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Gray Wolf and Krill Herd optimizations: Performance analysis and comparison

Gri Kurt ve Kril Sürü optimizasyonları: Performans analizi ve karşılaştırması

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Abstract

Herding behavior is defined as a group of animals of similar size that migrate in the same direction and hunt together. Gray wolves are usually seen in packs. Each gray wolf in the herd has a distinct duty and a distinct name that reflects the task. Krill swarms form the basis of ocean ecology. There are two reasons for the movement of the Krill herd. The first reason is that difficult for other organisms to prey on Krill living in herds. Another compelling factor is the way Krill form vast herds and effortlessly seize their prey. Gray Wolf Optimization (GWO) is inspired by gray wolf herding behavior, while Krill Herd Optimization (KHO) is based on krill herding. In this study, GWO and KHO algorithms are examined in detail and it is decided whether they had sufficient success. The fact that the GWO and KHO algorithms are swarm-based is accepted as a common feature of the two algorithms. However, compared with GWO and KHO analysis, as well as 23 single-mode, multi-modal, and fixed-size multimodal benchmarking optimization tests. In another hand, the success of the algorithms has been demonstrated by running them on various dimensions ({10, 20, 30, 50, 100, 500}). Additionally, the performances of the GWO and KHO are compared with Tree Seed Algorithm (TSA), Particle Swarm Algorithm (PSO), Jaya algorithm, Arithmetic Optimization Algorithm (AOA), Evolutionary Mating Algorithm (EMA), Fire Hawk Optimizer (FHO), Honey Badger Algorithm (HBA) algorithms. Moreover, all of the analyses are obtained in detail, complete with statistical tests and figures. As a result, while GWO and KHO algorithms show superior success in different test problems with their own characteristics, they are at a competitive level with many old and newly proposed algorithms today. In order to determine the success of the GWO and KHO algorithms, not only the classical test functions but also two different benchmark test sets are used. These are the CEC-C06 2019 functions and the big data problem, which is a current problem today. The same algorithms are run for both problems and rank values are obtained according to the average results. In CEC-C06 2019 functions, KHO achieved good results, while in big data problems, GWO achieved good results. In this study, the success of the GWO and KHO algorithms are examined in detail in three different experimental sets and it sheds light on researchers who will study with GWO and KHO algorithms.

Keywords: Gray wolf, Krill herd, Optimization algorithm.

Öz

Sürü davranışı, aynı yönde göç eden ve birlikte avlanan benzer büyüklükteki bir grup hayvan olarak tanımlanmaktadır. Gri kurtlar, genellikle sürüler halinde yaşamaktadırlar. Sürüdeki her gri kurdun ayrı bir görevi ve görevine göre aldığı farklı bir ismi bulunmaktadır. Diğer yandan Kril sürüleri, ekosistemin temelini oluşmaktadır. Kril sürüsünün hareketi iki sebebi bulunmaktadır. Birinci sebep, diğer canlılar için sürüler halinde yaşayan Kril'in avlanması ve yakalanmasının zor olmasıdır. Diğer sebebi ise, Kril sürüleri avlarını sürü hareketiyle kolayca yakalayabilmektedir. Gri Kurt Optimizasyonu (GWO) gri kurt sürü davranışından ilham alınırken, Kril Sürü Optimizasyonu (KHO) Kril sürü davranışından esinlenmiştir. Bu çalışmada GWO ve KHO algoritmaları detaylı bir şekilde incelenmiş ve yeterli bir başarıya sahip olup olmadıklarına karar verilmiştir. GWO ve KHO algoritmalarının sürü tabanlı olması, iki algoritmanın ortak bir özeliği olarak kabul edilmektedir. Ayrıca, GWO ve KHO performans analizinin yanı sıra 23 tek modlu, çok modlu ve sabit boyutlu çok modlu kıyaslama optimizasyon testleri ile karşılaştırılmıştır. Algoritmaların başarısı, çeşitli boyutlarda ({10, 20, 30, 50, 100, 500}) çalıştırılarak gösterilmiştir. İlaveten, GWO ve KHO algoritmaları Ağaç Tohum Algoritması (TSA), Parçacık Sürü Algoritması (PSO), Jaya algoritması, Aritmetik Optimizasyon Algoritması (AOA), Evrimsel Çiftleşme Algoritması (EMA), Ateş Şahini Optimize edicisi (FHO), Bal Porsuğu Algoritması (HBA) algoritmalarının performansı ile de karşılaştırılmıştır. Elde edilen tüm sonuçlar, istatistiksel testler ve şekillerle detaylı olarak gösterilmektedir. Sonuç olarak GWO ve KHO algoritmaları kendine öz özellikleri ile farklı test problemlerinde üstün başarı gösterirken, eski ve günümüzde yeni önerilmiş birçok algoritma ile de yarışır düzeydedir. GWO ve KHO algoritmalarının başarılarını tespit etmek için sadece klasik test fonksiyonları değil iki farklı kıyaslama test seti de kullanılmıştır. Bunlar CEC-C06 2019 fonksiyonları ve günümüzde güncel bir problem olan büyük veri problemidir. Aynı algoritmalar her iki problem içinde çalıştırılmış ve ortalama sonuçlara göre rank değerleri elde edilmiştir. CEC-C06 2019 fonksiyonlarında KHO iyi sonuçlar elde ederken büyük veri problemlerinde GWO iyi sonuçlar elde etmiştir. Bu çalışmada GWO ve KHO algoritmalarının başarıları üç farklı deneysel sette detaylı bir şekilde incelenmiş ve GWO ve KHO algoritmaları ile çalışacak araştırmacılar için ışık tutmaktadır.

Anahtar kelimeler: Gri kurt, Krill sürüsü, Optimizasyon algoritması.

1 Introduction

Many scientists research and seek to solve today's challenges by employing both traditional and novel methodologies and answers. In its most fundamental form, optimization is the most effective use of a huge amount of data [1]. To begin with, the objective function must always be defined in optimization problems [2]. To address concerns identified in the literature, many ways have been created. Swarm-based optimization

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algorithms have victim. Metaheuristic algorithms proposed in recent years; Slime Mould Algorithm (SMA) [3], Whale Optimization Algorithm (WOA) [4], Harris Hawks Optimization (HHO) [5], Crow Search Algorithm [6], Dolphin Echolocation Algorithm (DEA) [7], Spotted Hyena Optimizer (SHA) [8], Salp Swarm Algorithm (SSA) [9], Artificial Algae Algorithm (AAA) [10], Dragonfly Algorithm (DA) [11], Red Fox Optimization Algorithm (RFOA) [12], Moth-Flame Optimization Algorithm (MFOA) [13], Elephant Herding Optimization (EHO) [14], Aquila Optimizer (AO) [15], Mountain Gazelle Optimizer (MGO) [16], Horse Herd Optimizer (HHO) [17], Seagull Optimizer (SO) [18], Jellyfish Search Optimizer (JSO) [19], Monarch Butterfly Optimizer (MBO) [20], Grasshopper Optimizer (GO) [21], Butterfly Optimizer (BO) [22], Sailfish Optimizer (SO) [23], Arithmetic Optimization Algorithm (AOA) [24], Evolutionary Mating Algorithm (EMA) [25], Fire Hawk Optimizer (FHO) [26], Honey Badger Algorithm (HBA) [27], Gray Wolf Optimization (GWO) [28], and Krill Herd Optimization (KHO) [29].

Many research papers in the literature utilize the GWO and KHO algorithms. GWO has been developed to improve performance by combining through reinforcement learning techniques with neural networks [30]. The impact maximization problem is solved using an optimization problem that involves cost functions such as node efficiency and distance between them, according to GWO [31]. A chaotic Krill Swarm algorithm is proposed [32]. KHO is used in combination with a new hybrid differential evolution operator. Global numerical optimizations on a global scale are resolved [33]. The Lévy flight distribution and utilization elitism chart were proposed as an improvement to the KHO [34]. The monkey algorithm and KHO are used to create a hybrid feature selection system [35].

The way enormous prey is pursued in herds serves as the basis for the GWO swarm intelligence algorithm based on intelligence. KHO is a herd intelligence-based system inspired by Krill herd feeding behavior. GWO and KHO are algorithms that operate on the same basis. In this study, GWO and KHO are tested, and performance analyses are performed and compared to each other and other algorithms.

There are several aims of this study.

a) In this study, two algorithms based on herd intelligence proposed in the literature is examined in detail. In both algorithms, low, medium, and large dimensional 23 classical benchmark functions are compared. Thus, the performances of both algorithms not only in low but also, in large-scale (10, 20, 30, 50, 100, 500) dimensions have examined and compared,

b) GWO and KHO are compared with seven different heuristic algorithms (PSO, TSA, Jaya, AOA, HBA, FHO, EMA), including newly proposed algorithms in recent years. Thus, the competitiveness of the achievements of GWO and KHO with current algorithms has been shown,

c) GWO and KHO have been tested not only on classical benchmarks, but also on ten different CEC-C06 2019 functions and big data optimization problems (for six different big EEG data sets), which is one of the popular problems today. Thus, the successes of GWO and KHO are shown on different problems.

Thanks to this study, the achievements of GWO and KHO were examined in detail. In this respect, the study shows originality. The competitiveness and performance of GWO and KHO with existing and new heuristic algorithms has been demonstrated. Whether a superior hybrid GW-HB algorithm can be created by combining the positive characteristics of GWO and KHO sheds light on the researchers in the literature demonstrated by this study. In this study, although GWO and KHO have been run with classical benchmarks many times in the literature, a one-to-one comparison of the two has not been made. In addition, the performance of GWO and KHO in big data optimization problems and CEC-C06 2019 functions has not been demonstrated, and their success has been examined for the first time in this study.

According to the results, GWO achieved better results than KHO on the 23 different classic benchmark function. PSO, TSA, Jaya, GWO, and KHO are compared under equal conditions on the 23 different classic benchmark functions and the most successful average results belong to GWO. AOA, EMA, HBA, FHO, GWO, and KHO are compared under equal conditions on the 23 different classic benchmark function and KHO and GWO are the second-best algorithms after HBA. This shows that although GWO and KHO are old algorithms, they still have success to compete with newly developed algorithms in recent years. PSO, TSA, Jaya, GWO, and KHO are compared under equal ten different CEC-C06 2019 functions and KHO is the second-best algorithm after PSO. AOA, EMA, FHO, HBA, GWO, and KHO are compared under equal ten different CEC-C06 2019 functions and the most successful average results belong to KHO. This results shows that KHO has achieved with good success on CEC-C06 2019 functions. Finally, the same functions are run again for six different EEG data sets on a current problem defined as the big data optimization problem. The best results by average result rank are obtained by GWO. As a result, GWO and KHO are still successful algorithms that can compete with old and new optimization problems on different problems.

The remainder of this study is structured as follows: In Section 2, the Gray Wolf Optimizer (GWO) and Krill Herd Optimizer (KHO) algorithms are explained in detail. In Section 3, GWO and KHO algorithms are run on various unimodal and multimodal test functions in six different small, medium, and large-scale dimensions, and their successes are compared. Then, Tree Seed Algorithm (TSA), Particle Swarm Algorithm (PSO), Jaya algorithm, Arithmetic Optimization Algorithm (AOA), Evolutionary Mating Algorithm (EMA), Fire Hawk Optimizer (FHO), Honey Badger Algorithm (HBA), GWO, and KHO results were compared. The obtained results are analyzed by performing statistical tests. The algorithms' convergence graphs are generated, and their successes are illustrated using figures. Finally, the performance of the same algorithms was proved by picking two separate challenges (CEC-C06 2019 test functions and large data problems for six different massive EEG data sets). The findings are discussed in Section 4.

2 Material and method

All living things try to find food from nature to survive. The lifestyle of the gray wolf was used to model the GWO algorithm. The KHO algorithm was inspired by the lives of Cyrillic creatures and herds in the herd. The common feature of GWO and KHO was that they live in flocks managed by leaders. The common feature of GWO and KHO is that they live in herds managed by leaders. In this section, GWO and KHO are explained in detail [28], [29].

2.1 Gray Wolf optimization (GWO)

Gray wolves are excellent hunters. Gray wolves use their method to catch prey. Gray wolves often hunt in herds of 5-12.

The hunt is managed by a single leader wolf. The GWO is explained in further detail in the following section. Gray wolves encircle and immobilize their prey. When the hunt begins, the victim is surrounded by the wolves [28].

Gray wolf herds have specific duties. Each wolf herds given a different name. Alpha, Beta, Delta, and Omega wolves are called by their hunting duties. Alpha (α) refers to the leader wolf that establishes the hierarchy first level. The Alfa wolf discovers a haven for the flock, decides to awaken, and decides to hunt. The other wolves consider Alfa's judgments to be orders. Betas (β), the second level in the hierarchy, are the wolves who help the alpha with decisions. The other level of the hierarchy is called Delta (δ), and it is in charge of the herd's scouting and hunting. If the wolf herd is neither Alpha, Beta, nor Delta, it is believed to be at the bottom of the hierarchy and is known as Omega (ω). Omega is responsible for the wolf herd's care. Figure 1 shows the gray wolf hierarchy [28].

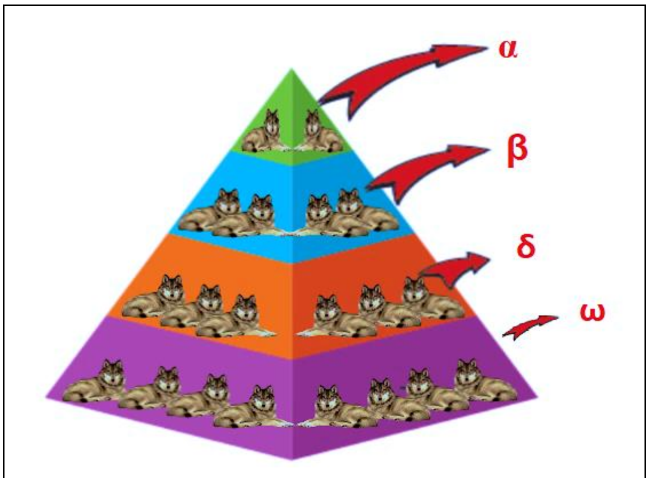

Figure 1. The gray wolf hierarchy.

