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Value-at-Risk Analysis for the Tunisian Currency Market: A Comparative Study

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ABSTRACT: The main purpose of this paper is to compare empirically four Value-at-Risk simulation methods, namely, the Variance-Covariance, the Historical Simulation, the Bootstrapping and the Monte Carlo. We tried to estimate the VaR associated to three currencies and four currency portfolios in the Tunisian exchange market. Data covers the period between 01-01-1999 and 31-12-2007. Independently of the used technique, the Japanese Yen seems to be the most risky currency. Moreover, the portfolio diversification reduces the exchange rate risk. Lastly, the number of violations, when they exist, does not generally differ between the simulation methods. Recent evaluation tests were applied to select the most appropriate technique predicting precisely the exchange rate risk. Results based on these tests suggest that the traditional Variance-Covariance is the most appropriate method.

Keywords: Value-at-Risk; Tunisian currency market; Monte Carlo simulation
JEL Classifications: C14; G32; F37

1. Introduction

The concept of risk has been extensively used in finance. Significant divergence on the exact definition of this term exists, making it difficult to give a common answer to an apparently simple question: *what's risk?*

Several definitions were proposed in the literature. Pioneer theoretical models were proposed by Roy (1952) and Markowitz (1952). During the sixties, Sharpe (1964) and Lintner (1965) have proposed the Capital Asset Pricing Model (CAPM). This model is considered as a mono-factorial model, in the sense that it considers only one factor in the explanation of assets risk, i.e. the correlation between the performance of the asset and the one relative to the market portfolio. This type of risk is called systematic risk or asset market risk, a category of risk which is not diversifiable. Asset unsystematic or specific risk is equal to its *beta*, a relative measure of risk determined by comparison with the *beta* of the market portfolio, which is equal to one. In the next decade, an alternative model of risk, based on the absence of arbitrage, is developed: the arbitrage pricing theory. This model assumes that risk is a multidimensional phenomenon, which makes it a multi-factorial one. The *beta* of an asset for a given factor is the relative sensitivity of its performance to this factor. The limit of this model is that it does not provide any explanation of factors that determine the asset return.

On behalf of the investor, another important question has always been asked: *what is the maximum loss i have to support, if it exists?*

The concept of Value-at-Risk has been proposed in 1996 to suggest responses to this question. It consists in a simple determination of risk by giving it an exact value. Jorion (1996) advances that the VaR describes the worst expected loss for a given horizon and confidence level. This concept is more and more used in the financial institutions management. The Value-at-Risk is generally employed to evaluate the asymmetric risk, such as that associated with options, which cannot be calculated by the standard errors or *the beta*. Thus, this approach is considered as a source of innovation, due to the development of various related methods, such as the Historical Simulation, the Variance Covariance, the Monte Carlo simulation, the expansion of Cornish-Fisher, the Cluster method and other complementary techniques (the extreme values theory and the stress-testing).

However, the existence of several methods for calculating the VaR makes it difficult to select the most appropriate one predicting the risk associated with a particular financial asset or financial assets portfolio. In this paper, we search to fill this gap, by comparing empirically various risk measurement models, namely, the Historical Simulation, the Variance-Covariance, the Bootstrapping and the Monte Carlo simulation. On the other hand, we will use some validation tests, to select the best technique which predicts the foreign exchange risk in the Tunisian exchange market. The originality of this paper resides in the application of the Bootstrapping and the Monte Carlo simulations. In the best of our knowledge, there has been no prior attempt to test the application of these techniques to estimate the Value-at-Risk for currencies or currency portfolios in the Tunisian exchange market. Using data from the Tunisian exchange market, we will calculate the Value-at-Risk associated to the main three currencies in the economy (US Dollar, Euro and Japanese Yen), as well as to a currency portfolio composed of two currencies or more. In addition to measuring the VaR, we will try to test the effectiveness of the different estimation methods by comparing the VaR output with the real gain and loss in the first step and by applying the back testing technique later.

The remainder of this paper is structured as follows: section 2 provides a brief overview of both data and different techniques we will use in the empirical investigation. Empirical results and discussion will be presented in the section 3. Section 4 concludes the paper.

2. Research Methodology and Data

The VaR estimation is based on two essential elements, namely, the confidence level and the window of observations. In our study, we will retain three confidence levels: 95%, 97.5% and 99% and a rolling window of 250 observations (one financial year or 250 days). As mentioned previously, the Variance-Covariance, the Historical Simulation, the Bootstrapping and the Monte Carlo simulation will be used to calculate the VaR associated to currencies and currency portfolios. Prior to start our empirical investigation, a brief discussion of these different techniques is required.

In general, the calculation of the Value-at-Risk can be performed if some assumptions are verified. The first one is associated with the normality of the returns distribution. The second one consists in the stationarity of the return. The calculation of the VaR using the Variance-Covariance essentially requires the estimation of volatility through the Generalized Autoregressive Conditional Heterocedasticity process (GARCH (p,q)), the Exponentially Weighted Moving Average (EWMA), the I-GARCH (p,q) or the Equally Weighted Moving Average process. In our study, the GARCH (1,1) model will be used, allowing us to predict the conditional variance of profit and loss distribution (Engle, 2001). It assumes that returns are distributed according to a normal distribution, which will be identified through the knowledge of its average (the return average) and its variance (the return standard deviation). Value-at-Risk is then deducted from this distribution, through the assumptions about the conditional distribution of returns. Distributions are generally those of Gauss or Student.

To forecast the Value-at-Risk using the GARCH model, we proceed according to the following steps:

i. Formulating the hypothesis of the conditional distribution of returns. We assume that returns follow the Gauss's law;
ii. Estimating the GARCH parameters over the period, ranging from 1 to T, using the Maximum Likelihood method;
iii. Predicting the conditional variance based on the estimated GARCH (1,1); and
iv. Deducting from (i) and (iii) the forecasted profit and loss conditional distribution fractile, valid for the date T +1, i.e. the Value-at-Risk.

Various empirical studies have shown that the GARCH (1,1) is the best method to predict the volatility of financial series. Among others, Alexander (1996) used the GARCH (1,1) on equity and currency assets for the period of one year. Sarma *et al. (*2001) applied this model to the estimation of S&P500 and NSE-50 indexes volatility. Recently, the GARCH (1,1) was applied by Bredin and Hyde (2004) to estimate the volatility of currency portfolios.

The second method used in this study is the Historical Simulation. It calculates the VaR on the basis of historical behaviors of the asset and the asset portfolio. This method avoids making restrictive assumptions. However, the application of the Historical Simulation needs large data to make precise estimates of the risk. The ability to consider only events that appear in the observed time series is considered as the second limitation of this technique. In fact, this method is based on the assumption that past return may be reproduced in the future. It proceeds by classifying the daily historical returns in ascending order to identify, depending on the chosen confidence level, the maximum loss over the previous period. The Value-at-Risk corresponds to the empirical fractiles of past returns. For example, for a sample of 250 return historical observations and a confidence level of 95%, the VaR is given by the value of output corresponding to the 13th highest loss, obtained by multiplying 250 days by 5%, i.e. 250 x 0.05 = 13.

Given the shortcomings and limitations of the Historical Simulation, the Bootstrapping is proposed to improve it, by estimating the VaR based on data simulated by Bootstrap. It consists in resampling with replacement of return historical data. The final technique we have the intention to use is the Monte Carlo simulation. This technique allows us to generate returns in a random way and to deduct the profit and loss distribution used for the fractals estimation. Indeed, when we obtain the average and the standard deviation of returns, we can make random selection. One can note that modeling in finance generally used normal random variables. In the case of this method, the VaR is determined using the same way as for the Historical Simulation's VaR, but based on simulated sample. The disadvantage of this method consists in the number of selection, which increases with the standard deviation of the simulated distribution.

