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Long Memory Analysis: An Empirical Investigation

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ABSTRACT: This study is an attempt to review the theory and applications of autoregressive fractionally integrated moving average (ARFIMA) and fractionally integrated generalized autoregressive conditional heteroskedasticity (FIGARCH) models, mainly for the purpose of the description of the observed persistence in the mean and volatility of a time series. The long memory feature in FIGARCH models makes them a better candidate than other conditional heteroskedasticity models for modeling volatility in financial series. ARFIMA model also has a considerable capacity for modeling the return behavior of these time series. The daily data related to Tehran Stock Exchange (TSE) index was used for the purpose of this study. Considering the fact that the existence of conditional heteroskedasticity effects were confirmed in the stock return series, robust regression technique was used for estimation of different ARFIMA models. Furthermore, different GARCH-type models were also compared. The results of ARFIMA model are indicative of the absence of long memory in return series of the TSE index and the results from FIGARCH model show evidence of long memory in conditional variance of this series.

Keywords: stock market; long memory; ARFIMA; FIGARCH.
JEL Classification: C13; C59; G10; G17

1. Introduction

Today, with the increasing growth in financial markets, the changes in these markets have had great influences on the countries' economy (Chuang et al., 2012). Accordingly, achieving a sustainable economic growth requires equipment and optimal allocation of resources at the national level and the realization of this may not be possible without the help of financial markets because any recession or booms in the larger markets can impress not only the national economies but also the world economy (Eizaguirre et al., 2009). On the other hand, there is an interaction between economic growth and the markets development (Bumann et al., 2013).

Basically, due to the clarity of information, fluidity and also the existence of speculators and investors with different decisions, there have been complicated behaviors and many volatilities; this situation makes managing and controlling them even more difficult. One of the reasons is that the price of a property is dependent on its risk or conditional variance (Conrad et al., 2011). So, by modeling price volatilities of a property such as the shares, firstly, brokers can determine the appropriate wage rates, secondly the assets managements in the firms can also prevent the losses and damages caused by the high volatility in assets return and thirdly it makes it possible for the investors to avoid probable losses in the future by investigating how the current volatilities impact upon the future volatilities (Mun and Brook, 2012).

On the other hand, EMH built based on Random Walk Models has been one of the basic challenges facing financial analysts since according to this hypothesis, the complex behavior of the financial markets cannot be modeled and predicted. By finding the roots of this issue, it can be found that the basic assumptions of the EMH cannot take into account all the elements involved in the financial markets. The most important assumption is that markets do not have memory in the sense that yesterday's happenings will not influence today's events and that investors are risk averse and always carefully consider all the information in the market (Burton, 1987). However, the results of many applied studies are indicative of the fact that majority of the investors are under the influence of the happenings in the market and form their expectations of the future stock prices in keeping with their experiences. This fact points to the conclusion that markets have memory (Granger and Joyeux, 1980). In addition, one cannot make a confident assertion that all the investors in the financial markets behave logically but that they may do trading and favor risking without paying attention to the market information because always some investors may make a profit and some may sustain losses. Therefore, although based on the assumptions of EMH, financial markets are apparently unpredictable, the fact is that this is not the case (Sowell, 1992). Thus, the assumptions of the EMH were faced with such criticisms and "Fractal Market Hypothesis" (FMH) was proposed which was able to provide a more comprehensive analysis of the markets. This hypothesis, in fact, implied the existence of a market composed of numerous investors pursuing their goals with different investment horizons. The types of information important to each one of these investors is different. On this basis, as long as the market sustains its fractal structure, it will stay stable without considering time scale of the investment horizons. On the other hand, when all the investors in the market have the same time horizon, the stability of the market will be undermined because people will do trading drawing on similar market information (Baillie, 1996).