The lead wolf decides how the prey will be encircled by wolves, and the other wolves just follow suit. Scientists see working with and hunting gray wolves as an essential activity. Equation 1 and Equation 2 are formulas for surrounding and arranging the prey [28].

$$\vec{D} = |\vec{C} \cdot \vec{X}_P(t) - \vec{X}(t)| \quad (1)$$

$$\vec{X}(t+1) = \vec{X}_P(t) - \vec{A} \cdot \vec{D} \quad (2)$$

where $\vec{X}(t)$ is the position of the wolf, and t is the time. $\vec{X}_P$, hunt position. $\vec{D}$, is calculated by using Equation 1. $\vec{A}$ and $\vec{C}$ coefficient values are calculated according to Equation 3 and Equation 4 [28].

$$\vec{A} = 2\vec{\alpha}.\vec{r}_1 - \vec{\alpha} \quad (3)$$

$$\vec{C} = 2.\vec{r}_2 \quad (4)$$

where r_1 and r_2 values are random numbers between 0 to 1 that are utilized to determine the optimal solution. Vector A represents a randomly chosen value within the range [-α, α]. The parameter α gradually diminishes, and a random value within the range [-1, 1] is selected. The subsequent position lies somewhere between its current position and the target location, as explained in reference [28]. This process is executed by incorporating information from vectors α, β, and δ, crucial for tracking the prey's position, as illustrated in Equations 5-7, which calculate the D vectors. Equations 8-10 are responsible for computing the X vectors. Furthermore, Equation 11 computes the vector $\vec{X}_{(t+1)}$ [28].

$$\vec{D}_\alpha = |\vec{C}_1.\vec{X}_\alpha - \vec{X}| \quad (5)$$

$$\vec{D}_\beta = |\vec{C}_2.\vec{X}_\beta - \vec{X}| \quad (6)$$

$$\vec{D}_\delta = |\vec{C}_3.\vec{X}_\delta - \vec{X}| \quad (7)$$

$$\vec{X}_1 = |\vec{X}_\alpha - \vec{A}_1.(\vec{D}_\alpha)| \quad (8)$$

$$\vec{X}_2 = |\vec{X}_\beta - \vec{A}_2.(\vec{D}_\beta)| \quad (9)$$

$$\vec{X}_3 = |\vec{X}_\delta - \vec{A}_3.(\vec{D}_\delta)| \quad (10)$$

$$\vec{X}_{(t+1)} = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \quad (11)$$

where $\vec{D}_\alpha$ represents the distance between $\vec{C}_1$ and $\vec{X}_\alpha$. $\vec{D}_\beta$ represents the distance between $\vec{C}_2$ and $\vec{X}_\beta$. $\vec{D}_\delta$ represents the distance between $\vec{C}_3$ and $\vec{X}_\delta$. $\vec{X}_1$ signifies the relationship between $\vec{X}_\alpha$ and Dα. In other words, $\vec{X}_1$ expresses a scaled version of $\vec{X}_\alpha$ by $\vec{D}_\alpha$. $\vec{X}_2$ signifies the relationship between $\vec{X}_\beta$ and $\vec{D}_\beta$. In other words, $\vec{X}_2$ expresses a scaled version of $\vec{X}_\beta$ by $\vec{D}_\beta$. $\vec{X}_3$ signifies the relationship between $\vec{X}_\delta$ and $\vec{D}_\delta$. In other words, $\vec{X}_3$ expresses a scaled version of $\vec{X}_\delta$ by $\vec{D}_\delta$. $\vec{X}_{(t+1)}$ denotes the arithmetic average of $\vec{X}_1$, $\vec{X}_2$, and $\vec{X}_3$. In other words, this expression generates a new $\vec{X}_{(t+1)}$ vector by averaging $\vec{X}_1$, $\vec{X}_2$, and $\vec{X}_3$. Figure 2 displays a flowchart of the GWO [28].

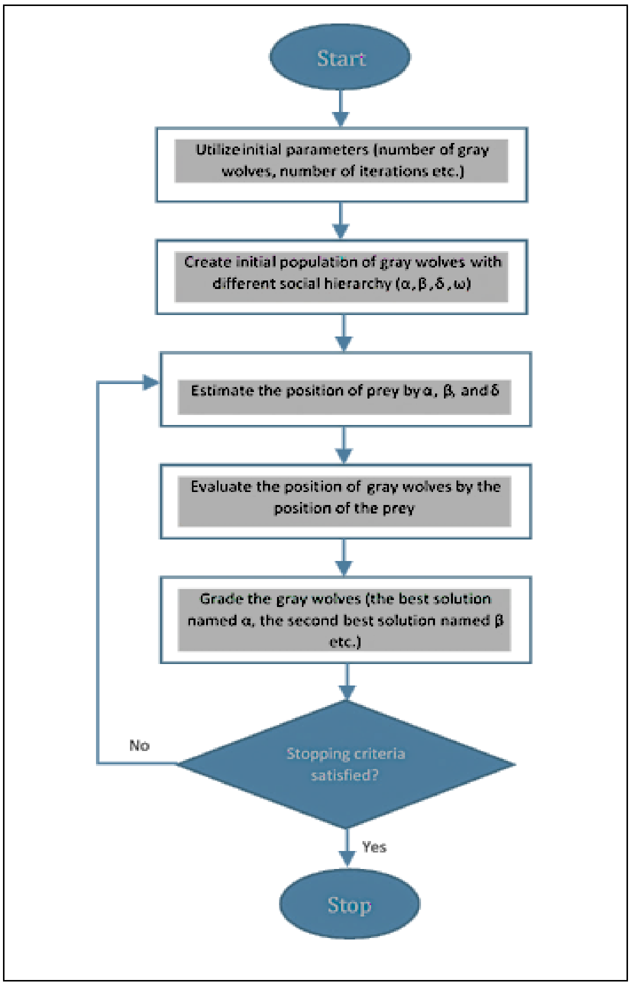

Figure 2. The flowchart of the GWO.

2.2 Krill Herd optimization (KHO)

Krill is a French word meaning "baby fish". Krill is one of the most common crustaceans on the planet. The Krill herd is an important part of the ecosystem. Krill is the food source for many animals such as whales, seals, fish, squid, penguins, and seabirds. Krill is 5 to 10 cm long and live for 2 to 6 years [29], [43].

Digestive systems are viewed from the outside due to the body's design of black eyes and a transparent body. Krill is similar to a shrimp because of the scab. Krill is detected prey by moving to the highest density. In other words, the closer one is to high-density food, the better. Figure 3 shows the body structure of the swarm's Krill herd [29].

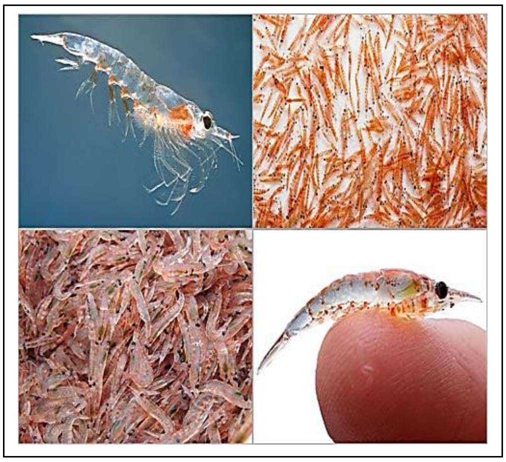

Figure 3. Swarm krill herd and body structure.

In the ecosystem, Krill herds play a crucial role. Krill herds can be found in oceans as deep as 91 meters and 304 meters. Krill move in herds and construct protective structures. The distribution of living Krill herds in the ocean is influenced by the temperature of the water in the area where they live. Krill live in large herds. As a result, other marine creatures have a tough time consuming the Krill herds. Krill move in herds and construct protective structures. Every herd of Krill has a leader. Krill decides on activities like live feeding, breeding, and protection from other living beings as a leader. The judgments taken are carried out without being challenged by other Krill in the herd. Krill live in flocks. Therefore, it is difficult for other creatures to eat the Krill herds [29].

KHO is a strong algorithm for exploitation (i.e., local search), but it may sometimes become stuck in some local optima, making it incapable of doing global search well. KHO is inspired by the Krill herd's way of existence. Krill herds individual positions on a two-dimensional surface at the end of three basic actions [29].

Time-dependent positions in the n-dimensional decision space (*Xi*), Equation 12 is provided by the Lagrangian model. Equations 13 and 14 describe the movement caused by others [29], [43].

$$\frac{dX_i}{dt} = N_i + F_i + D_i \tag{12}$$

$$N_i^{new} = N^{max}\,\alpha_i + \omega_n\,N_i^{old} \tag{13}$$

$$where\ \ \alpha_i = \alpha_i{}^{local} + \alpha_i{}^{target} \tag{14}$$

where N is the speed and is denoted as 0.01 m/s. a_i the direction of movement. N_i^{old} is represented next position, F_i, the hunting movement, D_i, the actual spread of *ith* Krill individuals, a_i^{local} neighbor's effects and a_i^{target} the target chosen by Krill herds. The effect of Krill movement on individual Krill movement is represented by Equations 15, 16, and 17 [29], [43].

$$a_i^{local} = \sum_{j=1}^{NN} \hat{K}_{i,j}\,\hat{X}_{i,j} \tag{15}$$

$$\hat{X}_{i,j} = \frac{x_j - x_i}{\|x_j - x_i\| + \varepsilon} \tag{16}$$

$$\hat{K}_{i,j} = \frac{K_i - K_j}{K^{worst} - K^{best}} \tag{17}$$

The suitability values of Krill herds are calculated and the best optimum value is K^{best}, best and the worst optimum value is K^{worst}, worst. The purpose function value of Krill herds is K_i, the goal function value of neighboring Krill creatures is K_j parameter. N, represents the total number of neighbors, location of Krill herds X. Neighbor election d_s, is represented by the sensing distance and is shown in Figure 4 [29].

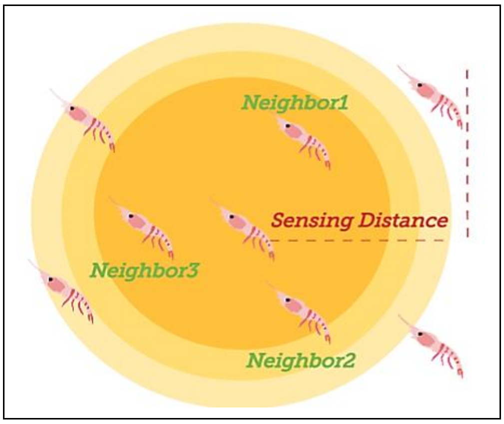

Figure 4. Krill herd is detected by neighbors around.

If the distance between two Krill individuals is reduced, it is concluded that the Krill individuals are neighbors. Krill is the individual with the highest objective function value. Individual krill impact is represented by Equation 18 [29],[43].

$$d_{s,i} = \frac{1}{5N}\sum_{j=1}^{N} \|X_i - X_j\| \tag{18}$$

where $d_{s,i}$ shows the sensed distance, N gives the total number of Krill individuals. Equation 19 is used to predict the effect on the individual Krill, Equation 20 yields the effect coefficient C^{best} [29], [43].

$$\alpha_i^{target} = C^{best}\,\hat{K}_{i,best}\,\hat{X}_{i,best} \tag{19}$$

$$C^{best} = 2\left(rand + \frac{I}{I_{max}}\right) \tag{20}$$

where *rand*, is the random number in [0.1] is the number of I loops and the maximum number of I_{max} loops. Two activities are carried out during foraging motion. Krill creatures are primarily concerned with seeking a food supply. Experience is the other process. Such qualities are expressed by the Equation 21 to Equation 25 [29],[43].

$$F_i = V_f\,\beta_i + \omega_f\,F_i^{old}$$
$$where\ \beta_i = \beta_i{}^{food} + \beta_i{}^{best} \tag{21}$$
$$\beta_i{}^{food} = C^{\,food}\,\hat{K}_{i,food}\,\hat{X}_{i,food}$$

$$X^{food} = \frac{\sum_{i=1}^{N} (1/K_i)\, X_i}{\sum_{i=1}^{N} (1/K_i)} \quad (22)$$

$$\beta_i^{\ food} = C^{\ food}\, \widehat{K}_{i,food}\, \widehat{X}_{i,food} \quad (23)$$

$$C^{\ food} = 2\left(1 - \frac{I}{I_{max}}\right) \quad (24)$$

$$\beta_i^{\ best} = \widehat{K}_{i,ibest}\, \widehat{X}_{i,ibest} \quad (25)$$

where V_f symbolizes foraging speed and is 0.02 (m/s) [29]. The parameter ω_f, foraging movement inertia weight between 0 and 1. The parameter F_i^{old}, the final foraging move. The parameter $\beta_i^{\ food}$, food attraction and $\beta_i^{\ best}$ is the optimum value that is that Krill person ever. The fitness value of the best previously visited location is $K_{i,ibest}$ [29],[43].

Members of the Krill herd are distributed at random throughout the water. The Krill herd migrates in a random direction, and its speed is referred to as the maximum diffusion rate. Physical propagation of a random process by Equation 26 and Equation 27 [29], [43].

$$D_i = D^{max}\delta \quad (26)$$

$$D_i = D^{max}\left(1 - \frac{I}{I_{max}}\right)\delta \quad (27)$$

The maximum propagation speed is [0.002, 0.010] (m/s) and δ, random faceted vector and values are in the range [1.1]. *Dmax,* maximum diffusion rate. *d,* parameter of the randomly selected direction. *D*, randomly selected from the range [-1,1]. Later in the solution, the $\left(1 - \frac{I}{I_{max}}\right)$ is used to reduce the random impact. KHO is doing a random search for physical diffusion. Equations 28 and 29 are represented a Krill position vector in the time span t to $y + \Delta y$ [29], [43].

$$X_i\,(t + \Delta t) = X_i\,(t) + \Delta t\, \frac{dX_i}{dt} \quad (28)$$

$$\Delta t = C_t \sum_{j=1}^{NV} (UB_j - LB_j) \quad (29)$$

Position parameters are y to $t + \Delta t$. *NV* represents the total number of variables. The variables and boundaries are represented by LB_j lower and UB_j upper, whereas C_t represents the constant value. C_t's value is set to 0.5. Figure 5 depicts the GWO flowchart [29].

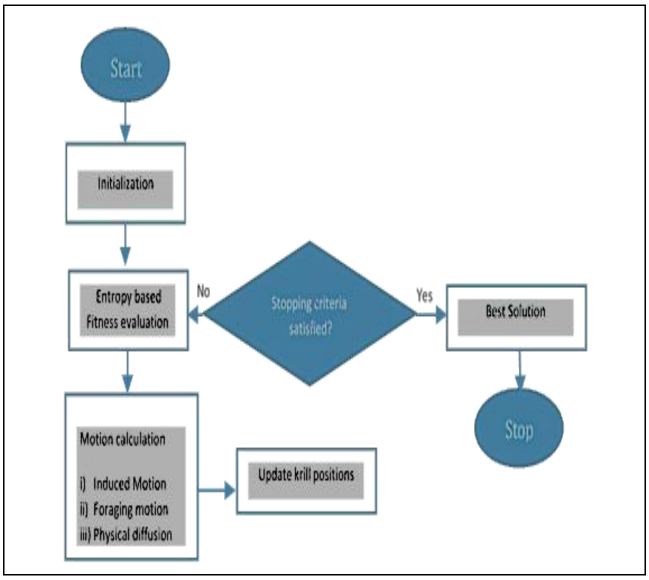

Figure 5. The flowchart of the KHO.

3 Results and discussion

3.1 GWO and KHO algorithms for thirteen unimodal and multimodal benchmark functions

Matlab is used in this study to code the GWO and KHO algorithms. GWO and KH original codes were taken from Mathworks library (https://ww2.mathworks.cn/en/). The algorithms are tested on 23 benchmark functions with varying features, including unimodal and multimodal functions. The variables for algorithms are shown in Table 1.

Table 1. The parameters for GWO and KHO

Dimension size	Population size	Maximum iteration
{10, 20, 30, 50, 100, 500}	30	1000

The mathematical definitions and details of the test problems are given in Tables 2-4 [3]. Because unimodal functions do not include local optima, they are used to test the capability of an algorithm's exploitation process. The multimodal functions are being used to test an algorithm's exploration ability since they feature numerous local optimum points at which it becomes trapped. The outcomes of the GWO and KHO algorithms are compared in Tables 5-6. Each algorithm ran each function independently 20 times and calculated the best, mean (Avrg.), and standard deviation (Stdd.) values on the results obtained.