In this paper, we will use daily data on exchange rate to estimate the VaR of a currency or a currency portfolio. To do this, we will calculate the VaR associated with each one of three foreign exchange rates: Tunisian Dinar/Euro, Tunisian Dinar/US Dollar and Tunisian Dinar/Japanese Yen. In the second step, we will calculate the VaR for different currency portfolios composed by, at least, two currencies. Finally, some validation tests will be performed.

3. Empirical Results and Discussion

3.1. Value-at-Risk associated to international currencies

We begin by estimating the Value-at-risk for the following exchange rates: the TND/US Dollar, the TND/Euro and the TND/Japanese Yen, using the Variance-Covariance. As mentioned previously, we assume that the GARCH (1,1) is the most appropriate model predicting the conditional volatility of financial assets. The estimation of GARCH (1,1) model was conducted using the Maximum Likelihood procedure. Daily data concerning the US Dollar, the Euro and the Japanese Yen returns, covering the period 01-01-1999 to 31-12-2007, were used. Results from the GARCH (1,1) estimation are presented in the table I.

Table 1. Conditional volatility estimation using the GARCH (1,1) model

Exchange rate	Constant		Squared residuals (-1)		GARCH (-1)		Log Likelihood
	Coeff. (10^{-6})	p-value	Coeff.	p-value	Coeff.	p-value	
TND/US$	0.122	0.011	0.0317	0.000	0.963	0.000	9084.6
TND/Euro	0.039	0.010	0.0314	0.000	0.962	0.000	10478.8
TND/Yen	1.060	0.000	0.0688	0.000	0.920	0.000	7518.9.9

At a first glance, one can note that all parameters are statistically significant at 1% level. To get more in-depth idea about the magnitude of the volatility of each currency, we reproduce in the figure 1 the estimated values of the conditional variance. It was determined using the equation of the conditional variance, written as follows:

$$\hat{\sigma}_t^2 = \hat{\alpha}_0 + \hat{\alpha}_1 \varepsilon^2_{t-1} + \hat{\beta}_1 \hat{\sigma}^2_{t-1} \quad (1)$$

As shown in the figure 1, the most volatile currency is the Japanese Yen, which will induce, as we will see later, the highest Value-at-Risk.

In general, the Value-at Risk, for a currency, is given by:

$$VaR_t = \alpha \times \sigma_t \quad (2)$$

Where σ_t represents the currency conditional volatility, obtained from the GARCH (1,1) model, α depends on the confidence level.

Figure 1. Adjusted volatility using the GARCH (1,1) model

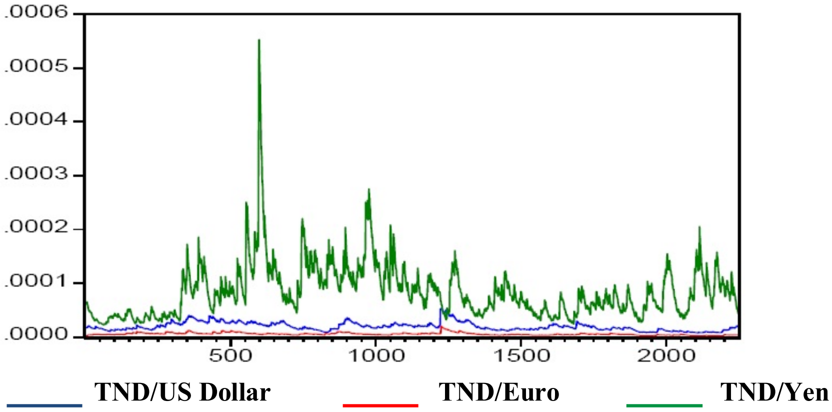


Table 2 summarizes the estimation of the Value-at-Risk for three different currencies, using the mentioned methods. According to this table, one can note that the various VaR simulations have all negative signs, since they represent losses on currencies detention. The analysis of the Value-at-Risk will be carried out for these amounts, but in absolute values. At the same time, under the normality assumption, the VaR estimation depends significantly on the confidence level (95%, 97.5% or 99%). The confidence levels allow us to control for the probability that the investor will obtain a return higher or equal to the Value-at-Risk. A VaR, calculated for a confidence level of 95% (α =5%), will be equal to 1.65[1] σ_t, i.e. there is a probability of 95% that the asset return will be, at least, equal to -1.65σ_t in the end of the period. The VaR, defined for 99% confidence level (α = 1%) and 97.5% (α =2.5%), will be equal, respectively, to -2.33 σ_t and -1.96 σ_t. One can notice that the VaR rises with the increase in the confidence level. This is perfectly consistent, because as the confidence level increases, it will bring closer to the 100% level, which represents the total loss. In general, if the confidence level is high, the VaR rank will be less and thus the VaR becomes higher. These findings confirm results of previous works, such as the one by Hendricks (1996), in which he realized an in-depth study on the performance of different VaR models in estimating the risk associated to currency portfolio. The Value-at-Risk was calculated using different approaches (the Historical Simulation, the Equally Weighted Moving Average and the Exponentially Weighted Moving Average) and for different confidence levels (95% and 99%). Empirical results reveal the existence of positive relationship between the estimated VaR and the confidence level. The author concludes that the choice of the latter extremely affects the performance of the VaR.

In our case, using the Variance-Covariance technique for a confidence level of 95%, the Japanese Yen seems to be the most risky currency, while the Euro is considered as the less risky one. The US Dollar is found in an intermediary position. These remarks still true, even when we change the confidence level and the time horizon. With regard to Historical Simulation, we can note that, for a 95% confidence level, the highest daily loss at January 1st, 2008 cannot exceed 1.82% for the Japanese Yen, 0.49% for the US Dollar and 0.26% for the Euro. Indeed, the Euro is less risky than the Dollar. The Japanese Yen is the riskiest currency.

[1] 1.65 is the tabulated value of normal law at 95% confidence level.

Table 2. Value-at-Risk estimation for three currencies using different approaches

