



Do Structural Breaks Affect Portfolio Designs and Hedging Strategies? International Evidence from Stock-Commodity Markets Linkages

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ABSTRACT

The present paper studies stock-commodity markets linkage using vector autoregression generalized autoregressive conditional heteroskedasticity (VAR-GARCH) approach for the period spanning from January 3, 2000 to March 12, 2014. The analysis has been performed through three competing specifications; the VAR-constant conditional correlation-GARCH, the VAR-Baba, Engle, Kroner, Kraft-GARCH, and the VAR-dynamic conditional correlation-GARCH, ignoring and accounting for structural breaks in volatility to look at the impact of the breaking events on volatility spillovers and its persistence as well as the implications on portfolio management. We found significant interdependency in first and second conditional moments. The structural break dates help forecast current conditional volatility and define its persistence. Their effects have been found slight on optimal weights, miscellaneous on hedge ratios but important on hedging effectiveness. We consider that our findings open up new insights for managerial and governmental policy purposes.

Keywords: Volatility Spillovers, Structural Breaks, Portfolio Designs and Hedging

JEL Classifications: F3, F15, G12, Q43

1. INTRODUCTION

It is by common consent that the increasing International Financial Integration (Agenor, 2003; Lane and Milesi-Ferretti, 2003; Vo, 2005b; Bekaert et al., 2007; Vo and Daly, 2007; Teng et al., 2014, Rejeb and Arfaoui, 2015) is attributed to the development of economic and financial cross-border relationships and synchronization of business cycles in the new context of financial globalization (Bordo and Helbling, 2003; Imbs, 2004; Schiavo, 2008; Arfaoui and Abaoub, 2010; Berger and Pozzi, 2013). In that framework, markets interdependence becomes a determining factor for portfolio investment decisions and international diversification.

Recent empirical literature has documented that the interdependence makes use of several channels through which innovations are

transmitted between stock exchange and other markets (Arouri et al., 2011; 2012). The channels such as oil prices, foreign exchanges, interest rates as discount rate, corporate cash-flows are establishing linkages between markets (return dynamics) but also leading volatility spillovers (Baffes, 2011; Arouri et al., 2012; Arfaoui and Ben Rejeb, 2015).

In the presence of both the sustainable financial integration trend and the high level of volatility spillovers, the behavior of equity and commodity prices becomes more sensitive to innovations. Indeed, important and serious information flows are likely to form breaking events which amends portfolio parameters and investment decisions. Mensi et al. (2013), say that ignoring structural changes begets spurious long memory effects in the data. Large variance persistence may be induced by the failure to identify sudden changes. That is to say, the inclusion of information

regarding structural changes reduces volatility persistence and improves the understanding of volatility transmission (Kang et al., 2011; Edwin and Malik, 2013). Consequently, structural changes are a fundamental part for international market linkages study as well as for managerial implications.

The volatility is then puzzling for both academicians and practitioners especially in the presence of break dates. In that framework, several empirical methods have been implemented to study the volatility spillovers between interdependent markets. In fact, a multivariate GARCH (MGARCH) model has been employed to analyze market linkages through interdependent conditional returns and volatilities. We use a vector autoregression generalized autoregressive conditional heteroskedasticity (VAR-GARCH) approach to perform this analysis and commit to memory that this method was initiated by Ling and McAleer (2003) and has been commonly used in recent empirical literature for various subject such as oil and equity markets (Arouri et al., 2010; Sadorsky, 2012), equity sectors (Hammoudeh et al., 2009), commodity markets (Mensi et al., 2013), oil and exchange rates (Harri et al., 2009), and commodity market breaks (Vivian and Wohar, 2012) and recently stock markets and forex (Arfaoui and Ben Rejeb, 2015).

The aim of this study is to shed light on stock-commodity market linkages through conditional return dynamics and conditional volatility spillovers. Stock market is approximated by the Standard and Poor (SP) 500 index and commodity markets are crude oil (Brent and West Texas Intermediate [WTI]), food (wheat and barley) and precious metals (gold and silver) markets. We consider that the selected markets represent a wide range of economic sectors and provide useful support for practical and managerial analysis.

We perform the analysis using VAR-constant conditional correlation (CCC)-GARCH model of Bollerslev (1990), VAR-dynamic conditional correlation (DCC)-GARCH of Engle (2002) and VAR-Baba, Engle, Kroner, Kraft (BEKK)-GARCH of Engle and Kroner (1995) specifications ignoring and accounting for structural breaks in volatility series. One of the main advantages of those specifications is that they make it possible to investigate the inter-markets return dynamics, the dynamics of conditional volatility and volatility spillovers and to shed light on break-date implications on conditional volatility and conditional correlation parameters. Additionally, the model provides meaningful estimates of the unknown parameters which tell about innovations and effects of shock transmissions. It allows us to detect the outcome of commodity market events on the SP 500 index returns, on commodity market returns and on both stock-commodity inter-market. On the whole, we beg the following questions: Do structural changes affect stock-commodity markets linkages? And do they matters for portfolio management?

Results from estimated specification models have been operated to present their managerial usefulness for portfolio investment. Indeed, we estimate optimal weights (Kroner and Ng, 1998) hedge ratios (Kroner and Sultan, 1993) and hedging effectiveness (Ku et al., 2007).

Our finding show the evidence of real effect of past on current daily behavior within each market, on the one hand, and a mutual interference between stock and commodity markets, on the other hand. We note that one lagged daily stock market return affect the whole current commodity-markets return which inform about behavioral factors effect on stock markets daily pricing of listed firms. In the opposite direction, only wheat and gold daily past returns help predict today's stock return. The current conditional volatility (GARCH terms) of stock market is significantly affected by both own past volatility and the 1 day lagged volatility of commodity markets in absence and presence of dummy volatility break dates except for wheat and precious metals. Reciprocally, past stock market's volatility help significantly predict current commodity markets GARCH terms except for Brent, barley, gold and silver. The dummy break dates as exogenous factor help forecast present conditional volatility of both stock and commodity markets and the conditional volatility is persistent for long time.

This present research differs from previous studies in at least four aspects. First, although studies of markets interdependency focus on both return and volatility spillover channels they almost make use of simple VAR-GARCH specification model. In addition, given that the estimated CCCs for returns across markets were very weak and not statistically significant, we support that the cross-markets correlation of conditional shocks were absent. At the same line, we find that the estimates of the dynamic conditional correlations are significant for all time, which is far from supporting empirically the assumption of CCC. This highlights the evidence of dynamic conditional correlations between the selected markets. Consequently, we try here to run three competitive specifications to perform our findings. Second, we rely on the information content of breaking events and market efficiency and incorporate dummy break dates in volatility series to observe the conditional moment's behavior. As stated in empirical literature, the structural breaks as dummy variables allows avoiding overestimation of persistence in the GARCH models and ensure adjusted parameters. Third, we project results on reality and demonstrate their managerial usefulness. Indeed, we test the effect of structural breaks on portfolio designs and hedging effectiveness. Finally, the study provides an implicit service making it possible to compare between stock and commodity markets for managerial and governmental executive purposes.

The remainder of the paper is organized as follows. Section 2, presents a brief literature review. Section 3, outlines the empirical methodology. Section 4, describes the data and their statistical properties and discusses empirical results. Section 5, presents the implications for portfolio management and Section 6 concludes the study.

2. LITERATURE REVIEW

Previous empirical researches admit that the increasing trend of financial globalization and market integration allows to a sustainable interdependence between financial and real markets. Studies' focusing on commodity markets state that volatility is persistent and affecting markets behavior through two ways: corporate cash flows and stock prices.

Initially, Jones and Kaul (1996) investigated the reaction of four developed stock markets (Canada, Japan, UK and USA) to oil shocks on the basis of standard cash-flow dividend valuation model and found evidence of significant impacts of oil price changes on corporate cash flows on the American and Canadian markets. We cite another studies such as Lanza et al. (2005), Henriques and Sadorsky (2008), Aloui et al. (2008), Aloui and Jammazi (2009), Arouri et al. (2012), etc. Furthermore, commodity returns share the same statistical properties as stock market returns. Indeed, non-normality, skewed and fat-tail distributions, autocorrelation, structural changes are frequently observed (for instance, Vivian and Wohar, 2012). We refer to Chkili et al. (2014), who state that commodity returns deserves to be experimented using different volatility models. Consequently, news might be transmitted from oil, food and agricultural commodities and metal markets to equity markets with a possible feedback through conditional correlations, return dynamics and volatility spillovers.

Being expressed in US dollars, crude oil has acquired a global extent. For instance, Malik and Hammoudeh (2007) observed significant volatility spillovers between the US equity market and global oil markets using MGARCH model on daily data spanning from 1994 to 2001. Their findings revealed as well that equity markets in the Gulf countries take delivery of volatility spillovers from the oil market. Park and Ratti (2008) used VAR model to shed light on the relationship between oil-price shocks and stock exchange returns in the USA and 13 other European countries for the period 1986-2005. They establish that oil-price shocks affect stock returns except for the USA. In the same line, Malik and Ewing (2009) employed bivariate GARCH models on daily data from January 1, 1992 to April 30, 2008 to test the volatility transmission between oil market and five American sector indexes. The authors find out significant transmission of shocks and volatility between oil prices and a number of market sectors. More recently, Arfaoui and Abaoub (2010) estimated a time-varying two-factor intertemporal capital asset pricing model with exchange risk, using weekly data over the period 1999 to 2008 and pointed out that oil prices affect equity systematic risk of listed companies.

Vivian and Wohar (2012) detected structural breaks in volatility series of spot return for 28 commodities from January 1985 to July 2010, and showed persistence of volatility for many commodity returns, even after structural breaks.

Lately, Arouri et al. (2012) analyzed data from January 1998 to December 2009 using a VAR-GARCH specification model and found significant volatility spillovers between oil prices and stock sector returns. More recently, Mensi et al. (2013) used VAR-GARCH model and daily data for the period 2000-2011, and found significant transmission among the SP 500 and commodity markets and that past shocks and volatility of the SP 500 impinge well on the oil and gold markets. Ewing and Malik (2013) used univariate and bivariate GARCH models on daily returns from July 1, 1993 to June 30, 2010, to examine the volatility spillovers between gold and oil futures incorporating structural changes. The authors supported strong evidence of significant volatility transmission between gold and oil returns in presence of structural breaks in variance.