Although rejecting the EMH implies non-randomness and, as a consequence, predictability of different series, this result is achieved because EMH has been formed based on the Random Walk Model and consequently the existence of a linear structure in the behavior of the market (Brock et al., 1992). On the other hand, with regard to the financial markets which mainly have a complex and chaotic structure, FMH analyzes and assesses the issue of predictability from the perspective of nonlinear models (Vacha and Vosvrda, 2005). Although accepting the dependence of the behavior of a financial market on the FMH is a confirmation of the use of different non-linear models in consonance with the feature of long memory (e.g., Auto Regressive fractionally Integrated Moving Average or ARFIMA model) and also different types of neural network models (e.g., Nonlinear Neural network Auto Regressive or NNAR model as a dynamic model), it should also be noted that the fact that the inherent features of the mentioned markets (e.g., long memory) can improve the results of modeling should not be overlooked.

In this way, with the development of financial markets, an increase in the number of investors in these markets, and existence of a close relationship between these markets and macroeconomic variables during the last two decades, prediction of the financial assets prices behavior in the dynamic field of economy and capital markets has promoted into one of the most important issues in financial sciences so that, this issue guides the policy-makers, planners, researchers, and investors in exact and efficient assessment of assets pricing, optimal allocation of financial resources and performance evaluation of risk management. In line with this, many of the studies during the recent years have been focused on this issue which helps improve traditional linear and nonlinear models in forecasting and making more accurate predictions. Traditionally, Auto Regressive Conditional Heteroskedasticity models were used for modeling return volatilities because these models have theoretical, financial and economical foundations. At the same time, these models have not yielded favorable results in forecasting financial markets because they do not take into consideration the long memory feature which is one of the most important features in modeling and forecasting of the financial markets. Therefore, during the recent years, Auto Regressive Fractional Integrated Conditional Heteroskedasticity models which are based on long memory have been an appropriate response to eliminate the mentioned limitations and for this reason they are frequently used, see e.g. Kasman et al. (2009), Conrad et al. (2011), Aye et al. (2012) and Tan et al. (2012).

Hence, the present study attempts not only to use the different models based on long memory (ARFIMA) but also model Tehran Stock Exchange Price Index volatility by using the Conditional Heteroskedasticity models especially FIGARCH in forecasting the dependent variable. For these purposes, we will utilize the data related to daily time series from 25/3/2009 to 22/10/2011 (616 observations) out of which 555 observations (about 90% of the observations) were used for modeling and 60 observations for out-of-sample forecasting.

2. Methodology

After many important studies were conducted on the existence of Unite Root and Cointegration in time series starting in 1980, econometrics experts examined other types and subtypes of non-stationary and approximate persistence which explain the processes existing in many of the financial and economic time series. Today, different studies have been and are being conducted on these processes including "Fractional Brownian Motion" and "Fractional Integrated Process" and the "processes with long memory" (Lento, 2009). Hurst (1951) for the first time found out about the existence of processes with long memory in the field of hydrology. After that, in early 1980s econometricians such as Granger and Joyex (1980) and Hosking (1981) developed econometric models with long memory and specified the statistical properties of these models. During the last three decades, numerous theoretical and empirical studies have been done in this area. For example, (Mandelbrot, 1999; Lee et al. 2006; Onali and Goddard 2009)'s studies can be mentioned as among the most influential in this regard.

The concept of long memory includes a strong dependency between outlier observations in time series which, in fact, means that if a shock hits the market, the effect of this shock remains in the memory of the market and influences market activists' decisions; however, its effect will disappear after several periods of time (in the long term). Thus, considering the nature and the structure of financial markets such as the stock market, which are easily and quickly influenced by different shocks (economic, financial and political), it is possible to analyze the effects of these shocks and in a way determine the time of their disappearance by observing the behavior of these markets (Los and Yalamova, 2004). Meanwhile, the long memory will be used as a means of showing the memory of the market. By examining the long memory, the ground will also be prepared for improvement of financial data modeling.

