International Journal of Economics and Financial Issues
Vol. 2, No. 3, 2012, pp.272-280
ISSN: 2146-4138
www.econjournals.com

Oil and S&P 500 Markets: Evidence from the Nonlinear Model

Yen-Hsien Lee
Department of Finance, Chung Yuan Christian University,
200 Chung Pei Road. Chung Li 32023, Taiwan, R.O.C.
Phone: 886-3-2655711. E-mail: yh@cycu.edu.tw

Fang Hao, corresponding author
Graduate Institute of Assets and Property Management,
Hwa Hsia Institute of Technology, No. 111, Gong Jhuan Rd.,
Chung Ho, Taipei, Taiwan, 23568, R.O.C. Email: marian0307@yahoo.com.tw

ABSTRACT: This study begins by using a MTAR model to explore the asymmetric effects of error corrections between oil prices in the U.S.A and S&P 500 prices under different regimes. After confirming the lead/lag relationship between the S&P 500 and oil prices, we employ a STECM to analyze the short-run return dynamics when there are deviations from the equilibrium between the two variables. Our empirical evidence shows that an asymmetric co-integration relationship exists between the S&P 500 and oil prices. In addition, the results of the Granger causality test based on the TECM document the unidirectional relationship from the oil price to the S&P 500 price. Moreover, the short-run adjustments of the mean reversion to equilibrium follow the LSTECM. The contribution of this study might be in that the LSTECM-GARCH model is well suited to describing the short-run return dynamics of the disequilibrium between the oil prices and S&P 500 prices in the U.S.A.

Keywords: Threshold Co-integration Test; Threshold Error-Correction Model; Stock Market; Oil Market; STECM-GARCH Model
JEL Classifications: C13; C22; C32; G18; G10; Q42

1. Introduction

Petroleum occupies an increasing share of the consumption of global energy resources, having already reached 38.5%.[1] Because the supply of petroleum is unable to meet the demand for it, and the reduction in the scale of storage of petroleum goes beyond market expectation, global oil prices have continuously risen in recent years. Londarev and Balan (2005) found that the biggest obstacle in using the crude oil will not be its availability, at least in the short run, but the ever fast increasing price. The changes in oil prices have a direct bearing on the performance of the macroeconomics, and the stock index is the leading indicator that directly reflects the macroeconomic variables. That is, in theory at least, a continuous increase in oil price would result in a decrease in global stock prices. In addition, the U.S.A. ranks number one in the world in terms of its consumption of oil, and the S&P500 index that simultaneously takes price and volume into consideration can better reflect the change in the overall value in of the U.S. stock market. However, past empirical findings on the lead/lag relationship between the oil price and the S&P 500 index have not resulted in consistent conclusions.[2] Thus, there is much value in exploring how the change in the oil price in the U.S.A. influences the S&P 500 return.

Most of the previous studies analyzed the relationship between the oil price and macroeconomic variables, while few studies have investigated the relationship between the oil price and the stock price.

---

[1] The practical value is obtained from BP Statistical Review of World Energy 2002, and the predicted value from the International Energy Outlook 2002, EIA.

[2] Huang, et al., (1996) found that oil futures returns significantly influence the stock returns of individual oil companies, but that there is no obvious lead/ lag relationship between oil futures returns and the S&P 500 stock index return. By contrast, Sadorsky (1999) demonstrated that the change in oil price negatively and obviously influences the S&P 500 stock index return. Basher and Sadorkey (2006) documented that the change in oil price significantly impacts stock returns in emerging markets.

These latter studies such as Papapetrou (2001) and Huang et al., (2005) found that there is a long-term equilibrium relationship between oil and stock prices. Moreover, Enders and Granger (1998) and Enders and Siklos (2001) proposed that if two series exhibit an asymmetric effect, the results of traditional co-integration tests exhibit low power. The results of Jones and Kaul (1996), Sadorsky (1999) and Papapetrou (2001) also showed the empirical evidence of asymmetric impact of oil price shocks on stock markets. Error corrections could not occur in partial regimes resulting from small deviations when certain factors related to oil or stock prices such as trading costs are considered; that is, co-integration could only exist in some regimes. The process of error corrections can be asymmetric, which implies that the adjustment direction provides more momentum than another direction. We therefore first follow Enders and Granger (1998) and Caner and Hansen (2001) to adopt the Momentum-Threshold Autoregressive (M-TAR) model to distinguish the relationship between the oil price and the S&P 500 price under different regimes. We then use the Threshold Error-Correction Model (TECM) to catch the asymmetric-adjustment threshold co-integration relationship between the two variables under certain regimes. That is, we first examine whether there exists a "co-integration asymmetry" between S&P 500 price and oil price.