Moreover, despite that several studies draw on the long run and short run relationship between markets and on the existence of co-integration for long-run diversification potentials, the instability of the long-run relationships has not yet received much attention. We state here that, in the framework of the sustained financial integration trend, international markets are conceivably potential sources of structural breaks in the asset pricing process. The market linkages through return dynamics and volatility transmissions deserve further investigations when incorporating the informational content of structural breaks. Pragmatically, investors and portfolio managers need to be adequately informed about the structure (bilateral or multilateral) of market co-movements. They need to be informed about the stability of such co-movements.

Kang et al. (2011), examine the impact of structural changes on volatility persistence and then incorporate that impact into bivariate estimation to understand the information flow and volatility transmission. The authors use the bivariate GARCH framework with and without structural changes and show that the degree of volatility persistence was reduced by incorporating the structural changes into the volatility model. They help improve the understanding of volatility transmission but do not show managerial implications for international portfolio investment.

We consider that stock-commodities markets linkages deserves further investigations when considering information flows about the structure of market co-movements such as instability, interdependency, breaks and structural changes.

3. EMPIRICAL METHODOLOGY

It is now well established in the empirical literature that information flows across markets get their delivery from correlation in the second moment more better that correlation in the first moment. Furthermore, the better proxy for information is volatility (Clark, 1973; Tauchen and Pitts, 1983; Ross, 1983). So GARCH-type approach will be a fitting specification. However, in the framework of our aims, we study the interdependence between stock market and each other markets such as oil, agricultural and food commodities, metal while interesting in the possible feedback. Consequently, we consider that a VAR specification is relevant for the specific purpose of our study.

It is commonly accepted that MGARCH specifications with conditional correlation and dynamic covariance such as the CCC-MGARCH, the BEKK-MGARCH and the DCC-MGARCH models are more relevant than univariate models to study volatility-spillovers issue. We intend to make use of the VAR-GARCH model, initiated by Ling and McAleer (2003) and applied by Chan et al. (2005; 2011) and Hammoudeh et al. (2009), Arouri et al. (2012), Mensi et al. (2014) for various economic topics, to explore the interdependence and volatility transmission between different markets. Since we are interesting in the implications of structural changes in weighting and hedging position of portfolio investment, it will be possible to introduce structural breaks dummies as exogenous variable in the GARCH model. Lamoureux and Lastrapes (1990), show that standard GARCH

models overestimate the underlying volatility persistence and structural breaks should be incorporated into a GARCH model to get reliable parameter estimates. In that framework, we have to pay attention to excessive parameters that may encounter convergence and estimation processes.

Moreover, using the Akaike information criterion and Bayesian information criteria, we select one lag for both conditional mean and variance equations for the studied market pairs.

In what follows, we present separately the conditional mean equation and the conditional variance equation in the multivariate framework. The former describe the return channel spillover while the former is considered for the variance spillover with three competitive models: The CCC-, the DCC- and the BEKK-GARCH(1,1).

3.1. The Conditional Mean Equation

The return channel spillover is empirically represented by a vector autoregressive (VAR) model. We declare the VAR(1) representation as follows:

$$Y_t = c + \Phi Y_{t-1} + \varepsilon_t \tag{1}$$

Where,

- $Y_t = (r_t^S, r_t^C)'$. r_t^S and r_t^C are the logarithmic returns on stock market index and returns on commodity markets at time t , respectively. Commodity markets are oil, food and precious metal markets;
- Φ is (2×2) matrix of coefficients to be estimated of the form $\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{21} \\ \Phi_{12} & \Phi_{22} \end{pmatrix}$;
- The coefficients Φ_{11} and Φ_{22} provide the measures of own-mean spillovers, while the coefficients Φ_{21} and Φ_{12} measure the cross-mean spillovers.
- $\varepsilon_t = (\varepsilon_t^S, \varepsilon_t^C)'$, ε_t^S and ε_t^C are residuals of the mean equations for stock and commodity returns, respectively. They are assumed to be serially uncorrelated but with non-nul covariances ($E(\varepsilon_t^S \varepsilon_t^C) \neq 0$).

3.2. The Conditional Variance Equation

The volatility spillovers are modeled by three MVGARCH class models. The first model includes the multivariate CCC-GARCH of Bollerslev (1990) which allow to an easy estimation and inference of the conditional volatility and the conditional correlation. The second specification is the full BEKK-GARCH model of Engle and Kroner (1995), which consider volatility persistence of each market and volatility spillover between markets. The third specification is the DCC-GARCH model of Engle (2002), as a generalization of the CCC model, which allow obtaining different perspectives of correlation via modeling wide variance-covariance matrices and time-varying cross-market comovements.

The residuals of the mean equation are defined as follows:

$$\varepsilon_t = \sqrt{h_t} \eta_t \sim N(0, h_t) \tag{2}$$

$$h_t = c + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} + d_1 D_1 + \dots + d_n D_n \tag{3}$$

- $\eta_t \equiv (\eta_t^S, \eta_t^C)'$ refers to (2×1) vector iid random vectors;
- $\sqrt{h_t} = \text{diag}(\sqrt{h_t^S}, \sqrt{h_t^C})$, with h_t^S and h_t^C are the conditional variances of r_t^S and r_t^C respectively augmented by accounting for break dates dummies as identified by the Bai and Perron (2003) test for multiple structural breaks which are given by Equation (4) and Equation (5):

$$h_t^S = c_S + \alpha_S (\varepsilon_{t-1}^S)^2 + \beta_S h_{t-1}^S + \alpha_C (\varepsilon_{t-1}^C)^2 + \beta_C h_{t-1}^C + \sum_{i=1}^n d_i D_i \tag{4}$$

$$h_t^C = c_C + \alpha_C (\varepsilon_{t-1}^C)^2 + \beta_C h_{t-1}^C + \alpha_S (\varepsilon_{t-1}^S)^2 + \beta_S h_{t-1}^S + \sum_{i=1}^n d_i D_i \tag{5}$$

Where, $D_i (i = 1, \dots, n)$, are the set of dummy variables which takes a value of one for each break point in variance and zero somewhere else (Lamoureux and Lastrapes, 1990; Aggarwal et al., 1999). In our bivariate representation, D_i is a 2×2 square diagonal matrix of parameters and d_i is a 1×2 row vector of volatility break variables.

In matrix, it will be:

$$\begin{pmatrix} h_t^S \\ h_t^C \end{pmatrix} = \begin{pmatrix} c_S \\ c_C \end{pmatrix} + \begin{pmatrix} \alpha_{S1} & \alpha_{S2} \\ \alpha_{C2} & \alpha_{C1} \end{pmatrix} \times \begin{pmatrix} (\varepsilon_{t-1}^S)^2 \\ (\varepsilon_{t-1}^C)^2 \end{pmatrix} + \begin{pmatrix} \beta_{S1} & \beta_{S2} \\ \beta_{C2} & \beta_{C1} \end{pmatrix} \times \begin{pmatrix} h_{t-1}^S \\ h_{t-1}^C \end{pmatrix} + \begin{pmatrix} d_{S1} & d_{S2} \\ d_{C1} & d_{C2} \end{pmatrix} \times \begin{pmatrix} D_S \\ D_C \end{pmatrix} \tag{6}$$

Equations (4) and (5) show how volatility is transmitted through time and across markets. The cross value of the error terms $(\varepsilon_{t-1}^S)^2$ and $(\varepsilon_{t-1}^C)^2$ represents return innovations on the corresponding markets at time $(t-1)$ and represents short run persistence (or the ARCH effect of past shocks), which captures the impact of the direct effects of shock transmission. The presence of (h_{t-1}^S) and (h_{t-1}^C) captures the volatility spillovers between stock market and each corresponding commodity markets. It accounts for the long-run persistence (or the GARCH effects of past volatilities). We remember that reciprocal effects allow to volatility of one market to be affected by its own past shock and volatility but also by past shock and volatility of other markets. The conditional covariance between stock returns and commodity index returns may be derived as follows:

$$H_t = D_t R_t D_t; \quad D_t = \text{diag}(\sqrt{h_t^{SS}}, \sqrt{h_t^{CC}}) \tag{7}$$

Where, $R_t = \rho_t^{S,C}$ is the conditional constant correlation (CCC). We note that the CCC is a restrictive assumption in so far as correlation coefficient is time-varying according to changes in economic and market circumstances.

The DCC-GARCH of Engle (2002) remedies the restrictive assumption of the CCC by allowing the conditional correlation matrix to vary over time. Consequently, R_t is the matrix of time-varying conditional correlations given by:

$$R_t = (\rho_t^{S,C}) = [\text{diag}(Q_t)]^{-\frac{1}{2}} \times Q_t \times [\text{diag}(Q_t)]^{-\frac{1}{2}} \tag{8}$$

R_t is the (2×2) symmetric positive-definite matrix which depends on squared standardized residuals $(\eta_t / \varepsilon_t = \sqrt{h_t} \times \eta_t)$, their unconditional variance-covariance matrix (\bar{Q}) and its own lagged value as represented as follows:

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha\eta_{t-1}\eta'_{t-1} + \beta Q_{t-1} \quad (9)$$

Where, α and β are non-negative scalars as it is $\alpha + \beta < 1$.

Subsequently, the conditional variance-covariance matrix of the DCC-GARCH(1,1) specification (Equation 7) will be:

$$H_t = D_t R_t D_t = \begin{pmatrix} h_t^{SS} & h_t^{SC} \\ h_t^{CS} & h_t^{CC} \end{pmatrix} = \begin{pmatrix} h_t^{SS} & \rho_t^{S,C} \sqrt{h_t^{SC} \times h_t^{CS}} \\ \rho_t^{S,C} \sqrt{h_t^{CS} \times h_t^{SC}} & h_t^{CC} \end{pmatrix} \quad (10)$$

The BEKK-GARCH class model defines the conditional variance-covariance matrix (H_t) as follows:

$$H_t = CC' + A\epsilon_{t-1}\epsilon'_{t-1}A' + BH_{t-1}B' \quad (11)$$

The element C is a (2×2) upper triangular matrix of constants for the pair of markets; A is a (2×2) matrix of coefficients that capture the effects of own and cross-market interdependencies; and B is a (2×2) matrix of coefficients that capture the own volatility persistence and the volatility transmissions between stock and commodity markets.

The estimation of the conditional variances and covariances allows computing optimal weights of a stock-commodity portfolio as well as the optimal hedge ratios. We note that in order to take into account the fact that normality condition is often rejected for economic and financial series we follow Ling and McAleer

(2003), and use the quasi-maximum likelihood estimation method to estimate parameters of the model.

