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PREDICTION OF CENTRAL GOVERNMENT BUDGET TAX REVENUES USING MARKOV MODEL

Merkezi Bütçe Vergi Gelirlerinin Markov Modeli ile Tahmin Edilmesi

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ABSTRACT

The aim of this paper is to describe the behavior of the sample data and to predict the realization rates of tax revenues by one step stochastic Markov chain model. The realization rates of the tax revenues are estimated by using 2000-2014 gross annual data extracted from TR Revenue Administration. Four Markov models are constructed for the realization rates of every tax revenue. The realization probabilities for the year 2016 are predicted by constructing probability matrices of transitions between classes described for every model. Revenues are also forecasted by the product of the initial probability matrix and transition probability matrix. Limiting matrix of predictions are found. The best Markov model was found by estimating the sum of mean square errors for every model. The results are compared and interpreted.
Keywords: Tax Revenues, Transition Probabilities, Markov Analysis, Budget Forecast

ÖZET

Bu makalenin amacı örnek verinin davranışını tanımlamak ve bir adımlı stokastik Markov zinciri modeli ile vergi gelirleri kalemlerinin gerçekleşme oranlarını tahmin etmektir. Gelir İdaresi Başkanlığı 2000-2014 brüt yıllık verileri kullanılarak vergi gelirlerinin gerçekleşme olasılığı hesaplanmıştır. Her verginin gerçekleşme oranları için dört Markov modeli oluşturulmuştur. Her model için belirlenen sınıflar arası geçiş olasılıkları matrisleri oluşturularak 2016 yılı gerçekleşme olasılıkları tahmin edilmiştir. Ayrıca gelirler başlangıç matrisi ve geçiş olasılıkları matrisinin çarpımı ile tahminlenmiştir. Tahminlerin limit matrisleri bulunmuştur. En iyi Markov modeli hata karelerinin ortalamasının hesaplanmasıyla bulunmuştur. Sonuçlar karşılaştırılarak yorumlanmıştır.
Anahtar Kelimeler: Vergi gelirleri, Geçiş Olasılıkları, Markov Analizi, Bütçe Tahmin

Introduction

Prediction of central government budget tax revenues has a great importance in planning the distribution of revenues to public expenditures. Tax revenues are generated from taxes collected from income, property, goods, services and foreign trade. The proportion of tax incomes in general budget revenues has been increasing

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(www.ekodialog.com>konular>genel_butce 8.12.2015). Even though tax incomes have increased over time, the realization rates show a decreasing to stationary or increasing to stationary behavior. Since public expenditures are also increasing by time, an increase in realization rates is also expected. Otherwise, indirect tax items would be increased to cover increasing public expenditures, which brings a heavy load to public. Tax increase and revaluation rates are determined by Turkish Statistical Institute (TUIK) from twelve month mean of domestic producer price index (dppi) in October (http://www.zaman.com.tr/ekonomi_2015-yilinda-vergiler-yuzde-1011-oraninda-artacak_2255169.html 10.12.2015). As of January 1st 2016, motor vehicle tax, stamp duty tax, environmental tax, fees, traffic fines and tax fines will increse by the revaluation rate 5.58% unless Council of Ministers increase or decrease this rate (http://www.hurriyet.com.tr/2016-vergi-artis-oranlari-belli-oldu-40009417 9.12.2016). Sub-items of tax revenues are individual income tax, corporate income tax, property tax, inheritance and gift tax, motor vehicle tax, value added tax, special consumption tax, banking and insurance transaction taxes, tax on wagering, special communication tax, tax on customs, VAT on imports and stamp duty tax. In this paper, tax revenues are analyzed and predicted by four Markov Models. The best of the four has the least sum of the mean square errors. Predictions of tax revenues are expected to be stationary and to have a limiting matrix.

Literature

Baasch et. al (2010) used Markov models to quantify transitions between successional stages. They presented a solution for converting multivariate ecological time series into transition matrices and demonstrate the applicability of this approach for a data set that resulted from monitoring the succession of sandy dry grassland in a post-mining landscape. They analyzed five transition matrices, four one-step matrices referring to specific periods of transition (1995–1998, 1998–2001, 2001–2004, 2004–2007), and one matrix for the whole study period (stationary model, 1995–2007).

Büyüktatlı et. al (2013) used initial allocations of investment program with actual spending percentages from the years of 1998-2009 of Turkish Atomic Energy Institute (TAEK) to predict annual allowances from Ministry of Energy and Natural Resources. An estimated percentage of realization of investment program for 2011 and results are interpreted with Markov analysis.

Cavers and Vasudevan (2015) directed graph representation of a Markov chain model to study global earthquake sequencing leads to a time series of state-to-state transition probabilities that includes the spatio-temporally linked recurrent events in the recordbreaking sense. A state refers to a configuration comprised of zones with either the occurrence or non-occurrence of an earthquake in each zone in a pre-determined time interval.

Grimshaw and Alexander (2011) used a Markov chain model to forecast outstanding balance of loans in each delinquency state. For that they used a markov chain Xn as the delinquency state of a loan in month n and a Markov Chain model for loan accounts that are ‘current’ this month having a probability of moving next month into ‘current’, ‘delinquent’ or ‘paid-off’ states. They forecasted ‘one month ahead’

portfolio delinquency balance for a portfolio of loans where each loan is n_i months from origination this month i=1,...,N.

Lazri et.al (2015) adopted a Markovian approach to discern the probabilistic behaviour of the time series of the drought. A transition probability matrix was constructed from drought distribution maps. Markov transition probability formula for four states and a simulation model with an initial probability vector was used to calculate the drought distribution area in the future.