year	TND/US Dollar			TND/Euro			TND/Yen		
	99 %	**97.5 %**	**95 %**	**99 %**	**97.5 %**	**95 %**	**99 %**	**97.5 %**	**95 %**
				Variance-Covariance					
2000	-0,0105	-0,0088	-0,0074	-0,0060	-0,0050	-0,0042	-0,0129	-0,0108	-0,0091
2001	-0,0120	-0,0101	-0,0085	-0,0072	-0,0061	-0,0051	-0,0221	-0,0186	-0,0157
2002	-0,0097	-0,0082	-0,0069	-0,0057	-0,0048	-0,004	-0,0331	-0,0278	-0,0234
2003	-0,0094	-0,0079	-0,0066	-0,0045	-0,0038	-0,0032	-0,0286	-0,0241	-0,0203
2004	-0,0153	-0,0128	-0,0108	-0,0084	-0,0070	-0,0059	-0,0156	-0,0132	-0,0111
2005	-0,0089	-0,0075	-0,0063	-0,0047	-0,0040	-0,0033	-0,0214	-0,018	-0,0151
2006	-0,0105	-0,0089	-0,0075	-0,0053	-0,0045	-0,0038	-0,0180	-0,0151	-0,0127
2007	-0,0078	-0,0066	-0,0055	-0,0041	-0,0035	-0,0029	-0,0252	-0,0212	-0,0179
2008	-0,0102	-0,0086	-0,0072	-0,0046	-0,0038	-0,0032	-0,0152	-0,0128	-0,0108
				Historical Simulation					
2000	-0.0105	-0.0076	-0.0063	-0.0057	-0.0049	-0.0044	-0.0106	-0.0088	-0.0078
2001	-0.0122	-0.0112	-0.0093	-0.0093	-0.0059	-0.0049	-0.0231	-0.0202	-0.0137
2002	-0.0111	-0.0094	-0.0077	-0.0054	-0.0044	-0.0039	-0.0492	-0.0283	-0.0198
2003	-0.0108	-0.0076	-0.0063	-0.0061	-0.0048	-0.0043	-0.0296	-0.0249	-0.0172
2004	-0.0108	-0.0086	-0.0078	-0.0076	-0.0051	-0.0044	-0.0257	-0.0178	-0.0172
2005	-0.0115	-0.0088	-0.0072	-0.0055	-0.0044	-0.0038	-0.0173	-0.0172	-0.0169
2006	-0.0093	-0.0079	-0.0071	-0.0048	-0.0045	-0.0034	-0.0117	-0.0088	-0.0086
2007	-0.0081	-0.0061	-0.0053	-0.0047	-0.0040	-0.0033	-0.0183	-0.0177	-0.0172
2008	-0.0081	-0.0064	-0.0048	-0.0042	-0.0035	-0.0025	-0.0192	-0.0186	-0.0181
				Bootstrapping					
2000	-0,0104	-0,0086	-0,0069	-0,0055	-0,0047	-0,0039	-0,0236	-0,0184	-0,0144
2001	-0,0107	-0,0084	-0,0068	-0,0055	-0,0046	-0,0038	-0,0217	-0,0179	-0,0155
2002	-0,0110	-0,0091	-0,0074	-0,0580	-0,0048	-0,0040	-0,0215	-0,0174	-0,0136
2003	-0,0110	-0,0085	-0,0070	-0,0057	-0,0047	-0,0041	-0,0228	-0,0185	-0,0156
2004	-0,0113	-0,0088	-0,0071	-0,0054	-0,0047	-0,0040	-0,0213	-0,0178	-0,0159
2005	-0,0110	-0,0087	-0,0071	-0,0060	-0,0047	-0,0039	-0,0229	-0,0178	-0,0160
2006	-0,0107	-0,0084	-0,0071	-0,0062	-0,0048	-0,0040	-0,0214	-0,0177	-0,0145
2007	-0,0104	-0,0090	-0,0071	-0,0057	-0,0047	-0,0039	-0,0222	-0,0183	-0,0157
2008	-0,0104	-0,0085	-0,0070	-0,0059	-0,0047	-0,0040	-0,0216	-0,0178	-0,0148
				Monte Carlo					
2000	-0,0118	-0,0096	-0,0079	-0,0064	-0,0052	-0,0043	-0,0245	-0,0199	-0,0163
2001	-0,0098	-0,0086	-0,0073	-0,0053	-0,0047	-0,0040	-0,0203	-0,0177	-0,0152
2002	-0,0106	-0,0082	-0,0078	-0,0058	-0,0045	-0,0042	-0,0220	-0,0169	-0,0161
2003	-0,0106	-0,0084	-0,0077	-0,0058	-0,0046	-0,0042	-0,0219	-0,0174	-0,0160
2004	-0,0088	-0,0079	-0,0070	-0,0048	-0,0043	-0,0038	-0,0182	-0,0164	-0,0146
2005	-0,0095	-0,0087	-0,0077	-0,0052	-0,0047	-0,0042	-0,0197	-0,0180	-0,0160
2006	-0,0094	-0,0073	-0,0062	-0,0051	-0,0040	-0,0034	-0,0195	-0,0150	-0,0128
2007	-0,0094	-0,0084	-0,0076	-0,0051	-0,0046	-0,0042	-0,0195	-0,0174	-0,0157
2008	-0,0087	-0,0082	-0,0078	-0,0047	-0,0045	-0,0042	-0,0180	-0,0170	-0,0161

The next technique we have used is considered as an improved version of the Historical Simulation. In fact, the application of the Bootstrapping allows us to create 50 samples of simulated returns, where each observation is obtained by random selection from the original sample of 2248 observations. Each new sample obtained by this procedure allows us to get an estimated VaR by the Historical Simulation. The estimates average, based on resampling the initial sample, represents the global VaR. Applying this procedure on a sample composed by three different currencies gives us the estimated Value-at-Risk. Results using the Bootstrapping technique are presented in the table II. The Japanese Yen remains, by far, the riskiest currency, independently of the confidence level and the time horizon. Monte Carlo simulation represents the last and most sophisticated technique used in the VaR estimation. By exploiting the return historical data, we will calculate the average and the standard deviation for each currency. Using these elements, we simulate the returns by doing random selection. Thus, based on the following procedure, Monte Carlo simulations of daily returns observed between 01/01/1999 and 31/12/2007 are presented in the table 3.

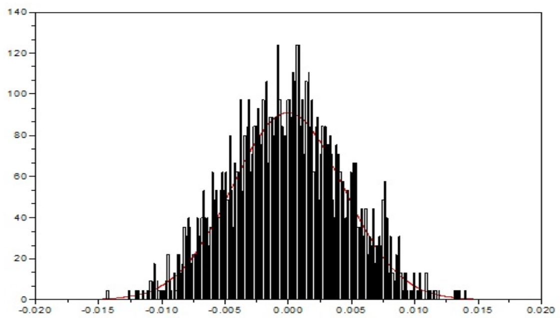
Table 3. Average and standard errors of daily returns (01/01/1999 - 31/12/2007)

	Average (%)	Standard Error (%)
US Dollar	-0.00461	0.4386
Euro	-0.01470	0.2337
Yen	-0.00998	0.9051

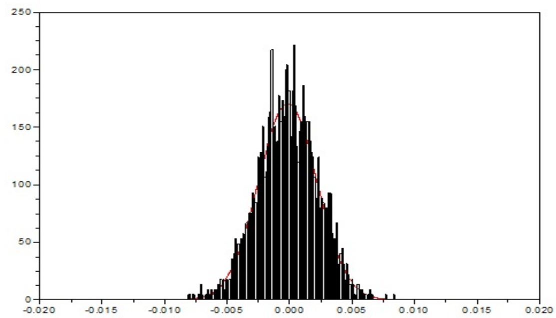
The figure 2 presents the simulated return for each currency, using the Monte Carlo method.