4. RESULTS AND DISCUSSION

4.1. Sources of Data and Descriptive Statistics

We use daily data spanning from January 3, 2000 to March 12, 2014. The data are SP 500 for stock market, the two crude oil benchmark: Europe Brent and Cushing WTIs, two food commodities: Wheat and barley, the two precious metals: Gold and silver. The data have been respectively sourced from the US Energy Information Administration, International Grains Council, Data Stream and SP 500 websites.

We use daily frequency in so far as the daily data allows considering information flows and illustrating some volatility transmission mechanism in spite of the presence of some flaws such as to non-synchronous trading days. We prove the chosen sample period by the intention to cover the global recessions and special events, to account for several sub-periods of economic growth and, of course, the recent global financial crisis which mark an observable separate dynamic pattern since 2007. Moreover, stock and commodity returns are computed by taking the natural logarithm of the ratio of two consecutive prices.

Statistical properties of daily return series are summarized in Table 1. By describing these statistics, we note that average daily returns vary between 0.007, for SP 500 and 0.0427, for gold. The lowest median value is equal to 0.0038, for wheat and the highest one is observed for WTI return series with a value equal to 0.1229.

Table 1: Statistical properties for daily return series

Portfolio	SP 500	Brent	WTI	Wheat	Barley	Gold	Silver
Panel A: Basic statistics							
Mean	0.0070	0.0412	0.0382	0.0256	0.0265	0.0427	0.0388
Median	0.0557	0.0864	0.1229	0.0038	0.0240	0.0615	0.0811
Standard deviation	1.3090	2.2864	2.4669	0.8500	0.9605	1.1838	2.1163
Skewness	-18.1811	-0.2941	-0.2392	0.1379	0.2458	-0.3541	-1.0793
Kurtosis	10.7688	8.7328	7.8531	6.9047	30.3263	8.7679	12.5309
Jarque-Bera	8977.039	4929.042	3529.576	2274.238	110862.6	5017.793	14189.51
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
LB. Q (z) ₁₂	59.824	18.887	10.429	420.130	506.260	26.345	14.036
	(0.0000)	(0.0910)	(0.0000)	(0.0000)	(0.0000)	(0.0101)	(0.2980)
LB. Q (z) ₁₈	103.420	31.322	45.1920	458.710	524.880	30.373	17.689
	(0.0000)	(0.0260)	(0.0000)	(0.0000)	(0.0000)	(0.034)	(0.4760)
Corr.	1	0.117	0.196	0.124	0.054	0.043	0.132
# usable Obs.	3562	3562	3562	3562	3562	3566	3566
Unit root test							
ADF, I (0)	-46.6527 ^a	-59.4957 ^a	-61.2694 ^a	-27.8148 ^a	-23.5944 ^a	-60.7217 ^b	-62.3780 ^a
	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0001)	(0.0001)
PP, I (0)	-65.5279 ^a	-59.4966 ^a	-61.5114 ^a	-50.1741 ^a	-51.3883 ^a	-60.7693 ^b	-62.3783 ^a
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
ZA, I (0)	-29.0794 ^a	-59.5744 ^a	-28.0033 ^a	-24.5312 ^c	-21.3007 ^c	-43.6350 ^a	-62.5271 ^c
	(0.0037)	(0.0195)	(0.0132)	(0.0000)	(0.0028)	(0.0269)	(0.0127)
F-statistics	263.2190	35.1297	209.8040	93.4705	86.8356	28.4981	122.7434
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
LM-statistics (T×R ²)	646.7952	34.8058	198.2235	338.7245	316.8240	28.2873	118.7202
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Q (z)_{12,18} note the Ljung-Box test for 12 and 18 orders serial autocorrelation. Corr. denotes the unconditional correlation coefficients with SP 500. Numbers in parenthesis are P values of the test. The 1% critical values are -2.5656 (a) and -3.432 (b) for the ADF and PP tests, respectively. ^aModel with neither constant nor deterministic trend, ^bModel with constant and without deterministic trend. ZA denotes Zivot-Andrews (1992) test. WTI: West Texas Intermediate, ADF: Augmented Dickey-Fuller, PP: Phillips-Perron

The less volatile daily return is observed for wheat (0.8500) and the most volatile is the one associated with WTI (2.4669), whereas the highest Sharpe ratio is observed for gold with a value of 0.0361.

Skewness values are both positive and negative. The positively skewed daily returns are observed for food commodities, while the negatively skewed returns are observed for the SP 500, oil and precious metal return indexes. Accordingly, investors are expected to see higher positive returns from the food indices rather than from stock market and other commodity indices. Moreover, the whole kurtosis coefficients are over three times of the normal distribution which allows rejecting the normality condition. All of the return indices exhibit as well significant departures from the normal distribution regarding the Jarque–Bera test. The Ljung–Box test show evidence of significant autocorrelations in all of the cases with the exception of Silver. We note that the LB statistics for gold return indices is significant at the 5% level.

The unconditional correlation coefficients between the daily returns of SP 500 and the commodity markets show that they are low and ranging from 0.043 (for gold) to 0.196 (for WTI). We note the correlation coefficients inform about the short run benefits of diversification strategy. A portfolio which is structured with low correlated assets lead to positive returns and is expected to have a lower variance.

As reported in Panel B, we test for the presence of unit roots in the daily return indices using the augmented Dickey-Fuller (ADF), Phillips–Perron and Zivot-Andrews tests. The results show that all

series are a stationary process and integrated of order zero. We try to perform the test to the presence of potential structural breaks, so we implement the Zivot-Andrews (1992) unit root test which considers endogenously break dates within the model. Results make it possible to confirm the stationary process in level. Furthermore, results confirm the presence of ARCH effects at the 1% level and therefore estimation of a GARCH model is appropriate.

4.2. Structural Breaks and Information Flows

Break dates are structural changes in volatility series which has been generated from the GARCH(1,1) process¹. In fact, changes in the GARCH component of the model set up the parameterization of time-varying moments as new arrival information to the market.

Table 2 reports results of parameter estimation of the GARCH(1,1) model for individual markets, and detailed analysis of volatility series. The parameters of the conditional variance equation are positive and statistically significant at 1% and satisfy the theoretical conditions of stability ($\omega > 0, \alpha \geq 0, \beta \geq 0$). Furthermore, the conditional volatility is persistent for all markets, since the risk premium measured by $(\alpha + \beta)$ is superior to 0.9.

The diagnostic of volatility series show that they have common features. Indeed, they are significantly departing from normality

1. The choice of the GARCH(1,1) model is made after a comparison with a non-linear EGARCH specification. The criteria used to determine the optimal lag include the information criteria of Akaike and Schwarz and the log-likelihood value.

Table 2: Estimation of GARCH (1,1) process and statistical properties of volatility series

Portfolio	SP 500	Brent	WTI	Wheat	Barley	Gold	Silver
Panel I: Parameters of variance equation							
ω	0.0164 (0.0000)	0.0175 (0.0064)	0.0438 (0.0001)	0.0024 (0.0000)	0.0889 (0.0000)	0.0158 (0.0000)	0.0593 (0.0000)
α	0.0886 (0.0000)	0.0477 (0.0000)	0.0558 (0.0000)	0.0505 (0.0000)	0.1782 (0.0000)	0.0458 (0.0000)	0.1045 (0.0000)
β	0.8997 (0.0000)	0.9501 (0.0000)	0.9377 (0.0000)	0.9475 (0.0000)	0.7344 (0.0000)	0.9425 (0.0000)	0.8889 (0.0000)
$(\alpha + \beta)$	0.9883	0.9978	0.9935	0.8980	0.9126	0.9883	0.9934
Log-likelihood	-5191.614	-7647.692	-7857.525	-3972.692	-4337.749	-5359.125	-7225.339
Panel II: Statistical properties of volatility series							
Mean	1.6937	5.4001	6.1685	0.7386	0.9536	1.3985	4.7459
Median	0.9446	4.1964	4.5016	0.5251	0.5418	1.0669	3.1158
Standard deviation	2.5955	4.6909	6.0258	0.6463	1.8373	1.0387	5.5270
Maximum	27.9612	38.1892	49.4728	4.9615	40.4883	8.8018	57.0915
Minimum	0.2760	0.9578	1.2890	0.0930	0.1062	0.4383	0.7149
Skewness	5.4212	3.2923	3.5016	2.1834	11.2657	3.0137	4.1512
Kurtosis	39.4923	16.6632	17.1156	9.4115	180.9758	14.5844	26.1505
Jarque-Bera	215091.9 (0.0000)	34141.66 (0.0000)	36851.44 (0.0000)	8931.181 (0.0000)	4776499.0 (0.0000)	25351.85 (0.0000)	89924.93 (0.0000)
LB. Q (12)	0.847***	0.885***	0.868***	0.973***	0.163***	0.860***	0.600***
LB. Q (18)	0.759***	0.839***	0.815***	0.820***	0.103***	0.809***	0.486***
ADF test	-4.9614 (0.0000)	-4.2458 (0.0006)	-4.4382 (0.0003)	-4.8231 (0.0000)	-13.9924 (0.0000)	-5.0306 (0.0000)	-9.0818 (0.0000)
LM statistics (T×R ²)	8.7207 (0.0031)	1.3627 (0.2431)	9.0472 (0.0026)	1.3792 (0.2402)	0.1789 (0.6723)	5.1049 (0.0239)	9.4919 (0.0021)
#Obs.	3562	3562	3562	3562	3562	3568	3568

The variance equation for the GARCH (1,1) model is $h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$. *** and ** Translate significance at 10%, 5% and 1% levels, respectively and indicate that the null hypothesis (normality, no autocorrelation, no stationarity and homogeneity) is rejected. GARCH: Generalized autoregressive conditional heteroskedasticity, WTI: West Texas Intermediate, ADF: Augmented Dickey-Fuller

according to the Jarque–Bera test, stationary through the ADF unit root test at the 1% confidence level, not auto-correlated and reject the null hypothesis of no ARCH effect as aid by the Engle’s (1982) test for conditional heteroscedasticity.

Tests for parameters instability and structural change in regression models have been an important part especially since Bai (1997) and Bai and Perron (1998; 2003a) provide theoretical and computational results that further extend the Quandt-Andrews framework by allowing for multiple unknown breakpoints. We then stake on the technique of Bai and Perron (1998; 2003) in so far as in Monte Carlo experiments, Bai and Perron (2006) find that this method is well-fitting to pinpoint unknown structural breaks. However, Bai and Perron (2006) state that the possible values of break date must be asymptotically distinct and bounded by the borders of the sample. Indeed, different thresholds (trimming parameters) were imposed for the estimation of their model [$\varepsilon = (0.25; 0.15; 0.10; 0.05)$], with $\varepsilon = h/T$, where, T is the sample size and h is the minimal permissible length of a segment. We note that the trimming value implies that each regime is restricted to have at least 15 observations. They recommend to not using a trimming parameter below 5% when bearing in mind of statistical properties such as heteroscedasticity and serial correlation. We really keep the threshold of 5% and, lastly, consider the information criteria to select the number of breaks. The minimized Schwarz and LWZ values are then shaded for easy identification. Table 3 summarize main findings of the conducted analysis.