Lukić et. al. (2013) used the stochastic method based on a Markov chain model to predict the annual precipitation in the territory of South Serbia for the period 2009-2013. For this purpose, the precipitation data rainfall recorded on the four synoptic stations were used for the period 1980-2010.

Usher (1979) discussed that complex non-random or Markovian processes are likely to characterize ecological successions, the transition probability matrix elements not being constant but being functions either of the abundance, or of the rate of change of abundance, of a recipient class.

Methodology: Markov Model

Markov chain is a stochastic process which is described by a transition matrix of transition probabilities from one state into another state (Vantika and Pasaribu, 2014, p.2).

A discrete time process $\{X_n, n = 0,1,2,...\}$ with discrete state space $X_n \in \{0, 1, 2, ...\}$ is a Markov chain if it has the Markov property: $P(X_{n+1}=j \mid X_n=i, X_{n-1} = i_{n-1}, ...,X_0 = i_0) = P(X_{n+1}=j|X_n=i) = p(i,j)$ where $p(i,j)$ depends only on the states i,j, and not on the time n or the previous states" (dept.stat.lsa.umich.edu/~ionides/620/notes/markov _chains.pdf 15.12.2015). The numbers $p(i,j)$ are called the transition probabilities of the chain. (galton.uchicago.edu/~lalley/Courses/312/MarkovChains.pdf 15.12.2015).

One step probability is $p_{ij}=P(X_1=j \mid X_0=i)$ (İlarslan, 2014, s.6190). In a first order Markov chain, the state at any time instant depends only on the state immediately preceding it, and hence is defined as a single-dependence chain and m step probability is $p^m_{ij} = P(X_m=j \mid X_0=i)$.

Construction of Transition Probabilities

Transition probability matrices are estimated for 2000-2014 for sub-items of tax revenues. The estimator of the transition probabilities is the relative frequency of the actual transitions from phase i to phase j, i.e. the observed transitions have to be divided by the sum of the transitions to all other phases (Lipták, 2011, p.141)

In this paper, $P_{ij} = n_{ij} / \sum_j n_{ij}$ where i, j = A, B, C, D, E and n_{ij} is the number of observed transitions from i to j and $\sum_j n_{ij}$ is the sum of observed transitions from i to j.

Frequency distribution of the realization rate intervals must be mutually exclusive (nonoverlapping) and class width must be equal for each interval (Bluman, 2014, p.45-46). Transition probabilities from X_i to X_j, $i, j = 0,1,2,\ldots,n$, can be constructed as the following matrix (Taha, 2000, p.726)

$$P_{ij} = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ P_{n1} & P_{n2} & \dots & P_{nn} \end{bmatrix}$$

Since p_{ij} are constant and independent of time (time homogeneous), matrix $P_{ij} = P$ is called a stochastic matrix. P_{ij} probabilities must satisfy the following conditions:

$$P_{ij} \geq 0 \quad \forall i, j = 1,2,...,n \qquad \sum_i p_{ij} = 1 \quad \forall i = 1,2,...,n$$

Prediction

Given that data at time n is in state X_0 and that the data will be in one of states $X_n \in \{0,1,2,...\}$ at time n+1, then the data at time n+2 can be predicted. Given initial probability $P(X_0 = i) = p_i$ for every i, the required probability is matrix multiplication $p_i \sum_k P_{ik} P_{kj}$. Equivalently, next year's probability distribution matrix can be predicted by

$$Q_{n+1} = Q_n P \qquad n = 0,1,2,3,\dots \qquad (1)$$

Initial probability matrices for four Markov models are 1*x*j row matrices. Stationary prediction matrices Q_{n+1} have a limiting matrix Q, which can be written as $\lim_{n\to\infty} Q_n = Q \cdot$

Best of Four Markov Models

For every year of the sample and for every Markov model, mean square error (mse) is calculated by $\frac{1}{n}\sum_{i=1}^{n}(r_i - \hat{r}_i)^2$ where i is the number of states, $\hat{Q}_{n+1} = Q_n \cdot P_n = [\hat{r}_1 \quad \hat{r}_2 \quad \hat{r}_3 \cdots \hat{r}_n]$ is predicted realization rate at time n+1 and $Q_n = [r_1 \quad r_2 \cdots r_n]$ is observed realization rate at time n. The least mse gives the best Markov model.

Statistical Significance of the Models

Variations between observed and expected frequencies can be tested by constructing a contingency table of frequency distribution of transitions between the states at 0,05 significance level with a degree of freedom.

To validate Markov model, for every year, the value of the χ^2 statistic is computed based on the null hypothesis, H_0: model is valid. At 0,05 level of significance and with the degrees of freedom, the χ^2 critical value and χ^2 test value are estimated. The null hypothesis is not rejected whenever χ^2 test value is less than the critical value. Test values are calculated by $\chi^2 = \sum_i (r_i - \hat{r}_i)^2 / \hat{r}_i$ where i is the number of categories, and r_i and $\hat{r}_i$ are the actual and estimated values, respectively.

Income Tax

Income tax targeted, collected (http://www.gib.gov.tr/sites/default/files/fileadmin/user_upload/VI/GBG/Tablo_47.xls.htm, http://www.gib.gov.tr/sites/default/files/fileadmin/user_upload/VI/GBG/Tablo_44.xls.htm, http://www.gib.gov.tr/sites/default/files/fileadmin/user_upload/VI/GBG/Tablo_46.xls.htm, http://www.gib.gov.tr/sites/default

/files/fileadmin/user_upload/VI/GBG/Tablo_45.xls.htm 5.12.2015) and realization rates for the years 2000-2014 are given in Table 2. In the last fifteen years the highest rate in income tax realized was 113,7% in 2001 and the lowest realized was 85.1% in 2009. Targeted income tax has increased every year except the year 2010. Tax collection has increased every year between 2000-2014. While targeted income tax was increasing 4,47 billion TL per year on average, tax collection was also increasing 4,88 billion TL per year on average.