2.1. ARFIMA Model

One of the most popular and most flexible models with the long memory is the ARFIMA model in which fractional cointegration degree (*d*) is representative of the long memory parameter because it is indicative of the features of the long memory in the time series of the related variable. After making sure about the existence of this feature in a time series using ACF[1] tests, classic R/S[2] analysis and also semi-parametric methods such as GPH[3], MRS[4], etc. (Xiu and Jin, 2007), the most important stage in the process of estimation of these models is the "fractional differencing" stage; economists, however, used first-time differencing in their empirical analyses due to its ease of use (in order to avoid the problems of spurious regression in non-stationary data and the difficulty of fractional differencing). Undoubtedly, this replacement (of first-time differencing with fractional differencing) leads to over- or under-differencing and consequently loss of some of the information in the time series (Huang, 2010). On the other hand, considering the fact that majority of the financial and economic time series are non-stationary and of the Differencing Stationary Process (DSP[5]) kind, in order to eliminate the problems related to over differencing and to obtain stationary data and get rid of the problems related to spurious regression, we can use Fractional Integration. Another interesting point is that Fractional Integration can assume different values, but a specific value for this parameter (*d*) is indicative the long memory feature. Two conditions need to be met for assuming these values. Firstly, if $-0.5< d< 0.5$, a series exhibits a stationary and invertible ARMA process with geometrically bounded autocorrelations. In other words, when $0 < d < 0.5$, the autocorrelation function decreases

[1] Auto Correlation Function
[2] Rescaled Range Analysis
[3] Geweke and Porter-Hudak
[4] Modified Rescaled Range
[5] And some are also trend stationary processes

hyperbolically and the related process is a stationary long memory process meaning that the autocorrelations decay to zero and will not be summable. When $-0.5 < d < 0$, the long memory process will be invoked. The medium-term memory shows that the related variable has been over-differenced and under such conditions, the reverse autocorrelation function decreases hyperbolically. The second condition is that a non-stationary is exhibited by the series if 0.5≤d<1 (Hosking, 1981). Finally, it is worth mentioning that spurious long memory should not be overlooked; in fact, spurious long memory happens as the result of structural breaks and inattention to nonlinear transformations (Kuswanto and Sibbertsen, 2008). Therefore, based on the concepts introduced, we can correctly model the behavior of a variable using this model. The general form of the model *ARFIMA(p,d,q)* is as follows:

$$\phi(L)(1-L)^d(y_t-\mu_t)=\theta(L)\varepsilon_t \qquad t=1,2,3,...,T \tag{1}$$

In which $\phi(L)$ is polynomial autocorrelation, $\theta(L)$ represents moving average polynomial, L is Lag Operator, μ_t is the mean of y_t. Besides, in this equation, $Z_t = y_t - \mu_t$ and is cointegrated with rank *d*. Features of Z are dependent on the *d* value. If $d < 0.5$, covariance of the model will be fixed and if $d > 0$, it will have long memory feature (Hosking, 1981). *p* and *q* are integers and *d* is a long memory parameter. $(1-L)^d$ represents a fractional differencing operator which is calculated using the following formula:

$$(1-L)^d = \sum_{j=0}^{\infty}\delta_j L_j = \sum_{j=0}^{\infty}\binom{d}{j}(-L)^j \tag{2}$$

In the above equation, it has been hypothesized that $\varepsilon_t \sim N(0,\sigma^2_{\varepsilon_t})$ and also ARMA section of the model are reversible (Aye et al., 2012).

2.2. Tests Used for Identifying the Long Memory Features

The most important step in estimating a model with long memory feature is examination of the existence of this feature in the return and volatility of the mentioned series. Identifying the existence of long memory feature via techniques such as ACF test, GPH test, etc. is possible; in the following section.

2.2.1. ACF Test

This method is one of the most popular tests identifying the long memory feature first introduced by Ding and Granger (1996). In this test, autocorrelation graph decreases from a certain value very slowly or hyperbolically (not exponentially). Therefore, such time series have long memory feature. It means that these processes cannot be produced by determined and specific AR and MA lags because in these series, AR and MA have infinite order (Xio and Jin, 2007).