The Bai and Perron (1998; 2003) test for multiple structural breaks involves a sequential testing process. The test begins with a test for a single structural break. If the null hypothesis of no structural break is rejected, the sample is split in two and the test is repeated for each sub-sample. The sequential process continues until the hypothesis could not be rejected. The test is based on minimizing the sum of squared errors for the sub-samples and identifying a global minimum.

4.3. Return Dynamics and Volatility Transmissions: Results and Discussion

We present and discuss here the empirical results of markets interdependencies mechanism and volatility transmission. The analysis is conducted on seven return indices and estimate six

bivariate VAR-GARCH models (systems). Each system consists in the stock versus one commodity market return indice.

Tables 4-9 present results of the three competing VAR-GARCH class model ignoring and incorporating structural breaks in volatility series. For each model, we discuss interdependencies and volatility spillovers between pairs of markets while peeking at the effect of structural breaks in variance on markets linkages. Tables 4 and 5 summarize estimation results of the VAR-CCC-GARCH model. The computed CCC between the stock and commodity markets daily return are all significantly positive at the 1% level. It ranges between 16.75% (16.78%) for WTI and 2.88% (3.11%) for gold without structural breaks (with structural breaks). This evidence suggests real and mutual interference between markets but it allows profit margin for portfolio diversification.

Through the conditional mean equation, we observe that both stock and commodity return depends on their own one lagged period return (with the exception of Brent and silver). However, in presence of structural breaks in variance, gold return will be independent of its own past. So predictability of gold return behavior becomes hard to achieve with break dates in its own volatility. In contrast, stock market return is predictable in absence and in presence of structural breaks in gold volatility. What deserves to be mentioned here is that one lagged daily stock market return affect the whole of today’s commodity-markets return. We explain this evidence by behavioral factors on stock markets pricing. In the opposite direction, only wheat and gold daily past returns help predict current stock market return. Unexpectedly, we observe that 1 day lagged Brent and silver returns do not affect their own current returns. This evidence point out that both London oil and silver markets do not confirm the weak-form of efficiency. For the other markets, the finding inform about short-term predictability in stock and commodity daily price changes and corroborate statement of a number of recent studies.

On the subject of the conditional variance equation, frequent patterns are observed for both stock and commodity markets. In fact, ARCH and GARCH coefficients are highly significant for nearly all cases. The current conditional volatility (GARCH terms) of stock market is significantly affected by both own past volatility and those of commodity markets in absence and in

Table 3: Detected structural break dates, Bai and Perron’s (2003) test, ($\varepsilon=0.05$)

SP 500	Brent	WTI	Wheat	Barley	Gold	Silver
9	9	13	12	6	12	11
19/10/2001	04/01/2001	03/08/2001	17/07/2002	22/01/2003	22/09/2000	22/09/2000
08/07/2002	25/09/2001	25/04/2002	23/10/2003	02/01/2004	04/02/2003	12/01/2004
11/04/2003	11/06/2002	08/01/2003	12/10/2004	17/07/2008	19/07/2004	20/01/2005
27/07/2007	17/03/2003	23/09/2003	11/04/2006	01/04/2009	13/01/2006	08/02/2006
16/09/2008	06/05/2005	28/10/2004	24/05/2007	07/07/2010	17/10/2006	23/10/2006
02/06/2009	03/09/2008	14/07/2005	04/02/2008	19/07/2011	12/11/2007	05/11/2007
16/09/2010	19/05/2009	01/11/2007	04/02/2009	-	05/08/2008	11/08/2008
01/06/2011	10/09/2010	18/09/2008	19/10/2009	-	21/04/2009	27/04/2007
14/02/2012	07/12/2012	04/06/2009	16/07/2010	-	17/03/2010	03/05/2011
-	-	21/07/2010	29/07/2011	-	08/08/2011	17/01/2012
-	-	06/05/2011	08/10/2012	-	23/04/2012	15/04/2013
-	-	20/01/2011	26/06/2013	-	12/04/2013	-
-	-	12/12/2012	-	-	-	-

WTI: West Texas Intermediate

Table 4: Estimation of VAR-CCC-GARCH model

Portfolio	Stock versus Brent		Stock versus WTI		Stock versus Wheat		Stock versus Barley		Stock versus Gold		Stock versus Silver	
	SP500	Brent	SP500	WTI	SP500	Wheat	SP500	Barley	SP500	Gold	SP500	Silver
Conditional mean equation												
Constant	0.0498*** (0.0010)	0.0608** (0.0503)	0.0518*** (0.0006)	0.0706*** (0.0300)	0.0508*** (0.0008)	0.0138 (0.1861)	0.0506*** (0.0008)	-0.0192* (0.0965)	0.0498*** (0.0009)	0.0464*** (0.0063)	0.0513*** (0.0007)	0.0225 (0.4004)
Stock {1}	-0.0546*** (0.0045)	0.1294*** (0.0000)	-0.0542*** (0.0048)	0.0937*** (0.0016)	-0.0548*** (0.0041)	0.0104 (0.2554)	-0.0522*** (0.0058)	0.0309*** (0.0000)	-0.0588*** (0.0020)	0.0213 (0.1109)	-0.0540*** (0.0055)	0.1181*** (0.0000)
Commodity {1}	0.0007 (0.9225)	-0.0058 (0.7477)	-0.0047 (0.5254)	-0.0377** (0.0169)	0.0119 (0.5366)	0.2591*** (0.0000)	0.0340** (0.0332)	0.2817*** (0.0000)	0.0725*** (0.0000)	-0.0307* (0.0773)	-0.0090 (0.2087)	-0.0097 (0.5693)
Conditional variance equation												
Constant	0.0145*** (0.0000)	0.0108* (0.0834)	0.0117*** (0.0011)	0.0362*** (0.0015)	0.0132*** (0.0000)	0.0014** (0.0159)	0.0178*** (0.0000)	0.0557 (0.0000)	0.0207*** (0.0000)	0.0171*** (0.0000)	0.0192*** (0.0000)	0.0647*** (0.0000)
$(\varepsilon_{t-1}^{stock})^2$	0.0905*** (0.0000)	-0.0043 (0.2338)	0.0904*** (0.0000)	-1.0018 (0.6101)	0.0905*** (0.0000)	-0.0272** (0.0174)	0.0936*** (0.0000)	-0.0200* (0.0837)	0.0883*** (0.0000)	0.0023 (0.7470)	0.0901*** (0.0000)	-0.0084*** (0.0052)
$(\varepsilon_{t-1}^{Commodity})^2$	-0.0341*** (0.0000)	0.0458*** (0.0000)	-0.0381*** (0.0000)	0.0570*** (0.0000)	-0.0081*** (0.0011)	0.0494*** (0.0000)	-0.0757*** (0.0000)	0.2272*** (0.0000)	-0.0059 (0.2280)	0.0449*** (0.0000)	-0.0147* (0.0698)	0.1045*** (0.0000)
h_{t-1}^{stock}	0.8912*** (0.0000)	0.0427 (0.2064)	0.8811*** (0.0000)	0.0632*** (0.0086)	0.8942*** (0.0000)	0.1461* (0.0976)	0.8948*** (0.0000)	0.0085 (0.9331)	0.9076*** (0.0000)	-0.4003 (0.2386)	0.9031*** (0.0000)	-0.0193 (0.3071)
$h_{t-1}^{Commodity}$	0.2486*** (0.0000)	0.9416*** (0.0000)	0.1950*** (0.0032)	0.9258*** (0.0000)	0.0254 (0.1194)	0.9480*** (0.0000)	1.0034*** (0.0001)	0.6751*** (0.0000)	-0.1253 (0.2616)	0.9461*** (0.0000)	-0.0598 (0.1643)	0.8925*** (0.0000)
$(\alpha+\beta)$	0.9817 (0.1120***)	0.9874 (0.0000)	0.9715 (0.0000)	0.9828 (0.0000)	0.9847 (0.0000)	0.9974 (0.0000)	0.9884 (0.0000)	0.9023 (0.0000)	0.9959 (0.0288*)	0.9910 (0.0849)	0.9932 (0.0000)	0.9970 (0.1386***)
CCC			0.1675*** (0.0000)		0.0874*** (0.0000)		0.0605*** (0.0000)					
Diagnostic tests												
Log-likelihood	-12,783.925		-12,975.996		-9033.9023		-9402.9394		-10,523.1561		-12,359.5979	
AIC	7.1895		7.2974		5.0833		5.2906		5.9098		5.9098	
LB1 Q (12)	13.8546 (0.3101)		13.7904 (0.3148)		13.3891 (0.3414)		14.2512 (0.2850)		15.6115 (0.2097)		14.5093 (0.2694)	
LB2 Q (12)	3.5656 (0.9901)		5.6784 (0.9314)		23.4651 (0.0240)		80.1395 (0.0000)		15.8171 (0.1998)		10.0038 (0.6156)	
McLeod-L11 (12)	32.1915 (0.0013)		33.4376 (0.0008)		23.4651 (0.0240)		24.8782 (0.0154)		31.7898 (0.0015)		31.0262 (0.0020)	
McLeod-L12 (12)	15.6150 (0.2095)		23.9546 (0.0206)		11.5537 (0.4821)		6.1336 (0.9092)		8.7834 (0.7213)		17.0640 (0.1472)	
McLeod-L112 (12)	3.4837 (0.9901)		5.3885 (0.9437)		1.8511 (0.9996)		2.0727 (0.9993)		5.7649 (0.9275)		3.4621 (0.9913)	
McLeod-L122 (12)	6.6343 (0.8808)		1.1583 (0.9992)		4.9352 (0.9601)		0.1061 (0.9999)		67.8886 (0.0000)		12.3654 (0.9936)	
Usable Obs.	3562		3562		3561		3561		3567		3567	

***, **, * Show the significance at 10%, 5% and 1% levels, respectively, WTI: West Texas Intermediate, CCC: Constant conditional correlation, GARCH: Generalized autoregressive conditional heteroskedasticity, VAR: Vector autoregression, AIC: Akaike information criterion

Table 5: Estimation of VAR-CCC-GARCH model incorporating structural breaks in volatility series