Income Tax Markov Models and Transition Probability Matrices

Income tax realization rates from smallest to largest are classified as E,D,C,B, A in model 1, D, C, B, A in model 2, C, B, A in model 3 and B, A in model 4. For years between 2000 and 2014 table 2 shows that realization rates are over 100% in three categories of model 1, in two categories of model 2, in two categories of model 3.

Table 1. Income Tax Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
108.3 ≤ r	A	0	1/2	0	0	1/2
102.5 ≤ r ≤ 108.2	B	1/5	2/5	2/5	0	0
96.7 ≤ r ≤ 102.4	C	2/5	2/5	0	0	1/5
90.9 ≤ r ≤ 96.6	D	0	0	0	0	0
r ≤ 90.8	E	0	0	1	0	0

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
106.7 ≤ r	A	1/3	1/3	0	1/3
99.5 ≤ r ≤ 106.6	B	1/3	1/2	0	1/6
92.3 ≤ r ≤ 99.4	C	1/3	2/3	0	0
r ≤ 92.2	D	0	0	1	0

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
104.3 ≤ r	A	2/5	2/5	1/5
94.7 ≤ r ≤ 104.2	B	4/7	2/7	1/7
r ≤ 94.6	C	0	1	0

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
99.5 ≤ r	A	7/9	2/9
r ≤ 99.4	B	3/5	2/5

For the years 2000-2014, the realization rates of income tax, classes and transitions for four Markov models are shown in Table 2.

Tablo 2. Income Tax (000 TL) and Classification of Realization Rates and Transitions

Year	**Targeted**	**Collected**	**Real. Rate (%)**	**Class**	**M1**	**Class**	**M2**	**Class**	**M3**	**Class**	**M4**
2000	6.276.000	6.212.977	99	C		C		B		B	
2001	10.186.000	11.579.424	113,7	A	CA	A	CA	A	BA	A	BA
2002	15.401.000	13.717.660	89,1	E	AE	D	AD	C	AC	B	AB
2003	17.196.918	17.063.761	99,2	C	EC	C	DC	B	CB	B	BB
2004	18.655.000	19.689.593	105,5	B	CB	B	CB	A	BA	A	BA
2005	21.170.000	22.817.530	107,8	B	BB	A	BA	A	AA	A	AA
2006	29.071.000	31.727.644	109,1	A	BA	A	AA	A	AA	A	AA
2007	36.922.897	38.061.543	103,1	B	AB	B	AB	B	AB	A	AA
2008	38.780.119	39.249.867	101,2	C	BC	B	BB	B	BB	A	AA
2009	46.598.274	39.668.595	85,1	E	CE	D	BD	C	BC	B	AB
2010	42.927.809	41.969.451	97,8	C	EC	C	DC	B	CB	B	BB
2011	48.951.204	51.092.935	104,4	B	CB	B	CB	A	BA	A	BA
2012	56.710.510	58.797.752	103,7	B	BB	B	BB	B	AB	A	AA
2013	65.483.652	65.914.727	100,7	C	BC	B	BB	B	BB	A	AA
2014	73.289.337	79.451.776	108,4	A	CA	A	BA	A	BA	A	AA

Prediction For Income Tax Realization Rate

Given that 2014 income tax realization rate 108.4% is in state A and that income tax will be in one of states A, B, C, D or E in 2015, income tax realization rate for 2016 is predicted.

Tablo 3. Income Tax Realization Rates Predictions for 2016

Realization Interval (%)	M1 Pred.	Realization Interval (%)	M2 Pred.	Realization Interval (%)	M3 Pred.	Real. Int. (%)	M4 Pred.
$r \leq 90.8$	0	$r < 92.2$	0.17	$r \leq 94.6$	0,14	$r \leq 99.4$	0,26
$90.9 \leq r \leq 96.6$	0	$92.3 \leq r \leq 99.4$	0,33	$94.7 \leq r \leq 104.2$	0,47	$99.5 \leq r$	0,74
$96.7 \leq r \leq 102.4$	0,70	$99.5 \leq r \leq 106.6$	0,28	$104.3 \leq r$	0,39		
$102.5 \leq r \leq 108.2$	0,20	$106.7 \leq r$	0,22				
$108.3 \leq r$	0,10						

Stationarity of Income Tax Predictions

Predictions are estimated in Excel by formula (1) for all models. According to four models, all probabilities become stationary in 2031, 2026, 2022 and 2020 respectively.

Statistical Significance of The Model For Income Tax

In model 1 of income tax, variations between observed and expected frequencies can be tested by constructing a contingency table of frequency distribution of transitions between the states at 0,05 significance level with 16 df. Since chi square test value 11,23 is less than critical value 26,296, H_0 is not rejected. This shows that there is no significant variations. The values in paranthesis in the Table 4 are expected frequencies which are found from (row sum X column sum)/total. Table 4 shows that in model 1 transitions in higher realization states are stable and in lower states rates are improving.

Table 4. Contingency Table of Observed and Expected Income Tax Rates of Model 1.

	A	B	C	D	E	Total
A	0 (0,43)	1 (0,71)	0 (0,57)	0	1 (0,29)	2
B	1 (1,07)	2 (1,79)	2 (1,43)	0	0 (0,71)	5
C	2 (1,07)	2 (1,79)	0 (1,43)	0	1 (0,71)	5
D	0	0	0	0	0	0
E	0 (0,43)	0 (0,71)	2 (0,57)	0	0 (0,29)	2
Total	3	5	4	0	2	14

Corporate Tax

Corporate tax targeted, collected and realization rates for years 2000-2014 are given in Table 6. In the last fifteen years the highest corporate tax rate realized was 175% in 2001 when targeted at the lowest and the lowest realized was 71,2 in 2000. Targeted corporate tax has increased every year except the years 2001, 2005, 2007 and 2010. Tax collection has increased every year between 2000-2014 except in 2013 when it had a slight decrease. While targeted corporate tax was increasing 2 billion TL per year on average, tax collection was also increasing 2,2 billion TL per year on average.