Many past studies used the linear model and found that the results as to whether the changes in oil prices significantly and negatively influence stock returns differ according to the evidence provided by different countries, industries and time series. Sadorsky (1999) used VAR to demonstrate that the change in oil prices had a larger negative and evident impact on stock returns than the interest rate and industry index, and the positive change in oil price could better explain the expected error variance of the stock return than the negative change in oil price. On average, the absolute value of a negative impulse is about 20% larger than that of a positive impulse, which makes the impact of the change in the oil prices on the stock returns asymmetric.[3] Papapetrou (2001) employed the VECM and found that the changes in the oil prices had a significant impact on the changes in the stock price, and that a positive increase in the oil prices would reduce the stock returns. However, these studies did not model the fact that small and large deviations from the equilibrium between the oil price and stock price may exhibit significantly different return dynamics. Huang et. al. (2005) applied the multivariate threshold model to investigate the impacts of an oil price change and its volatility on changes in industrial production and real stock returns. Their results suggested that the change in the oil price or its volatility has a limited impact on the economy if the change is below the threshold levels, while in the case where the change is above the threshold levels, it appeared that the change in the oil price better explains the macroeconomic variables than the volatility in the oil price. However, the impact of the change in the oil price on the stock return should not be applied in an abrupt transiting threshold model. Basher and Sadorkey (2006) used an international multi-factor model that allows for both unconditional and conditional risk factors to find strong evidence that a change in oil prices impacts stock returns in emerging markets. Also, Basher and Sadorsky (2006) demonstrate that the impact of the positive change in the oil price on stock return is more significant than the negative change in the oil price, which is consistently with the result of Sadorsky (1999). Thus, it is necessary to clarify whether there is an "asymmetry of the positive-and-negative impulse" of the oil prices on the S&P500 return. Furthermore, presenting the relationship between the stock returns and the change in the oil prices is probably non-linear, and it is suitable to use the error correction model to describe the adjustment behavior in terms of the deviations in the co-integration relationship between the oil price change and the stock return. Besides, there may be an asymmetry of small and large deviations. Thus, we use the smooth transition error correction model (STECM) to describe the non-linear dynamic correlation between the oil price and the stock price and the "asymmetry revision processes between the small and large deviations" from the equilibrium following the gradual transitions.

When a smooth transition model is used to forecast the S&P 500 and oil returns under highly frequent daily data, the estimates of the variance often does not converge as a result of residual heterogeneity. This study follows Chan and McAleer (2003) and Lee and Chiu (2010) to cause the residuals in the smooth transition model to follow the GARCH process. In addition, we add the threshold value to the Granger causal relationship between the two variables so as to consider the same

[3] Kaul and Seyhun (1990) deemed that the oil price negatively and significantly affects the stock return. The empirical results of Jones and Kaul (1996) indicated that the stock price is evidently influenced by the change in the oil price, but the extent of the impact differs from one country to another.

threshold regimes. The remainder of this paper is organized as follows. Section 2 introduces the methodology. Section 3 then describes the data and the methodology and analyzes the empirical findings. Finally, the conclusions are presented in Section 4.

2. Methodology

2.1 Threshold Co-integration and Threshold Granger-Causality Tests

By using the threshold co-integration of Enders and Granger (1998) and the first regression takes the form

$$y_t = \alpha + \beta x_t + \varepsilon_t , \tag{1}$$

where ε_t is the stochastic disturbance term. A regression of the form

$$\Delta\varepsilon_t = I_t \rho_1 \varepsilon_{t-1} + (1 - I_t)\rho_2 \varepsilon_{t-1} + \sum_{i=1}^{l} \gamma_i \Delta\varepsilon_{t-i} + \mu_t , \tag{2}$$

is then taken, where $\{\varepsilon_t\}$ contains the regression residuals from eq. (2), μ_t is an i.i.d. disturbance with zero mean, and I_t is the Heaviside indicator such that

$$I_t = \begin{cases} 1 & if \quad \varepsilon_{t-1} \ge 0 \\ 0 & if \quad \varepsilon_{t-1} < 0 \end{cases} \quad \text{or} \quad I_t = \begin{cases} 1 & if \quad \varepsilon_{t-1} \ge \tau \\ 0 & if \quad \varepsilon_{t-1} < \tau \end{cases} , \tag{3}$$