Portfolio	Stock versus Brent		Stock versus WTI		Stock versus Wheat		Stock versus Barley		Stock versus Gold		Stock versus Silver	
	SP 500	Brent	SP500	WTI	SP 500	Wheat	SP 500	Barley	SP 500	Gold	SP 500	Silver
Conditional mean equation												
Constant	0.0503*** (0.0008)	0.0609* (0.0511)	0.0523*** (0.0006)	0.0723** (0.0263)	0.0508*** (0.0008)	0.0143 (0.1718)	0.0498*** (0.0009)	-0.0197* (0.0896)	0.0494*** (0.0010)	0.0459 (0.0072)	0.0513*** (0.0007)	0.0206 (0.4448)
Stock {1}	-0.0536*** (0.0055)	0.1306*** (0.0000)	-0.0535*** (0.0056)	0.0926*** (0.0018)	-0.0544*** (0.0044)	0.0106 (0.2499)	-0.0501*** (0.0084)	0.0328*** (0.0000)	-0.0588*** (0.0021)	0.0215 (0.1106)	-0.0532*** (0.0068)	0.1191*** (0.0000)
Commodity {1}	0.0008 (0.9101)	-0.0064 (0.7237)	-0.0048 (0.5179)	-0.0382** (0.0163)	0.0127 (0.5112)	0.2593*** (0.0000)	0.0344** (0.0296)	0.2793*** (0.0000)	0.0721*** (0.0000)	-0.0282 (0.1188)	-0.0088 (0.2368)	-0.0099 (0.5693)
Conditional variance equation												
Constant	0.0142*** (0.0000)	0.0096* (0.0785)	0.0119*** (0.0009)	0.0345*** (0.0019)	0.0130*** (0.0000)	0.0014*** (0.0125)	0.0185*** (0.0000)	0.0550*** (0.0000)	0.0187*** (0.0000)	0.0140*** (0.0000)	0.0191*** (0.0000)	0.0617*** (0.0000)
$(\varepsilon_{t-1}^{stock})^2$	0.0901*** (0.0000)	-0.0049 (0.1772)	0.0899*** (0.0000)	-0.0023 (0.5178)	0.0889*** (0.0000)	-0.0259** (0.0225)	0.0902*** (0.0000)	-0.0216* (0.0579)	0.0862*** (0.0000)	0.0031 (0.6704)	0.0887*** (0.0000)	-0.0093*** (0.0021)
$(\varepsilon_{t-1}^{Commodity})^2$	-0.0318*** (0.0000)	0.0403*** (0.0000)	-0.0382*** (0.0000)	0.0569*** (0.0000)	-0.0074*** (0.0024)	0.0473*** (0.0000)	-0.0757*** (0.0000)	0.2266*** (0.0000)	-0.0058 (0.2575)	0.0383*** (0.0000)	-0.0150* (0.0650)	0.1028*** (0.0000)
h_{t-1}^{stock}	0.8913*** (0.0000)	0.0439 (0.1815)	0.8820*** (0.0000)	0.0602** (0.0105)	0.8962*** (0.0000)	0.1228 (0.1629)	0.8991*** (0.0000)	-0.0626 (0.5329)	0.9082*** (0.0000)	-0.3018 (0.2749)	0.9048*** (0.0000)	-0.0228 (0.2307)
$h_{t-1}^{Commodity}$	0.2359*** (0.0000)	0.9483*** (0.0000)	0.1895*** (0.0040)	0.9273*** (0.0000)	0.0196 (0.2125)	0.9499*** (0.0000)	1.0444*** (0.0001)	0.6745*** (0.0000)	-0.2209 (0.1159)	0.9549*** (0.0000)	-0.0625 (0.1458)	0.8945*** (0.0000)
SB stock	0.2965 (0.2038)	-0.4079 (0.3245)	0.2523 (0.2837)	0.1934 (0.7519)	0.1994 (0.3916)	0.0728 (0.2705)	0.3467 (0.1612)	-0.3806*** (0.0077)	0.2889 (0.1973)	0.9166*** (0.0000)	0.3233 (0.1321)	0.5546 (0.2345)
SB commodity	-0.0994 (0.6493)	-1.2839*** (0.0000)	-0.0028 (0.9838)	-0.7903 (0.1232)	0.1752 (0.2512)	0.0441 (0.5281)	0.7180 (0.1012)	-0.0379 (0.9272)	0.1451 (0.2183)	0.5004*** (0.0006)	0.0427 (0.7156)	0.1705 (0.7527)
($\alpha + \beta$)	0.9814 (0.1130***)	0.9886 (0.0000)	0.9719 (0.1678***)	0.9842 (0.0000)	0.9852 (0.0872***)	0.9972 (0.0000)	0.9893 (0.0605***)	0.9011 (0.0002)	0.9944 (0.0311*)	0.9932 (0.0606)	0.9935 (0.1378***)	0.9973 (0.0000)
CCC												
Diagnostic tests												
Log-likelihood	-12.776.5675		-12.972.1508		-9032.2172		-9399.1907		-10.502.9262		-12.354.6656	
AIC	7.1896		7.2995		5.0846		5.2907		5.90237		6.9409	
LB1 Q (12)	13.8333 (0.3115)		13.6995 (0.3203)		13.3791 (0.3421)		14.7123 (0.2575)		15.5161 (0.2144)		14.3862 (0.2767)	
LB2 Q (12)	3.3040 (0.9930)		5.7056 (0.9302)		49.3889 (0.0000)		79.6031 (0.0000)		16.4913 (0.1698)		10.0914 (0.6079)	
McLeod-L11 (12)	33.2157 (0.0009)		33.9167 (0.0007)		23.1306 (0.0266)		23.5511 (0.0234)		34.4251 (0.0012)		32.4619 (0.0012)	
McLeod-L12 (12)	17.4985 (0.1318)		24.9799 (0.0149)		12.1325 (0.4351)		6.0337 (0.9144)		10.5268 (0.5698)		17.5902 (0.1287)	
McLeod-L112 (12)	3.6120 (0.9895)		5.4042 (0.9431)		1.4661 (0.9999)		1.7031 (0.9997)		5.0267 (0.9569)		3.6456 (0.9890)	
McLeod-L122 (12)	6.1462 (0.9085)		1.2187 (1.0000)		5.2205 (0.9502)		0.1039 (1.0000)		32.4470 (0.0002)		3.6791 (0.9886)	

*** ** * Show the significance at 10%, 5% and 1% levels, respectively, WTI: West Texas Intermediate, CCC: Constant conditional correlation, GARCH: Generalized autoregressive conditional heteroskedasticity, VAR: Vector autoregression, AIC: Akaike information criterion

Table 6: Estimation of VAR-DCC-GARCH model

Portfolio	Stock versus Brent		Stock versus wti		Stock versus Wheat		Stock versus Barley		Stock versus Gold		Stock versus Silver	
	SP 500	Brent	SP 500	WTI	SP 500	Wheat	SP 500	Barley	SP 500	Gold	SP 500	Silver
Conditional mean equation												
Constant	0.0519*** (0.0005)	0.0503* (0.0943)	0.0553*** (0.0002)	0.0587* (0.0608)	0.0517*** (0.0006)	0.0115 (0.2689)	0.0506*** (0.0008)	-0.0203* (0.0834)	0.0497*** (0.0009)	0.0468*** (0.0057)	0.0487*** (0.0011)	0.0084 (0.7497)
Stock {1}	-0.0676*** (0.0003)	0.1307*** (0.0000)	-0.0580*** (0.0016)	0.0945*** (0.0004)	-0.0569*** (0.0026)	0.0120 (0.1862)	-0.0562*** (0.0032)	0.0331*** (0.0000)	-0.0588*** (0.0020)	0.0212*** (0.1111)	-0.0447* (0.0151)	0.1185*** (0.0000)
Commodities {1}	0.0011 (0.7535)	-0.0139 (0.4331)	-0.0051 (0.4654)	-0.0457*** (0.0024)	0.0122 (0.5233)	0.2549*** (0.0000)	0.0344** (0.0305)	0.2834*** (0.0000)	0.0725*** (0.0000)	-0.0302* (0.0827)	-0.0087 (0.2357)	-0.0108 (0.5205)
Conditional variance equation												
Constant	0.0165*** (0.0000)	0.0207*** (0.0025)	0.0173*** (0.0000)	0.0563*** (0.0000)	0.0161*** (0.0000)	0.0022*** (0.0000)	0.0176*** (0.0000)	0.0546*** (0.0000)	0.0207*** (0.0000)	0.0170*** (0.0000)	0.0185*** (0.0000)	0.0504*** (0.0000)
$(\mathcal{E}_{t-1}^{stock})^2$	0.0893*** (0.0000)	-0.0069 (0.1130)	0.0898*** (0.0000)	-0.0024 (0.5841)	0.0919*** (0.0000)	-0.0285** (0.0184)	0.0942*** (0.0000)	-0.0237* (0.0600)	0.0882*** (0.0000)	0.0023 (0.7647)	0.0897*** (0.0000)	-0.0025 (0.5702)
$(\mathcal{E}_{t-1}^{Commodity})^2$	-0.0141* (0.0972)	0.0482*** (0.0000)	0.0175 (0.1182)	0.0587*** (0.0000)	-0.0113*** (0.0000)	0.0506*** (0.0000)	-0.0779*** (0.0000)	0.2169*** (0.0000)	-0.0058 (0.2415)	0.0449*** (0.0000)	-0.0091 (0.4862)	0.1051*** (0.0000)
h_{t-1}^{stock}	0.8990*** (0.0000)	0.0083 (0.1746)	0.8974*** (0.0000)	0.0052 (0.3282)	0.8965*** (0.0000)	0.0350 (0.1384)	0.8933*** (0.0000)	0.0388 (0.7624)	0.9078*** (0.0000)	-0.2816 (0.1144)	0.9008*** (0.0000)	-0.0113 (0.1120)
$h_{t-1}^{Commodity}$	-0.0021 (0.8661)	0.9502*** (0.0000)	-0.0454*** (0.0012)	0.9354*** (0.0000)	0.0197*** (0.0053)	0.9458*** (0.0000)	0.9851*** (0.0000)	0.6924*** (0.0000)	-0.0873 (0.1392)	0.9462*** (0.0000)	0.0025 (0.9045)	0.8916*** (0.0000)
$(\alpha+\beta)$	0.9883	0.9984	0.9872	0.9941	0.9884	0.9964	0.9875	0.9093	0.9960	0.9911	0.9905	0.9967
DCC	0.0178*** (0.0000)	0.0000 (0.0000)	0.0273*** (0.0000)	0.0000 (0.0000)	0.0065*** (0.0029)	0.0029 (0.0000)	0.0042* (0.0658)	0.0000 (0.0000)	0.0098*** (0.0003)	0.0003 (0.0003)	0.0418*** (0.0000)	0.0000 (0.0000)
k1	0.9805*** (0.0000)	0.0000 (0.0000)	0.9708*** (0.0000)	0.0000 (0.0000)	0.9923*** (0.0000)	0.0000 (0.0000)	0.8357*** (0.0000)	0.0000 (0.0000)	0.5742 (0.9986)	0.0000 (0.0000)	0.9464*** (0.0000)	0.0000 (0.0000)
Diagnostic tests	-12,710.5195 (7.1488)	0.0000 (0.2711)	-12,850.2230 (7.2273)	0.0000 (0.2968)	-9018.653 (5.0753)	0.0000 (0.3599)	-9400.9075 (5.29004)	0.0000 (0.2774)	-10,523.4350 (5.9105)	0.0000 (0.2096)	-12,285.9625 (6.8987)	0.0000 (0.2744)
Log-likelihood	14.4806	14.4806	14.0619	14.0619	13.1267	13.1267	14.3752	14.3752	15.6125	15.6125	14.4256	14.4256
AIC	5.0146	5.0146	6.8506	6.8506	52.3311	52.3311	80.8370	80.8370	31.8521	31.8521	6.2356	6.2356
LB1 Q (12)	32.2386	32.2386	30.9026	30.9026	23.4951	23.4951	24.7185	24.7185	30.1458	30.1458	29.5385	29.5385
LB2 Q (12)	18.2031	18.2031	23.6587	23.6587	11.8248	11.8248	6.4319	6.4319	11.6255	11.6255	5.8028	5.8028
McLeod-L11 (12)	3.2005	3.2005	2.9395	2.9395	1.7350	1.7350	1.9921	1.9921	5.8028	5.8028	4.3258	4.3258
McLeod-L12 (12)	9.4076	9.4076	1.2909	1.2909	5.2479	5.2479	0.1152	0.1152	16.2385	16.2385	0.9650	0.9650