But, the realization rate was decreasing by approximately 1,5% per year in the given period.

Corporate Tax Transition Probability Matrices

In four categories of model 1, in three categories of model 2, in all categories of model 3 and model 4, realization rates are over 100% between 2000 and 2014.

Table 5. Corporate Tax Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
154,4 ≤ r	A	.5	0	0	.5	0
133,6 ≤ r ≤ 154,3	B	0	0	0	0	0
112,8 ≤ r ≤ 133,5	C	0	0	1/3	1/3	1/3
92 ≤ r ≤ 112,7	D	0	0	1/6	4/6	1/6
r ≤ 91,9	E	1/3	0	1/3	1/3	0

Markov Model 2		**Transition Matrix**			
Real. Interval (%)		A	B	C	D
149,2 ≤ r	A	.5	0	.5	0
123,2 ≤ r ≤ 149,1	B	0	0	0	1
97,2 ≤ r ≤ 123,1	C	0	1/7	5/7	1/7
r ≤ 97,1	D	1/4	0	3/4	0

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
140,8 ≤ r	A	.5	0	.5
105,8 ≤ r ≤ 140,7	B	0	.5	.5
r ≤ 105,7	C	1/6	1/2	1/3

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
123,2 ≤ r	A	1/3	2/3
r ≤ 123,1	B	2/11	9/11

For the years 2000-2014, the realization rates of corporate tax, classes and transitions for four Markov models are shown in Table 6.

Table 6. Corporate Tax (000 TL) and Classification of Realization Rates and Transitions

Year	**Targeted**	**Collected**	**Real. Rate**	**Class**	**MM1**	**Class**	**MM2**	**Class**	**MM3**	**Class**	**MM4**
2000	**3.309.000**	**2.356.787**	71,2	E		D		C		B	
2001	**2.100.000**	**3.675.665**	175	A	EA	A	DA	A	CA	A	BA
2002	**3.595.000**	**5.575.495**	155,1	A	AA	A	AA	A	AA	A	AA
2003	**8.918.160**	**8.645.345**	96,9	D	AD	D	AD	C	AC	B	AB
2004	**9.335.000**	**9.619.359**	103	D	DD	C	DC	C	CC	B	BB
2005	**8.890.000**	**11.401.986**	128,3	C	DC	B	CB	B	CB	A	BA
2006	**14.756.000**	**12.447.354**	84,4	E	CE	D	BD	C	BC	B	AB
2007	**14.410.186**	**15.718.474**	109,1	D	ED	C	DC	B	CB	B	BB
2008	**16.976.161**	**18.658.195**	109,9	D	DD	C	CC	B	BB	B	BB
2009	**22.611.359**	**20.701.805**	91,6	E	DE	D	CD	C	BC	B	BB
2010	**20.071.108**	**22.854.846**	113,9	C	EC	C	DC	B	CB	B	BB
2011	**25.359.580**	**29.233.725**	115,3	C	CC	C	CC	B	BB	B	BB
2012	**30.035.121**	**32.111.820**	106,9	D	CD	C	CC	B	BB	B	BB
2013	**32.043.560**	**31.434.581**	98,1	D	DD	C	CC	C	BC	B	BB
2014	**33.892.413**	**35.163.517**	103,8	D	DD	C	CC	C	CC	B	BB

Realization Rates and Transition Probability Matrices of Other Tax Revenues

Table 7. Property Tax Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
134,5 ≤ r	A	0	0	0	.5	.5
123,1 ≤ r ≤ 134,4	B	0	0	0	0	0
111,7 ≤ r ≤ 123	C	0	0	0	1	0
100,3 ≤ r ≤ 111,6	D	0	0	0	.4	.6
r ≤ 100,2	E	1/6	0	1/6	1/6	3/6

Markov Model 2		**Transition Matrix**			
Real. Interval (%)		A	B	C	D
r ≤ 103	A	0	0	.5	.5
103,1 ≤ r ≤ 117,2	B	0	0	0	0
117,3 ≤ r ≤ 131,4	C	0	0	.25	.75
131,5 ≤ r	D	1/8	0	1/8	6/8

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
r ≤ 103	A	0	0	1
103,1 ≤ r ≤ 117,2	B	0	0	1
117,3 ≤ r ≤ 131,4	C	1/11	1/11	9/11

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
117,3 ≤ r	A	0	1
r ≤ 117,2	B	1/12	11/12

Table 8. Inheritance and Gift Tax Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
125,7 ≤ r	A	1/3	0	2/3	0	0
108,6 ≤ r <125,6	B	0	1/3	1/3	0	1/3
91.5 ≤ r < 108,5	C	2/5	1/5	2/5	0	0
74,4 ≤ r < 91,4	D	0	0	0	0	0
r < 74,3	E	0	1/3	1/3	0	1/3

Markov Model 2		**Transition Matrix**			
Real. Interval (%)		A	B	C	D
121,5 ≤ r	A	.25	.75	0	0
100,1 ≤ r ≤ 121,4	B	.2	.4	.2	.2
78,7 ≤ r ≤ 100	C	1	0	0	0
r ≤ 78,6	D	0	1/3	1/3	1/3

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
114,1 ≤ r	A	1/3	1/2	1/6
85,8 ≤ r ≤ 114,2	B	3/5	2/5	0
r ≤ 85,7	C	1/3	1/3	1/3