2.2.2. The GPH Test (Spectral Density Method)

This method is based on Frequency Domain Analysis. In the framework of spectral and frequency domain analysis, the observed time series is weighted summation of the underlying time series which have different periodical patterns. Periodogram technique is used for differentiating between short and long memories. This technique was proposed by Gewek and Porter-Hudak (1983) and is often known as the GPH estimator. Overall, GPH statistics estimates the long memory parameter (*d*) which is based on the following periodogram regression:

$$\ln[I(w_j)] = \beta_0 + \beta_1 \ln\lfloor 4\sin(w_j/2)\rfloor + e_j \tag{3}$$

In which $w_j = 2\pi j/T$, $j = 1,2,...,n$ and e_j represent residuals of the model and w_j refers to Fourier Frequency Transformation ($n=\sqrt{T}$). Finally, $I(w_j)$ is a simple periodogram which is defined as follows:

$$I(w_j) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T}\varepsilon_t e^{-w_j t}\right| \tag{4}$$

Thus, the GPH statistic equals $-\hat{\beta}_1$.

2.3. Different Types of ARCH Models

Auto Regressive Conditional Heteroskedasticity (ARCH) models first proposed by Engel (1982) later on expanded by Borlerslev (1986) include the kind of models that are used for explaining the volatilities of a time series. Following that different types of ARCH models were introduced. They are divided into two groups: Linear (IGARCH and GARCH) and nonlinear models (EGARCH, TGARCH, PGARCH, FIGARCH, etc.).

2.3.1. Linear GARCH Models

Borlerslev (1986) started introducing the generalized model of ARCH, i.e., GARCH model based on Engel's ARCH model. The distinguishing factor between these two models is the existence of variance lags in the conditional variance equation. In fact, GARCH model has a similar structure to ARMA. Stipulated forms of this model include:

$$\begin{aligned} M_t &= \mu_t + \varepsilon_t \\ \varepsilon_t &= z_t \sqrt{h_t} \ , \quad z_t \sim N(0,1) \end{aligned} \tag{5}$$

$$\begin{aligned} h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \\ h_t &= \sigma_t^2 \end{aligned} \tag{6}$$

Equation (5) above is a mean equation which includes two sections; one of them is μ_t, which should be an appropriate structure for explaining mean equation, and the other is ε_t, which is indicative of residuals in the model above which has heteroskedasticity variance and is consisted of two normal elements (z_t and conditional standard deviation ($\sqrt{h_t}$)). As a matter of fact, h_t is a conditional variance equation that is estimated along with the mean equation to eliminate the problems related to the heteroskedasticity variance ε_t. In the equation (6), ω is the average of σ_t^2, the ε_{t-1}^2 coefficient indicates the effects of ARCH and h_{t-1} coefficient represents the effects of GARCH (Chang et al., 2009). One of the most important features of this model is the existence of temporary shocks imposed on the time series under investigation (Kittiakarasakun and Tse, 2011).

Furthermore, the results of Engel and Borlerslev's (1986) studies show that in some of the cases the GARCH equation mentioned above has a unit root. It means that, for example, in GARCH(1,1), the $\alpha_1 + \beta_1$ value is very close to one. In this case, the GARCH model is cointegrated and is called IGARCH. In these models, if there is a shock to the time series under investigation, it will have lasting effects and become noticeable in the long term (Poon and Granger, 2003).

2.3.2. Nonlinear GARCH Models or the FIGARCH Model

FIGARCH model was first proposed by Baillie (1996). In this model, a variable has been defined as fraction differencing, which ranges from zero and one. A General form of the FIGARCH(p,d,q) is as follows:

$$(1-L)^d \Phi(L)\varepsilon_t^2 = \omega + B(L)\upsilon_t \tag{7}$$

In equation (7), $\Phi(L)$ is the function of appropriate lag (q), $B(L)$ is the function of appropriate lag (p), L is the lag operator, and d represents fraction differencing parameter. If *d=0,* the FIGARCH model will turn into GARCH, and if *d=1*, it will turn into IGARCH. It should be noted that in these models, the effects of the shocks are neither lasting as in IGARCH models nor temporary as in GARCH models; the effects are between these two extremes meaning that the effects of the shocks will decrease at a hyperbolic rate.