where τ is the threshold value. When $\varepsilon_{t-1} > \tau$, en. (2) becomes $\Delta\varepsilon_t = I_t \rho_1 \varepsilon_{t-1} + \sum_{i=1}^{l} \gamma_i \Delta\varepsilon_{t-i} + \mu_t$, otherwise $\Delta\varepsilon_t = \rho_2 \varepsilon_{t-1} + \sum_{i=1}^{l} \gamma_i \Delta\varepsilon_{t-i} + \mu_t$ is used. This representation not only captures the asymmetric effect, but can also test the long-run relationship between x_t and y_t. Enders and Granger (1998) and Caner and Hansen (2001) claim that it is also possible to allow the Heaviside indicator to depend on the change in ε_{t-1} (namely, $\Delta\varepsilon_{t-1}$) rather than on the level of ε_{t-1}. This leads to the Momentum-Threshold Autoregressive (M-TAR) model. The difference between TAR and M-TAR model is that there is a co-integration relationship while the change in ε_{t-1} or the level of ε_{t-1} is larger / smaller than a specific threshold. The Heaviside indicator of eq. (3) then becomes,

$$I_t = \begin{cases} 1 & if \quad \Delta\varepsilon_{t-1} \ge 0 \\ 0 & if \quad \Delta\varepsilon_{t-1} < 0 \end{cases} \quad \text{or} \quad I_t = \begin{cases} 1 & if \quad \Delta\varepsilon_{t-1} \ge \tau \\ 0 & if \quad \Delta\varepsilon_{t-1} < \tau \end{cases} \tag{4}$$

M-TAR implies that the adjustment mechanism of ε_t is dynamic, since the momentum of the series is greater in one direction than the other. Thus, for any large and smooth changes, M-TAR can explain the series more efficiently.

The transmissions are tested using the threshold error-correction model (TECM). The TECM can be expressed as $\Delta Y_{it} = \alpha + \gamma_1 Z_{t-1}^{+} + \gamma_2 Z_{t-1}^{-} + \sum_{i=1}^{k_1} \delta_i \Delta Y_{1t-i} + \sum_{i=1}^{k_2} \theta_i \Delta Y_{2t-i} + v_t$, where Y_i = (S&P 500 and Oil), $Z_{t-1}^{+} = I_t \hat{u}_{t-1}$, $Z_{t-1}^{-} = (1 - I_t)\hat{u}_{t-1}$ such that $I_t = 1$ if $u_{t-1} \ge \tau$, $I_t = 0$ if $u_{t-1} \le \tau$ and v_t is a white-noise disturbance. From the system, the Granger-causality tests are examined by testing whether all the coefficients of $\Delta Y_{1,t-i}$ or $\Delta Y_{2,t-i}$ jointly differ statistically from zero based on a standard F-test and/or whether the γ_j coefficients of the error-correction are also significant.[4]

[4] If $H_0 : \theta_1 = \theta_2 = \gamma_1 = 0$ or $\theta_1 = \theta_2 = \gamma_2 = 0$ is rejected, it implies that the oil price is granger-caused the S&P 500 price; otherwise, the oil price is not granger-caused the S&P500 price. If $H_0 : \delta_1 = \delta_2 = \gamma_1 = 0$ or $\delta_1 = \delta_2 = \gamma_2 = 0$ is rejected, it implies that the S&P500 price is granger-caused the oil price; otherwise, the S&P500 price is not granger-caused the oil price. Thus, there are four testable hypotheses in the Granger-causality tests. Furthermore, if the former is rejected, it also implies that there exists a co-integration

2.2 The Nonlinear Smooth Transition Autoregression Model

The STAR model can be represented as follows:

$$Y_t = \alpha_0 + \alpha_1' W_t + (\beta_0 + b_1' W_t) G(Z_{t-d};\gamma,\tau) + \nu_t,\ \nu_t \sim n.i.d.(0,\sigma^2) \tag{5}$$