*, **, ***: S how the significance at 10%, 5% and 1% levels, respectively. , WTI: West Texas Intermediate, CCC: Constant conditional correlation, GARCH: Generalized autoregressive conditional heteroskedasticity, VAR: Vector autoregression, AIC: Akaike information criterion, DCC: Dynamic conditional correlation

Table 7: Estimation of VAR-DCC-GARCH model incorporating structural breaks in volatility series

Portfolio	Stock versus Brent		Stock versus WTI		Stock versus Wheat		Stock versus Barley		Stock versus Gold		Stock versus Silver	
	SP 500	Brent	SP 500	WTI	SP 500	Wheat	SP 500	Barley	SP 500	Gold	SP 500	Silver
Conditional mean equation												
Constant	0.0521*** (0.0005)	0.0507* (0.0922)	0.0516*** (0.0005)	0.0449 (0.1249)	0.0517*** (0.0006)	0.0120 (0.2508)	0.0501*** (0.0009)	-0.0204* (0.0829)	0.0493*** (0.0010)	0.0464*** (0.0068)	0.0489*** (0.0010)	0.0066 (0.8014)
Stock {1}	-0.0668*** (0.0004)	0.1321 (0.0000)	-0.0501*** (0.0081)	0.0324 (0.1810)	-0.0566** (0.0028)	0.0120 (0.1858)	-0.0535*** (0.0052)	0.0345*** (0.0000)	-0.0589*** (0.0021)	0.0214 (0.1145)	-0.0440** (0.0172)	0.1194*** (0.0000)
Commodity {1}	0.0013 (0.8529)	-0.0145 (0.4118)	-0.0018 (0.7938)	0.2464*** (0.0000)	0.0132 (0.4908)	0.2553*** (0.0000)	0.0349 (0.0261)	0.2814*** (0.0000)	0.0720*** (0.0000)	-0.0277 (0.1296)	-0.0083 (0.2676)	-0.0109 (0.5242)
Conditional variance equation												
Constant	0.0162*** (0.0000)	0.0182** (0.0031)	0.0166*** (0.0000)	0.0313*** (0.0002)	0.0156*** (0.0000)	0.0022*** (0.0000)	0.0186*** (0.0000)	0.0542*** (0.0000)	0.0187*** (0.0000)	0.0140*** (0.0000)	0.0184*** (0.0000)	0.0471*** (0.0000)
$(\varepsilon_{t-1}^{stock})^2$	0.0887*** (0.0000)	-0.0075* (0.0937)	0.0887*** (0.0000)	-0.0067 (0.1641)	0.0905*** (0.0000)	-0.0262** (0.0314)	0.0908*** (0.0000)	-0.0225* (0.0631)	0.0860*** (0.0000)	0.0031 (0.6752)	0.0885*** (0.0000)	-0.0026 (0.5447)
$(\varepsilon_{t-1}^{Commodity})^2$	-0.0143* (0.0734)	0.0443*** (0.0000)	-0.0106 (0.2595)	0.0550*** (0.0000)	-0.0106*** (0.0002)	0.0490*** (0.0000)	-0.0776*** (0.0000)	0.2181*** (0.0000)	-0.0058 (0.3171)	0.0382*** (0.0000)	-0.0101 (0.4364)	0.1032*** (0.0000)
h_{t-1}^{stock}	0.8995*** (0.0000)	0.0079 (0.1956)	0.8987*** (0.0000)	0.0079 (0.2268)	0.8979*** (0.0000)	0.0270 (0.2698)	0.8980*** (0.0000)	-0.0612 (0.6257)	0.9084*** (0.0000)	-0.2285 (0.1815)	0.9019*** (0.0000)	-0.0131* (0.0732)
$h_{t-1}^{Commodity}$	0.0002 (0.9853)	0.9550*** (0.0000)	-0.0076 (0.5868)	0.9419*** (0.0000)	0.0177** (0.0111)	0.9473*** (0.0000)	1.0339*** (0.0000)	0.6896*** (0.0000)	-0.1670*** (0.0000)	0.9550*** (0.0000)	0.0035 (0.8679)	0.8934*** (0.0000)
SB stock	0.3080 (0.1875)	-0.1868 (0.6752)	0.3021 (0.1970)	-0.3144 (0.4852)	0.1552 (0.5152)	0.0549 (0.4235)	0.3606 (0.1574)	-0.3361** (0.0349)	0.2895 (0.1971)	0.9178*** (0.0000)	0.3433 (0.1171)	0.6426 (0.1672)
SB commodities	-0.0594 (0.7872)	-1.2267 (0.0001)	-0.0274 (0.8407)	-0.6741* (0.0817)	0.1931 (0.2144)	0.0353 (0.6214)	0.6807 (0.1166)	-0.0449 (0.9108)	0.1465 (0.2137)	0.5026 (0.6044)	0.0192 (0.8544)	0.3079 (0.5620)
$(\alpha+\beta)$	0.9882 (0.0000)	0.9993 (0.0000)	0.9874 (0.0000)	0.9969 (0.0000)	0.9884 (0.0000)	0.9963 (0.0000)	0.9888 (0.0000)	0.9077 (0.0000)	0.9944 (0.0000)	0.9932 (0.0000)	0.9904 (0.0000)	0.9966 (0.0000)
DCC	0.0185*** (0.0000)	0.0185*** (0.0000)	0.0163*** (0.0000)	0.0065*** (0.0036)	0.0065*** (0.0036)	0.0065*** (0.0036)	0.0037** (0.0945)	0.0037** (0.0945)	0.0000 (1.0000)	0.0000 (1.0000)	0.0419*** (0.0000)	0.0000 (0.0000)
k1	0.9797*** (0.0000)	0.9797*** (0.0000)	0.9822*** (0.0000)	0.9922*** (0.0000)	0.9922*** (0.0000)	0.9922*** (0.0000)	0.8357*** (0.0000)	0.8357*** (0.0000)	0.2891 (1.0000)	0.2891 (1.0000)	0.9463*** (0.0000)	0.9463*** (0.0000)
Diagnostic tests												
Log-likelihood	-12.7060288	-12.7060288	-12.5846399	-9017.3337	-9017.3337	-9017.3337	-9397.5545	-9397.5545	-10.5031128	-10.5031128	-12.280778	-12.280778
AIC	7.1485	7.1485	7.0804	5.0768	5.0768	5.0768	5.2904	5.2904	5.9030	5.9030	6.9000	6.9000
LB1 Q (12)	14.5428 (0.2674)	14.5428 (0.2674)	13.9934 (0.3011)	13.1525 (0.3581)	13.1525 (0.3581)	13.1525 (0.3581)	14.7593 (0.2549)	14.7593 (0.2549)	15.5187 (0.2143)	15.5187 (0.2143)	14.3488 (0.2790)	14.3488 (0.2790)
LB2 Q (12)	4.8600 (0.9625)	4.8600 (0.9625)	81.4998 (0.0000)	51.8812 (0.0000)	51.8812 (0.0000)	51.8812 (0.0000)	80.3005 (0.0000)	80.3005 (0.0000)	16.4731 (0.1705)	16.4731 (0.1705)	10.2835 (0.5911)	10.2835 (0.5911)
McLeod-L11 (12)	33.0397 (0.0010)	33.0397 (0.0010)	32.3635 (0.0012)	23.0770 (0.0271)	23.0770 (0.0271)	23.0770 (0.0271)	23.5627 (0.0233)	23.5627 (0.0233)	32.4985 (0.0012)	32.4985 (0.0012)	30.6260 (0.0022)	30.6260 (0.0022)
McLeod-L12 (12)	19.9057 (0.0689)	19.9057 (0.0689)	7.5755 (0.8174)	12.1711 (0.4320)	12.1711 (0.4320)	12.1711 (0.4320)	6.3291 (0.8986)	6.3291 (0.8986)	10.5755 (0.5656)	10.5755 (0.5656)	17.0652 (0.1472)	17.0652 (0.1472)
McLeod-L112 (12)	3.2279 (0.9937)	3.2279 (0.9937)	3.1455 (0.9944)	1.3937 (0.9999)	1.3937 (0.9999)	1.3937 (0.9999)	1.7058 (0.9997)	1.7058 (0.9997)	5.0583 (0.9560)	5.0583 (0.9560)	2.9174 (0.9961)	2.9174 (0.9961)
McLeod-L122 (12)	8.7004 (0.7283)	8.7004 (0.7283)	3.4049 (0.9919)	5.3967 (0.9434)	5.3967 (0.9434)	5.3967 (0.9434)	0.1124 (0.9999)	0.1124 (0.9999)	123.9631 (0.0000)	123.9631 (0.0000)	3.5671 (0.9900)	3.5671 (0.9900)