Figure 2. Monte Carlo simulation of exchange rates daily returns (2250 selections)

TND/US Dollar



TND/Euro



TND/Japanese Yen

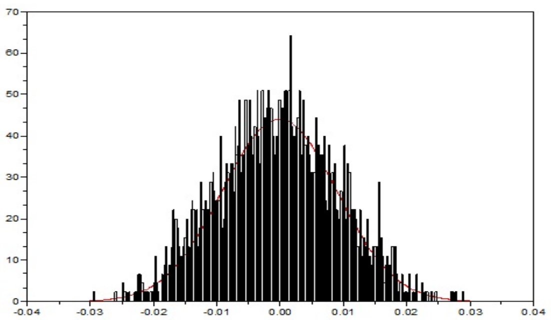
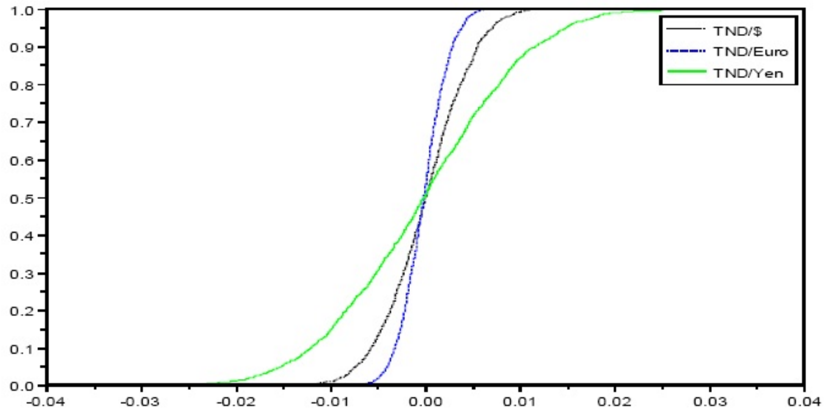


Figure 3 describes the cumulative distribution of simulated returns. We notice here that the stiffest slope of the cumulative distribution functions is the one relative to the Euro, which has the weakest standard deviation. On the other side, the weakest slope is observed for the Japanese Yen whose standard deviation is the highest.

Figure 3. Cumulative distribution of simulated returns using the Monte Carlo method



Once the daily returns of different exchange rates are calculated, we proceed to the calculation of the VaR relative to the Monte Carlo method. We will use the same methodology such as the Historical Simulation. Results, presented in table II, confirm the ones found using the previous techniques. In fact, we conclude that the relationship between the confidence level and the Value-at-Risk is positive. In the same way, the Euro seems to be the least risky currency, while the Japanese Yen is found to be the riskiest one.

3.2. Value-at-Risk associated to international currency portfolios

In this stage of the study, we will focus on the estimation of the Value-at-risk for different currency portfolios. As in the previous section, we consider three international currencies, i.e. the US Dollar, the Euro and The Japanese Yen. These currencies are the most commonly used in the Tunisian foreign transactions. So, the VaR will be calculated for four different portfolios, i.e. the Euro/US Dollar, the Euro/Japanese Yen, the US Dollar/Japanese Yen and the US Dollar/Euro/Japanese Yen. Recall that the techniques are the same we have used in the previous section. For a given observation window, portfolio return will be determined using the following equation:

$$R_{PF} = \sum_{i=1}^{N} W_i R_i \qquad (3)$$

Where R_{PF} is the portfolio return, R_i is the return associated to a currency i, N is the number of currencies in the portfolio, W_i is the weight associated to the currency i. As we can observe, the portfolio return is function of the weights assigned to the currency i and of its return. By referring to Cassidy and Gizky (1997), W_i is determined from the value of the exchange rate relative to the currency i in the last day of the historical observations. Accordingly, we will calculate the VaR relative to each currency portfolio at the first day of each year, by using a rolling window of 250 observations. Three confidence levels will be considered, namely 95%, 97,5% and 99%. With regard to the choice of the window of observations, we referred to the paper of Hendricks (1996), which stipulates that a window of 125 days is adequate to capture the short-run market movements, and that a window of 1250 days allows to obtain more precise VaR. Consequently, we choose to estimate the VaR based on an intermediate rolling window of 250 days. Value-at-Risk simulation results are presented in the table 4.

Table 4. Value-at-Risk estimation for various currency portfolio using different approaches