The success and performances of GWO and KHO algorithms are evaluated and compared to benchmark test optimizations. The outcomes of the GWO and KHO algorithms are compared in Tables 5-6 for various dimensions (10, 20, 30, 50, 100, and 500) on thirteen unimodal and multimodal test functions. According to the mean results, GWO outperformed most of the test functions. GWO excels not only in low-scale dimensions but also in large-scale dimensional problems.

3.2 GWO, KHO, PSO, TSA, Jaya algorithms for twenty-three unimodal, multimodal, and fixed-dimension multimodal benchmark functions

The success and performance of GWO and KHO algorithms are evaluated and compared to benchmark test optimizations. Additionally, GWO and KHO are compared to other well-known algorithms, such as Particle Swarm Optimization (PSO) [36], Tee Seed Algorithm (TSA) [37], and Jaya [38]. Table 7 shows the parameter settings for algorithms. Test results are shown in Table 8. The rank value was calculated for the algorithms according to the average result values.

According to Table 8, the most successful average results belong to GWO. The GWO excelled in 12 of the 23 test functions. According to the average results, TSA is the most successful algorithm after GWO. According to the standard deviation results of the GWO and TSA, they are equally successful (8 of the 23 test functions). When the best results are examined, it is seen that it achieves the best results in most of the GWO test functions. Of all the unimodal test functions except F5 and F6, GWO is the most successful algorithm. Similarly, the best results in all multimodal test functions except F8 and F13 belong to GWO. TSA showed superior success in all fixed-dimensional multimodal test functions except F15 and F17. Figure 6 shows the convergence graphs of the GWO, KHO, PSO, TSA, and Jaya algorithms at various test functions (F1, F2, F3, F4, F5, F8, F9, F10, F11, F12, F15, and F16).

Table 2. The unimodal benchmark test functions of the mathematical formulations [15].

No.	Dim	Range	F_{min}	Formulation
F1	30	[−100,100]	0	$f1(\vec{x}) = \sum_{i=1}^{D} x_i^2$
F2	30	[-10, 10]	0	$f2(\vec{x}) = \sum_{i=1}^{D} \lvert x_i\rvert + \prod_{i=1}^{D} \lvert x_i\rvert$
F3	30	[−100,100]	0	$f3(\vec{x}) = \sum_{i=1}^{D} ([x_i + 0.5])^2$
F4	30	[−100,100]	0	$f4(\vec{x}) = \max_i\{\lvert x_i\rvert, 1 \leqslant i \leqslant D\}$
F5	30	[−30,30]	0	$f5(\vec{x}) = \sum_{i=1}^{D-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$
F6	30	[−100,100]	0	$f6(\vec{x}) = \sum_{i=1}^{D} ([x_i + 0.5])^2$
F7	30	[−1.28,1.28]	0	$f7(\vec{x}) = \sum_{i=1}^{D} \mathrm{i}x_i^4 + random[0,1)$

Table 3. The multimodal benchmark test functions of the mathematical formulations [15].

No.	Dim	Range	F_{min}	Formulation
F8	30	[-500, 500]	-418,9829×5	$f8(\vec{x}) = \sum_{i=1}^{n} - x_i sin\left(\sqrt{\lvert x_i\rvert}\right)$
F9	30	[-5.12, 5.12]	0	$f9(\vec{x}) = \sum_{i=1}^{D} \left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$
F10	30	[-32,32]	0	$f10(\vec{x}) = -20\exp\left\{-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{D} x_i^2}\right\} - \exp\left\{\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right\} + 20 + e$
F11	30	[-600, 600]	0	$f11(\vec{x}) = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$
F12	30	[−50,50]	0	$f12(\vec{x}) = \frac{\pi}{D}\left\{10\sin^2(\pi y_1) + \sum_{i=1}^{D-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_D - 1)^2\right\} + \sum_{i=1}^{D} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{1}{4}(x_i + 1) u_{x_i,a,k,m} = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a \leqslant x_i \leqslant a \\ k(x_i - a)^m & x_i < -a \end{cases}$
F13	30	[−50,50]	0	$f13(\vec{x}) = \frac{1}{10}\left\{\sin^2(\pi x_1) + \sum_{i=1}^{D-1}(x_i - 1)^2[1 + \sin^2(3\pi x_{i+1})] + (x_n - 1)^2[1 + \sin^2(2\pi x_{i+1})]\right\} + \sum_{i=1}^{D} u(x_i, 5, 100, 4)$

Table 4. The Fixed-dimension Multimodal benchmark test functions of the mathematical formulations [15].

No.	Dim	Range	F_{min}	Formulation
F14	2	[-65, 65]	1	$f14(\vec{x}) = \left(\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j + \sum_{i=1}^{2}(x_i - a_{i,j})^6}\right)^{-1}$
F15	4	[-5, 5]	0,00030	$f15(\vec{x}) = \sum_{i=1}^{11}\left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$

Table 4. Continued.

No.	Dim	Range	F_{min}	Formulation
F16	2	[-5, 5]	-1,0316	$f16(\vec{x}) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$
F17	2	[-5, 5]	0,398	$f17(\vec{x}) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)cosx_1 + 10$
F18	2	[-2, 2]	3	$f18(\vec{x}) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$
F19	3	[-1, 3]	-3,86	$f19(\vec{x}) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2\right)$
F20	6	[0,1]	-3,32	$f20(\vec{x}) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2\right)$
F21	4	[0,10]	-10,1532	$f21(\vec{x}) = -\sum_{i=1}^{5}[(X - a_i)(X - a_i)^T c_i]^{-1}$
F22	4	[0,10]	-10,4028	$f22(\vec{x}) = -\sum_{i=1}^{7}[(X - a_i)(X - a_i)^T c_i]^{-1}$
F23	4	[0,10]	-10,5363	$f23(\vec{x}) = -\sum_{i=1}^{10}[(X - a_i)(X - a_i)^T c_i]^{-1}$

Table 5. The results of the GWO and KHO are compared for dimension as 10, 20, and 30.

No.		Dimension=10		Dimension=20		Dimension=30	
		GWO	KHO	GWO	KHO	GWO	KHO
F1	*Avrg.*	**2,30E-117**	5,64E-07	**1,80E-75**	8,59E-05	**6,39E-59**	8,00E-03
	Stdd.	**6,2316E-117**	1,81064E-06	**4,1747E-75**	0,00016809	**1,93E-58**	6,31E-03
	Best	**6,48E-123**	1,03E-07	**1,42E-79**	2,37E-05	**3,79E-61**	1,40E-03
F2	*Avrg.*	**6,61E-67**	2,40E-05	**1,22E-43**	4,22E-01	**1,65E-34**	9,30E+01
	Stdd.	**1,53627E-66**	2,69836E-05	**2,68234E-43**	1,76898775	**2,41E-34**	6,21E+01
	Best	**1,18E-69**	9,51E-06	**2,34E-45**	5,33E-03	**7,23E-36**	2,15E+01
F3	*Avrg.*	**2,70E-52**	7,05E-15	**6,97E-25**	3,24E-12	**2,92E-15**	1,93E-11
	Stdd.	**1,17254E-51**	1,47844E-14	**1,92141E-24**	1,2844E-11	**1,06E-14**	5,13E-11
	Best	**3,48E-65**	2,13E-15	**2,91E-30**	1,45E-14	**1,58E-19**	2,51E-13
F4	*Avrg.*	**1,31E-37**	2,86E-05	**7,63E-20**	6,29E-03	**1,68E-14**	3,82E-01
	Stdd.	**2,40469E-37**	1,38697E-05	**2,40037E-19**	0,00853484	**2,54E-14**	8,62E-02
	Best	**2,44E-39**	2,37E-05	**4,49E-22**	1,46E-03	**6,06E-16**	2,38E-01
F5	*Avrg.*	6,36E+00	**0,00000**	1,65E+01	**0,00000**	2,70E+01	**0,00000**
	Stdd.	0,46432389	**0,00000**	0,82186261	**0,00000**	8,35E-01	**0,00000**
	Best	5,28E+00	**0,00000**	1,51E+01	**0,00000**	2,52E+01	**0,00000**
F6	*Avrg.*	8,97E-07	**2,53E-07**	5,00E-02	**6,77E-05**	7,47E-01	**4,76E-03**
	Stdd.	2,87732E-07	**1,91582E-07**	0,10007389	**6,06518E-05**	3,80E-01	**2,94E-03**
	Best	4,44E-07	**1,61E-07**	**2,99E-06**	2,42E-05	2,51E-01	**1,18E-03**
F7	*Avrg.*	**3,37E-04**	6,16E-04	**5,76E-04**	2,33E-03	**9,13E-04**	1,21E-02
	Stdd.	**0,00018705**	0,00023673	0,00029014	**0,00019783**	**4,54E-04**	3,76E-03
	Best	**5,62E-05**	3,97E-04	**1,65E-04**	2,18E-03	**1,88E-04**	6,62E-03
F8	*Avrg.*	**-2,85E+03**	-3,95E+01	**-4,37E+03**	-3,14E+02	**-5,94E+03**	-5,17E+00
	Stdd.	317,2351239	**3,76822E-07**	501,9459737	**15,16749311**	6,39E+02	**4,34E+01**
	Best	**-3,40E+03**	-3,95E+01	**-5,54E+03**	-3,27E+02	**-7,67E+03**	-5,97E+00
F9	*Avrg.*	**0,000000**	-8,42E+01	**1,42E-15**	-1,64E+02	**1,04E-01**	-1,91E+00
	Stdd.	**0,000000**	1,843013691	**6,19437E-15**	7,563435423	**4,54E-01**	3,17E+01
	Best	**0,000000**	-8,60E+01	**0,00E+00**	-1,69E+02	**0,00E+00**	-2,41E+00
F10	*Avrg.*	**4,97E-15**	-2,20E+04	**1,05E-14**	-4,82E+08	**1,71E-14**	-9,75E+27
	Stdd.	**1,26857E-15**	1,264327874	**2,99357E-15**	428642,2668	**3,07E-15**	2,75E+26
	Best	**4,44E-15**	-2,20E+04	**7,99E-15**	-4,83E+08	**1,51E-14**	-1,03E+28
F11	*Avrg.*	1,07E-02	**8,39E-03**	5,55E-03	**5,53E-04**	**4,15E-03**	5,82E-03
	Stdd.	0,01072249	**0,00585519**	0,00940883	**0,00147128**	7,48E-03	**3,55E-03**
	Best	0,00000	**4,93E-03**	0,00000	**6,28E-05**	**0,00000**	2,70E+00
F12	*Avrg.*	5,90E-03	**2,23E-12**	2,28E-02	**1,21E-12**	**3,39E-02**	1,56E-01
	Stdd.	0,00901314	**2,80184E-12**	0,0124533	**1,45031E-12**	**1,94E-02**	6,95E-01
	Best	1,05E-07	**3,30E-14**	1,75E-06	**1,43E-13**	6,95E-03	**1,43E-13**
F13	*Avrg.*	5,00E-03	**7,31E-12**	2,14E-01	**1,38E-12**	4,44E-01	**1,28E-10**
	Stdd.	0,02179942	**1,27527E-11**	0,11077582	**5,24588E-12**	2,26E-01	**2,00E-10**
	Best	5,91E-07	**3,94E-14**	4,95E-06	**1,75E-13**	2,52E-05	**1,02E-13**

Table 6. The results of the GWO and KHO are compared for dimensions 50, 100, and 500.

No.		Dimension=50		Dimension=100		Dimension=500	
		GWO	KHO	GWO	KHO	GWO	KHO
F1	*Avrg.*	**1,25E-43**	4,66E-01	**1,62E-29**	6,67E+01	**1,53E-12**	5,13E+05
	Stdd.	**1,48771E-43**	0,15855295	**1,28343E-29**	11,4728047	**6,63448E-13**	2117,10406
	Best	**5,73E-46**	1,32E-01	**1,77E-30**	4,38E+01	**7,31E-13**	5,10E+05
F2	*Avrg.*	**3,63E-26**	5,43E+14	**6,54E-18**	4,80E+114	**5,96E-08**	Inf
	Stdd.	**3,21176E-26**	2,36536E+15	**3,04092E-18**	2,0912E+115	**1,36731E-08**	Inf
	Best	**5,86E-27**	4,51E+03	**2,61E-18**	3,64E+86	**3,30E-08**	Inf
F3	*Avrg.*	5,45E-06	**3,68E-14**	9,93E+00	**3,14E-12**	1,46E+05	**4,46E-12**
	Stdd.	1,54092E-05	**2,13731E-14**	19,5280381	**1,06208E-11**	50668,8862	**1,51366E-11**
	Best	2,00E-11	**1,07E-14**	1,10E-01	**1,38E-14**	6,32E+04	**5,23E-14**
F4	*Avrg.*	**1,99E-09**	2,06E+00	**1,78E-02**	1,00E+01	**6,18E+01**	6,99E+01
	Stdd.	**2,42533E-09**	0,30662599	**0,04449635**	1,17144501	**5,71755184**	17,7862347
	Best	**1,72E-10**	1,86E+00	**1,04E-04**	9,17E+00	**5,01E+01**	6,00E+01
F5	*Avrg.*	4,71E+01	**0,000000**	9,74E+01	**0,000000**	4,98E+02	**0,000000**
	Stdd.	0,81071158	**0,000000**	0,75145316	**0,000000**	0,14910443	**0,000000**
	Best	4,61E+01	**0,000000**	9,60E+01	**0,000000**	4,97E+02	**0,000000**
F6	*Avrg.*	2,22E+00	**3,03E-01**	**9,02E+00**	8,47E+01	**9,33E+01**	5,30E+05
	Stdd.	0,59837725	**0,10554444**	**0,85784928**	22,442505	**1,65602015**	58092,1681
	Best	1,00E+00	**2,35E-01**	**7,03E+00**	6,29E+01	**9,03E+01**	4,67E+05
F7	*Avrg.*	**1,32E-03**	6,01E-02	**2,87E-03**	2,22E+02	**1,05E-02**	1,81E+09
	Stdd.	**0,00066566**	0,01616069	**0,00121852**	195,441126	**0,00332996**	287546146,7
	Best	**3,31E-04**	4,99E-02	**1,33E-03**	9,75E+01	**4,75E-03**	1,67E+09
F8	*Avrg.*	**-8,81E+03**	-1,26E+03	**-1,63E+04**	-5,28E+03	-6,07E+04	**-9,11E+04**
	Stdd.	859,9022668	**139,594083**	1542,643308	**393,1746961**	**3745,05463**	26108,7137
	Best	**-1,06E+04**	-1,38E+03	**-1,89E+04**	-5,45E+03	-6,92E+04	**-1,07E+05**
F9	*Avrg.*	2,45E-01	**-2,70E+02**	**2,66E-01**	1,08E+03	**6,17E+00**	4,76E+05
	Stdd.	**1,06788837**	36,16641762	**1,15981062**	425,260789	**7,33422393**	62972,7727
	Best	**0,00E+00**	-2,70E+02	**1,14E-13**	8,84E+02	**6,55E-11**	4,22E+05
F10	*Avrg.*	3,39E-14	**-4,00E+21**	1,09E-13	**-1,74E+41**	5,04E-08	**-1,59E+212**
	Stdd.	**3,90799E-15**	4,39658E+20	**7,65896E-15**	7,3078E+40	1,49678E-08	**1,00678E-20**
	Best	2,93E-14	**-4,15E+21**	1,00E-13	**-2,05E+41**	2,85E-08	**-2,65E+212**
F11	*Avrg.*	**0,000000**	6,81E-02	**1,60E-03**	1,02E+00	**3,60E-03**	1,25E+02
	Stdd.	**0,000000**	0,01511773	**0,00484735**	0,00524309	**0,0131986**	8,15974082
	Best	**0,000000**	6,05E-02	**0,00E+00**	1,01E+00	**9,04E-14**	1,11E+02
F12	*Avrg.*	**8,19E-02**	1,56E-01	2,58E-01	**2,44E-06**	**7,27E-01**	7,86E+10
	Stdd.	**0,03110235**	0,67782417	0,0488396	**2,92482E-06**	**0,03265843**	10856377812
	Best	3,13E-02	**2,96E-16**	1,84E-01	**7,50E-10**	**6,38E-01**	7,31E+10
F13	*Avrg.*	1,93E+00	**4,05E-12**	6,35E+00	**5,25E-01**	**4,60E+01**	1,08E+11
	Stdd.	0,34593713	**1,11011E-11**	**0,41165541**	2,21485335	**0,51664896**	13629137587
	Best	1,20E+00	**1,01E-13**	5,59E+00	**7,63E-05**	**4,51E+01**	1,04E+11

Table 7. The parameter settings for algorithms.

