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Profit Maximizing Probabilistic Inventory Model under Trade Credit

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ABSTRACT

In the classical EOQ models it has been considered that demand is deterministic but in many practical situations it is not possible to have a fixed demand. This study discusses the more realistic overview of demand, as in realistic situation having dependent demand is difficult; it is possible only if you're supplying sub-assembly parts on contract basis. Therefore, this study considers stochastic demand. Here maximum demand is dependent on average yearly demand and prescribed demand function. Thus initial inventory level is taken to be maximum demand derived with the help of demand function and average demand. Demand pattern considered in this model was proposed by Naddor (1966) in his book "inventory systems" with various realistic factors. The realistic factors considered are selling price is always greater than cost price, permissible delay in payments and even the optimality of profit equation has been checked. This study proves by optimality conditions that the profit maximization equations derived in this model help to maximize profit.

Keywords: Probabilistic Demand, Trade Credit, Optimality, Convexity
JEL Classification: C

1. INTRODUCTION

In present business transactions, it is a common practice to provide a purchaser a fixed time period before they settle the account with the supplier. This provides an advantage to the buyer, since they do not have to pay the supplier immediately after receiving the items, but instead, can defer their payment until the end of the allowed period. In this way, paying later indirectly reduces the purchase cost of the items. On the other hand, the permissible delay in payments produces benefits to the supplier such as, it should attract new purchasers who consider it to be type of price reduction, it encourages the customer to order more and it may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices and thus introduce lasting price reductions. On the other hand, the policy of granting credit terms adds not only an additional cost but also an additional dimension of default risk to the supplier.

Goyal (1985) was the first to introduce the concept of permissible delay. He established an EOQ model when the supplier offers the retailer permissible delay in payments. Later Chand and Ward investigated the model of Goyal under the framework of classical EOQ model which was further investigated by Aggarwal and Jaggi (1996) for deteriorating items Sarkar et al. (2000) developed a supply chain model for perishable products under inflation and permissible delay in payment. Teng (2002) considered an EOQ under condition of permissible delay in payment, which was further extended by Chung and Huang (2004) for limited storage capacity. Teng et al. (2005) further developed an algorithm for a retailer to determine its optimal price and lot size simultaneously when the suppliers offer a permissible delay in payments. Goyal et al. (2007) has further extended the concept of permissible delay by developing an optimal ordering policies when the supplier provides a progressive interest scheme.

Mahapatra and Malti (2005) presented the multi objective and single objective, inventory models of stochastically deteriorating items, in which demand is a function of inventory level and selling price of the commodity. Production rate depends upon the quality level of the items produced and unit production cost is a function of production rate. Deterioration depends upon both the quality of the item and the duration of time for storage. The time-related deterioration function follows a two parameter Weibull distribution in time. Brander et al. (2005) examine that deterministic model can be used if demand is stationary stochastic. A dynamic programming approach from Bomberger and a heuristic method from Segerstedt are used to calculate lot sizes for four items. The production of these items is simulated with different variations in demand rates.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and Notations are used in this chapter.

2.1. Assumptions

1. Probabilistic demand pattern is considered.
2. Shortages are not allowed, hence initial level inventory is equal to the maximum level demand x_{max} (T) during the time period, so that there will be no shortages, hence $I_o = x_{max}(T)$.
3. The maximum and minimum value of probabilistic demand is known with certainty.
4. In this we have considered the effect of permissible and he pays only on the completion of cycle period.
5. Replenishment rate is infinite i.e., replenishment is instantaneous.
6. The inventory involves only a single item.
7. Lead time is zero.

2.2. Notations

I_o: Initial inventory level
I_e: The interest earned per rupees per year
I_p: The interest paid per rupees per year
Z: The total annual profit
f(x): The probability density function of demand for cycle period
r: Average demand rate.

3. MATHEMATICAL MODEL AND ANALYSIS

The average level of inventory in any cycle period, T during which there is a demand x is I_o-x, where be its mean and let the average rate of demand $r=\frac{\bar{x}(T)}{t}$ is known and constant. Hence the expected average amount in inventory is

$$I = \int_{min_{min}}^{x_{max}} \left(I_0 - \frac{x}{2}\right) f(x)dx = I_0 - \frac{\bar{x}(T)}{2} = I_0 - \frac{rT}{2} \tag{1}$$

3.1. The Total Annual Profit Consists of the Following

a. Sales revenue = $p\bar{x}(T)$

b. Cost of placing orders = $\frac{A}{T}$

c. Cost of purchasing = $\frac{cI_0}{T}$

d. Cost of carrying inventory = $i\left(I_0 - \frac{rT}{2}\right)$.