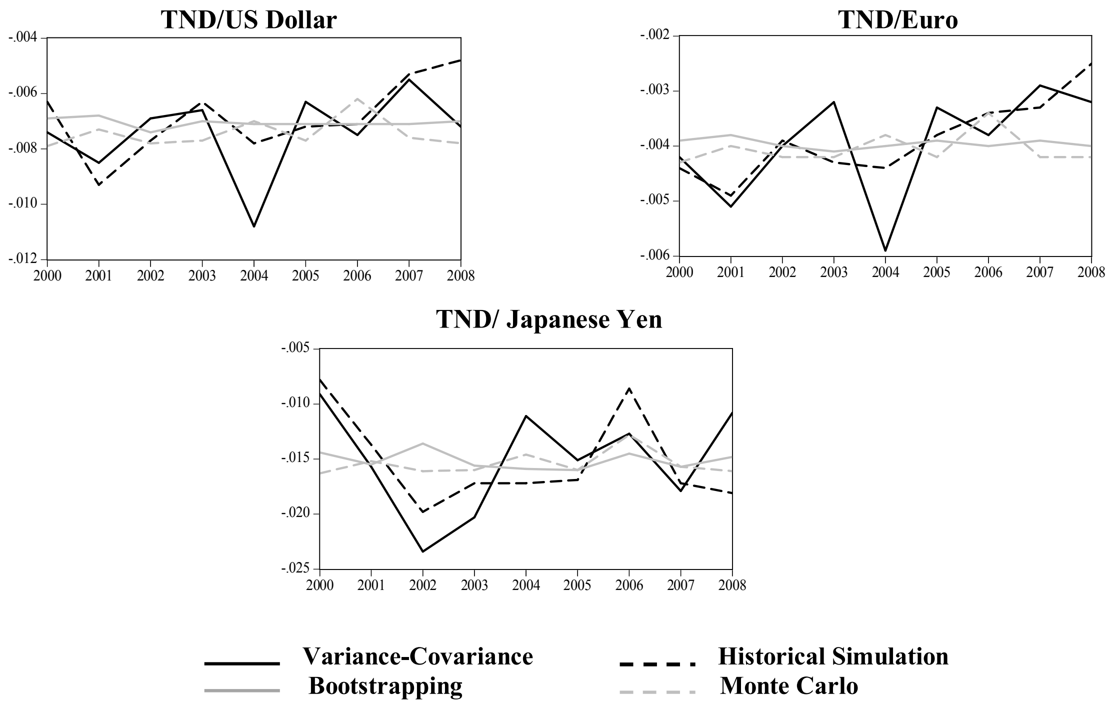
year	US Dollar/Euro			US Dollar/Yen			Euro/Yen			US Dollar/Euro/Yen		
	99%	97.5%	95%	99 %	97.5 %	95%	99 %	97.5 %	95 %	99%	97.5%	95%
Variance-Covariance												
2000	-0,0044	-0,0037	-0,0031	-0,0064	-0,0053	-0,0045	-0,0083	-0,0070	-0,0059	-0,0045	-0,0038	-0,0032
2001	-0,0057	-0,0048	-0,0041	-0,0153	-0,0129	-0,0109	-0,0146	-0,0123	-0,0104	-0,0116	-0,0098	-0,0082
2002	-0,0048	-0,0041	-0,0034	-0,0219	-0,0184	-0,0155	-0,0208	-0,0175	-0,0147	-0,0162	-0,0137	-0,0115
2003	-0,0025	-0,0021	-0,0017	-0,0146	-0,0123	-0,0104	-0,0156	-0,0131	-0,0111	-0,0099	-0,0083	-0,0070
2004	-0,0046	-0,0038	-0,0032	-0,0114	-0,0096	-0,0081	-0,0088	-0,0074	-0,0062	-0,0064	-0,0054	-0,0046
2005	-0,0036	-0,0030	-0,0025	-0,0094	-0,0079	-0,0066	-0,0135	-0,0113	-0,0095	-0,0072	-0,0061	-0,0051
2006	-0,0055	-0,0047	-0,0039	-0,0089	-0,0075	-0,0063	-0,0102	-0,0086	-0,0072	-0,0060	-0,0050	-0,0042
2007	-0,0029	-0,0025	-0,0021	-0,0161	-0,0135	-0,0114	-0,0140	-0,0118	-0,0099	-0,0109	-0,0092	-0,0078
2008	-0,0043	-0,0036	-0,0030	-0,0067	-0,0056	-0,0047	-0,0102	-0,0085	-0,0072	-0,0049	-0,0041	-0,0035
Historical Simulation												
2000	-0,0044	-0,0032	-0,0027	-0,0054	-0,0044	-0,0039	-0,0071	-0,0059	-0,0052	-0,0037	-0,0031	-0,0028
2001	-0,0063	-0,0052	-0,0044	-0,0160	-0,0141	-0,0100	-0,0158	-0,0132	-0,0092	-0,0124	-0,0106	-0,0075
2002	-0,0053	-0,0045	-0,0037	-0,0319	-0,0190	-0,0136	-0,0301	-0,0177	-0,0126	-0,0232	-0,0140	-0,0101
2003	-0,0025	-0,0016	-0,0012	-0,0152	-0,0127	-0,0088	-0,0164	-0,0137	-0,0097	-0,0102	-0,0087	-0,0060
2004	-0,0026	-0,0025	-0,0023	-0,0150	-0,0106	-0,0101	-0,0142	-0,0098	-0,0095	-0,0096	-0,0068	-0,0066
2005	-0,0048	-0,0036	-0,0029	-0,0074	-0,0074	-0,0074	-0,0114	-0,0110	-0,0107	-0,0057	-0,0057	-0,0057
2006	-0,0049	-0,0042	-0,0037	-0,0061	-0,0047	-0,0045	-0,0067	-0,0051	-0,0049	-0,0040	-0,0031	-0,0030
2007	-0,0029	-0,0021	-0,0019	-0,0126	-0,0115	-0,0110	-0,0098	-0,0095	-0,0094	-0,0082	-0,0076	-0,0074
2008	-0,0032	-0,0025	-0,0019	-0,0087	-0,0087	-0,0087	-0,0126	-0,0121	-0,0116	-0,0067	-0,0067	-0,0067
Bootstrapping												
2000	-0,0044	-0,0037	-0,0029	-0,0118	-0,0092	-0,0072	-0,0144	-0,0113	-0,0089	-0,0085	-0,0067	-0,0052
2001	-0,0050	-0,0040	-0,0032	-0,0149	-0,0122	-0,0104	-0,0140	-0,0116	-0,0099	-0,0111	0,0091	-0,0078
2002	-0,0298	-0,0045	-0,0036	-0,0154	-0,0125	-0,0098	-0,0336	-0,0115	-0,0091	-0,0246	0,0095	-0,0075
2003	-0,0027	-0,0020	-0,0016	-0,0118	-0,0095	-0,0080	-0,0129	-0,0105	-0,0089	-0,0078	0,0064	-0,0054
2004	-0,0037	-0,0027	-0,0021	-0,0129	-0,0106	-0,0093	-0,0118	-0,0098	-0,0088	-0,0083	0,0069	-0,0061
2005	-0,0044	-0,0035	-0,0029	-0,0099	-0,0077	-0,0070	-0,0147	-0,0115	-0,0102	-0,0077	0,0060	-0,0054
2006	-0,0057	-0,0045	-0,0038	-0,0105	-0,0087	-0,0071	-0,0122	-0,0101	-0,0082	-0,0071	0,0059	-0,0048
2007	-0,0039	-0,0034	-0,0026	-0,0154	-0,0129	-0,0108	-0,0118	-0,0097	-0,0084	-0,0101	0,0084	-0,0071
2008	-0,0040	-0,0033	-0,0027	-0,0097	-0,0080	-0,0066	-0,0143	-0,0118	-0,0098	-0,0074	0,0061	-0,0051
Monte Carlo												
2000	-0,0030	-0,0024	-0,0020	-0,0087	-0,0071	-0,0058	-0,0104	-0,0085	-0,0070	-0,0088	-0,0071	-0,0059
2001	-0,0027	-0,0023	-0,0020	-0,0097	-0,0084	-0,0072	-0,0094	-0,0082	-0,0071	-0,0103	-0,0090	-0,0077
2002	-0,0030	-0,0023	-0,0022	-0,0108	-0,0083	-0,0079	-0,0105	-0,0081	-0,0077	-0,0118	-0,0091	-0,0086
2003	-0,0016	-0,0013	-0,0012	-0,0076	-0,0060	-0,0055	-0,0086	-0,0068	-0,0062	-0,0075	-0,0060	-0,0055
2004	-0,0017	-0,0015	-0,0013	-0,0077	-0,0069	-0,0061	-0,0067	-0,0060	-0,0054	-0,0070	-0,0063	-0,0056
2005	-0,0024	-0,0022	-0,0019	-0,0062	-0,0056	-0,0050	-0,0083	-0,0075	-0,0067	-0,0066	-0,0061	-0,0054
2006	-0,0031	-0,0024	-0,0020	-0,0069	-0,0053	-0,0045	-0,0073	-0,0056	-0,0048	-0,0065	-0,0050	-0,0043
2007	-0,0021	-0,0019	-0,0017	-0,0100	-0,0089	-0,0081	-0,0069	-0,0061	-0,0055	-0,0089	-0,0080	-0,0072
2008	-0,0020	-0,0019	-0,0018	-0,0061	-0,0057	-0,0054	-0,0078	-0,0074	-0,0070	-0,0062	-0,0058	-0,0055

As mentioned previously, one can note that the absolute value of the VaR associated to a portfolio is positively linked to the confidence level. This positive relationship, found in the case of VaR associated to one currency, can be explained by the rank of the VaR. The examination of this table shows that an investor holding a portfolio composed by three currencies will make less loss than another one holding the portfolios composed by the US Dollar and the Japanese Yen or by the Euro and the Japanese Yen, for different confidence levels. This statement remains valid for the various techniques used for the VaR calculation. For example, using the Variance-Covariance in 2000, the VaR associated to a portfolio composed by three currencies indicates that the investor can lose 0.0045 and 0.0032 of the portfolio current market value, for a confidence level of 99% and 95% respectively. However, if he holds portfolio composed by the Euro and the Japanese Yen, losses will be 0.0083 at 99% level and 0.0059 at 95% level. Thus, we can say that the decline of the VaR is primarily attached to the degree of portfolio diversification. By diversifying his currency portfolio and choosing the suitable weight assigned to each currency, the investor can guarantee an optimum portfolio with minimum risk. Using the bootstrap simulations, the same conclusions remain generally valid. In fact, the lowest VaR is associated to the portfolio composed by the US Dollar and the Euro. For example, in 2000, the VaR, at 99% confidence level, was about -0.0044. The portfolio containing three currencies is placed in the second position, with a VaR equal to -0.0085 for the same confidence level and the same year. When estimating the VaR using the Monte Carlo, the risk associated to the US Dollar/Euro portfolio is still the lowest for all the period, but the second lowest risk becomes the one relative to the portfolio composed by the US Dollar and the Japanese Yen, and not the one composed by the three currencies. Except the Monte Carlo simulations, the other techniques confirm that the two portfolios containing the Japanese Yen are considered as the riskiest portfolios. These results are important, since portfolios diversification seems to reduce the risk, but the most diversified portfolios did not come up in the first position, as it is the case of the US Dollar/Euro/Japanese Yen portfolio. We remark that the raise or the decline of the VaR associated to a currency portfolio is strongly related to the investor's attitude towards risk. Indeed, if the investor is risk-averse, the anticipation of risk (and the estimated VaR) becomes higher. Otherwise, it will be lower. To conclude, one can say that the Variance-Covariance method seems to be relatively simple to implement, given the existence of several initial assumptions. These assumptions, such as the prices stationarity, the linear relationship between prices and risk factors, the normality of market factors fluctuation, are so restrictive, and are generally not verified for financial time series. The non-parametric methods, as the Historical Simulation or the Monte Carlo simulation, are more robust. In fact, these methods are rather based on various scenarios to evaluate the portfolio and estimate the VaR, which is the distributional quintile of the portfolio gains or losses. These techniques differ only in the way used to specify scenarios.