Dimension size	Population size	Maximum iteration
{30, 2, 3, 4, 6}	30	1000

Table 8. The results of the GWO and KHO are compared with other algorithms (PSO, TSA, Jaya) for dimension as 30.

No.		GWO	KHO	PSO	TSA	Jaya
F1	*Avrg.*	6,39E-59	8,00E-03	3,49E-08	6,18E+00	2,38E-01
	Stdd.	1,93E-58	6,31E-03	9,39E-08	1,95E+00	2,32E-01
	Best	3,79E-61	1,40E-03	7,47E-19	2,59E+00	1,43E-02
F2	*Avrg.*	1,65E-34	9,30E+01	4,83E-02	3,84E-01	3,94E+03
	Stdd.	2,41E-34	6,21E+01	6,29E-02	8,51E-02	1,66E+04
	Best	7,23E-36	2,15E+01	2,35E-03	2,48E-01	2,58E-01
F3	*Avrg.*	2,92E-15	1,93E-11	3,18E+00	3,32E+04	8,87E+04
	Stdd.	1,06E-14	5,13E-11	3,61E+00	5,34E+03	2,46E+04
	Best	1,58E-19	2,51E-13	1,30E-01	2,35E+04	5,16E+04
F4	*Avrg.*	1,68E-14	3,82E-01	6,04E-01	5,67E+01	6,59E+00
	Stdd.	2,54E-14	8,62E-02	4,57E-01	5,95E+00	4,37E+00
	Best	6,06E-16	2,38E-01	1,49E-01	4,33E+01	1,89E+00

Table 8. Continued.

No.		GWO	KHO	PSO	TSA	Jaya
F5	*Avrg.*	2,70E+01	0,00E+00	5,04E+01	2,00E+04	6,72E+01
	Stdd.	8,35E-01	0,00E+00	2,68E+01	6,15E+03	3,24E+01
	Best	2,52E+01	0,00E+00	1,63E+01	9,22E+03	2,38E+01
F6	*Avrg.*	7,47E-01	4,76E-03	6,33E-07	6,16E+00	6,78E+00
	Stdd.	3,80E-01	2,94E-03	2,40E-06	1,64E+00	2,27E+00
	Best	2,51E-01	1,18E-03	2,11E-14	3,80E+00	4,09E+00
F7	*Avrg.*	9,13E-04	1,21E-02	1,52E-02	1,78E-01	1,36E-01
	Stdd.	4,54E-04	3,76E-03	8,95E-03	4,85E-02	7,61E-02
	Best	1,88E-04	6,62E-03	7,97E-03	6,96E-02	3,57E-02
F8	*Avrg.*	-5,94E+03	-5,17E+00	-6,69E+03	-5,17E+03	-9,87E+03
	Stdd.	6,39E+02	4,34E+01	7,09E+02	3,33E+02	1,52E+03
	Best	-7,67E+03	-5,97E+00	-7,83E+03	-5,89E+03	-1,15E+04
F9	*Avrg.*	1,04E-01	-1,91E+00	4,82E+01	2,25E+02	2,36E+02
	Stdd.	4,54E-01	3,17E+01	1,21E+01	1,65E+01	5,61E+01
	Best	0,00E+00	-2,41E+00	2,79E+01	1,80E+02	1,04E+02
F10	*Avrg.*	1,71E-14	-9,75E+27	1,09E+00	2,84E+00	1,97E+01
	Stdd.	3,07E-15	2,75E+26	7,76E-01	2,31E-01	4,49E-01
	Best	1,51E-14	-1,03E+28	1,61E-07	2,49E+00	1,83E+01
F11	*Avrg.*	4,15E-03	5,82E-03	3,59E-02	1,05E+00	3,45E-01
	Stdd.	7,48E-03	3,55E-03	2,36E-02	1,30E-02	2,83E-01
	Best	0,00E+00	2,70E+00	2,70E-12	1,02E+00	2,91E-02
F12	*Avrg.*	3,39E-02	1,56E-01	2,75E-01	1,92E+03	9,45E+00
	Stdd.	1,94E-02	6,95E-01	7,64E-01	2,45E+03	4,29E+00
	Best	6,95E-03	1,43E-13	7,86E-24	2,80E+01	4,22E+00
F13	*Avrg.*	4,44E-01	1,28E-10	1,16E-01	3,50E+04	2,93E-01
	Stdd.	2,26E-01	2,00E-10	2,34E-01	5,80E+04	5,56E-01
	Best	2,52E-05	1,02E-13	2,75E-18	1,18E+03	1,86E-02
F14	*Avrg.*	4,82E+00	4,95E+00	3,35E+00	9,98E-01	1,00E+00
	Stdd.	4,20E+00	4,25E+00	3,49E+00	2,22E-16	4,43E-03
	Best	9,98E-01	9,98E-01	9,98E-01	9,98E-01	9,98E-01
F15	*Avrg.*	3,32E-03	4,00E-03	7,25E-04	6,91E-04	3,72E-04
	Stdd.	7,16E-03	9,25E-03	5,61E-04	7,94E-05	5,98E-05
	Best	3,07E-04	3,41E-04	3,07E-04	4,84E-04	3,09E-04
F16	*Avrg.*	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00
	Stdd.	5,95E-09	4,88E-09	0,00E+00	0,00E+00	2,37E-05
	Best	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00
F17	*Avrg.*	3,98E-01	7,78E+00	3,98E-01	4,98E-01	6,98E+00
	Stdd.	5,41E-05	9,11E-16	1,11E-16	5,55E-17	1,78E-15
	Best	3,98E-01	7,78E+00	3,98E-01	4,98E-01	6,98E+00
F18	*Avrg.*	7,05E+00	3,00E+00	3,00E+00	3,00E+00	3,00E+00
	Stdd.	1,77E+01	2,63E-09	8,88E-16	2,49E-15	5,39E-03
	Best	3,00E+00	3,00E+00	3,00E+00	3,00E+00	3,00E+00
F19	*Avrg.*	-3,86E+00	-3,86E+00	-3,86E+00	-3,86E+00	-3,86E+00
	Stdd.	3,09E-03	2,09E-02	8,88E-16	8,88E-16	8,88E-16
	Best	-3,86E+00	-3,86E+00	-3,86E+00	-3,86E+00	-3,86E+00

Table 8. Continued.

No.		GWO	KHO	PSO	TSA	Jaya
	Avrg.	-3,25E+00	-3,22E+00	-3,27E+00	-3,32E+00	-3,24E+00
F20	*Stdd.*	8,12E-02	8,12E-01	5,91E-02	4,44E-16	5,32E-02
	Best	-3,32E+00	-3,32E+00	-3,32E+00	-3,32E+00	-3,32E+00
	Avrg.	-9,14E+00	-8,14E+00	-6,77E+00	-1,01E+01	-5,41E+00
F21	*Stdd.*	2,02E+00	2,82E+00	3,48E+00	1,91E-01	1,50E+00
	Best	-1,02E+01	-1,02E+01	-1,02E+01	-1,02E+01	-8,98E+00
	Avrg.	-1,01E+01	-1,01E+01	-8,66E+00	-1,04E+01	-7,67E+00
F22	*Stdd.*	1,52E+00	1,55E+00	3,06E+00	9,80E-13	2,27E+00
	Best	-1,04E+01	-1,04E+01	-1,04E+01	-1,04E+01	-1,04E+01
	Avrg.	-1,03E+01	-1,02E+01	-5,18E+00	-1,04E+01	-7,99E+00
F23	*Stdd.*	1,18E+00	1,22E+00	3,58E+00	4,26E-01	2,61E+00
	Best	-1,05E+01	-1,05E+01	-1,05E+01	-1,05E+01	-1,05E+01
Rank		**1**	**3**	**3**	**2**	**3**

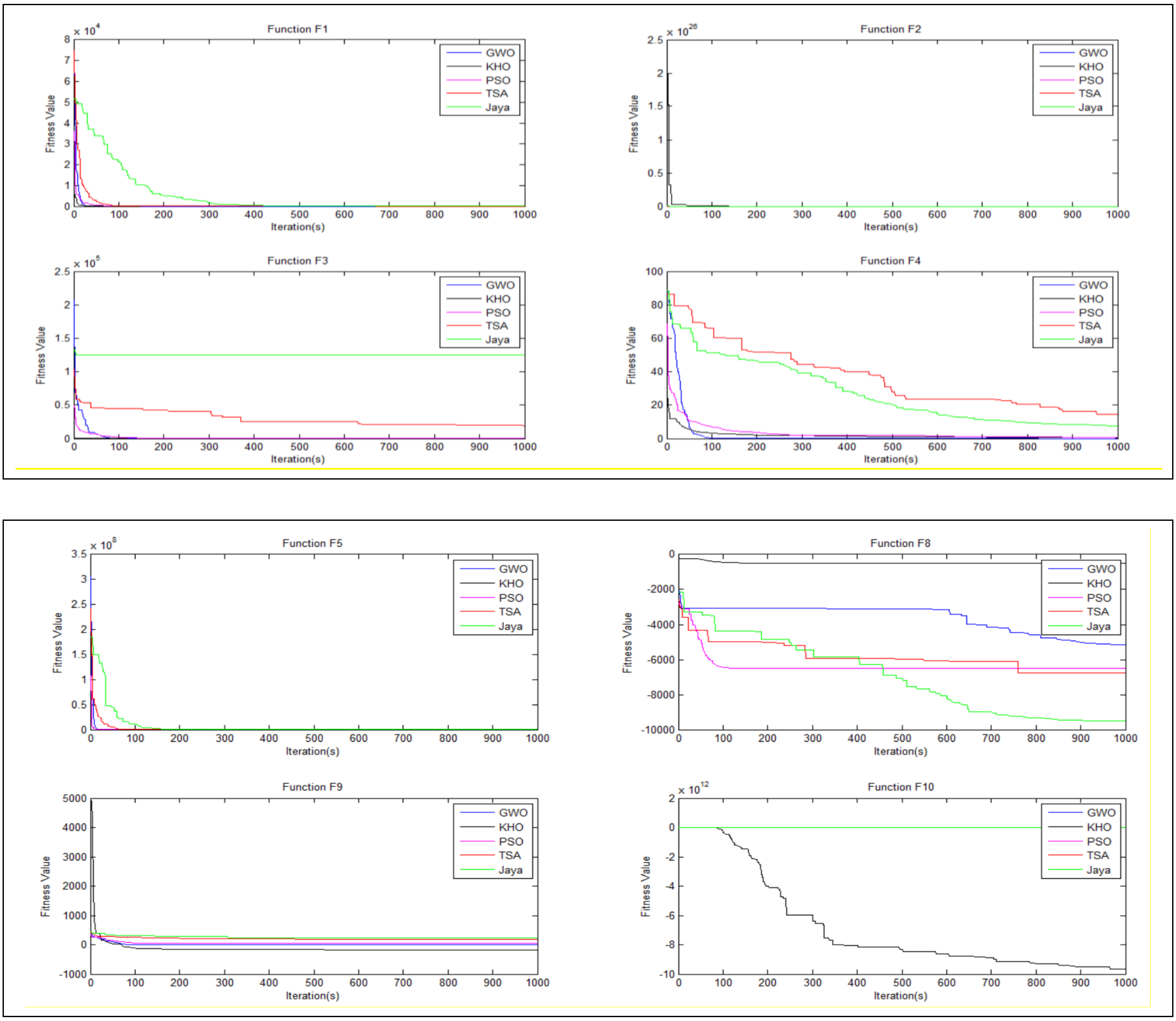

Figure 6. Convergence graphs for GWO, KHO, PSO, TSA, and Jaya.

Figure 6. Continued.

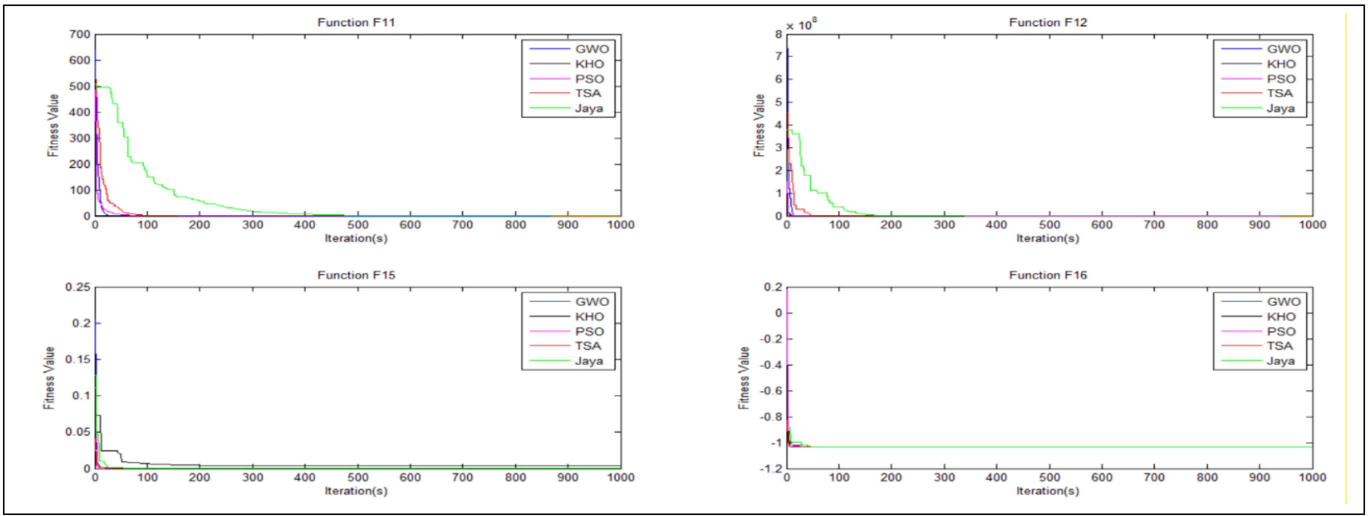

Figure 6. Convergence graphs for GWO, KHO, PSO, TSA, and Jaya.