Where $W_t = (Y_{i,t-1}, Y_{i,t-2}, \ldots, Y_{i,t-p})'$, $a_1 = (\alpha_1, \alpha_2, \ldots, \alpha_p)'$ and $b_1 = (\beta_1, \beta_2, \ldots, \beta_p)'$; $G(Z_{t-d};\gamma,\tau)$ is a continuous transition function with the transition variable Z_{t-d} and parameters (γ,τ) that provides a variety of nonlinear models. The parameter d is the delay parameter, γ is the smooth, or slope, parameter, and τ is the transition parameter. Eq. (5) is termed the logistic STAR (LSTAR) model if G has the form:

$$G(Z_{t-d};\gamma,\tau) = \left(1 + \exp\{-\gamma(Z_{t-d} - \tau)\}\right)^{-1},\ \gamma > 0 \tag{6}$$

This transition function is monotonically increasing in Z_{t-d}. It is worth noting that the slope parameter γ of G governs the transition speed from zero to unity, and the transition parameter τ determines the location of the transition. If G has the form:

$$G(Z_{t-d};\gamma,\tau) = 1 - \exp(-\gamma(Z_{t-d} - \tau)^2),\ \gamma > 0 \tag{7}$$

then eq. (5) is termed the exponential STR (ESTR) model. In particular, the parameters in eq. (7) change symmetrically about τ with Z_{t-d}.

The null hypothesis of linearity in eq. (5) is $H_0 : \gamma = 0$. Luukkonen, Saikkonen, and Terasvirta (1988) circumvented this problem via a third-order Taylor approximation to G about the null $\gamma = 0$. This approximation is written as:

$$y_t = \alpha_0 + \alpha_1' W_t + \psi_1' W_t Z_{t-d} + \psi_2' W_t Z_{t-d}^2 + \psi_3' W_t Z_{t-d}^3 + \xi_t \tag{8}$$

If the delay parameter d is assumed to be known, the linearity test is equivalent to the test of the hypothesis $H_0 : \psi_1 = \psi_2 = \psi_3 = 0$. Define an auxiliary regression,

$$\hat{u}_t = \alpha_0 + \alpha_1' W_t + \psi_1' W_t Z_{t-d} + \psi_2' W_t Z_{t-d}^2 + \psi_3' W_t Z_{t-d}^3 + \eta_t \tag{9}$$

where $\hat{u}_t$ is the residual obtained from the regression $Y_{ti,} = \alpha_0 + \alpha_1' W_t + u_t$ under the null hypothesis of linearity. The LM-type test of the linearity against the STAR model (including both the LSTAR and ESTAR models) is used to calculate the following statistic:

$$LM = \frac{(SSR_0 - SSR_1)/3m}{SSR_1/(T - 4m - 1)} \tag{10}$$

where SSR_0 and SSR_1 is the sum of the squared residuals $\hat{u}_t$ and η_t obtained from eq. (5) and (9). The statistic has an asymptotic F distribution with $3m$ and $T - 4m - 1$ degrees of freedom under the null hypothesis of linearity. A null hypothesis sequence is thus considered, i.e., $H_{01} : \psi_3 = 0$, $H_{02} : \psi_2 = 0 | \psi_3 = 0$ and $H_{03} : \psi_1 = 0 | \psi_2 = \psi_3 = 0$. When $\psi_3 \neq 0$ ($\psi_3 = 0$ and $\psi_2 \neq 0$). Therefore, the rejection of the null hypothesis H_{01} or the test results accept both H_{01} and H_{02} but reject H_{03} confirm the model to be a LSTAR model. Likewise, an ESTAR model can be selected if the test results accept H_{01} and reject H_{02}.

This study includes the error-correction item between the oil price and the S&P 500 price in the STAR-GARCH model to form the STECM-GARCH model. The STECM-GARCH model is constructed as follows:

$$Y_{1,t} = \left(\alpha_0 + \rho_1 Z_{t-1} + \sum_{j=1}^{p} \alpha_j Y_{i,t-j}\right) + \left(\beta_0 + \rho_2 Z_{t-1} + \sum_{j=1}^{p} \beta_j Y_{i,t-j}\right) G(Z_{t-d};\gamma,\tau) + \nu_t \tag{11}$$

when error term is larger than a threshold value. If the latter is rejected, it implies that there exists a co-integration when error term is smaller than a threshold value. Since Granger-causality tests are highly sensitive to lag length selection, this study uses the AIC criterion to determine the appropriate lag lengths and finds the lag lengths of both k_1 and k_2 to be equal to one ($k_1 = k_2 = 2$).

$$v_t|\Omega_{t-1} \sim N(0,h_t),\ h_t = \phi_0 + \phi_1\varepsilon_{t-1}^2 + \phi_2 h_{t-1} \quad (12)$$

where $G(Z_{t-d};\gamma,\tau)$ is a continuous transition function with the transition variable Z_{t-d} and parameters (γ,τ) that provides a variety of non-linear models. This study indicates that Z_{t-d} is an error correction term. In addition, h_t denotes the conditionally heterogeneous variance.