***, **, * Show the significance at 10%, 5% and 1% levels, respectively. WTI: West Texas Intermediate, CCC: Constant conditional correlation, GARCH: Generalized autoregressive conditional heteroskedasticity, VAR: Vector autoregression, AIC: Akaike information criterion, DCC: Dynamic conditional correlation

Table 8: Estimation of VAR-BEKK-GARCH model

Portfolio	Stock versus Brent	Stock versus WTI	Stock versus Wheat	Stock versus Barley	Stock versus Gold	Stock versus Silver
Conditional mean equation						
Constant	0.0507*** (0.0007)	0.0526*** (0.0003)	0.01518*** (0.0006)	0.0541*** (0.0005)	0.0492*** (0.0011)	0.0464*** (0.0017)
Stock {1}	-0.0726*** (0.0001)	-0.0612*** (0.0007)	-0.0551*** (0.0026)	-0.0522*** (0.0009)	-0.0601*** (0.0013)	-0.0454*** (0.0081)
Commodity {1}	0.0021 (0.7667)	-0.0046 (0.4995)	0.0134 (0.4820)	0.0388** (0.0241)	0.0756*** (0.0000)	-0.0086 (0.2455)
Constant	0.0454 (0.1328)	0.0496 (0.1097)	0.0157 (0.1386)	-0.0153 (0.1976)	0.0481*** (0.0048)	0.0085 (0.7442)
Stock {1}	0.1380*** (0.0000)	0.1008*** (0.0001)	0.0129 (0.1513)	0.0531*** (0.0000)	0.0232* (0.0743)	0.1164*** (0.0000)
Commodity {1}	-0.0160 (0.3503)	-0.0440*** (0.0016)	0.2538*** (0.0000)	0.2756*** (0.0000)	-0.0201 (0.2234)	-0.0112 (0.4967)
Conditional variance equation						
C (1,1)	0.1229*** (0.0000)	0.1224*** (0.0000)	0.1282*** (0.0000)	0.1186*** (0.0000)	0.1253*** (0.0000)	0.1281*** (0.0000)
C (2,1)	0.0325 (0.1791)	0.0864*** (0.0034)	-0.0067 (0.4653)	0.0514** (0.0270)	0.0124 (0.5616)	0.0224 (0.4480)
C (2,2)	0.1082*** (0.0000)	0.1492*** (0.0000)	0.0446*** (0.0000)	0.2455*** (0.0000)	0.1113*** (0.0000)	-0.2281*** (0.0000)
A (1,1)	0.2856*** (0.0000)	0.2916*** (0.0000)	0.2884*** (0.0000)	0.2570*** (0.0000)	0.2874*** (0.0000)	0.2729*** (0.0000)
A (1,2)	0.0178 (0.2766)	0.0372** (0.0224)	-0.0010 (0.8704)	-0.1289*** (0.0000)	0.0097 (0.3474)	-0.0236 (0.1337)
A (2,1)	-0.0055 (0.3338)	-0.0029 (0.5616)	-0.0167 (0.3294)	-0.0109 (0.5222)	0.0123 (0.2843)	0.0026 (0.5645)
A (2,2)	0.1920*** (0.0000)	0.2085*** (0.0000)	0.1936*** (0.0000)	0.4756*** (0.0000)	0.1807*** (0.0000)	0.3062*** (0.0000)
B (1,1)	0.9528*** (0.0000)	0.9518*** (0.0000)	0.9517*** (0.0000)	0.9617*** (0.0000)	0.9523*** (0.0000)	0.9560*** (0.0000)
B (1,2)	0.0095*** (0.0072)	-0.0159*** (0.0022)	0.0006 (0.7405)	0.0304*** (0.0000)	-0.0032 (0.3813)	0.0072 (0.1946)
B (2,1)	0.0018 (0.1739)	0.0004 (0.7877)	0.0060 (0.0995)	0.0027 (0.7687)	-0.0026 (0.4009)	-0.0004 (0.8054)
B (2,2)	0.9812*** (0.0000)	0.9763*** (0.0000)	0.9799*** (0.0000)	0.8455*** (0.0000)	0.9788*** (0.0000)	0.9496*** (0.0000)
Diagnostic tests						
Log-likelihood	12728.3323	12868.7235	-9052.2213	-9422.9335	-10553.9758	-12308.6015
AIC	7.1583	7.2371	5.0936	5.3018	5.9287	6.9128
LB1 Q (12)	15.0169 (2405)	14.1747 (0.2897)	13.2373 (0.3520)	14.4614 (0.2722)	15.2239 (0.2294)	14.6605 (0.2605)
LB2 Q (12)	5.3493 (0.9453)	6.4132 (0.8938)	55.8081 (0.0000)	85.1691 (0.0000)	15.0588 (0.2382)	9.7832 (0.6350)
McLeod-Li1 (12)	34.6431 (0.0005)	31.7506 (0.0015)	30.2650 (0.0025)	52.4782 (0.0000)	35.6685 (0.0004)	41.0887 (0.0000)
McLeod-Li2 (12)	24.8410 (0.0156)	32.2983 (0.0012)	18.2310 (0.1089)	7.9550 (0.7886)	21.5229 (0.0432)	21.9863 (0.0377)
McLeod-Li12 (12)	3.9409 (0.9845)	2.8236 (0.9967)	3.4106 (0.9919)	9.7666 (0.6364)	8.3651 (0.7560)	6.0721 (0.9124)
McLeod-Li22 (12)	10.1557 (0.6023)	2.1572 (0.9991)	7.9029 (0.7927)	0.1798 (0.9998)	149.0023 (0.0000)	4.9895 (0.9583)

*, **, ***: Show the significance at 10%, 5% and 1% levels, respectively., WTI: West Texas Intermediate, GARCH: Generalized autoregressive conditional heteroskedasticity, VAR: Vector autoregression, AIC: Akaike information criterion, BEKK: Baba, Engle, Kroner, Kraft

presence of dummy volatility break dates except for wheat and precious metals. Reciprocally, past stock markets' volatility help significantly predict current commodity markets GARCH terms except for Brent, barley, gold and silver.

Whereas, it affects solely WTI market in presence of structural breaks in volatility. Accordingly, we confirm the absence of mutual and direct effects between stock and precious-metals GARCH terms

($h_{t,j}$). For foods, the 1 day lagged GARCH term of barley affect positively and significantly pricing process on stock markets. This evidence is persisting in DCC and BEKK class models and might be explained by the strategic position of barley in worldwide food industry. ARCH terms exhibit significant coefficients in most cases and prove that current stock-market conditional volatility depends with own past shocks and commodity markets shocks except for gold

Table 9: Estimation of VAR-BEKK-GARCH model incorporating structural breaks in volatility series

Portfolio	Stock versus Brent	Stock versus WTI	Stock versus Wheat	Stock versus Barley	Stock versus Gold	Stock versus Silver
Conditional mean equation						
Constant	0.0501*** (0.0007)	0.0534*** (0.0002)	0.0522*** (0.0006)	0.0541*** (0.0006)	0.0490*** (0.0011)	0.0462*** (0.0018)
Stock {1}	-0.0716*** (0.0001)	-0.0606*** (0.0008)	-0.0549*** (0.0029)	-0.0507*** (0.0013)	-0.0592*** (0.0016)	-0.0446*** (0.0096)
Commodity {1}	0.0022 (0.7606)	-0.0045 (0.5019)	0.0148 (0.4405)	0.0383** (0.0329)	0.0747 (0.0000)	-0.0083 (0.2665)
Constant	0.0459 (0.1295)	0.0501 (0.1043)	0.0162 (0.1264)	-0.0162 (0.1725)	0.0479*** (0.0052)	0.0068 (0.7940)
Stock {1}	0.1379 (0.0000)	0.1011*** (0.0001)	0.0131 (0.1539)	0.0351*** (0.0001)	0.0247* (0.0716)	0.1169*** (0.0000)
Commodity {1}	-0.0158 (0.3576)	-0.0441*** (0.0019)	0.2544*** (0.0000)	0.2753*** (0.0000)	-0.0176 (0.3184)	-0.0113 (0.4917)
Conditional variance equation						
C (1,1)	0.1220*** (0.0000)	0.1224*** (0.0000)	0.1252*** (0.0000)	0.1158*** (0.0000)	0.1211*** (0.0000)	-0.1291*** (0.0000)
C (2,1)	0.0310 (0.2048)	0.0855*** (0.0038)	-0.0057 (0.5357)	0.0509** (0.0356)	0.0132 (0.5065)	-0.0203 (0.4969)
C (2,2)	-0.1102*** (0.0000)	0.1493*** (0.0000)	0.0413*** (0.0000)	0.2437*** (0.0000)	0.0997 (0.0000)	0.2189*** (0.0000)
A (1,1)	0.2841*** (0.0000)	0.2916*** (0.0000)	0.2864*** (0.0000)	0.2476*** (0.0000)	0.2840*** (0.0000)	0.2734*** (0.0000)
A (1,2)	0.0134 (0.4161)	0.0360** (0.0314)	-0.0006 (0.9265)	-0.1338*** (0.0000)	0.0078 (0.4933)	-0.0255 (0.1267)
A (2,1)	-0.0065 (0.2529)	-0.0038 (0.4474)	-0.0169 (0.3372)	-0.0115 (0.5254)	0.0184 (0.1015)	0.0009 (0.8813)
A (2,2)	0.1915*** (0.0000)	0.2091*** (0.0000)	0.1852*** (0.0000)	0.4766*** (0.0000)	0.1665*** (0.0000)	0.2987*** (0.0000)
B (1,1)	0.9530*** (0.0000)	0.9516*** (0.0000)	0.9520*** (0.0000)	0.9633*** (0.0000)	0.9530*** (0.0000)	0.9553 (0.0000)
B (1,2)	-0.0084 (0.1070)	0.0156*** (0.0030)	0.0004 (0.8397)	0.0308*** (0.0000)	-0.0021 (0.5776)	0.0067 (0.2599)
B (2,1)	-0.0020 (0.1408)	0.0005 (0.6925)	0.0052 (0.1687)	0.0018 (0.8543)	-0.0036 (0.1963)	-0.0002 (0.9879)
B (2,2)	0.9810*** (0.0000)	0.9762*** (0.0000)	0.9811*** (0.0000)	0.8445*** (0.0000)	0.9805*** (0.0000)	0.9514*** (0.0000)
SB stock11	0.5091*** (0.0083)	0.3827* (0.0662)	0.4771 (0.2145)	0.6416*** (0.0004)	0.3756* (0.0674)	-0.4816** (0.0167)
SB stock21	0.3346 (0.5150)	0.1053 (0.8741)	0.3883 (0.2597)	0.1056 (0.7826)	-0.1793 (0.7377)	-0.8995** (0.0419)
SB stock22	0.1066 (0.9986)	0.3691 (0.4738)	-0.0417 (0.9999)	-0.2437 (0.9998)	0.7566*** (0.0000)	-0.2190 (0.9995)
SB commodity11	0.1217 (0.9992)	-0.0482 (0.9579)	0.3860** (0.0105)	0.7609*** (0.0000)	0.2470 (0.1248)	-0.0153 (0.9671)
SB commodity21	-0.0282 (0.9997)	-0.0089 (0.9973)	0.0817 (0.6152)	0.4384 (0.4396)	-0.1878 (0.6714)	-1.0944 (0.7288)
SB commodity22	0.1103 (0.9999)	-0.1493 (0.9999)	-0.3495** (0.0038)	-0.2437 (0.9999)	0.6747*** (0.0000)	-0.2200 (0.9999)
Diagnostic tests						
Log-likelihood	-12,724.7943	-12,865.2729	-9046.2403	-9414.9757	-10,534.4543	-12,303.9924
AIC	7.1617	7.2406	5.0936	5.3022	5.9212	6.9136
LB1 Q (12)	14.9868 (0.2422)	14.1106 (0.2937)	13.2826 (0.3488)	15.1788 (0.2318)	15.1162 (0.2351)	14.7808 (0.2536)
LB2 Q (12)	5.3380 (0.9457)	6.2955 (0.9005)	54.0137 (0.0000)	84.8751 (0.0000)	15.7031 (0.2052)	9.8926 (0.6254)
McLeod-Li1 (12)	35.6229 (0.0004)	32.1537 (0.0013)	28.4038 (0.0048)	49.3255 (0.0000)	36.0354 (0.0003)	41.1548 (0.0000)
McLeod-Li2 (12)	24.7095 (0.0163)	31.8888 (0.0014)	19.1344 (0.0853)	7.9315 (0.7905)	22.7342 (0.0301)	23.2000 (0.0261)
McLeod-Li12 (12)	4.0218 (0.9830)	2.8435 (0.9966)	2.1152 (0.9992)	5.3008 (0.9472)	7.9520 (0.7889)	5.9968 (0.9162)
McLeod-Li22 (12)	9.9389 (0.6213)	2.0852 (0.9993)	8.7899 (0.7208)	0.1819 (0.9999)	226.1291 (0.0000)	4.7695 (0.9652)