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
100,1 ≤ r	A	7/9	2/9
r ≤ 100	B	3/5	2/5

Table 9. Motor Vehicle Tax Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
110,8 ≤ r	A	0	0	1	0	0
105,5 ≤ r ≤110,7	B	0	.5	.25	.25	0
100,2 ≤ r ≤105,4	C	0	0	4/6	2/6	0
94,9 ≤ r ≤ 100,1	D	1/3	0	1/3	0	1/3
r ≤ 94,8	E	0	1	0	0	0

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
109,7 ≤ r	A	0	0	1	0
103 ≤ r ≤109,6	B	0	4/5	1/5	0
96,3 ≤ r ≤ 102,9	C	0	0	4/6	2/6
r ≤ 96,2	D	.5	.5	0	0

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
107,4 ≤ r	A	0	1	0
98,5 ≤ r ≤ 107,3	B	1/9	6/9	2/9
r ≤ 98,4	C	1/3	1/3	1/3

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
102,9 ≤ r	A	4/6	2/6
r ≤ 102,8	B	2/8	6/8

Table 10. Value Added Tax Included Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
109.8 ≤ r	A	0	0	1	0	0
105.4 ≤ r ≤ 109.7	B	0	0	0	0	1
101 ≤ r ≤ 105.3	C	1/4	1/4	1/2	0	0
96.6 ≤ r ≤ 100.9	D	0	0	2/5	3/5	0
r ≤ 96.5	E	0	0	0	1/3	2/3

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
108.7 ≤ r	A	0	1/2	0	1/2
103.2 ≤ r ≤ 108.6	B	1/2	0	1/2	0
97.7 ≤ r ≤ 103.1	C	1/6	2/6	3/6	0
r ≤ 97.6	D	0	0	1/2	1/2

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
106.8 ≤ r	A	0	1/2	1/2
99.5 ≤ r ≤ 106.7	B	2/5	3/5	0
r ≤ 99.4	C	0	2/7	5/7

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
103.3 ≤ r	A	1/2	1/2
r ≤ 103.2	B	3/10	7/10

Table 11. Special Consumption Tax Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
102,6 ≤ r	A	1/3	0	2/3	0	0
98,7 ≤ r ≤ 102,5	B	0	1/3	1/3	0	1/3
95,4 ≤ r ≤ 98,6	C	2/5	1/5	2/5	0	0
91,8 ≤ r ≤ 95,3	D	0	0	0	0	0
r ≤ 91,7	E	0	1/3	1/3	0	1/3

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
101,7 ≤ r	A	2/3	1/3	0	0
97,2 ≤ r ≤ 101,6	B	1/4	1/4	1/2	0
92,7 ≤ r ≤ 97,1	C	0	1/3	1/3	1/3
r ≤ 92,6	D	1	0	0	0

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
100,2 ≤ r	A	1	0	0
94,2 ≤ r ≤ 100,1	B	0	4/5	1/5
r ≤ 94,1	C	½	0	1/2

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
97,2 ≤ r	A	5/7	2/7
r ≤ 97,1	B	2/4	2/4

Table 12. Banking and Insurance Transaction Tax Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
131,6 ≤ r	A	1/2	0	1/2	0	0
114,2 ≤ r ≤ 131,5	B	0	1/3	1/3	0	1/3
96,8 ≤ r ≤ 114,1	C	1/5	1/5	2/5	1/5	0
79,4 ≤ r ≤ 96,7	D	0	0	1/3	2/3	0
r ≤ 79,3	E	0	0	1	0	0

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
127,1 ≤ r	A	1/3	0	1/3	1/3
105,4 ≤ r ≤ 127	B	1/3	0	2/3	0
83,7 ≤ r ≤ 105,3	C	1/7	1/7	5/7	0
r ≤ 83,6	D	0	1	0	0

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
119,8 ≤ r	A	1/4	1/2	1/4
90,9 ≤ r ≤ 119,7	B	1/3	2/3	0
r ≤ 90,8	C	0	1	0

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
105,4 ≤ r	A	2/6	4/6
r ≤ 105,3	B	3/8	5/8

Table 13. Tax on Wagering Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
111,5 ≤ r	A	1/2	0	1/2	0	0
103,2 ≤ r ≤111,6	B	1/2	0	0	0	1/2
94,7 ≤ r ≤103,1	C	0	1	0	0	0
86, 2 ≤ r ≤ 94,6	D	0	1/2	12	0	0
r ≤ 86,1	E	0	0	1	0	0

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
109,5 ≤ r	A	1/3	0	1/3	1/3
98,9 ≤ r ≤ 109,4	B	1	0	0	0
88,3 ≤ r ≤ 98,8	C	1/4	1/2	1/2	0
r ≤ 88,2	D	0	0	1	0

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
105,9 ≤ r	A	1/4	1/2	1/4
91,8 ≤ r ≤ 105,8	B	1/2	1/2	0
r ≤ 91,7	C	0	1	0

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
98,9 ≤ r	A	1/2	1/2
r ≤ 98,8	B	3/5	2/5

Table 14. Special Communication Tax Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
125,9 ≤ r	A	0	0	1	0	0
111,9 ≤ r ≤ 125,8	B	0	0	0	0	0
97,9 ≤ r ≤ 111,8	C	0	0	1/4	1/2	1/4
83,9 ≤ r ≤ 97,8	D	1/7	0	4/7	2/7	0
r ≤ 83,8	E	0	0	0	1	0

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
122,4 ≤ r	A	0	1	0	0
104,9 ≤ r ≤ 122,3	B	0	0	1	0
87,4 ≤ r ≤ 104,8	C	1/9	1/9	2/3	1/9
r ≤ 87,3	D	0	0	1/2	1/2