3.3. Comparison of Value-at-Risk Methodologies at 95% confidence level

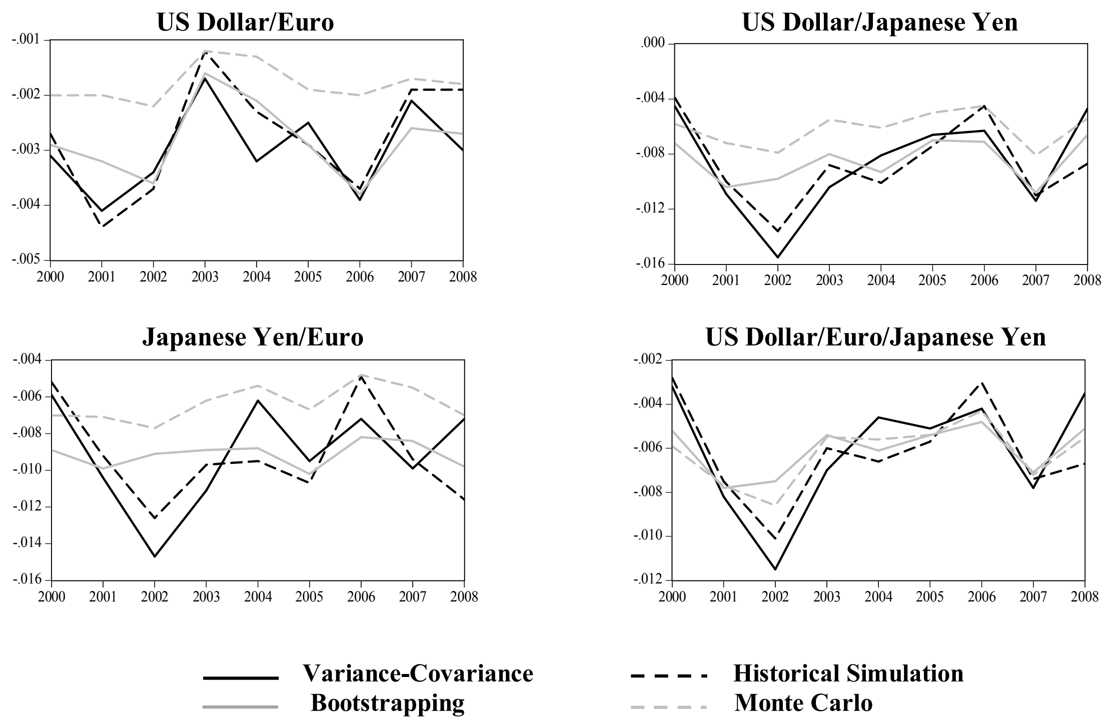
According to the previous investigation, it is clear that the VaR estimations may be occasionally different. The divergence of estimation results will induce some difficulty about the selection of the most adequate technique to calculate the risk, given that imprecise measures may cause inefficiencies for the investor. By applying the cited techniques to calculate the Value-at-Risk for the US Dollar, the Euro and the Japanese Yen, we conclude that this latter is the riskiest currency, since it represents the highest VaR. In addition, we note the existence of positive relationship between the confidence level and the estimated VaR associated to a currency or a portfolio currency. For the Historical Simulation, this relation is due to the VaR rank, which increases with the fall of the confidence level. For the Variance-Covariance method, the multiplier relating the VaR to the standard deviation is positively related to the confidence level, which raises the VaR. Finally, when calculating the Value-at-Risk associated to different currency portfolios, we find that the portfolio composed by the US Dollar and the Euro is the less risky portfolio and that the one composed by the US Dollar and the Japanese Yen is the riskiest. At the same time, the introduction of a third currency induces the reduction of risk. Diversification seems to be one of the factors reducing the risk associated to financial assets portfolios. Thus, we confirm the existence of the negative relationship between the number of financial assets and the estimated VaR. Figure 4 represents the VaR of every currency, using the different methods employed previously. The confidence level and the window of observations are set to be, respectively, 95% and 250 days.

Figure 4. Estimated VaR for different currencies at 95 % confidence level



Based on this figure, we notice the existence of strong similarity of the curve of the VaR relative to the three currencies, using the Bootstrapping and the Monte Carlo simulations. We also note that some resemblance between the VaR calculated using the Variance-Covariance and the Historical Simulation is observed. With regard to the currencies portfolios, we conclude that the Value-at-Risk at 95% confidence level varies depending on the simulation technique. This result confirms the existence of some difficulties on behind the investor when choosing the appropriate method estimating the precise Value-at-Risk. To illustrate the difference of results according to the techniques, we build the figure 5, representing the VaR of different currency portfolios at 95% confidence level, for an observation window of 250 days.

Figure 5. Estimted VaR for different currency portfolios at 95 % confidence level



The question which arises in this stage is: which methods is the most appropriate to measure the risk associated to currency or currency portfolio?

3.4. Validation of VaR forecasting methods

In this section, we attempt to apply some tests in order to choose the most appropriate technique allowing us to calculate precisely the VaR.

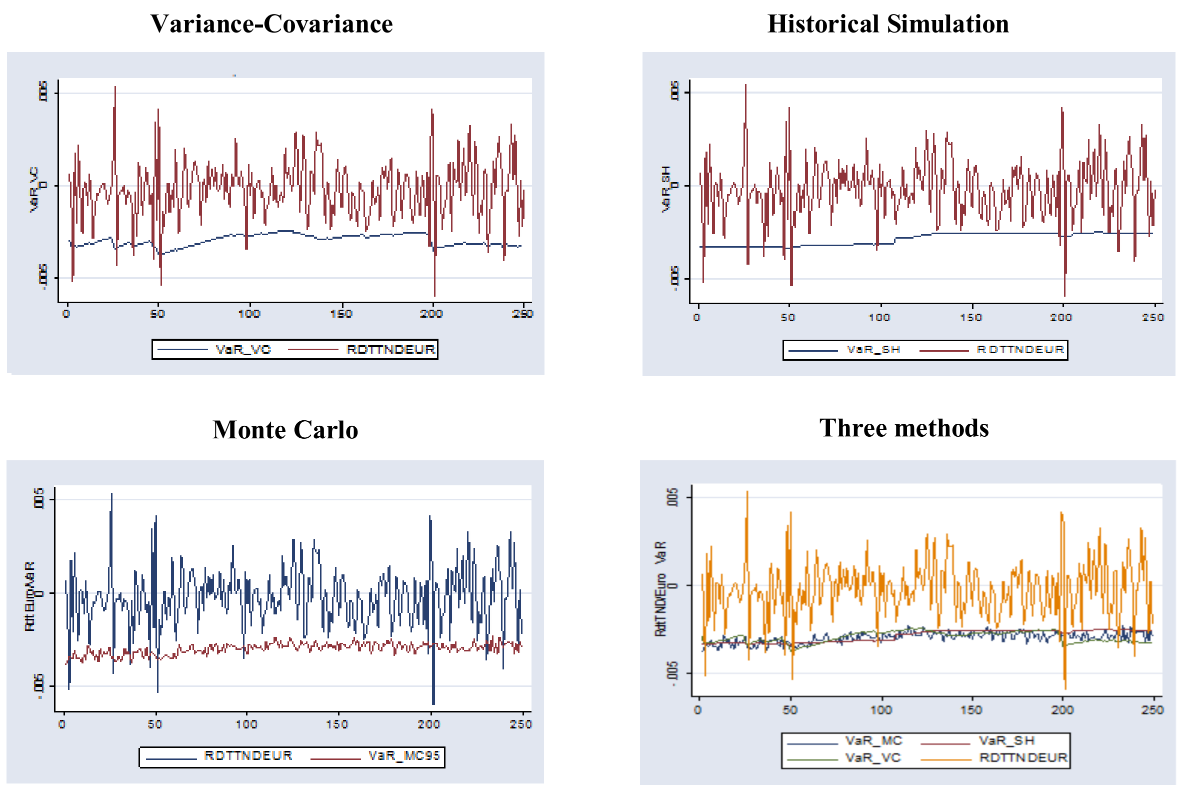
a. Comparison between the observed return and the simulated VaR