2.4. Criteria for Comparing Forecasting Performance

On the whole, MSE and RMSE criteria are among the most frequently used criteria for comparing forecasting accuracy of the models among other criteria for fitting the accuracy of prediction. In this study, we used the MSE criterion for comparing forecasting accuracy of the models because this criterion has important features among which is taking account of the outlying data in comparing forecasting accuracy of the models. Besides, this criterion has a higher accuracy as against RMSE which shows the error differences as lower (Swanson et al., 2011).

$$MSE = \frac{\sum(\hat{y}_t - y_t)^2}{n} = \frac{SSR}{n} \qquad (8)$$

3. Results Analysis

For the purpose of this study, we used daily data of the Tehran Stock Exchange (TSE) Index from 2009/25/03 to 2011/22/10. It should also be mentioned that the acronyms of the variables used in this study include: TEDPIX (Tehran Exchange Dividend Price Index) and DLTED, showing the difference of the logarithm (return) of the Dividend Price Index.

3.1. Descriptive Analysis of the Data

Considering the importance of the utilized data in this study, before modeling the mentioned index, a descriptive statistics related to the data will be analyzed first (see Table 1 for details):

Table 1. Descriptive Statistics

Criterions	Accounting Value	Criterions	Accounting Value
observations	616	Jarque- Bra	9953.99 (0.000)
Mean	0.00193	Box- Ljung Q(10)	108.81 (0.000)
S.D	0.00797	McLeod-Li $Q^2(10)$	241.25(0.000)
Skewness	2.2684	ARCH (10)	8.9832 (0.000)
Kurtosis	22.1799		

With a brief look at the above table, it can be found that the mean of time series return in Tehran Exchange Return in the period under investigation is 0.00193 and its standard deviation is 0.00797. By comparing these two, it can be realized that this time series has experience a high level of volatility during this period. The Jarque-Bera test indicates non-normal distribution of this time series. Besides, the kurtosis statistics also indicate that the distribution of the mentioned time series is fat tail. Observing the Liang-Box statistics (with ten lags), can find, the null hypothesis about the lack of a serial correlation between the terms of the time series be rejected. The McLeod-Lee statistics also reject the null hypothesis about the lack of Serial correlation between square of the time series return) which is, in fact, expressive of the existence of nonlinear effects in this time series. It should be mentioned that the results of Engel's test were consistent with McLeod-Lee's test and confirmed the hypothesis about conditional variance of the time series return.

3.2. Stationary Test

As the next step, stationary of the DLTED series (done to prevent creation of a spurious regression) will be assessed using different tests (see Table 2 for the results).

Table 2. The Results Related to Stationary of the Stock Return Series

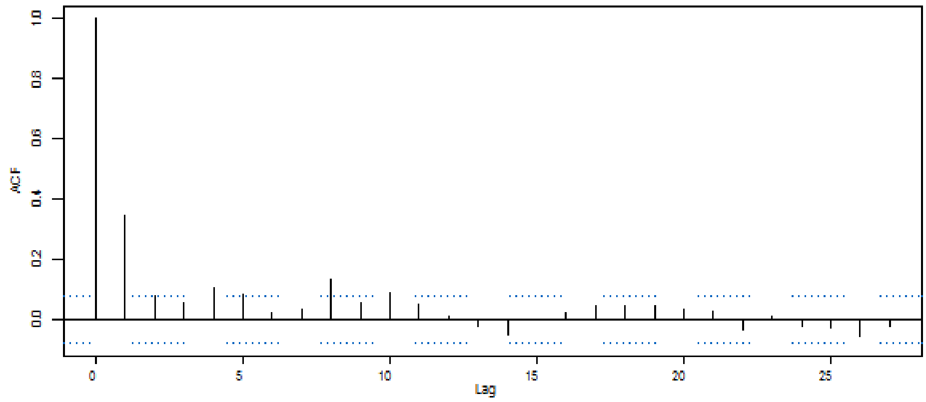
Test	Critical Stat.	Accounting Value	Result
ADF[6]	-1.9413	-16.586	Stationary
ERS[7]	3.2600	0.9403	Non-Stationary
PP[8]	-1.9413	-17.543	Stationary
KPSS[9]	0.4630	0.590	Non-Stationary