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
116,5 ≤ r	A	0	0	1
93,2 ≤ r ≤ 116,4	B	1/8	5/8	2/8
r ≤ 93,1	C	0	3/4	1/4

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
104,9 ≤ r	A	1/2	1/2
r ≤ 104,8	B	1/11	10/11

Table 15. Tax on Customs Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
116,1 ≤ r	A	0	1/3	1/3	1/3	0
104,6 ≤ r ≤ 116	B	2/3	0	0	1/3	0
93,1 ≤ r ≤ 104,5	C	0	0	1/3	0	2/3
81,6 ≤ r ≤ 93	D	2/3	1/3	0	0	0
r ≤ 81,5	E	0	1/2	0	1/2	0

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
113,3 ≤ r	A	0	2/3	1/3	0
98,9 ≤ r ≤ 113,2	B	2/5	0	2/5	1/5
84,5 ≤ r ≤ 98,8	C	2/4	1/4	0	1/4
r ≤ 84,4	D	0	1/2	1/2	0

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
108.5 ≤ r	A	2/5	2/5	1/5
89,3 ≤ r ≤ 108,4	B	1/5	1/5	3/5
r ≤ 89,2	C	3/4	1/4	0

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
98.8 ≤ r	A	1/2	1/2
r ≤ 98.7	B	4/6	2/6

Table 16. VAT on Imports Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
$130,3 \leq r$	A	0	0	0	1	0
$114,7 \leq r \leq 130,2$	B	0	1/2	0	1/2	0
$99,1 \leq r \leq 114,6$	C	0	0	3/5	2/5	0
$83,5 \leq r < 99$	D	0	0	3/5	1/5	1/5
$r \leq 83,4$	E	0	1	0	0	0

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
$126,4 \leq r$	A	0	0	0	1
$106,9 \leq r \leq 126,3$	B	0	1/4	3/4	0
$87,4 \leq r \leq 106,8$	C	0	2/7	4/7	1/7
$r \leq 87,3$	D	0	1/2	1/2	0

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
$119,7 \leq r$	A	0	1/2	1/5
$93,8 \leq r \leq 119,6$	B	0	3/7	4/7
$r \leq 93,7$	C	1/5	4/5	0

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
$106,8 \leq r$	A	1/5	4/5
$r \leq 106,7$	B	3/9	6/9

Table 17. Stamp Duty Tax Markov Models and Transition Probability Matrices

Markov Model 1		**Transition Matrix**				
Real.Interval (%)		A	B	C	D	E
$115,8 \leq r$	A	0	0	0	1/2	1/2
$108,2 \leq r \leq 115,7$	B	0	1/4	1/4	1/2	0
$100,6 \leq r \leq 108,1$	C	0	1	0	0	0
$93 \leq r \leq 100,5$	D	1/5	1/5	0	2/5	1/5
$r \leq 92,9$	E	0	1/2	0	1/2	0

Markov Model 2		**Transition Matrix**			
Real.Interval (%)		A	B	C	D
$113,9 \leq r$	A	0	1/4	1/4	2/4
$104,4 \leq r \leq 113,8$	B	0	0	1/2	1/2
$94,9 \leq r \leq 104,3$	C	2/3	0	1/3	0
$r \leq 94,8$	D	1/5	1/5	1/5	2/5

Markov Model 3		**Transition Matrix**		
Real. Interval (%)		A	B	C
$110,6 \leq r$	A	1/6	3/6	2/6
$98 \leq r \leq 110,5$	B	1/2	0	1/2
$r \leq 97,9$	C	1/2	0	1/2

Markov Model 4		**Transition Matrix**	
Real. Interval (%)		A	B
$104,3 \leq r$	A	1/6	5/6
$r \leq 104,2$	B	4/8	4/8

Predictions of Tax Revenues for 2016

Given that 2014 tax realization rate in a state and this tax will be in one of states A, B, C, D or E in 2015, realization rates matrices are predicted for 2016 by formula (1).

Table 18. 2016 Prediction of Tax Revenues

		A	B	C	D	E		A	B	C	D	E
Income Tax	M1	0,10	0,20	0,70	0,00	0,00	Coporate Tax	0,06	0	0,22	0,56	0,17
	M2	0,22	0,28	0,33	0,17			0,04	0,10	0,62	0,24	
	M3	0,39	0,47	0,14				0,14	0,42	0,44		
	M4	0,74	0,26					0,21	0,79			
Property Tax	M1	0,08	0,00	0,08	0,40	0,44	IGT	0,29	0,15	0,49	0,00	0,07
	M2	0,09	0,00	0,19	0,72			0,33	0,38	0,15	0,14	
	M3	0,07	0,07	0,86				0,44	0,46	0,10		
	M4	0,08	0,92					0,74	0,26			
MVT	M1	0,00	0,50	0,25	0,25	0,00	VAT Include	0,12	0,13	0,50	0,00	0,25
	M2	0,00	0,40	0,60	0,00			0,08	0,42	0,25	0,25	
	M3	0,15	0,67	0,19				0,24	0,56	0,20		
	M4	0,35	0,65					0,40	0,60			
Special Con Tax	M1	0,12	0,13	0,50	0,00	0,25	BITT	0,20	0,20	0,40	0,20	0,00
	M2	0,08	0,42	0,25	0,25			0,33	0,00	0,67	0,00	
	M3	0,24	0,56	0,20				0,33	0,67	0,00		
	M4	0,40	0,60					0,36	0,64			
Tax on Wagerin	M1	0,25	0,00	0,75	0,00	0,00	Special Com Tax	0,07	0,00	0,35	0,52	0,06
	M2	0,34	0,00	0,33	0,33			0,07	0,19	0,61	0,13	
	M3	0,50	0,38	0,12				0,08	0,58	0,34		
	M4	0,55	0,45					0,13	0,87			
Tax on Customs	M1	0,45	0,11	0,11	0,11	0,22	VAT on Imports	0,00	0,00	0,60	0,32	0,08
	M2	0,43	0,08	0,27	0,22			0,00	0,31	0,61	0,08	
	M3	0,39	0,29	0,32				0,11	0,64	0,25		
	M4	0,58	0,42					0,29	0,71			
Stamp Duty Tax	M1	0,08	0,23	0,05	0,46	0,18						
	M2	0,22	0,17	0,28	0,33							
	M3	0,33	0,25	0,42								
	M4	0,33	0,67									

A Better Model For Tax Revenues

Sum of mean square errors for a better model of each tax revenue is given in table 19. Values in bold indicates the better model.