3. Data and Empirical Analysis

3.1 Data

The sample period extends from January 1, 1992 to November 7, 2006. Daily S&P 500 and West Texas Intermediate (WTI) oil transaction data were collected and transformed into daily returns, yielding 3,716 observations. The daily data were obtained from the Bloomberg.

3.2 Summary Statistics

Table 1 lists the descriptive statistics for the S&P 500 and oil returns. The average returns were found to be 0.0323 and 0.0003.[5] The JB statistics indicate that the distribution of the S&P 500 and oil returns has a fatter tail and a sharper peak than a normal distribution. The statistics also show that the S&P 500 and oil returns are negatively skewed, and the leptokurtosis implies that the distribution of returns has a fatter tail than the normal distribution. Figure 1 shows that the stock prices and oil prices may appear to be non-stationary and that both tend to move more or less together over time, a phenomenon that is later confirmed via a co-integration technique.

Table 1. Summary Statistics of returns for S&P 500 and Oil

Iterms	S&P 500	Oil
Mean	0.0323	0.0003
SD	1.0078	0.0225
Maximum	5.5744	0.1331
Minimum	-7.1127	-0.1612
Skewness	-0.1221	-0.2684
Kurtosis	7.1435***	6.1537***
Jarque-Bera Test	2666.7470***	1584.1130***

Notes: 1. SD denotes standard error.
2. Jarque-Bera Test denotes the Jarque-Bera normality test.
3. *,** and *** denotes rejection of the hypothesis at the 10%, 5% and 1% level

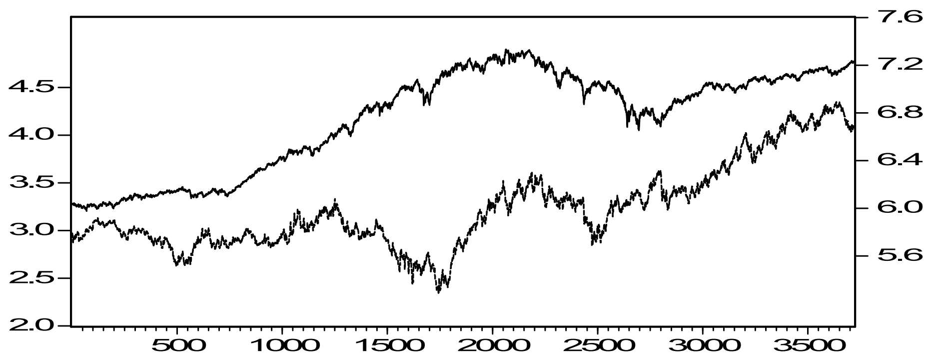

Figure 1. Up line is price of S&P 500 and down line is price of oil

[5] This study uses the Augmented Dickey-Fuller, Phillips and Perron and Kwiatkowski, et al. unit root tests on prices and their differentials with respect to the S&P 500 and oil. These tests are designed to indicate whether the S&P 500 and oil are non-stationary in terms of their levels and stationary in terms of their first differences. This study thus suggests that they are integrated of order one, i.e., I(1).

3.3. Threshold Co-integration Test

Table 2 shows that MTAR has better explanatory ability than TAR because the AIC and SBC values of MTAR are minimal. Since the null hypothesis of no co-integration and symmetry is rejected for the MTAR model over TAR model which exist a symmetric co-integration, an asymmetric co-integration relationship exists while error adjustment rather than error level between the S&P500 and WTI (oil price) is larger or smaller than a specific threshold. Moreover, this study further employs Granger-causality tests directly based on the TECM parameters to examine the lead/lag relationship between the S&P500 and oil prices. The unidirectional relationship from the oil price to the S&P 500 price is further confirmed, and is interpreted by the rejection of $\theta_1 = \theta_2 = \gamma_2 = 0$ for negative shocks of STAR. The above findings imply that, for the impact of changes in the oil price on the S&P500 returns, we should analyze their error correction behaviors as deviating from their co-integration relationship. Then, to explore the possible asymmetry between the small and large deviations from the equilibrium, we use the non-linearity test of linearity against the non-linear STECM model to examine the existence of asymmetry dynamic correlation between the S&P500 price and WTI.