If the long memory feature does not exist, it is expected that the series becomes stationary by first differencing, but the results of first differencing show that stock return series is stationary in ADF and PP tests while in the KPSS and also ERS test the results are indicative of non-stationary of the series (see Table 2 for the results). Such conditions might have been caused by the long memory feature in this series. For this reason, the long memory feature in the stock return series (by fractional differencing series) was further analyzed by the researchers. Besides, interpreting the Autocorrelation plot can also help to find if there is long memory in the stock return series; as shown in Fig. 1 below,

6 Augmented Dickey–Fuller
7 Elliott, Rothenberg and Stock
8 Philips-Prone
9 Kwiatkowski–Phillips–Schmidt–Shin

the autocorrelation between different lags in the time series has not disappeared even after about 30 periods and, in fact, these autocorrelations in the series are decreasing at a very slow rate. This is anomalous to the behavior of autocorrelation of the stationary series in which the autocorrelations between different lags in the series decrease exponentially.

Figure 1. ACF Graph for Stock Return Series



3.3. Examining the Fractal Market Hypothesis

Generally, dependence of the behavior of a market on the Efficient Market Hypothesis depends on the significance of long memory parameter in its time series. In general, models that are based on long memory are highly dependent on the value of long memory parameter and also attenuation of the autocorrelation functions. On this basis, in the following subsections, the values of long memory parameter are estimated using the GPH. On the whole, this test is conforms to the frequency domain analysis and uses the Log-Period gram technique; this technique is a means for differentiating short and the long memory processes. It should also mentioned that slope of the regression line resulting from applying the Log-Period gram technique gives us the long memory parameter and if significant, the significance of the related feature in the stock return series can be inferred and the fractal market hypothesis is confirmed. The results of this test have been provided in Table 3 below.

Table 3. Estimation of d Parameter Using GPH Test Based on the NLS Method

Series	d-Parameter	t-stat.	Prob.
Stock series	0.14088	3.13	0.002
Stock return series	1.04695	12.3	0.000

As shown in Table 3 above, the value for long memory parameter is non-zero (and also lower than 0.5) which is a confirmation of the existence of long memory in the stock return series. Therefore, two conclusions can be drawn from the above test: first, the fractal market hypothesis is supported. The second conclusion is that this series should be fraction differenced once again so that modeling can be done in conformity with it. Therefore, although the existence of the long memory feature was confirmed in the return index, in the following sections, we will also focus on the existence of this feature in the stock volatility series. On this basis, in the following sections, stock return series models will be focused on using the models that are based upon long memory.

3.4. Estimation of the ARFIMA Model

There are different methods for estimation of the ARFIMA model and *d* parameter including Approximate Maximum Likelihood (AML), Exact Maximum Likelihood (EML), Modified Profile Likelihood (MPL), and Non Linear Least Square (NLS) (Ooms and Doornik, 1998). In the present study, EML, MPL, NLS methods have been selected for estimating these types of models using Ox-Metrics software. Furthermore, based on the Akaike information criterion, a comparison was made between different models of ARFIMA and the model that is found to have the lowest score of the information criterion, will be the best model for explaining mean equation of the stock return series.