Tables 9-13 show the statistical results of the comparison algorithms using the Wilcoxon Signed-Rank (WSR) Test. If the p-value is less than 0.05, the sign is positive, otherwise, it is negative. The positive sign indicates that there is a significant difference between the compared algorithm results. Examining Tables 9-13 shows that the sign is positive for most results. This proves that the compared algorithms obtain very different results from each other.

3.3 AOA, EMA, FHO, HBA, GWO, KHO algorithms for twenty-three unimodal, multimodal, and fixed-dimension multimodal benchmark functions

The success and performance of GWO and KHO algorithms are evaluated and compared to benchmark test optimizations. Additionally, GWO and KHO are compared to other new algorithms in literature in recent years, such as Evolutionary Mating Algorithm (EMA) [25], Arithmetic Optimization Algorithm (AOA) [24], Fire Hawk Optimizer (FHO) [26], and Honey Badger Algorithm (HBA) [27]. For a fair comparison, the population size was determined as 30 and the maximum number of iterations is determined as 1000 in all algorithms. Each algorithm was run independently 20 times. Results are shown according to three different criteria (mean, standard deviation, and best). Test results are shown in Table 14. The rank value is calculated for the algorithms according to the average result values. After HBA, KHO and GWO are the second-best algorithms. This shows that although GWO and KHO are old algorithms, they still have the success to compete with newly developed algorithms in recent years. Tables 15-16 show the statistical results of the comparison algorithms using the Wilcoxon Signed-Rank (WSR) Test. Figure 7 shows the convergence graphs of the GWO, KHO, AOA, EMA, FHO, and HBA algorithms at various test functions (F1, F2, F3, F4, F5, F8, F9, F10, F11, F12, F15, and F16).

3.4 GWO, KHO, PSO, TSA, Jaya, AOA, EMA, FHO, HBA algorithms for CEC-C06-2019 benchmark functions

In this subsection, GWO and other comparison algorithms are compared on different new CEC-06 2019 test functions. In this subsection, the success of algorithms is also shown on different CEC test functions.

The typical parameter settings for comparison are shown in Table 17. The definitions for the CEC-C06 2019 test functions are shown in Table 18 [39], [40]. Tables 19 and 20 are displayed the comparison's findings. Each algorithm independently are executed each function 20 times before calculating the best, mean, and standard deviation values based on the outcomes. The algorithms are determined the rank value based on the typical outcome values. According to Table 19, the most successful average results belong to PSO. The PSO is excelled in 7 of the 10 test functions. According to the average results, KHO is the most successful algorithm after PSO. According to the standard deviation results, KHO is successful (5 of the 10 test functions). According to the standard deviation results, PSO is successful (4 of the 10 test functions). When the best results are examined, it is seen that it achieves the best results in most of the PSO test functions (8 of the 10 test functions). According to Table 20, When the rank results are evaluated according to Table 20, KHO is in the first place, followed by GWO and HBA. This showed that KHO and GWO continued their success in the CEC-C06 2019 functions series as well. In recent years, it has achieved competitive success with newly proposed algorithms.

Figure 8 shows the convergence graphs of the GWO, KHO, PSO, TSA, Jaya, AOA, EMA, FHO, and HBA algorithms on CEC-C06 2019 test functions (F1, F2, F3, F4, F5, F6, F7, F8, F9, and F10). Tables 21-27 present the statistical findings of the WSR test-based comparison algorithms. The sign is positive if the p-value is less than 0.05; otherwise, it is negative. The presence of a positive sign implies that the outcomes of the two algorithms are significantly different. Examining Tables 21-27 show that the sign is positive for most results. This proves that the compared algorithms obtain very different results from each other.

3.5 GWO, KHO, PSO, TSA, Jaya, AOA, EMA, FHO, HBA algorithms for EEG Signal-based datasets

In this subsection, the success of GWO, KHO, and other algorithms on six different data sets is shown. The datasets are large-dimension and contain noisy and noiseless EEG signals. Using these datasets for the first time in the literature, Goh and his friends recently introduced a major data optimization problem [41],[42]. The big data problem defines the following Equations 30 to 36.

$$X = G \times H \tag{30}$$

$$H = H_1 + H_2 \tag{31}$$

$$X = G \times H_1 + G \times H_2 \tag{32}$$

$$C = \frac{covar(G,\ G \times H_1)}{var(G) \times var(G \times H_1)} \tag{33}$$

$$Minimize \quad f_1 = \frac{1}{(S^2 - S)} \sum_{i \neq j} (C_{ij})^2 + \frac{1}{S} \sum_{i} (1 - C_{ij})^2 \tag{34}$$

$$Minimize \quad f_2 = \frac{1}{S \times T} \sum_{ij} (H_{ij} - H1_{ij})^2 \tag{35}$$

$$\begin{aligned} &Minimize && f_1 + f_2 \\ &Subject\ to && -8 \leq H_1 \leq 8 \end{aligned} \tag{36}$$

where G is an S × S dimensional matrix and H is an S × T dimensional matrix (S= the number of time series and T= the length for each time series) [41],[42].

Table 28 displays the algorithm parameter sets. Each algorithm is ran separately 20 times. Results are shown according to three different criteria (mean, standard deviation, and best). Table 29 presents the test results. The rank value is calculated for the algorithms according to the average result values. Tables 30-31 are showed the statistical results of the comparison algorithms using the WSR Test. In terms of average rank results, GWO is surpassed all EEG datasets. Based on the computed p values, GWO is found a semantic difference in all EEG data sets.

Table 9. The statistical results of the GWO and other algorithms using the WSR Test.

F	GWO-KHO		GWO-PSO		GWO-TSA		GWO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F1	0,00214	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F2	7,74E-06	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F3	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F4	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F5	0,00021	(+)	0,0400	(+)	2,57E-06	(+)	1,03E-04	(+)
F6	0,00021	(+)	0,00042	(+)	0,00042	(+)	0,00042	(+)
F7	0,00021	(+)	2,57E-06	(+)	6,66E-05	(+)	2,57E-06	(+)
F8	2,57E-06	(+)	0,0028	(+)	4,49E-04	(+)	1,20E-04	(+)
F9	2,57E-06	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F10	7,74E-06	(+)	0,00023	(+)	0,00021	(+)	0,00021	(+)
F11	0,2384	(-)	4,49E-04	(+)	0,00021	(+)	0,00021	(+)
F12	0,00021	(+)	0,3317	(-)	0,00021	(+)	0,00021	(+)
F13	0,00021	(+)	6,81E-04	(+)	6,81E-04	(+)	0,0206	(+)
F14	1	(-)	0,3317	(-)	1,22E-04	(+)	0,0017	(+)
F15	0,0730	(-)	0,8519	(-)	0,0731	(-)	0,0731	(-)
F16	0,00021	(+)	1,78E-03	(+)	1,78E-03	(+)	2,57E-06	(+)
F17	2,57E-06	(+)	0,00021	(+)	0,00021	(+)	2,57E-06	(+)
F18	2,57E-06	(+)	0,00021	(+)	0,00021	(+)	0,0015	(+)
F19	0,2627	(-)	6,81E-04	(+)	0,00021	(+)	6,66E-05	(+)
F20	0,0674	(-)	0,0793	(-)	0,00021	(+)	0,8813	(-)
F21	0,2471	(-)	0,1790	(-)	2,57E-06	(+)	0,0013	(+)
F22	0,00021	(+)	0,5257	(-)	2,57E-06	(+)	0,0090	(+)
F23	0,0015	(+)	0,0019	(+)	0,0013	(+)	0,0333	(+)

Table 10. The statistical results of the KHO and other algorithms using the WSR Test.

F	KHO-GWO		KHO-PSO		KHO-TSA		KHO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F1	7,74E-06	(+)	0,00021	(+)	0,00021	(+)	2,31E-05	(+)
F2	7,74E-06	(+)	0,00021	(+)	0,00021	(+)	0,0137	(+)
F3	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F4	0,00021	(+)	0,1084	(-)	0,00021	(+)	0,00021	(+)
F5	0,00021	(+)	2,31E-05	(+)	0,00021	(+)	0,00021	(+)
F6	0,00021	(+)	2,31E-05	(+)	0,00021	(+)	0,00021	(+)
F7	2,57E-06	(+)	0,0930	(-)	0,00021	(+)	0,00021	(+)

Table 10. Continued.

F	KHO-GWO		KHO-PSO		KHO-TSA		KHO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F8	2,57E-06	(+)	2,57E-06	(+)	0,00021	(+)	0,00021	(+)
F9	0,00214	(+)	0,00214	(+)	0,00214	(+)	0,00214	(+)
F10	0,00214	(+)	0,00214	(+)	0,00214	(+)	0,00214	(+)
F11	0,2384	(-)	3,90E-04	(+)	2,31E-05	(+)	0,00021	(+)
F12	2,31E-05	(+)	0,1258	(-)	2,31E-05	(+)	0,00021	(+)
F13	2,31E-05	(+)	1,39E-04	(+)	2,31E-05	(+)	0,00021	(+)
F14	1	(-)	0,0259	(+)	6,83E-05	(+)	0,00021	(+)
F15	0,0730	(-)	2,31E-05	(+)	2,31E-05	(+)	0,00021	(+)
F16	0,00021	(+)	6,83E-05	(+)	6,83E-05	(+)	0,00021	(+)
F17	0,00021	(+)	0,00052	(+)	0,00052	(+)	0,00052	(+)
F18	0,00021	(+)	1	(-)	1	(-)	8,86E-05	(+)
F19	0,2627	(-)	0,00052	(+)	0,00052	(+)	0,00052	(+)
F20	0,0674	(-)	0,0209	(+)	6,83E-05	(+)	0,3007	(-)
F21	0,2471	(-)	0,0579	(-)	2,31E-05	(+)	1,40E-04	(+)
F22	2,31E-05	(+)	3,68E-05	(+)	7,74E-06	(+)	8,43E-05	(+)
F23	0,0015	(+)	0,0016	(+)	3,93E-04	(+)	0,0221	(+)

Table 11. The statistical results of the PSO and other algorithms using the WSR Test.

F	PSO-GWO		PSO-KHO		PSO-TSA		PSO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F1	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F2	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F3	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F4	0,00021	(+)	0,1084	(-)	0,00021	(+)	1,03E-04	(+)
F5	0,0400	(+)	1,20E-04	(+)	0,00021	(+)	0,0569	(-)
F6	1,20E-04	(+)	1,41E-04	(+)	0,00021	(+)	1,21E-04	(+)
F7	1,20E-04	(+)	0,0930	(-)	0,00021	(+)	1,03E-04	(+)
F8	0,0028	(+)	1,20E-04	(+)	0,00021	(+)	1,63E-04	(+)
F9	0,00124	(+)	0,00125	(+)	0,00125	(+)	0,00125	(+)
F10	0,00125	(+)	7,74E-06	(+)	1,03E-03	(+)	1,03E-03	(+)
F11	4,49E-04	(+)	3,90E-04	(+)	0,00021	(+)	1,21E-04	(+)
F12	0,3317	(-)	0,1258	(-)	1,20E-04	(+)	0,00021	(+)
F13	6,81E-04	(+)	1,39E-04	(+)	1,20E-04	(+)	0,0620	(-)
F14	0,3317	(-)	0,0259	(+)	9,75E-05	(+)	0,0251	(+)
F15	0,8519	(-)	6,31E-05	(+)	0,6012	(-)	0,3703	(-)
F16	1,91E-04	(+)	7,74E-06	(+)	1	(-)	0,00021	(+)
F17	1,21E-04	(+)	6,84E-06	(+)	6,85E-06	(+)	6,85E-06	(+)
F18	1,20E-04	(+)	1	(-)	1	(-)	0,00021	(+)
F19	6,66E-05	(+)	7,74E-06	(+)	1	(-)	1	(-)
F20	0,0793	(-)	0,0209	(+)	0,0039	(+)	0,1762	(-)
F21	0,1790	(-)	0,0579	(-)	0,0020	(+)	0,1259	(-)
F22	0,5257	(-)	3,68E-05	(+)	0,0625	(-)	0,3811	(-)
F23	0,0019	(+)	0,0016	(+)	1,22E-04	(+)	0,0123	(+)

Table 12. The statistical results of the TSA and other algorithms using the WSR Test.

F	TSA-GWO		TSA-KHO		TSA-PSO		TSA-Jaya	
	p	sign	p	sign	p	sign	p	sign
F1	0,00021	(+)	0,00021	(+)	0,00021	(+)	1,20E-04	(+)
F2	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,0022	(+)
F3	0,00021	(+)	0,00021	(+)	0,00021	(+)	1,20E-04	(+)
F4	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F5	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F6	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,4553	(-)
F7	1,03E-04	(+)	1,03E-04	(+)	0,00021	(+)	0,0620	(-)
F8	4,49E-04	(+)	1,03E-04	(+)	0,00021	(+)	1,03E-04	(+)
F9	0,00021	(+)	1,03E-04	(+)	0,00021	(+)	0,2959	(-)
F10	0,00021	(+)	7,74E-06	(+)	0,00021	(+)	1,20E-04	(+)
F11	0,00021	(+)	1,03E-04	(+)	0,00021	(+)	1,03E-04	(+)
F12	1,03E-04	(+)	1,03E-04	(+)	0,00021	(+)	1,20E-04	(+)
F13	0,00031	(+)	0,00031	(+)	0,00031	(+)	0,00031	(+)
F14	1,22E-04	(+)	7,74E-06	(+)	9,75E-05	(+)	1,03E-04	(+)
F15	0,0731	(-)	1,03E-04	(+)	0,6012	(-)	1,03E-04	(+)
F16	1,91E-04	(+)	7,74E-06	(+)	1	(-)	1,03E-04	(+)
F17	0,00021	(+)	0,00031	(+)	0,00031	(+)	0,00031	(+)
F18	0,00021	(+)	1	(-)	1	(-)	1,20E-04	(+)
F19	0,00021	(+)	7,74E-06	(+)	1	(-)	1	(-)
F20	0,00021	(+)	7,74E-06	(+)	0,0039	(+)	2,18E-04	(+)
F21	0,00021	(+)	2,31E-05	(+)	0,0020	(+)	1,03E-04	(+)
F22	0,00021	(+)	7,74E-06	(+)	0,0625	(-)	1,22E-04	(+)
F23	0,0013	(+)	3,93E-04	(+)	1,22E-04	(+)	2,44E-04	(+)

Table 13. The statistical results of the Jaya and other algorithms using the WSR Test.

F	Jaya-GWO		Jaya-KHO		Jaya-PSO		Jaya-TSA	
	p	sign	p	sign	p	sign	p	sign
F1	0,00021	(+)	1,03E-04	(+)	0,00021	(+)	1,03E-04	(+)
F2	0,00021	(+)	0,0137	(+)	0,00021	(+)	0,0022	(+)
F3	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F4	0,00021	(+)	0,00021	(+)	1,03E-04	(+)	0,00021	(+)
F5	1,03E-04	(+)	0,00021	(+)	0,0569	(-)	0,00021	(+)
F6	0,00022	(+)	0,00021	(+)	0,00022	(+)	0,4553	(-)
F7	0,00042	(+)	0,00021	(+)	0,00042	(+)	0,0620	(-)
F8	1,20E-04	(+)	0,00021	(+)	1,63E-04	(+)	1,03E-04	(+)
F9	0,00021	(+)	0,00021	(+)	0,00022	(+)	0,2959	(-)
F10	0,00021	(+)	7,74E-06	(+)	0,00042	(+)	0,00022	(+)
F11	0,00021	(+)	6,66E-05	(+)	1,20E-04	(+)	0,00042	(+)
F12	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F13	0,0206	(+)	6,66E-05	(+)	0,0620	(-)	6,66E-05	(+)
F14	0,0017	(+)	6,66E-05	(+)	0,0251	(+)	6,66E-05	(+)
F15	0,0731	(-)	6,66E-05	(+)	0,3703	(-)	6,66E-05	(+)
F16	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F17	0,00021	(+)	0,00011	(+)	0,00011	(+)	0,00011	(+)
F18	0,0015	(+)	0,00011	(+)	0,00011	(+)	0,00011	(+)

Table 13. Continued.