Table 19. Tax Revenues SMSE

Tax Revenues	**Sum of Mean Square Errors (SMSE)**			
	Model 1	**Model 2**	**Model 3**	**Model 4**
Income tax	**2,61**	3,65	3,89	5,07
Corporate tax	3,52	**3,26**	4,80	3,97
Property Tax	2,62	2,36	**2,08**	2,69
Inheritence and Gift Tax	**3,21**	4,28	4,75	4,30
Motor Vehicle Tax	**3,04**	4,21	4,99	4,75
Vat Included	**2,91**	3,45	4,66	4,14
Special Consumption Tax	2,60	3,14	**2,50**	3,69
Banking and Insurance Tax	**3,02**	3,42	3,89	3,83
Tax on Wagering	**2,30**	2,79	3,14	2,80
Special Communication Tax	**2,12**	3,08	4,33	2,33
Tax on Customs	**3,83**	4,26	4,77	4,61
Vat on Imports	**2,61**	3,88	5,00	4,51
Stamp Duty Tax	**3,54**	4,15	4,22	3,67

Stationarity of Tax Revenues

Stationary matrices for tax revenues are found for every model and given in table 20. Q_0 is initial probability matrix for 2014 realization rates of tax revenues and *SY* is the stationarity year when probability matrix becomes stable.

Table 20. Stationary Matrices of Tax Revenues

	Markov Model 1			**Markov Model 2**		
Tax Revenues	Q_0	$\lim_{n\to\infty} Q_n = Q_\infty$	*SY*	Q_0	$\lim_{n\to\infty} Q_n = Q_\infty$	*SY*
Income tax	1 0 0 0 0	.19 .36 .30 0 .15	2031	1 0 0 0	.28 .40 .16 .16	2026
Corporate tax	0 0 0 1 0	.10 0 .21 .53 .16	2024	0 0 1 0	.09 .09 .64 .18	2024
Property Tax	0 0 0 0 1	.08 0 .08 .34 .50	2025	0 0 0 1	.09 0 .18 .73	2019
Vat Included	0 0 1 0 0	.09 .09 .35 .22 .26	2036	0 1 0 0	.18 .23 .41 .18	2032
Vat on Imports	0 0 1 0 0	0 .13 .48 .32 .07	2028	0 0 1 0	0 .29 .62 .09	2021
Special Consumption Tax	0 1 0 0 0	.27 .18 .46 0 .09	2032	1 0 0 0	.43 .29 .21 .07	2025
Communication Tax	0 0 1 0 0	.06 0 .41 .43 .10	2027	0 0 1 0	.07 .14 .65 .14	2026
Inheritence and Gift	0 0 1 0 0	.27 .18 .46 0 .09	2030	0 1 0 0	.29 .44 .13 .13	2023
Motor Vehicle Tax	0 0 0 0 1	.07 .14 .52 .20 .07	2029	0 0 0 1	.07 .36 .43 .14	2027
Tax on Wagering	0 1 0 0 0	.29 .28 .29 0 .14	2097	0 1 0 0	.38 .17 .33 .12	2038
Stamp Duty Tax	0 0 0 1 0	.08 .30 .07 .42 .13	2026	0 0 1 0	.26 .13 .29 .32	2023
Customs Tax	1 0 0 0 0	.30 .23 .15 .22 .10	2037	1 0 0 0	.27 .32 .28 .13	2036
Banking and Insurance Tax	0 0 0 0 1	.17 .12 .42 .25 .04	2029	0 0 0 1	.20 .15 .58 .07	2025

	Markov Model 3			**Markov Model 4**		
Tax Revenues	Q_0	$\lim_{n\to\infty} Q_n = Q_\infty$	*SY*	Q_0	$\lim_{n\to\infty} Q_n = Q_\infty$	*SY*
Income tax	1 0 0	.42 .44 .14	2022	1 0	0,73 0,27	2020
Corporate tax	0 0 1	.14 .43 .43	2020	0 1	0,21 0,79	2019
Property Tax	0 0 1	.08 .08 .84	2020	0 1	0,08 0,92	2018
Vat Included	0 1 0	.19 .48 .33	2027	0 1	0,38 0,62	2032
Vat on Imports	0 1 0	.07 .57 .36	2031	0 1	0,29 0,71	2019
Special Consumption Tax	1 0 0	1 0 0	2015	1 0	0,64 0,36	2020
Communication Tax	0 1 0	.08 .61 .31	2022	0 1	0,15 0,85	2023
Inheritence and Gift Tax	0 1 0	.45 .44 .11	2021	1 0	0,73 0,27	2020
Motor Vehicle Tax	0 0 1	.14 .64 .22	2022	0 1	0,43 0,57	2026
Tax on Wagering	0 1 0	.45 .44 .11	2025	1 0	0,53 0,45	2020
Stamp Duty Tax	0 1 0	.37 .19 .44	2023	0 1	0,37 0,63	2023
Customs Tax	1 0 0	.43 .30 .27	2022	1 0	0,57 0,43	2019
Banking and Insurance Tax	0 0 1	.29 .64 .07	2022	0 1	0,36 0,64	2017

Statistical Significance of Markov Model

In the present study, the validity of model is checked for the years 2013 and 2014. The degrees of freedom (df), χ^2 critical values and test values are given in table 21. The null hypothesis is not rejected since χ^2 test value is less than the critical value. The values of the χ^2 test are less than χ^2 critical values for the years 2013 and 2014, which implies that the estimated realization rates of revenues and the actual realization rates of revenues are not significantly different. Table 21 results show that Markov model is valid.