Table 2. The Threshold Co-integration Test and Granger-Causality tests based on TECM

Co-integration Test	TAR Model	MTAR Model
T	2.4230*	4.6644***
F	1.6840	6.7225***
AIC	-700.65607	-705.6018
SBC	-688.2158	-693.1621
Granger-Causality tests	S&P 500	Oil
$H_0: \theta_1 = \theta_2 = \gamma_1 = 0$	1.3199[0.26601]	
$H_0: \theta_1 = \theta_2 = \gamma_2 = 0$	2.3824[0.0675]*	
$H_0: \delta_1 = \delta_2 = \gamma_1 = 0$		1.544[0.2009]
$H_0: \delta_1 = \delta_2 = \gamma_2 = 0$		1.8582[0.1344]

Notes: 1. *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively. P value are in [ ].
2. F and T denote the null hypothesis of no co-integration and symmetry.
3. Threshold Error-Correction Model: $\Delta Y_{it} = \alpha + \gamma_1 Z_{t-1}^{+} + \gamma_2 Z_{t-1}^{-} + \sum_{i=1}^{k_1} \delta_i \Delta Y_{1t-i} + \sum_{i=1}^{k_2} \theta_i \Delta Y_{2t-i} + v_t$ where $Y_i =$ (S&P 500, Oil), $Z_{t-1}^{+} = I_t \hat{u}_{t-1}$, $Z_{t-1}^{-} = (1-I_t)\hat{u}_{t-1}$ such that $I_t = 1$ if $u_{t-1} \geq 0$, $I_t = 0$ if $u_{t-1} \leq 0$ and v_t is a white-noise disturbance.

Table 3. Non-linearity Test and LSTECM Model vs. ESTECM Model Test

Panel A: Non-linearity Test						
d	1	2	3	4	5	6
H_0 F Stat	1.864**,a	1.859**	1.855**	1.851**	1.845**	1.838**

Panel B: LSTECM Model vs. ESTECM Model Test			
d	H_{01} F Stat	H_{02} F Stat	H_{03} F Stat
1	0.6958	1.9800*	2.9153**,a
2	0.6804	1.9814*	2.9148**
3	0.6637	1.9877*	2.9118**
4	0.6465	1.9963*	2.9092**
5	0.6304	2.0029*	2.9019**
6	0.6144	2.0059*	2.8937**

Note: 1. *, **, *** denote significantly at the 10%, 5% and 1% level, respectively.
2. a indicates the minimum p-value in determining the optimal value.
3. d is the optimal lag length for the transition variable Z_{t-d}.

The results of the LM test of linearity against the non-linear STAR model in Panel A of Table 3 indicate significant evidence of non-linearity in the returns of the S&P500. We estimate a range of values for $d_{(1 \leq d \leq 6)}$, where the *F* statistics with the minimum *p*-values or the maximum *F* statistics determine the optimal value for *d*. The results in Panel B of Table 3 show that H_{03} is significantly rejected for *d*=1, indicating that LSTAR is a more appropriate model.

The results of the oil to S&P500 returns under LSTECM in Table 4 shows the estimated value of γ to be a significantly positive number, implying a rapid transition from one regime to the other. The estimated results of the smooth transition function of the S&P500 returns are listed in equation (13). Figure 2 shows that the smooth transition function designed to capture the interactions between the noise traders and informed traders follows the logistic transition type.

$$G(Z_{t-d};\gamma,\tau)=\left(1+\exp\left[-\underset{(1.182)}{6.5025}\times\left(\Delta Z_{t-1}-\underset{(0.0001)}{0.0035}\right)\right]\right)^{-1} \tag{13}$$