F	Jaya-GWO		Jaya-KHO		Jaya-PSO		Jaya-TSA	
	p	sign	p	sign	p	sign	p	sign
F19	6,66E-05	(+)	7,74E-06	(+)	1	(-)	1	(-)
F20	0,8813	(-)	0,3007	(-)	0,1762	(-)	2,18E-04	(+)
F21	6,66E-05	(+)	1,40E-04	(+)	0,1259	(-)	6,66E-05	(+)
F22	0,0090	(+)	8,43E-05	(+)	0,3811	(-)	1,22E-04	(+)
F23	0,0333	(+)	0,0221	(+)	0,0123	(+)	2,44E-04	(+)

Table 14. The results of the GWO and KHO are compared with other algorithms (AOA, EMA, FHO, HBA) for dimension as 30.

No.		GWO	KHO	AOA	EMA	FHO	HBA
F1	*Avrg.*	6,39E-59	8,00E-03	2,04E-23	4,04E-18	4,32E-80	2,59E-276
	Stdd.	1,93E-58	6,31E-03	8,87E-23	1,61E-17	1,88E-79	0,000000
	Best	3,79E-61	1,40E-03	1,59E-253	2,14E-22	9,20E-96	2,94E-288
F2	*Avrg.*	1,65E-34	9,30E+01	0,000000	1,26E-15	3,61E-20	2,19E-145
	Stdd.	2,41E-34	6,21E+01	0,000000	1,95E-15	1,31E-19	8,82E-145
	Best	7,23E-36	2,15E+01	0,000000	1,73E-17	6,97E-23	4,93E-151
F3	*Avrg.*	2,92E-15	1,93E-11	4,89E-03	1,47E+03	5,59E-77	5,19E-201
	Stdd.	1,06E-14	5,13E-11	9,64E-03	1,86E+03	2,44E-76	0,000000
	Best	1,58E-19	2,51E-13	8,46E-272	8,97E-01	2,41E-90	9,37E-216
F4	*Avrg.*	1,68E-14	3,82E-01	1,89E-02	7,62E-03	2,47E-34	2,63E-115
	Stdd.	2,54E-14	8,62E-02	2,09E-02	1,08E-02	4,82E-34	1,14E-114
	Best	6,06E-16	2,38E-01	2,50E-122	1,96E-04	9,73E-40	6,96E-122
F5	*Avrg.*	2,70E+01	0,000000	2,82E+01	2,68E+01	1,73E-01	2,17E+01
	Stdd.	8,35E-01	0,000000	3,47E-01	1,25E-01	9,36E-02	6,25E-01
	Best	2,52E+01	0,000000	2,75E+01	2,65E+01	4,90E-02	2,01E+01
F6	*Avrg.*	7,47E-01	4,76E-03	2,67E+00	8,44E-06	1,44E+00	1,74E-07
	Stdd.	3,80E-01	2,94E-03	1,89E-01	1,28E-05	2,34E+00	5,01E-07
	Best	2,51E-01	1,18E-03	2,34E+00	5,09E-07	1,26E-02	1,09E-09
F7	*Avrg.*	9,13E-04	1,21E-02	4,54E-05	3,07E-02	9,93E-04	1,56E-04
	Stdd.	4,54E-04	3,76E-03	4,91E-05	1,10E-02	4,04E-04	8,97E-05
	Best	1,88E-04	6,62E-03	1,74E-06	9,99E-03	3,86E-04	2,83E-05
F8	*Avrg.*	-5,94E+03	-5,17E+00	-5,77E+03	-8,56E+03	-1,26E+04	-8,75E+03
	Stdd.	6,39E+02	4,34E+01	3,77E+02	6,10E+02	3,24E-02	1,19E+03
	Best	-7,67E+03	-5,97E+00	-6,53E+03	-9,47E+03	-1,26E+04	-1,05E+04
F9	*Avrg.*	1,04E-01	-1,91E+00	0,000000	2,51E+00	0,000000	0,000000
	Stdd.	4,54E-01	3,17E+01	0,000000	6,33E+00	0,00E+00	0,000000
	Best	0,000000	-2,41E+00	0,000000	0,000000	0,00E+00	0,000000
F10	*Avrg.*	1,71E-14	-9,75E+27	8,88E-16	9,96E-11	8,88E-16	1,99E+00
	Stdd.	3,07E-15	2,75E+26	0,000000	1,26E-10	0,00E+00	5,98E+00
	Best	1,51E-14	-1,03E+28	8,88E-16	1,39E-12	8,88E-16	8,88E-16
F11	*Avrg.*	4,15E-03	5,82E-03	1,02E-01	4,07E-03	0,000000	0,000000
	Stdd.	7,48E-03	3,55E-03	8,74E-02	8,85E-03	0,000000	0,000000
	Best	0,000000	2,70E+00	1,34E-03	0,000000	0,000000	0,000000
F12	*Avrg.*	3,39E-02	1,56E-01	4,01E-01	1,90E-05	1,62E-03	**1,52E-08**
	Stdd.	1,94E-02	6,95E-01	4,06E-02	8,24E-05	1,21E-03	2,34E-08
	Best	6,95E-03	1,43E-13	3,07E-01	8,84E-09	6,39E-04	9,13E-10

Table 14. Continued.

No.		GWO	KHO	AOA	EMA	FHO	HBA
F13	*Avrg.*	4,44E-01	1,28E-10	2,81E+00	2,55E-02	1,23E-02	1,06E-01
	Stdd.	2,26E-01	2,00E-10	1,06E-01	3,90E-02	6,81E-03	1,41E-01
	Best	2,52E-05	1,02E-13	2,61E+00	2,53E-07	2,67E-03	1,06E-07
F14	*Avrg.*	4,82E+00	4,95E+00	9,79E+00	2,43E+00	1,68E+00	1,10E+00
	Stdd.	4,20E+00	4,25E+00	4,20E+00	2,44E+00	8,33E-01	4,32E-01
	Best	9,98E-01	9,98E-01	9,98E-01	9,98E-01	9,98E-01	9,98E-01
F15	*Avrg.*	3,32E-03	4,00E-03	2,06E-02	1,78E-03	1,53E-03	6,80E-03
	Stdd.	7,16E-03	9,25E-03	3,54E-02	4,27E-03	1,43E-03	9,63E-03
	Best	3,07E-04	3,41E-04	3,32E-04	3,89E-04	3,42E-04	3,07E-04
F16	*Avrg.*	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00
	Stdd.	5,95E-09	4,88E-09	9,06E-08	0,000000	2,06E-05	0,00E+00
	Best	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00	-1,03E+00
F17	*Avrg.*	3,98E-01	7,78E+00	4,05E-01	Inf	Inf	Inf
	Stdd.	5,41E-05	9,11E-16	8,02E-03	Inf	Inf	Inf
	Best	3,98E-01	7,78E+00	3,99E-01	Inf	Inf	Inf
F18	*Avrg.*	7,05E+00	3,00E+00	1,55E+01	4,35E+00	3,00E+00	8,40E+00
	Stdd.	1,77E+01	2,63E-09	2,19E+01	5,88E+00	1,83E-03	1,83E+01
	Best	3,00E+00	3,00E+00	3,00E+00	3,00E+00	3,00E+00	3,00E+00
F19	*Avrg.*	-3,86E+00	-3,86E+00	-3,85E+00	-3,86E+00	-3,82E+00	-3,86E+00
	Stdd.	3,09E-03	2,09E-02	3,05E-03	1,72E-03	7,18E-02	2,81E-03
	Best	-3,86E+00	-3,86E+00	-3,86E+00	-3,86E+00	-3,86E+00	-3,86E+00
F20	*Avrg.*	-3,25E+00	-3,22E+00	-3,06E+00	-3,25E+00	-3,19E+00	-3,24E+00
	Stdd.	8,12E-02	8,12E-01	1,11E-01	5,82E-02	9,84E-02	7,93E-02
	Best	-3,32E+00	-3,32E+00	-3,20E+00	-3,32E+00	-3,30E+00	-3,32E+00
F21	*Avrg.*	-9,14E+00	-8,14E+00	-3,87E+00	-4,52E+00	-9,10E+00	-9,31E+00
	Stdd.	2,02E+00	2,82E+00	1,09E+00	2,60E+00	1,04E+00	2,53E+00
	Best	-1,02E+01	-1,02E+01	-6,40E+00	-1,02E+01	-1,00E+01	-1,02E+01
F22	*Avrg.*	-1,01E+01	-1,01E+01	-4,36E+00	-6,81E+00	-9,27E+00	-9,64E+00
	Stdd.	1,52E+00	1,55E+00	1,31E+00	3,33E+00	1,04E+00	2,31E+00
	Best	-1,04E+01	-1,04E+01	-7,95E+00	-1,04E+01	-1,01E+01	-1,04E+01
F23	*Avrg.*	-1,03E+01	-1,02E+01	-4,03E+00	-6,00E+00	-9,47E+00	-7,86E+00
	Stdd.	1,18E+00	1,22E+00	1,83E+00	3,43E+00	4,94E-01	3,67E+00
	Best	-1,05E+01	-1,05E+01	-8,91E+00	-1,05E+01	-1,03E+01	-1,05E+01
Rank		**2**	**2**	**4**	**5**	**3**	**1**

Table 15. The statistical results of the GWO and other algorithms (AOA, EMA, FHO, HBA) using the WSR Test.

F	GWO-AOA		GWO-EMA		GWO-FHO		GWO-HBA	
	p	sign	p	sign	p	sign	p	sign
F1	0,0137	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F2	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F3	1	(-)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F4	0,1454	(-)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F5	3,38E-04	(+)	0,2043	(-)	0,00021	(+)	0,00021	(+)
F6	0,00021	(+)	0,00021	(+)	0,9405	(-)	0,00021	(+)
F7	0,00021	(+)	0,00021	(+)	0,4781	(-)	0,00021	(+)

Table 15. Continued.

F	GWO-AOA		GWO-EMA		GWO-FHO		GWO-HBA	
	p	sign	p	sign	p	sign	p	sign
F8	0,6012	(-)	0,00021	(+)	0,0010	(+)	0,00021	(+)
F9	0,0313	(+)	0,0801	(-)	0,0313	(+)	0,0313	(+)
F10	4,26E-05	(+)	0,0010	(+)	4,26E-05	(+)	0,0113	(+)
F11	0,00021	(+)	1	(-)	0,0625	(-)	0,0625	(-)
F12	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F13	0,00021	(+)	0,00021	(+)	1,03E-04	(+)	1,63E-04	(+)
F14	0,0136	(+)	0,0527	(-)	0,0206	(+)	2,44E-04	(+)
F15	0,0028	(+)	0,0569	(-)	0,0731	(-)	0,7172	(-)
F16	2,92E-04	(+)	1,91E-05	(+)	0,00021	(+)	1,91E-04	(+)
F17	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,0010	(+)
F18	0,7256	(-)	0,0013	(+)	0,0015	(+)	0,0111	(+)
F19	0,0010	(+)	0,0010	(+)	0,0304	(+)	0,0090	(+)
F20	4,49E-04	(+)	0,5016	(-)	0,0930	(-)	0,2180	(-)
F21	1,03E-04	(+)	0,0010	(+)	0,1913	(-)	0,0137	(+)
F22	1,03E-04	(+)	0,0479	(+)	0,0015	(+)	0,0124	(+)
F23	0,0010	(+)	0,0040	(+)	0,0015	(+)	0,6012	(-)

Table 16. The statistical results of the KHO and other algorithms (AOA, EMA, FHO, HBA) using the WSR Test.

F	KHO-AOA		KHO-EMA		KHO-FHO		KHO-HBA	
	p	sign	p	sign	p	sign	p	sign
F1	0,00254	(+)	1,19E-05	(+)	0,00254	(+)	0,00254	(+)
F2	0,00254	(+)	0,00475	(+)	0,00254	(+)	0,00254	(+)
F3	1	(-)	0,00475	(+)	0,00254	(+)	0,00254	(+)
F4	6,94E-05	(+)	0,00475	(+)	0,00254	(+)	0,00254	(+)
F5	0,00253	(+)	0,00475	(+)	0,00475	(+)	0,00475	(+)
F6	0,00475	(+)	0,00475	(+)	0,00475	(+)	0,00475	(+)
F7	0,00475	(+)	1,03E-04	(+)	0,00475	(+)	0,00475	(+)
F8	0,00475	(+)	0,00487	(+)	0,00254	(+)	0,00245	(+)
F9	0,0015	(+)	0,0015	(+)	0,00254	(+)	0,00254	(+)
F10	0,0015	(+)	0,0015	(+)	0,00254	(+)	0,00254	(+)
F11	1,03E-04	(+)	0,0015	(-)	0,00254	(+)	0,00254	(+)
F12	0,0015	(+)	0,0015	(+)	0,00475	(+)	0,00475	(+)
F13	0,0015	(+)	0,00478	(+)	0,00475	(+)	0,00475	(+)
F14	5,91E-04	(+)	0,0015	(+)	0,00254	(+)	0,00254	(+)
F15	0,2627	(-)	0,0015	(+)	2,19E-04	(+)	1	(-)
F16	0,00475	(+)	0,0015	(+)	0,0015	(+)	0,00254	(+)
F17	0,00475	(+)	0,0015	(+)	0,0015	(+)	0,00254	(+)
F18	0,00475	(+)	1	(-)	0,00475	(+)	0,5000	(-)
F19	0,00245	(+)	3,93E-04	(+)	0,9108	(-)	0,0529	(-)
F20	0,00475	(+)	0,2974	(-)	0,4330	(-)	0,0135	(+)
F21	0,00475	(+)	0,00254	(+)	0,0015	(+)	0,0069	(+)
F22	0,00475	(+)	0,00254	(+)	0,0015	(+)	1,71E-05	(+)
F23	0,00475	(+)	0,0038	(+)	0,0015	(+)	0,5890	(-)