Table 21. Validity of Tax Revenues

	2013				2014			
Tax Revenues	df	$X^2_{0,05}$ Crit.V	$X^2_{0,05}$ Test V	H_0: Valid	df	$X^2_{0,05}$ Crit.V	$X^2_{0,05}$ Test V	H_0: Valid
Income tax	2	5,991	1,12	Accept	2	5,991	2,03	Accept
Corporate tax	1	3,841	0,17	Accept	2	5,991	4	Accept
Property Tax	2	5,991	0,29	Accept	2	5,991	0,25	Accept
Vat Included	1	3,841	2	Accept	1	3,841	3,5	Accept
Vat on Imports	1	3,841	0,75	Accept	1	3,841	0,75	Accept
Special Consumption Tax	1	3,841	0,67	Accept	1	3,841	0,5	Accept
Communication Tax	2	5,991	1	Accept	2	5,991	0,75	Accept
Inheritence and Gift Tax	1	3,841	0,4	Accept	1	3,841	0,33	Accept
Motor Vehicle Tax	1	3,841	0,5	Accept	1	3,841	0,4	Accept
Tax on Wagering	1	3,841	2	Accept	1	3,841	1	Accept
Stamp Duty Tax	1	3,841	1	Accept	1	3,841	0,5	Accept
Customs Tax	1	3,841	1	Accept	1	3,841	0,5	Accept
Banking and Insurance Tax	1	3,841	0,8	Accept	1	3,841	0,5	Accept

Findings, Discussion and Results

According to transition matrices, transitions of tax revenues are declining in higher states and improving in lower states. 2016 predictions with respect to middle state using the better models are given in table 22.

Table 22. Tax Revenue Predictions For 2016 According to Better Models

Tax Revenues	**Better Markov Model**	**Realization Rate r (%)**	**Predictions For 2016 According to Better Models**	
			Probability (%)	**1 – Probability (%)**
Income tax	1	C or higher 96,7 ≤ r	100	0
Corporate tax	2	C or higher 97,2 ≤ r	76	24
Property Tax	3	B or higher 103,1 ≤ r	14	86
Inheritence and Gift Tax	1	C or higher 91,5 ≤ r	93	7
Motor Vehicle Tax	1	C or higher 100,2 ≤ r	75	25
Vat Included	1	C or higher 101 ≤ r	75	25
Special Consumption Tax	3	B or higher 94,2 ≤ r	80	20
Banking and Insurance Tax	1	C or higher 96,8 ≤ r	80	20
Tax on Wagering	1	C or higher 94,7 ≤ r	100	0
Special Communication Tax	1	C or higher 97,9 ≤ r	42	58
Tax on Customs	1	C or higher 93,1 ≤ r	67	33
Vat on Imports	1	C or higher 99,1 ≤ r	60	40
Stamp Duty Tax	1	C or higher 100,6 ≤ r	36	74

According to model 1 of income tax, the probabilities of five states will be stable in 2031. Income tax rate more likely will be realized at 102.5% or higher in the long run. Probability of income tax rate greater than 108.3% is improving from 10% in 2016 to a stable 19.05%. Probability of income tax rate between 102.5% and 108.2% is improving from 20% in 2016 to a stable 35.71%. Probability of $96.7 \leq r \leq 102.4$ is decreasing from 70% to a stable 29.76%. Other tax revenues predictions are compared in table 23.

Table 23. Comparison of 2016 Predictions To Stationary Matrices According to Better Models

Tax Revenues	Better Markov Model	Comparison of 2016 Predictions To Stationary Matrices According to Better Models		
		2016 Prediction	Stationary Matrix	SY
Income tax	1	.10 .20 .70 0 0	.19 .36 .30 0 .15	2031
Corporate tax	2	.04 .10 .62 .24	.09 .09 .64 .18	2024
Property Tax	3	.07 .07 .86	.08 .08 .84	2020
Inheritence and Gift Tax	1	.29 .15 .49 0 0	.27 .18 .46 0 .09	2030
Motor Vehicle Tax	1	0 .50 .25 .25 0	.07 .14 .52 .20 .07	2029
Vat Included	1	.12 .13 .50 0 .25	.09 .09 .35 .22 .26	2036
Special Consumption Tax	3	.24 .56 .20	1 0 0	2015
Banking and Insurance Tax	1	.20 .20 .40 .20 0	.17 .12 .42 .25 .04	2029
Tax on Wagering	1	.25 0 .75 0 0	.29 .28 .29 0 .14	2097
Special Communication Tax	1	.07 0 .35 .52 .06	.06 0 .41 .43 .10	2027
Tax on Customs	1	.45 .11 .11 .11 .22	.30 .23 .15 .22 .10	2037
Vat on Imports	1	0 0 .60 .32 .08	0 .13 .48 .32 .07	2031
Stamp Duty Tax	1	.08 .23 .05 .46 .18	.08 .30 .07 .42 .13	2026

This study can be used to predict the other sub-items of tax revenues. Central government can take the advantages of this study in the planning and improvement of tax collection process.

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