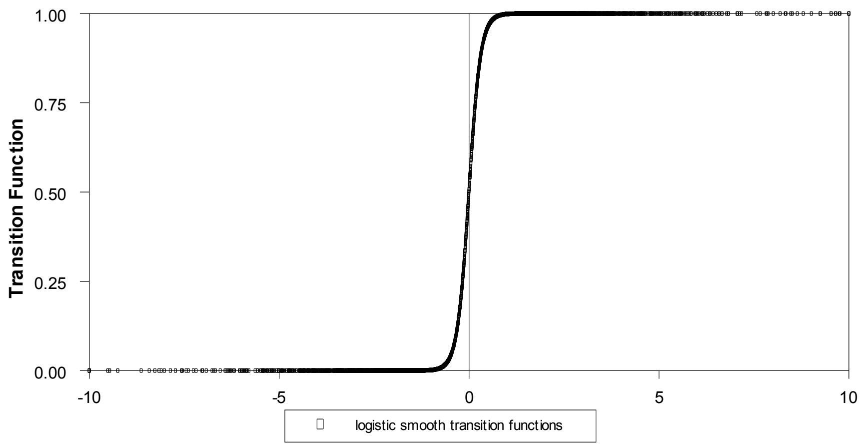

Figure 2. Logistic the smooth transition function

The smooth transition from the lower regime to the asymmetric upper regime is almost instantaneous at the threshold values of Z_{t-1}= 0.003 to 0.004.[7] The short-run adjustment speeds for the positively and negatively large deviations of Z_{t-1} are unequal, and the coefficients of Z_{t-1} for oil returns with their large negative and positive deviations are -0.00024 and -0.00008 respectively, showing that there are mean reversions to equilibrium as large negative and positive deviations ($(Z_{t-1}-\tau)\rightarrow-\infty$ and $(Z_{t-1}-\tau)\rightarrow+\infty$) occur. The results reveal that there is evidence of mean reversion behavior in large negative and positive deviations as a result of the co-movement between S&P 500 and oil returns in the U.S.A. In particular, there is a quick and evident mean reversion to equilibrium while large negative deviations exist.[8] More specifically, when there are large deviations from equilibrium, the arbitrageurs have more confidence in driving the market in the appropriate direction, and thus the S&P 500 price will quickly reverse to equilibrium. On the contrary, when there are small deviations from equilibrium, the

---

[7] The short-run dynamics of the S&P500 returns reach the lower regime as $(Z_{t-1}-\tau)\rightarrow-\infty$ and $F(Z_{t-1};\gamma,\tau)\rightarrow 0$, whereas they reach the upper regime as $(Z_{t-1}-\tau)\rightarrow+\infty$ and $F(Z_{t-1};\gamma,\tau)\rightarrow 1$.

[8] When the S&P 500 price is significantly higher than the oil price, say $(p_{S\&P500}/p_{oil})>>k$ (when large positive deviations exist), informed traders will tend to purchase relatively cheaper oil and the inducement to buy oil increases; thus, in the situation where $(p_{S\&P500}/p_{oil})>>k$, the adjustment speed of the S&P 500 price reversing to equilibrium is smaller than that of the oil price. On the contrary, when the S&P 500 price is lower than the oil price, say $(p_{S\&P500}/p_{oil})<<k$ (when large negative deviations exist), informed traders will tend to purchase the relatively cheaper S&P 500 index and the inducement to buy the S&P 500 index will thus be increased. Therefore, in the situation where $(p_{S\&P500}/p_{oil})<<k$, the adjustment speed of the S&P 500 price reversing to equilibrium is greater than that of the oil price.

arbitrageurs can be exposed to greater price risks, and thus there is price persistence of the S&P500 price.

The estimated parameters of ϕ_1 and ϕ_2 show that the impacts of current and previous news in the oil market on the volatility of S&P500 returns are all obvious. The results of the sign bias test and the joint test in Table 4 demonstrate that the positive and negative volatilities in the original residuals of S&P500 returns are not asymmetric. The S&P500 returns under LSTECM appear to be without any evidence of the ARCH effect and error autocorrelation, and the likelihood value is acceptable. These tests show that the fitted level of LSTECM-GARCH model is good.

Table 4. LSTECM-GARCH Model

Variable	Coefficient	Std
α_0	0.13133***	0.0055
ρ_1	-0.00024***	0.0000
α_1	-0.01708**	0.0081
α_2	-0.02301***	0.0076
α_3	0.01784***	0.0029
α_4	0.00075	0.0029
β_0	-0.06093***	0.0010
ρ_2	0.00016***	0.00000001
β_1	0.02357***	0.0007
β_2	0.01778***	0.0049
β_3	-0.01039***	0.0007
β_4	0.0002***	0.000087
γ	6.50248***	1.8882
τ	-0.00354***	0.0002
ϕ_0	0.00750***	0.0003
ϕ_1	0.07398***	0.0007
ϕ_2	0.92072***	0.0006
Likelihood	-2675.5990	
Diagnosis of Model		
Ljung-Box Q(10)	14.7582	
Ljung-Box Q(10)	8.2783	
ARCH(1)	0.6916	
ARCH(4)	2.8098	
Sign Bias Test	-0.0142	
Negative Sign Bias Test	-0.0136	
Positive Sign Bias Test	0.0316	
Joint Test of Sign and Size Bias Test	2.3139	