To validate the methods of the Value-at-Risk calculation and forecasting, we decompose the whole sample into two sub-samples. The first is qualified as the estimation sub-sample, with a size of T-N. The second is considered as a forecasting sub-sample. Its size is set to be N. This decomposition allows us to obtain the forecasting sequence of the VaR. The next step is to compare these values to the historical returns, and to make conclusions about the consistency of each method. Thus, starting from the T-N observation relative to the first sub-sample, we make a forecasting of the first observation relative the second sub-sample, which allows us to obtain the forecasted VaR for the date T-N+1. The reproduction of this procedure gives us a sequence of N forecasts. Methodologically, we proceeded as follows:

- **The Variance-Covariance:** In the case of the GARCH model, instead of estimating successively 250 models (corresponding to the number of days in 2007), we constructed a set of forecasts by estimating all the model parameters based on 80% of observations (2000 observations for the period from 01/01/1999 to 31/12/2006). Then, we determine a sequence of conditional variances for the remaining sample (250 observations, relative to 2007).
- **The Historical Simulation:** In this case, the VaR is determined starting from a sliding window of 250 observations, by taking the 13th lowest return among the 250 most recent observations.
- **The Monte Carlo:** To estimate the Value-at-Risk, the following steps were undertaken. Firstly, we calculate 250 daily averages and 250 daily standard deviations of return, based on sliding window over the period 2006-2007. Secondly, we will simulate a sequence of 250 returns for every day in 2007. Then, returns will be classified by ascending order. The final step consists in selecting the 13th lowest value, corresponding to the Value-at-Risk at 95%.

Figure 6 provides both the observed return and the forecasted VaR associated to the Euro at confidence level of 95%, obtained from a sample of 250 observations.

Figure 6. VaR for the TND/Euro exchange rate estimated by different methods 95% confidence level



Based on this figure, one can note that for some points, the observed return takes place below the calculated VaR, which weakens the forecasting method.

b. The backtesting techniques

The VaR violations

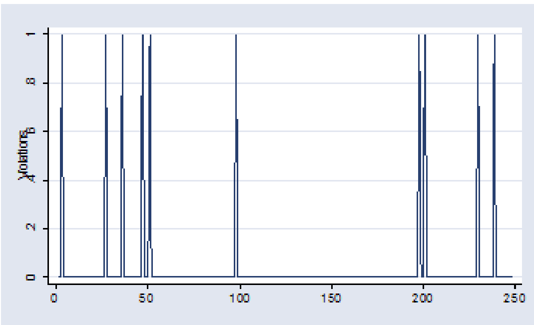
Several tests were proposed to evaluate the validity of the VaR measures and forecasts. The majority of these validation tests are based on the occurrence of the VaR violations (Campbell, 2006). Violations may be defined as a situation in which we observe a loss more important (in absolute value) than the simulated VaR. In practice, we often define an indicator variable, noted $I_t(\alpha)$, associated to the occurrence of a violation:

$$I_t(\alpha) = \begin{cases} 1 \;\; if \;\; R_t < VaR_{t/t-1}(\alpha) \\ \quad 0 \;\; otherwise \end{cases} \qquad (4)$$

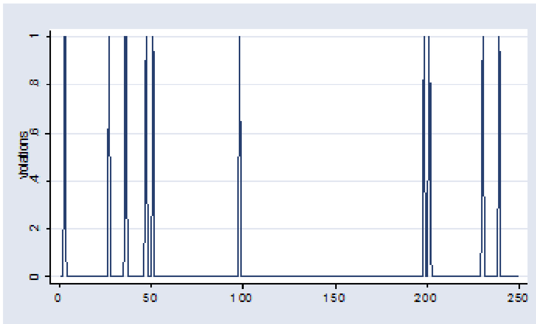
By comparing the observed returns to the VaR forecasted via different methods, we identified a sequence of violations, allowing us to define periods for which we find a loss more important, in absolute value, than the forecasted VaR. Figure 7 summarizes the comparison between the anticipated VaR and the losses and gains in 2007. The X-axis indicates the VaR violations points, for which we detect a loss more important than the calculated VaR.

Figure 7. VaR viloaltions for the TND/Euro in 2007

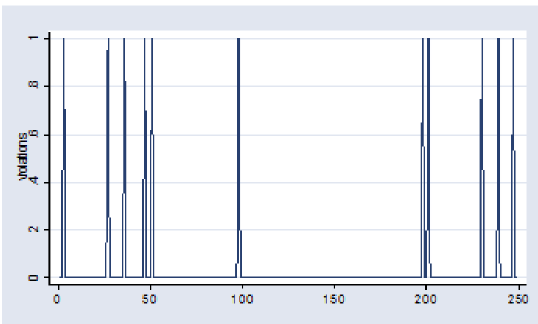
Monte Carlo



Variance-Covariance



Historical Simulation



Based on this graph, one can advance that the number of exceptions is almost identical for the three methods of VaR forecasting, which represents a good indicator. Likewise, the occurrence of violations, used for the validation tests, is generally observed in the same dates, independently of the retained forecasting method.