For permissible delay, there are two distinct cases in this inventory of inventory system:

1. Payment at or before the total depletion of inventory, T>M and
2. After depletion payment, T≤M.

3.2. Case I T>M

This situation indicates that the permissible payment time expires on or before the inventory depleted completely to zero. As a result, the total cost is comprised of the sum of ordering cost, carrying cost, and the interest payable minus the interest earned. Here the buyer sells r*M units in total by the end of the permissible delay M, and has c*r*M to pay the supplier. The items in stock are charged at interest rate I_p by the supplier starting at time M. Therefore, the interest payable per cycle for the inventory not sold after due date M is given by:

$$cIp\int_M^T I\, {dt}/{T} == c\frac{I_p}{T}\left[I_0 - \frac{rT}{2}\right](T-M) \tag{2}$$

But, during the period of permissible delay, the buyer sells the product and the revenue from the sales can be used to earn interest, therefore the interest earned during the positive inventory is given by:

$$pI_e\int_0^M \bar{x}(T)t\, {dt}/{T} = pI_e\frac{r}{2}M^2 \tag{3}$$

Therefore, the total annual profit Z is given by:

$$Z{=}p\bar{x}(T) - \frac{A}{T} - \frac{cI_0}{T} - i\left(I_0 - \frac{rT}{2}\right) - p\frac{I_p}{T}\left[I_0 - \frac{rT}{2}\right](T-M) - pI_e\frac{r}{2}M^2 \tag{4}$$

3.3. Case II

In this case customer sells as customer sells all the items before expiration of permissible delay; hence no interest is paid only interest is earned on the given inventory. hence, the interest earned per year is:

$$pI_e\left[\int_0^T \bar{x}tdt + \bar{x}T(M-T)\right] = pI_e\bar{x}\left(M - {T}/{2}\right) \tag{5}$$

Hence, total profit is given by:

$$Z{=}p\bar{x}(T) - \frac{A}{T} - \frac{cI_0}{T} - i\left(I_0 - \frac{rT}{2}\right) - pI_e\bar{x}\left(M - {T}/{2}\right) \tag{6}$$

Now to decide the cycle period we have to find the value of I_o or x_{max} (T) suppose.

$$x_{max} = \bar{x}(T)G(T) = rtG(T) \tag{7}$$

Where G (T) is some function relating the maximum demand during any period T to the average demand during that period. Obviously,

$$G(T) \geq 1 \quad (8)$$

We can consider different values of G (T),

- Case I G (T) = K, when ratio of maximum demand and average demand to be a constant, substituting these values in equation (7), we get the required profit maximization equation,
- Case II G (T) = 1+a/T, generally it is found that the ratio of maximum demand to average demand during T would generally depend on T. i.e., the larger the value of T, the smaller is the ratio, substituting these values in equation (7), we get the required profit maximization equations.

Differentiating equation (4) with respect to T, we obtain:

$$\frac{\partial Z}{\partial T} = \frac{A}{T^2} + \frac{cI_0}{T^2} + \frac{ir}{2} - \frac{pI_pI_0M}{T^2} \quad (9)$$

Again differentiating equation (9) with respect to T, we get:

$$\frac{\partial^2 Z}{\partial T^2} = -\frac{2A}{T^3} - \frac{cI_0}{T^3} + \frac{2pI_pI_oM}{T^3} < 0 \quad (10)$$

Which is obviously less than zero as interest payable cannot be equal to the total cost price of the inventory, hence above equation is strictly concave function? Hence we can easily obtain the optimal value of T, which maximizes the total annual profit.

Differentiating equation (6) with respect to T, we have:

$$\frac{\partial Z}{\partial T} = \frac{A}{T^2} + \frac{cI_0}{T^2} + \frac{r}{2} - \frac{pI_e}{2}\vec{x} \quad (11)$$

Again differentiating with respect to T, we get.

$$\frac{\partial^2 Z}{\partial T^2} = \frac{-2A}{T^3} - \frac{cI_0}{T^3} < 0 \quad (12)$$

Which is obviously less than zero as cost price and ordering cost both cannot be negative, hence the equation (10) and (6) is strictly concave, thus equation (4) and (6) provides the profit maximization equation.

4. CONCLUSION

This study considered the effect of trade credit with stochastic demand, which has not been used earlier, as both these are realistic concepts without considering these two factors, we can't think of any practically applicable inventory model. The concavity of the profit equation has been verified and which shows that given model is helpful for finalizing the ordering policy for any supplier/retailer and different pattern of the given demand function are also considered.

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