Notes: *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

4. Conclusion

This study first uses a M-TAR model to distinguish the asymmetric process of error corrections between the oil price in the U.S.A. and the S&P 500 price in different regimes. We then adopt a TECM to catch the threshold co-integration relationship between the two variables in these regimes, and employ the Granger-causality test to examine the lead/lag relationship between the S&P 500 and oil prices based on the TECM. Subsequently, in order to capture the different return dynamics of both of the small and large deviations from the co-movement between the S&P 500 and oil prices, this study applies a STECM to allow for a smooth transition for different types of return behavior under different regimes.

The empirical results indicate that the MTAR model has better explanatory power than the TAR, implying that an asymmetric co-integration relationship exists between the S&P 500 and oil prices. Then, based on the results of the Granger-causality test, we confirm the unidirectional relationship from the oil prices to the S&P 500 prices. The results of the LM test find significant evidence of non-linearity in the S&P500 returns, and the model choice of Terasvirta (1994) document that the short-run dynamic adjustments of the S&P 500 and oil prices follow the logistic transition function. In, addition, the mean reversion behavior occurs separately as evidenced by large negative and positive deviations from the co-movement. In particular, as large negative deviations exist, there is a larger and evident mean reversion to equilibrium. This study demonstrates that the GARCH model can really resolve the inefficiency problem due to the heterogeneity of the residual variances when the LSTECM is estimated. Finally, the LSTECM-GARCH model is well fitted to capture the short-run return dynamics of deviations from the equilibrium between the prices of oil and the S&P 500 in the U.S.A.

References

Basher, S.A., Sadorkey, P. (2006), *Oil Price Risk and Emerging Stock Markets*. Global Finance Journal, 17(2), 224-251.

Caner, M., Hansen, B. (2001), *Threshold Autoregression with a Unit Root.* Econometrica, 69(6), 1555-1596.

Chan, F., McAleer, M. (2003), *Estimating Smooth Transition Autoregressive Models with GARCH Errors in the Presence of Extreme Observations and Outliers*. Applied Financial Economics, 13(18), 581-592.

Enders, W., Granger, C.W.F. (1998), *Unit-Root Tests and Asymmetric Adjustment with an Example Using the Term Structure of Interest Rates.* Journal of Business Economics & Statistics, 16, 304-311.

Enders, W., Siklos, P.L. (2001), *Cointegration and threshold adjustment.* Journal of Business Economics & Statistics, 19, 166-176.

Huang, B. N., Hwang, M. J., Peng, H.P. (2005), *An Application of The Multivariate Threshold Model.* Energy Economics, 27(3), 455-476.

Huang, R. D., Masulis, R.W., Stoll, H. R. (1996), *Energy Shocks and Financial Markets.* Journal of Futures Markets, 16, 1−27.

Jones, C. M.,Kaul, G. (1996), *Oil and the Stock Markets.* Journal of Finance, 51 (2), 463-491.

Kaul, G., Seyhun, H.N. (1990), *Relative Price Variability, Real Shock, and the Stock Market.* Journal of Finance, 45, 479-496.

Lee, Y.H., Chiu, C.L. (2010), *Nonlinear Adjustment of Short-Term Deviations Impacts on the US Real Estate Market.* Applied Economics Letter, 17(6), 597- 603

Londarev, A., Balan, P. (2005), T*he Oil Market: the Important Phenomena of the Globalization of the World Economy?*. Journal of Economics, 9, 861-879.

Luukkonen, R., Saikkonen, P., Terasvirta, T. (1988), *Testing Linearity against Smooth Transition Autoregressive Models.* Biometrika, 75, 491–499.

Papapetrou, E. (2001), *Oil Price Shocks, Stock Market, Economic Activity and in Employment Greece.* Energy Economics, 23(5), 511-532.

Sadorsky, P. (1999), *Oil Price Shocks and Stock Market Activity*. Energy Economics, 21, 449−469.

Terasvirta, T. (1994), *Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models.* Journal of American Statistic Association, 89, 281-312.