil than natural gas.

Table 8. hedge ratios and optimal portfolio weights base on VAR (1)-diagonal VECH

Portfolio	Average $w_{12,t}$	Average β_t
Crude oil/gasoline	0.64	0.27
Crude oil/heat oil	0.23	0.86
Crude oil/natural gas	0.78	0.03
Gasoline/heat oil	0.24	0.55
Heat oil/natural gas	0.82	0.03

We now follow the example given in Kroner and Sultan (1993) regarding risk-minimizing hedge ratios and apply it to our petroleum. In order to minimize risk, a long (buy) position of one dollar taken in one petroleum should be hedged by a short (sell) position of β_t in another petroleum at time t. The rule to have an effective hedge is to have an inexpensive hedge. The β_t is given by:

$$\beta_t = \frac{h_{12,t}}{h_{22,t}} \tag{11}$$

where β_t is the risk minimizing hedge ratio for two petroleum, $h_{12,t}$ is the conditional covariance between petroleum 1 and 2 and $h_{22,t}$ is the conditional variance of the second petroleum.

The second column of Table 8 reports the average values of β_t. The results show that the most effective hedging among all the petroleum is hedging long (buy) crude oil position by shorting (selling) natural gas. The least effective hedging among all the petroleum is hedging long (buy) crude oil position using (selling) heat oil.

7. Conclusion

This paper investigates volatility comovements and spillovers for crude oil, gasoline, heat oil and natural gas future. The results of volatility analysis are used to calculate the optimal two-petroleum portfolio weights and hedging ratios. In addition, this paper estimated three popular multivariate GARCH models, namely the VAR (1) - diagonal VECH, the VAR (1) - diagonal BEKK and the VAR (1)-CCC model, for the four petroleum future returns.

The empirical results overall showed that the estimates of the multivariate GARCH parameters are statistically significant in almost all cases except in the case of RGASOLINE with RNG. This indicates that the short run persistence of shocks on the dynamic conditional correlations is greatest for RCRUDE with RHEATOIL, while the largest long run persistence of shocks to the conditional correlations for RCRUDE with RGASOLINE.

The next step, we will choose the best model by considering the value of log-likelihood, AIC, SIC and HQ. We found that the best model in volatility and hedging ratios analysis is the VAR (1)-diagonal VECH model.

The results from these optimal portfolio weights base on the VAR (1)-diagonal VECH estimates suggest that investors should have more heat oil than crude oil and other petroleum in their portfolio to minimize risk without lowering the expected return. Such results can be useful as the management the volatility of the petroleum for investors.

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