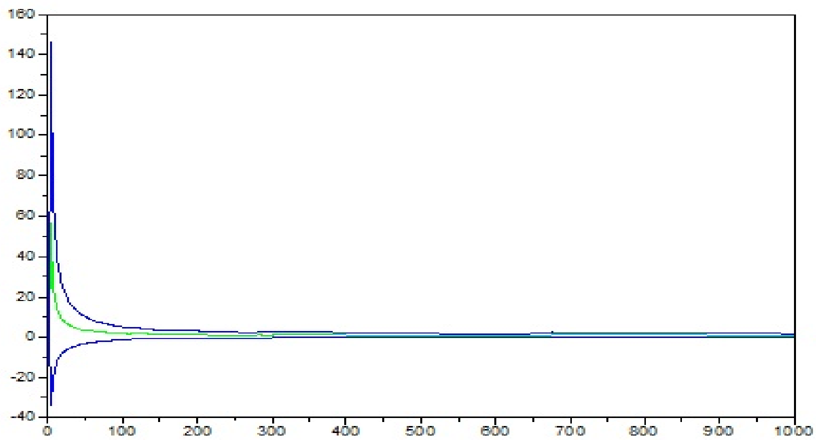
Table 5. Comparison between the estimated VaR and the observed loss and gain in 2007

Dates of violations		Historical Simulation	Variance-Covariance	Monte Carlo	
Order number	Date of occurence			250 selections	500 selections
3	04/01/2007	+	+	+	+
27	08/02/2007	+	+	+	+
36	21/02/2007	+	+	+	+
47	08/03/2007	+	+	+	+
51	14/03/2007	+	+	+	+
98	24/05/2007	+	+	+	+
198	16/10/2007	+	+	+	+
201	19/10/2007	+	+	+	+
230	30/11/2007	+	+	+	+
239	13/12/2007	+	+	+	+
247	27/12/2007	+	-	-	-
Number of violations		11	10	10	10

(+) denotes the existence of violation in this date, (-) denotes the absence of violation.

Thus, as shown in the table 5, the three methods of simulation give us the same empirical repartition function of violations. In fact, with the exception of the 247th day, the dates of violations are the same for the three methods. Furthermore, the number of violations for the Monte Carlo has not changed, whether we use 250 or 500 selections. This may reveal the robustness of our forecasting results. The figure 8, drawn by varying the number of simulation, demonstrates the high convergence of the Monte Carlo method, shown by the narrowness of the VaR confidence level starting from the 250th observation.

Figure 8. VaR confidence level at 95%, estimated by the Monte Carlo method (TND/Euro)



The expected shortfall

In the case of violation, it may be insufficient to use only an indicator variable taking the value one as information. Various measures were proposed to take into account the extent of the loss beyond the calculated VaR, in the case of the violations. An obvious measure, named the expected shortfall, is defined as the expected loss in the case of violation, and represented by the average of extreme losses. The values of the expected shortfall, calculated in our case, are presented in the table 6.

As shown in the table, it seems that the three approaches underestimate the Value-at-Risk. The lowest loss is found using the Variance-Covariance method, then the Monte Carlo and finally the Historical Simulation.

Table 6. Expected shortfall for different methods

Methods	Historical Simulation	Variance-Covariance	Monte Carlo
Expected shortfall	-0.01289	-0.01048	-0.01227

Evaluation tests based on the relative size and the variability

These tests consist in determining if a given VaR simulation method provides estimations with higher risk than the other methods, by evaluating the relative size and the variability of the Value-at-Risk. This evaluation is particularly based on 2 tests proposed by Hendr icks (1996), i.e. the Mean Relative Bias (MRB) and the Root Mean Squared Relative Bias (RMSRB).

The MRB statistics is calculated to capture the difference in extent that the different approaches produce identical average size estimations. It is given by:

$$MRB_i = \frac{1}{T}\sum_{t=1}^{T}\frac{VaR_{it} - \overline{VaR_t}}{\overline{VaR_t}} \qquad (5)$$

where VaR_{it} denotes the VaR relative to method *i* in year *t*, $\overline{VaR_t}$ is the average of VaRs calculated by all methods in year *t*, *T* represents the time horizon.

By calculating this statistics associated to the three methods used to calculate the VaR relative to the Euro (the Variance-Covariance, the Historical Simulation and the Monte Carlo), we obtain results presented in table 7.

Table 7. Mean Relative Bias statistics for different methods

Methods	Historical Simulation	Variance-Covariance	Monte Carlo
MRB	-0,02062	0,00663	0,01399

The Mean Relative Bias associated to the Variance-Covariance method is equal to 0.663%. This means that the MRB is superior by about 0.663% to the VaR average, which induces an overestimation of the estimated risk. This overestimation of risk may be explained by the fact that the Variance-Covariance method is essentially based on the return normality assumption. This assumption may induce some errors by overestimating the risk, given the leptokurtic characteristic of return distribution. By applying the same test for the Monte Carlo, we find that the VaR overestimation reaches 1.39%, exceeding the one relative to the Variance-Covariance. The MRB relative to the Historical Simulation demonstrates the existence of the VaR underestimation, superior in absolute value of the two previous methods. Consequently, the MRB test allows us to conclude that the Historical Simulation produce less reliable results, when compared to the other simulation methods, since it underestimates considerably the extent of the risk. Thus, this approach is considered as conservative approach. The Variance-Covariance approach is more effective than the Monte Carlo, given that it produces the lowest overestimation.

The second test we have the intention to use is the Root Mean Squared Relative Bias (RMSRB). This test offers information about the extent of overestimation or underestimation relative to each method. It is calculated as follows:

$$RMSRB_i = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{VaR_{it} - \overline{VaR_t}}{\overline{VaR_t}}\right)^2} \qquad (6)$$

Results of the RMSRB test, presented in the table 8, show that the lowest dispersion around the average VaR is captured by the Monte Carlo method, which seems to be the most precise method. The Historical Simulation and the Variance-Covariance methods are, respectively, in the second and the third places, according to this statistics.

Table 8. Root Mean Squared Relative Bias statistics for different methods

Method	Historical Simulation	Variance-Covariance	Monte Carlo
RMSRB	0,06665	0,07281	0,05904

In order to compare the three simulation methods, we present in the table 9, containing a summary of tests conducted in this section.

Table 9. Summary of validation tests

Validation tests	Historical Simulation		Variance-Covariance		Monte Carlo	
Number of violations	11	-	10	+	10	+
Expected Shortfall	-0.01289	-	-0.0104854	+	-0.012271	-
MRB	-0,02062	-	0,00663	+	0,01399	-
RMSRB	0,06665	-	0,07281	-	0,05904	+

(+) indicates the most appropriate method to predict risk using different validation tests, (-) indicates the contrary case.

This table shows that validation tests relative to the simulation methods did not provide us the same conclusions. However, for a given significance criterion, we can classify the various simulation methods. It seems that the Variance-Covariance is the most consistent method to estimate correctly the Value-at-Risk. The Monte Carlo approach comes in the second place, and finally we find the Historical Simulation.

4. Concluding Remarks

Throughout this paper, we tried to estimate the Value-at-Risk relative to the most representative currencies in the Tunisian exchange market (i.e. the Euro, The US Dollar and the Japanese Yen) and to different currency portfolios, composed by, at least, two currencies. To do this, we applied four different methods, namely the Variance-Covariance, the Historical Simulation, the Bootstrapping and the Monte Carlo. The next objective was to apply several validation tests, which allows us to choose the appropriate technique to estimate the currency risk. The VaR estimations are based on daily returns observed between 01/01/1999 and 31/12/2007. The rolling window is set to be 250 observations. Our results reveal that the Euro is the least risky currency. However, the Japanese Yen is considered as the most risky currency. With regard to the currency portfolios, we found that the one composed by the US Dollar and the Japanese Yen is the riskiest. At the same time, portfolio diversification seems to reduce risk, since the inclusion of the Euro in this portfolio shrinks risk. We also noticed the existence of a positive effect of the increase in the confidence level on the Value-at-Risk associated to both currency and currency portfolio.

The comparison of returns associated to the Euro in 2007 with the forecasted VAR at 95% confidence level provides us with the same distribution of violations empirical repartition. However, the position of the VAR compared to the returns at the dates of violations occurrence is not necessarily the same. Indeed, different tests associated to various VaR estimation methods did not give the same conclusions, which confirms the results highlighted by several authors, such as Beder (1995).

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