Table 4. The Results of Estimation of Different Models of ARFIMA

Models	Akaike Information Criterion			ARCH test
	ML	NLS	EML	
ARFIMA(1,0.14,1)	-7.2126	-7.3241	-7.3235	4.8(0.02)
ARFIMA(1,0.14,2)	-7.2153	-7.3289	-7.3242	3.9(0.04)
ARFIMA(2,0.14,1)	-7.2124	-7.3234	-7.3226	4.4(0.03)
ARFIMA(2,0.14,2)	-7.2125	-7.3250	-7.3237	5.7(0.01)

According to Table 4, it can be concluded that ARFIMA(1,0.14,2) has the lowest Akaike information criteria score and has the best performance (see Table 5 for specifications).

Table 5. The results of estimation for ARFIMA(1,0.14,2)

Variables	Coefficient	T-Stat.	Prob.
Constant	0.0316	2.21	0.002
d-ARFIMA	0.1408	3. 13	0.000
AR(1)	0.8541	31.41	0.000
MA(1)	0.6163	18.67	0.000
MA(2)	0.2358	3.53	0.004
Dummy(1)	0.079	7.28	0.000
Dummy(2)	0.0519	8.73	0.000

It is worth mentioning that, the dummy variables introduced in the above equation can be defined as the following: Dummy(1) are related to the financial crisis in 2007-2008 and Dummy(2) is related to transferring the shares of Telecommunication Company of Iran in the stock in line with the implementation of Article 44. Additionally, considering the fact that diagnostic tests conducted on residuals of the related model are indicative of the existence of conditional variance heteroskedasticity effects (in Table 5), Robust Regression was used for estimating this model.

3.5. Estimating the Different types of GARCH Models

According to the results provided in Table 4, the ARFIMA (1,0.14,2) model, based on the Akaike statistics, has the best performance and based on the ARCH test, the effects of the ARCH (conditional heteroskedasticity variance) are confirmed to exist in residuals of these models and consequently, in order to eliminate the problems associated with heteroskedasticity variance, models of the ARCH family can be used. Therefore, in the next part, not only the long memory feature will be tested in the stock volatility index, there will also be a focus on modeling variance equation of the series using GARCH models including those with long memory (fractal) and the non-fractal ones. The results related to different forms have been presented in Table 6.

Table 6. Different types of GARCH Models

Models	ARFIMA(1,1)		ARFIMA(1,2)		ARFIMA(2,1)		ARFIMA(2,2)	
	AIC	SBC	AIC	SBC	AIC	SBC	AIC	SBC
GARCH	-7.3182	-7.2523	-7.3243	-7.2501	-7.3172	-7.2430	-7.3133	-7.2309
EGARCH	-6.9688	-6.8864	-6.9667	-6.8761	-6.9651	-6.8744	-6.9618	-6.8629
GJR-GARCH	-7.3221	-7.2478	-7.3349	-7.2525	-7.3244	-7.2420	-7.3209	-7.2302
APGARCH	-7.3341	-7.2518	-7.3308	-7.2402	-7.3333	-7.2426	-7.3271	-7.2281
IGARCH	-7.3125	-7.2548	-73122	-7.2463	-7.3114	-7.2455	-7.3075	-7.2333
FIGARCH(BBM)	-7.3126	-7.2384	-7.3343	-7.2588	-7.3088	-7.2264	-7.3073	-7.2166
FIGARCH(Chang)	-7.2991	-7.2250	-7.2981	-7.2155	-7.2976	-7.2151	-7.2937	-7.2031

All the proposed models shown in Table 6 have been based on different mean equations with long memory and as shown in this table, different combinations include three general parts: the first part of that (at the top of the table) includes different non-fractal models of conditional heteroskedasticity variance, the second part includes the combination of a conditional heteroskedasticity variance with unit root model (IGARCH), and finally, the third part (down the table) includes the different types of fractal conditional heteroskedasticity variance models (FIGARCH).