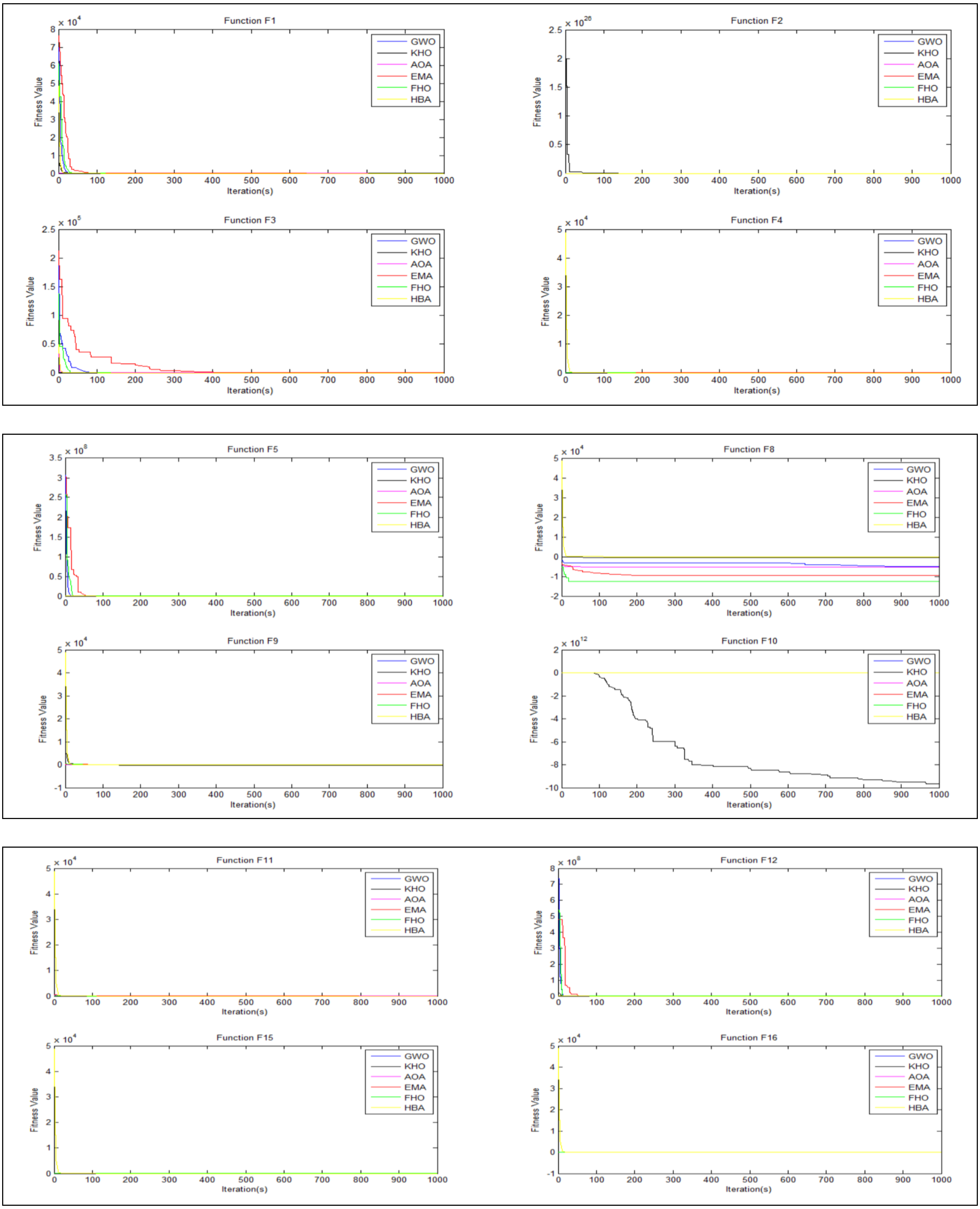

Figure 7. Convergence graphs for GWO, KHO, AOA, EMA, FHO, and HBA.

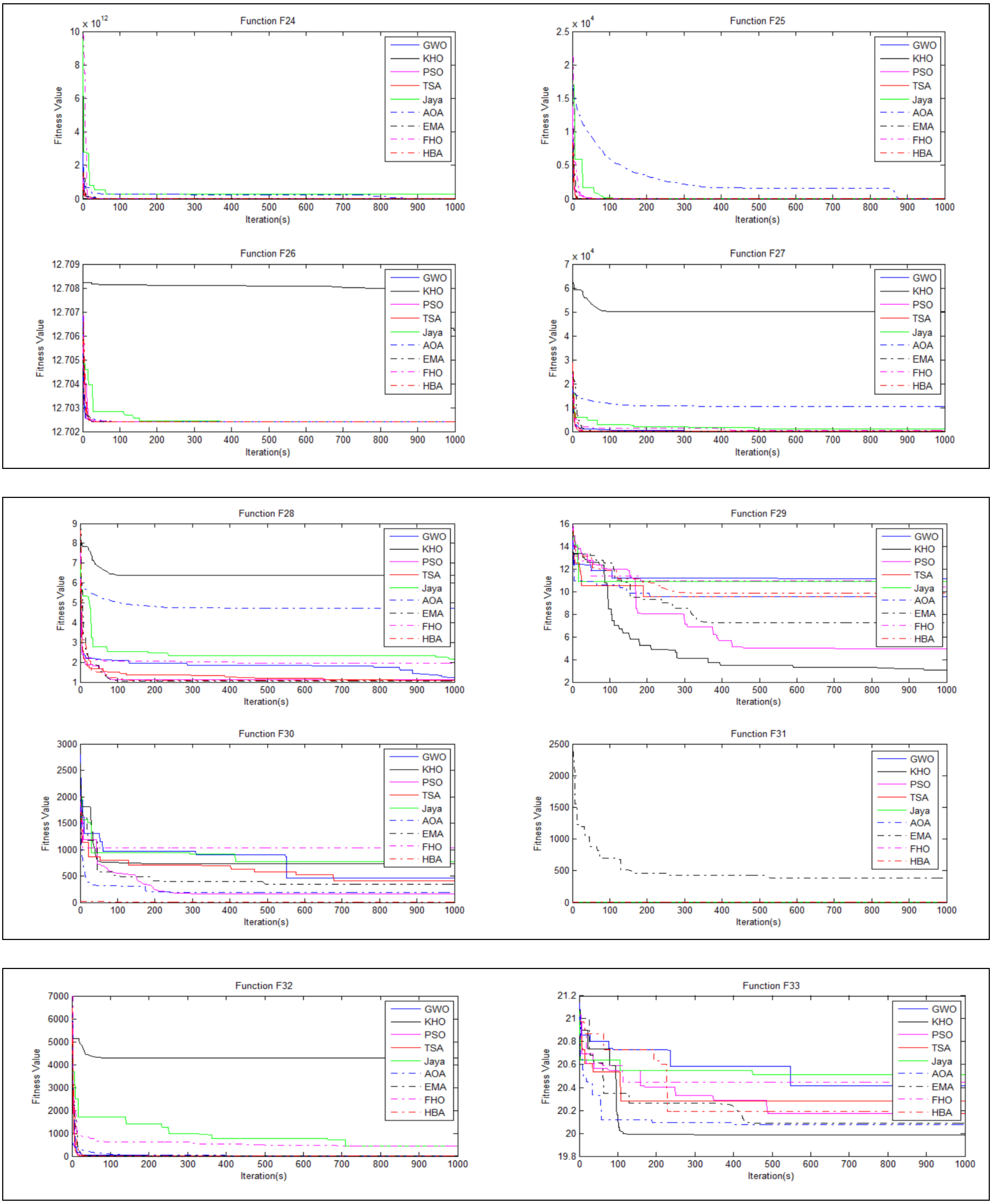

Figure 8. Convergence graphs for GWO, KHO, PSO, TSA, Jaya, AOA, EMA, FHO, and HBA on CEC-C06 2019 test functions.

Table 17. The parameter settings for algorithms.

Dimension size	Population size	Maximum iteration
{9, 10, 16, 18}	30	1000

Table 18. Description of CEC-C06 2019 Test Functions [40].

ID	Functions	Dimension	Range	f_{min}
F24	Storn's Chebyshev Polynomial Fitting Problem	9	[-8192, 8192]	1
F25	Inverse Hilbert Matrix Problem	16	[-16384, 16384]	1
F26	Lennard-Jones Minimum Energy Cluster	18	[-4, 4]	1
F27	Rastrigin's Function	10	[-100, 100]	1
F28	Griewank's Function	10	[-100, 100]	1
F29	Weierstrass Function	10	[-100, 100]	1
F30	Modified Schwefel's Function	10	[-100, 100]	1
F31	Expanded Schaffer's F6 Function	10	[-100, 100]	1
F32	Happy Cat Function	10	[-100, 100]	1
F33	Ackley Function	10	[-100, 100]	1

Table 19. The results of the GWO and KHO are compared with other algorithms (PSO, TSA, Jaya) on CEC-C06 2019 test functions.

No.		GWO	KHO	PSO	TSA	Jaya
	Avrg.	9,46E+07	4,64E+04	1,48E+08	8,68E+09	1,43E+11
F24	*Stdd.*	1,75E+08	7,67E+03	1,59E+08	3,04E+09	8,24E+10
	Best	3,90E+04	4,33E+04	1,07E+07	3,31E+09	2,23E+10
	Avrg.	1,73E+01	1,73E+01	1,73E+01	1,73E+01	1,75E+01
F25	*Stdd.*	1,35E-04	8,72E-06	0,000000	0,000000	4,82E-02
	Best	1,73E+01	1,73E+01	1,73E+01	1,73E+01	1,74E+01
	Avrg.	1,27E+01	1,27E+01	1,27E+01	1,27E+01	1,27E+01
F26	*Stdd.*	2,24E-04	6,64E-09	5,33E-15	2,99E-08	2,85E-04
	Best	1,27E+01	1,27E+01	1,27E+01	1,27E+01	1,27E+01
	Avrg.	1,48E+02	5,00E+04	2,24E+01	3,68E+01	1,89E+03
F27	*Stdd.*	2,82E+02	1,46E-11	1,16E+01	6,01E+00	8,29E+02
	Best	2,13E+01	5,00E+04	7,96E+00	1,98E+01	9,72E+02
	Avrg.	1,34E+00	6,21E+00	1,09E+00	1,36E+00	2,21E+00
F28	*Stdd.*	2,21E-01	8,97E-02	5,86E-02	9,40E-02	1,20E-01
	Best	1,07E+00	6,16E+00	1,01E+00	1,13E+00	2,06E+00
	Avrg.	1,08E+01	2,06E+00	5,76E+00	1,06E+01	1,06E+01
F29	*Stdd.*	5,34E-01	7,49E-01	1,53E+00	7,00E-01	7,61E-01
	Best	9,56E+00	1,53E+00	3,31E+00	8,79E+00	8,82E+00
	Avrg.	3,37E+02	2,34E+02	1,87E+02	6,57E+02	7,46E+02
F30	*Stdd.*	1,66E+02	3,39E+01	1,52E+02	1,63E+02	1,38E+02
	Best	6,94E+01	2,14E+02	-5,25E+01	3,83E+02	4,36E+02
	Avrg.	4,58E+00	5,80E+00	4,85E+00	5,82E+00	6,31E+00
F31	*Stdd.*	8,19E-01	5,36E-02	7,94E-01	3,44E-01	2,55E-01
	Best	3,26E+00	5,74E+00	3,21E+00	5,06E+00	5,83E+00
	Avrg.	4,41E+00	4,29E+03	2,37E+00	2,42E+00	3,98E+02
F32	*Stdd.*	7,48E-01	1,12E-07	2,34E-02	3,19E-02	1,59E+02
	Best	3,24E+00	4,29E+03	2,34E+00	2,36E+00	1,31E+02
	Avrg.	2,04E+01	2,00E+01	1,90E+01	2,04E+01	2,05E+01
F33	*Stdd.*	7,48E-02	1,14E-04	4,36E+00	7,09E-02	8,39E-02
	Best	2,03E+01	2,00E+01	1,51E-14	2,03E+01	2,03E+01
Rank		**3**	**2**	**1**	**4**	**5**

Table 20. On the test functions for CEC-C06 2019, the performance of the GWO and KHO is compared with that of other algorithms (AOA, EMA, FHO, and HBA).

No.		GWO	KHO	AOA	EMA	FHO	HBA
F24	*Avrg.*	9,46E+07	4,64E+04	2,91E+06	2,16E+09	7,33E+04	4,02E+04
	Stdd.	1,75E+08	7,67E+03	5,54E+05	1,80E+09	6,64E+04	4,18E+02
	Best	3,90E+04	4,33E+04	9,99E+05	1,22E+08	3,57E+04	3,84E+04
F25	*Avrg.*	1,73E+01	1,73E+01	1,91E+01	1,73E+01	1,74E+01	1,73E+01
	Stdd.	1,35E-04	8,72E-06	3,80E-01	0,000000	1,53E-02	0,000000
	Best	1,73E+01	1,73E+01	1,84E+01	1,73E+01	1,74E+01	1,73E+01
F26	*Avrg.*	1,27E+01	1,27E+01	1,27E+01	1,27E+01	1,74E+01	1,27E+01
	Stdd.	2,24E-04	6,64E-09	7,48E-04	2,50E-08	3,74E-02	5,33E-15
	Best	1,27E+01	1,27E+01	1,27E+01	1,27E+01	1,74E+01	1,27E+01
F27	*Avrg.*	1,48E+02	5,00E+04	1,75E+04	1,24E+02	1,74E+01	2,75E+01
	Stdd.	2,82E+02	1,46E-11	1,19E+03	1,14E+01	1,66E-02	1,27E+01
	Best	2,13E+01	5,00E+04	1,51E+04	1,08E+02	1,74E+01	1,39E+01
F28	*Avrg.*	1,34E+00	6,21E+00	3,70E+00	1,35E+00	1,74E+01	3,04E+01
	Stdd.	2,21E-01	8,97E-02	9,07E-01	1,13E-02	1,58E-02	2,06E+01
	Best	1,07E+00	6,16E+00	2,08E+00	1,25E+00	1,74E+01	6,96E+00
F29	*Avrg.*	1,08E+01	2,06E+00	8,28E+00	7,02E+00	1,11E+01	9,49E+00
	Stdd.	5,34E-01	7,49E-01	7,19E-01	1,38E+00	8,31E-01	1,74E+00
	Best	9,56E+00	1,53E+00	6,57E+00	4,66E+00	9,11E+00	5,28E+00
F30	*Avrg.*	3,37E+02	2,34E+02	4,13E+02	4,49E+02	1,16E+03	4,66E+02
	Stdd.	1,66E+02	3,39E+01	1,09E+02	2,14E+02	2,07E+02	4,03E+02
	Best	6,94E+01	2,14E+02	1,00E+02	7,16E+01	7,75E+02	-8,91E+01
F31	*Avrg.*	4,58E+00	5,80E+00	5,48E+00	5,44E+00	1,16E+03	5,16E+00
	Stdd.	8,19E-01	5,36E-02	5,32E-01	6,30E-01	2,66E+02	7,71E-01
	Best	3,26E+00	5,74E+00	4,53E+00	4,13E+00	6,32E+02	3,78E+00
F32	*Avrg.*	4,41E+00	4,29E+03	2,35E+02	2,40E+00	1,11E+03	2,37E+00
	Stdd.	7,48E-01	1,12E-07	1,82E+02	2,82E-02	2,52E+02	2,32E-02
	Best	3,24E+00	4,29E+03	6,43E+00	2,36E+00	5,47E+02	2,34E+00
F33	*Avrg.*	2,04E+01	2,00E+01	2,01E+01	2,00E+01	1,22E+03	2,03E+01
	Stdd.	7,48E-02	1,14E-04	7,79E-02	6,33E-02	2,76E+02	1,39E-01
	Best	2,03E+01	2,00E+01	1,98E+01	2,00E+01	7,75E+02	2,00E+01
Rank		**2**	**1**	**4**	**3**	**4**	**2**

Table 21. The statistical results of the GWO and other algorithms (PSO, TSA, Jaya) using the WSR Test.