By comparing information criteria related to different types of GARCH models, it can be easily found that the ARFIMA (1,2)-FIGARCH(BBM) model has the lowest Akaike and Schwarz information criteria, so, it is the best model for explaining the behavioral pattern of volatility in the stock series (see Table 7 for the coefficients for variables of this model and the statistics related to significance of these coefficients). Another conclusion to be drawn from the results shown in the table is the existence of the long memory feature in the stock volatility series.

Furthermore, statistics related to examining the existence of heteroskedasticity variance in residuals of this model (statistics related to Liang-Box, McLeod-Lee and ARCH) have also been presented the table 7 with the estimation of this model.

Table 7. Estimating ARFIMA-FIGARCH Model Results

Mean Equation				
Variables	Coefficient	Standard Error	T-Statistic	Probability
C	0.002	0.0008	2.56	0.010
d-ARFIMA	0.18	0.014	12.85	0.000
AR(1)	0.28	0.073	3.93	0.000
MA(1)	-0.09	0.008	-12.09	0.000
MA(2)	-0.11	0.016	-6.47	0.000
Dummy(1)	0.06	0.009	6.16	0.000
Dummy(2)	0.04	0.005	7.84	0.000
Variance Equation				
Variables	Coefficient	Standard Error	T-Statistic	Probability
C	1.94	0.776	2.51	0.006
d-FIGARCH	0.31	0.031	10.06	0.000
ARCH	0.56	0.259	2.19	0.028
GARCH	0.75	0.154	4.85	0.000
Log likelihood	1891.932	Box- Ljung Q(10)	12.06 (0.098)	
Akaike	-7.334374	McLeod-Li $Q^2(10)$	4.87 (0.771)	
Schwarz	-7.258863	ARCH(10)	0.0031 (0.955)	

Source: The Findings of the Study

According to the Table 7, there are some points worth mentioning. First of all, the dummy variables introduced in the mean equation of the above model indicate the existence of unusual shocks to the time series under investigation. Furthermore, in the model under investigation all the coefficients (except the constant) are significant at .95 level of confidence. The results of Liang-Box test show no sign of serial correlation in the residuals of this model. The existence of heteroskedasticity variables in the residuals was also negated based on the results from McLeod-Lee and ARCH test.

4. Conclusions and Implications

The present study evaluated different models (both fractal and non-fractal) for modeling volatilities of the TSE index. Accordingly, first of all the existence of long memory feature in this series was considered and on this basis, the existence of the long memory feature was confirmed in the return and volatility of the stock, and, consequently, ARFIMA(1,0.14,2) model was selected as the best explainer of the behavior of mean equation. Then the conditional heteroskedasticity was tested and modeled (and confirmed) in different mean equations with long memory. The results of this study confirmed the existence of the long memory feature in the In the mean and variance equation of the mentioned series and finally ARFIMA(1,2)-FIGARCH(BBM) model was selected as the best. Based on the findings, the behavior of the stock index has modeling capability and, accordingly, the Efficient market Hypothesis about it will be rejected. On the other hand, due to the existence of long memory property in both return and volatility indices, Fractal Market Hypothesis is confirmed about it and it can be concluded that nonlinear models and those dealing with long memory have higher level of forecasting accuracy in comparison with their counterparts.

Some suggestions can also be made based on the results. One is that as regards the existence of long memory feature in Tehran Stock Exchange Price Index return, paying attention to this fact can help improve the results of modeling and consequently economic predictions because this feature suggests that although the current shocks will have their effects at that very time or at the most with a small delay, a remarkable part of these shocks can influence the behavior of the time series with such feature in the future; therefore, as it was confirmed in this study and in many other studies, taking this feature into account will lead to improved performance in the models; investors and decision-makers in the financial markets and macro-economy can be informed about this finding. Secondly, considering hybrid models have become very popular during the recent years, the claim that 'combining the complicated (nonlinear) methods and the issue of long memory feature can yield better results' can be further investigated in future studies. Finally, the use of these models in other volatile markets can also be tested and investigated.

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