F	GWO-KHO		GWO-PSO		GWO-TSA		GWO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F24	1,20E-04	(+)	0,0276	(+)	0,00021	(+)	0,00021	(+)
F25	0,00042	(+)	0,00042	(+)	0,00021	(+)	0,00021	(+)
F26	1	(-)	0,1250	(-)	0,7480	(-)	0,0013	(+)
F27	0,00042	(+)	5,17E-04	(+)	0,0072	(+)	0,00021	(+)
F28	0,00021	(+)	1,89E-04	(+)	0,7369	(-)	0,00021	(+)
F29	0,00021	(+)	0,00042	(+)	0,7089	(-)	0,6813	(-)
F30	0,0251	(+)	0,0051	(+)	5,17E-04	(+)	1,20E-04	(+)
F31	2,93E-04	(+)	0,2471	(-)	3,90E-04	(+)	1,03E-04	(+)
F32	1,63E-04	(+)	0,00021	(+)	0,00023	(+)	1,63E-04	(+)
F33	1,63E-04	(+)	0,00021	(+)	0,8228	(-)	0,1560	(-)

Table 22. The statistical results of the GWO and other algorithms (AOA, EMA, FHO, HBA) using the WSR Test.

F	GWO-KHO		GWO-PSO		GWO-TSA		GWO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F24	0,0100	(+)	0,00758	(+)	0,00758	(+)	1,03E-04	(+)
F25	0,00758	(+)	0,00758	(+)	0,00758	(+)	0,0354	(+)
F26	0,0048	(+)	0,3125	(-)	0,00758	(+)	0,1250	(-)
F27	0,00758	(+)	0,0137	(+)	0,00758	(+)	0,0017	(+)
F28	0,00758	(+)	0,5503	(-)	0,00475	(+)	0,0245	(+)
F29	0,00758	(+)	0,00241	(+)	0,1913	(-)	0,0206	(+)
F30	0,1354	(-)	0,1084	(-)	0,00758	(+)	0,5016	(-)
F31	0,0019	(+)	0,0022	(+)	0,00758	(+)	0,0137	(+)
F32	0,00758	(+)	0,00758	(+)	0,00758	(+)	0,00654	(+)
F33	0,00758	(+)	0,00758	(+)	0,00758	(+)	0,0206	(+)

Table 23. The statistical results of the KHO and other algorithms (PSO, TSA, Jaya) using the WSR Test.

F	GWO-KHO		GWO-PSO		GWO-TSA		GWO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F24	1,20E-04	(+)	0,00042	(+)	0,00042	(+)	0,00021	(+)
F25	0,00042	(+)	4,38E-05	(+)	4,37E-05	(+)	0,00021	(+)
F26	1	(-)	0,0156	(+)	0,2969	(-)	0,00021	(+)
F27	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F28	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F29	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F30	0,0251	(+)	0,1169	(-)	0,00021	(+)	0,00021	(+)
F31	2,93E-04	(+)	1,63E-04	(+)	0,7369	(-)	0,00021	(+)
F32	0,00023	(+)	0,00023	(+)	0,00021	(+)	0,00021	(+)
F33	0,00023	(+)	0,0015	(+)	0,00021	(+)	0,00021	(+)

Table 24. The statistical results of the KHO and other algorithms (AOA, EMA, FHO, HBA) using the WSR Test.

F	GWO-KHO		GWO-PSO		GWO-TSA		GWO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F24	5,10E-05	(+)	0,00124	(+)	0,0045	(+)	0,00745	(+)
F25	0,00124	(+)	0,00458	(+)	0,00475	(+)	0,00456	(+)
F26	0,00124	(+)	0,2344	(-)	0,00124	(+)	0,0156	(+)
F27	0,00745	(+)	0,00245	(+)	0,00214	(+)	0,00124	(+)
F28	0,00124	(+)	0,00654	(+)	0,00124	(+)	0,00124	(+)
F29	0,00124	(+)	0,00124	(+)	0,00124	(+)	0,00124	(+)
F30	0,00124	(+)	0,0011	(+)	0,00124	(+)	0,0228	(+)
F31	0,0479	(+)	0,0333	(+)	0,00124	(+)	0,0036	(+)
F32	0,00124	(+)	0,00124	(+)	0,00124	(+)	0,00124	(+)
F33	0,00475	(+)	0,00142	(+)	0,00124	(+)	0,00241	(+)

Table 25. The statistical results of the PSO and other algorithms using the WSR Test.

F	GWO-KHO		GWO-PSO		GWO-TSA		GWO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F24	0,0276	(+)	0,00042	(+)	0,00042	(+)	0,00021	(+)
F25	0,00042	(+)	4,38E-05	(+)	1	(-)	0,00021	(+)
F26	0,1250	(-)	0,0156	(+)	0,0313	(+)	0,00021	(+)
F27	5,17E-04	(+)	0,00021	(+)	0,0017	(+)	0,00021	(+)
F28	1,89E-04	(+)	0,00021	(+)	1,20E-04	(+)	0,00021	(+)
F29	0,00033	(+)	0,00021	(+)	0,00033	(+)	0,00033	(+)

Table 25. Continued.

F	GWO-KHO		GWO-PSO		GWO-TSA		GWO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F30	0,0051	(+)	0,1169	(-)	1,03E-04	(+)	1,03E-04	(+)
F31	0,2471	(-)	1,63E-04	(+)	5,93E-04	(+)	0,00021	(+)
F32	0,00021	(+)	0,00033	(+)	1,89E-04	(+)	0,00021	(+)
F33	0,00021	(+)	0,0015	(+)	0,00033	(+)	0,00021	(+)

Table 26. The statistical results of the TSA and other algorithms using the WSR Test.

F	GWO-KHO		GWO-PSO		GWO-TSA		GWO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F24	0,00021	(+)	0,00086	(+)	0,00086	(+)	0,00021	(+)
F25	0,00021	(+)	4,37E-05	(+)	1	(-)	0,00021	(+)
F26	0,7480	(-)	0,2969	(-)	0,0313	(+)	0,00021	(+)
F27	0,0072	(+)	0,00021	(+)	0,0017	(+)	0,00021	(+)
F28	0,7369	(-)	0,00021	(+)	1,20E-04	(+)	0,00021	(+)
F29	0,7089	(-)	0,00021	(+)	0,00086	(+)	0,4781	(-)
F30	5,17E-04	(+)	0,00021	(+)	1,03E-04	(+)	0,1354	(-)
F31	3,90E-04	(+)	0,7369	(-)	5,93E-04	(+)	4,49E-04	(+)
F32	0,00042	(+)	0,00022	(+)	1,89E-04	(+)	0,00021	(+)
F33	0,8228	(-)	0,00022	(+)	0,00022	(+)	0,0438	(+)

Table 27. The statistical results of the Jaya and other algorithms using the WSR Test.

F	GWO-KHO		GWO-PSO		GWO-TSA		GWO-Jaya	
	p	sign	p	sign	p	sign	p	sign
F24	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F25	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F26	0,0013	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F27	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F28	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F29	0,6813	(-)	0,00021	(+)	0,00021	(+)	0,4781	(-)
F30	1,20E-04	(+)	0,00021	(+)	1,03E-04	(+)	0,1354	(-)
F31	1,03E-04	(+)	0,00021	(+)	0,00021	(+)	4,49E-04	(+)
F32	0,00021	(+)	0,00021	(+)	0,00021	(+)	0,00021	(+)
F33	0,1560	(-)	0,00021	(+)	0,00021	(+)	0,0438	(+)

Table 28. Parameter settings.

Parameters	Values
Population size	30
The number of time series (S)	4, 12, 16
Time series of length (T)	256
Maximum iteration	300

Table 29. The results of the GWO and KHO are compared with other algorithms (PSO, TSA, Jaya, AOA, EMA, FHO, HBA) on EEG signal datasets.

Datasets	PSO	TSA	Jaya	AOA	EMA	FHO	HBA	GWO	KHO
EEG1									
Best	1,55E+00	1,93E+01	1,28E+01	1,45E+00	1,80E+00	1,79E+00	1,13E+00	9,25E-01	2,07E+00
Avrg	1,72E+00	2,18E+01	1,37E+01	1,50E+00	2,26E+00	1,84E+00	1,22E+00	**1,01E+00**	2,11E+00
Stdd	9,28E-02	7,27E-01	5,84E-01	2,20E-02	2,66E-01	3,15E-02	3,98E-02	4,81E-02	8,26E-02

Table 29. Continued.

Datasets	PSO	TSA	Jaya	AOA	EMA	FHO	HBA	GWO	KHO
EEG2									
Best	1,52E+00	2,01E+01	1,29E+01	1,45E+00	2,01E+00	1,77E+00	1,12E+00	8,89E-01	2,14E+00
Avrg	1,72E+00	2,18E+01	1,38E+01	1,51E+00	2,24E+00	1,84E+00	1,21E+00	**9,91E-01**	2,17E+00
Stdd	1,23E-01	6,52E-01	3,90E-01	3,01E-02	2,08E-01	5,03E-02	4,98E-02	3,31E-02	3,96E-02
EEG3									
Best	3,11E+00	2,22E+01	1,37E+01	1,65E+00	2,50E+00	1,85E+00	1,42E+00	1,22E+00	2,42E+00
Avrg	3,48E+00	2,25E+01	1,42E+01	1,72E+00	1,70E+01	1,91E+00	1,49E+00	**1,30E+00**	2,50E+00
Stdd	2,19E-01	1,91E-01	3,30E-01	2,59E-02	8,44E+00	2,32E-02	3,73E-02	3,17E-02	5,12E-02
EEG4									
Best	3,05E+00	2,20E+01	1,35E+01	1,69E+00	2,44E+00	1,87E+00	1,42E+00	1,20E+00	2,46E+00
Avrg	3,45E+00	2,25E+01	1,40E+01	1,72E+00	1,90E+01	1,92E+00	1,49E+00	**1,27E+00**	2,47E+00
Stdd	2,95E-01	1,63E-01	2,91E-01	2,80E-02	7,23E+00	2,66E-02	3,24E-02	4,75E-02	2,75E-02
EEG5									
Best	3,59E+00	2,22E+01	1,36E+01	1,67E+00	1,11E+01	1,90E+00	1,46E+00	1,36E+00	2,53E+00
Avrg	3,98E+00	2,27E+01	1,40E+01	1,75E+00	2,21E+01	1,94E+00	1,56E+00	**1,43E+00**	2,60E+00
Stdd	2,63E-01	1,99E-01	1,85E-01	4,44E-02	2,53E+00	1,85E-02	5,10E-02	2,53E-02	2,27E-02
EEG6									
Best	3,46E+00	2,25E+01	1,38E+01	1,66E+00	2,25E+01	1,91E+00	1,51E+00	1,37E+00	2,49E+00
Avrg	3,99E+00	2,27E+01	1,42E+01	1,73E+00	2,27E+01	1,94E+00	1,57E+00	**1,43E+00**	2,49E+00
Stdd	3,05E-01	1,14E-01	2,31E-01	4,64E-02	1,47E-01	1,58E-02	3,61E-02	3,90E-02	4,44E-16
Rank	2	2	2	2	2	2	2	1	2

Table 30. The statistical results of the KHO and other algorithms (GWO. PSO. TSA. Jaya. AOA. EMA. FHO. HBA) using the WSR Test.

F	EEG1		EEG2		EEG3		EEG4		EEG5		EEG6	
	p	*sign*	*p*	*sign*	*p*	*sign*	*p*	*sign*	*p*	*sign*	*p*	*sign*
KHO -GWO	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
KHO -PSO	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
KHO -TSA	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
KHO Jaya	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
KHO -AOA	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000452	(+)	0,000248	(+)	0,000148	(+)
KHO -EMA	0,0522	(-)	0,3905	(-)	0,00214	(+)	0,009051	(+)	0,000248	(+)	0,000248	(+)
KHO -FHO	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
KHO -HBA	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)

Table 31. The statistical results of the GWO and other algorithms (KHO. PSO. TSA. Jaya. AOA. EMA. FHO. HBA) using the WSR Test.

F	EEG1		EEG2		EEG3		EEG4		EEG5		EEG6	
	p	*sign*	*p*	*sign*	*p*	*sign*	*p*	*sign*	*p*	*sign*	*p*	*sign*
GWO-KHO	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
GWO-PSO	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
GWO-TSA	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
GWO-Jaya	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
GWO-AOA	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
GWO-EMA	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
GWO-FHO	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)
GWO-HBA	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)	0,000248	(+)

4 Conclusion

Gray wolves prowl in large herds to get easy food. The development of Gray Wolf Optimizer (GWO) involved simulating gray wolves' foraging tactics. Krill swarms are important aquatic creatures for the ecosystem. The Krill Herd Optimizer (KHO) is based on krill herd foraging and hunting behavior. Swarm optimization is used by both GWO and KHO. GWO and KHO are both capable of solving a wide range of problems. In this study. The performance of the GWO and KHO algorithms are compared. GWO is an old successful algorithm that has been studied extensively in the literature. KHO, on the other hand, is an algorithm that has remained in the background in the literature and has not been studied much. In this study, the success degrees of GWO and KHO will be determined and it will form the basis of a future GWO-KHO hybrid study. In addition, although GWO and KHO are old algorithms, the level of competition with various old and newly proposed algorithms has been tested in detail in this study on 23 separate unimodal, multimodal, and fixed-dimension multimodal test functions in six distinct small, medium, and large scale dimensions (10, 20, 30, 50, 100, and 500). GWO and KHO algorithms are performed and their results are compared. The findings indicated that GWO outperformed KHO in the majority of the test functions. Then, Tree Seed Algorithm (TSA), Particle Swarm Algorithm (PSO), Jaya algorithms, Arithmetic Optimization Algorithm (AOA), Evolutionary Mating Algorithm (EMA), Fire Hawk Optimizer (FHO), Honey Badger Algorithm (HBA), GWO, and KHO results are compared. Obtained results are analyzed by author contribution statements performing statistical tests. The algorithms' convergence graphs are depicted and success rates are illustrated in depth using figures. According to PSO, TSA, Jaya, GWO, KHO result, the most successful average results belong to GWO. According to the average results, TSA is the most successful algorithm after GWO. According to AOA, EMA, HBA, FHO, GWO, KHO result, KHO and GWO are the second-best algorithms after HBA. This shows that although GWO and KHO are old algorithms, they still have the success to compete with newly developed algorithms in recent years. Then, by choosing a different benchmark test function (CEC-C06 2019), the identical algorithms are compared and their efficacy is demonstrated. According to mean results of PSO, TSA, Jaya, GWO, KHO, the most successful average results belong to KHO after PSO. According to mean results of AOA, EMA, FHO, HBA, GWO, KHO, the most successful average results belong to KHO. This shows that KHO has achieved good success on CEC-C06 2019 functions. Finally, the big data optimization problem is applied to the same functions running on six different EEG data sets. The best results by average result rank are obtained by GWO. As a result. GWO and KHO are still successful algorithms that can compete with old and new optimization problems on different problems.

In future studies. GWO and KHO algorithms will be hybridized to show their success in real-world problems.

5 Author contribution statements

In the scope of this study. The Emine BAS in the formation of the idea, the writing of codes, and the design. Aysegul IHSAN in the assessment of obtained results. Supplying the used and examining the results other than the literature review.

6 Ethics committee approval and conflict of interest statement

There is no need to obtain permission from the ethics committee for the article prepared. There is no conflict of interest with any person/institution in the article prepared.

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