*, **, ***: Show the significance at 10%, 5% and 1% levels, respectively, , WTI: West Texas Intermediate, GARCH: Generalized autoregressive conditional heteroskedasticity, VAR: Vector autoregression, AIC: Akaike information criterion, BEKK: Baba, Engle, Kroner, Kraft

both in presence and absence of structural breaks. Likewise, lagged stock market shocks significantly affect current commodity markets conditional variance except for the two strategic commodities;

oil and gold. We state that past own and bidirectional shocks are leading volatility spillovers between stock and commodity markets in absence and presence of structural breaks in volatility series.

In the same line, the dummy break dates as exogenous factor (Table 5) help forecast present conditional volatility of both stock and commodity markets. Indeed, structural breaks in SP 500 volatility have a significant instantaneous positive (negative) effect on gold (barley) markets. In the same way, shocks in conditional volatility are persistent for long time in so far as the risk premium ($\alpha + \beta$) is closer to unity and ranging from 0.9023 (barley) and 0.9974 (wheat).

Comparing to GARCH terms, ARCH coefficients have small size which makes it possible to infer that conditional volatility does not react simultaneously to impulses on own and bidirectional shocks. They are likely to progress steadily over time regarding substantial effects of past volatility, as indicated by the large values of GARCH terms. Roughly speaking, the results for stock and commodity sectors put forward interesting insights. In fact, stock market is diffuser of changes (shock and volatility) on commodity markets except for oil and gold. The Brent and silver markets are weakly efficient and pricing processes are led by behavioral and speculative factors other than economic and financial. The current findings seem to be plausible and corroborate previous recent studies. We cite, *inter alia*, Arouri et al. (2011a; 2011b; 2012), Chang et al. (2011), Mensi et al. (2013; 2014), etc.

As a final point for CCC specification, the results of diagnostic tests based on standardized residuals are shown in each estimation table. We find that departure from normality and autocorrelation are reduced to a great extent than those presented in Table 1, of statistical properties of daily return series. More importantly, standardized residuals do not exhibit remaining ARCH effects. Therefore, the bivariate VAR(1)-GARCH (1,1) class model fits better to capture the bidirectional dynamics stock-commodity markets both in presence and absence of structural breaks in volatility series.

For comparison and flawlessness intent, we estimate the VAR-DCC-GARCH model of Engle (2002) and report results in Tables 6 and 7. The foremost idea is that the DCC parameters (k_1 and k_2) are statistically significant in all cases, leading to the rejection of the hypothesis of CCC for every part of conditioning information to daily returns. The values are ranging between 0.0042 (barley) and 0.0418 (silver) for short-run persistence of shocks on the DCC and between 0.9923 (wheat) and 0.5742 (for gold) for largest long-run persistence of shocks on the DCC. When we consider structural breaks, values vary between 0.0022 (gold) and 0.0419 (silver) for short-run persistence of shocks and between 0.9922 (wheat) and 0.2891 (gold) for the long-run persistence.

Interpretation of the conditional mean equation makes it possible to confirm obtained results from the CCC specifications with slightly superior effect regarding values of the significant coefficients. On the subject of the conditional variance equation, the DCC specification confirms the effect of commodity's past volatility on current stock market's volatility. The exception which deserves to be declared is that the effect of lagged volatility, on current stock market volatility, of Brent (barley) appears (disappears) in CCC but disappears (appears) in the DCC class model.

In Tables 8 and 9, we recapitulate the estimates of the VAR-BEKK-GARCH class model both without and with break dates effect. Taking a close look at the mean equation, the current daily stock return depends on the own one lagged return and those of barley and gold. Reciprocally, commodity current daily return depends on own past return except for Brent, precious metals and those of stock market except for wheat. When incorporating the dummy structural breaks in volatility, we find the same results for stock markets return. This result illustrates the evidence of short-term predictability in some commodity price changes through time. As for the cross-markets interdependencies in mean, we state that gold and barley, as strategic commodities, help predict stock market pricing behavior. On the contrary, wheat pricing process is independent from stock market daily news. Furthermore, London oil market and precious metals confirm the weak-form of informational efficiency. This statement corroborates those observed by Mensi et al. (2014) using DCC-GARCH and BEKK-GARCH class model for dynamic spillovers between oil and cereals pricing.

Regarding the conditional variance equations, we declare that the current conditional volatility of the stock and commodity markets is well depending on both own past shocks (a_{11} and a_{22}) and past conditional volatilities (b_{11} and b_{22}). For spillover mechanisms, the cross-market shock effects (a_{12} and a_{21}) is found for stock market which affect WTI and barley (a_{12}) and, surprisingly, for commodity market shocks which does not affect stock market current daily pricing (a_{21}). In contrast, conditional volatility of wheat is the solely that affect current stock market pricing volatility (b_{21}). Reciprocally, stock market conditional volatility affects commodities except for wheat and precious metals (b_{12}).

When structural breaks in volatility are incorporated, we observe the evidence of some significant effects. Indeed, structural breaks in stock markets variance affect daily conditional variance of precious metals. Reciprocally, only the structural breaks of wheat have a significant outcome on stock market current volatility. Moreover, the own shocks persist within stock-commodity markets. For cross-markets effects, shocks transmit from stock market to both WTI and barley current conditional variances. Inversely, and unexpectedly, commodity shocks do not affect conditional variance on stock markets.

At the same, the own conditional variances are still influencing on both stock and commodity markets. Stock market conditional volatility still remains without effect on wheat and precious metals but newly on Brent (b_{12}). In opposite, the effect of wheat's conditional volatility disappears and we did not mark a substantial effect from commodity on current stock market volatility.

5. IMPLICATIONS FOR PORTFOLIO MANAGEMENT: DESIGNS AND HEDGING

Estimation results will be used to highlight the managerial implications in view of an international investor. We then compute optimal portfolio weights and hedge ratios and try to assess the diversification strategy through the hedging effectiveness statistics while controlling for the effect of structural breaks on that statistics.

5.1. Portfolio Designs

According to Kroner and Ng (1998), the optimal weight of holding stock markets in the SP 500 index and the commodity index is given by:

$$w_t^{COM,SP500} = \frac{h_t^{SP500} - h_t^{COM,SP500}}{h_t^{COM} - 2h_t^{COM,SP500} + h_t^{SP500}} \quad (12)$$

$$w_t^{COM,SP500} = \begin{cases} 0 & \text{if } w_t^{COM,SP500} < 0 \\ w_t^{COM,SP500} & \text{if } 0 \leq w_t^{COM,SP500} \leq 1 \\ 1 & \text{if } w_t^{COM,SP500} > 1 \end{cases} \quad (13)$$

Where, $w_t^{COM,SP500}$ denotes the weight of commodity market index in the one-dollar portfolio of two assets at time t . h_t^{SP500} and h_t^{COM} refer to conditional variances of stock market in the SP 500 index and commodity return indices, respectively. The term $h_t^{COM,SP500}$ is the conditional covariance between the SP 500 and commodity indexes at time t . The weight of the stock market index in the considered portfolio is obtained by computing the $(1 - w_t^{COM,SP500})$. Statistics for the portfolio weights are computed from fitting the cited three VAR(1)-GARCH(1,1) class models.

5.2. Hedging Strategy

Portfolio designs might be learned as an early hedging strategy against adverse evolution of asset pricing process. A timely strategy is also available for the investor in so far as he can decide on the optimal hedge ratio for his portfolio. In that framework, the hedging question consists of identifying how much a long position (buy) in one dollar in the SP 500 index should be hedged by a short position (sell) in β_t dollar in the commodity market index. We follow Kroner and Sultan (1993) and use the hedge ratio which takes the following form:

$$\beta_t = \frac{h_t^{COM,SP500}}{h_t^{SP500}} \quad (14)$$

Table 10 sum up the statistics of portfolio designs and hedge ratios for the three competing specifications of VAR-GARCH class model.

As shown in Table 10, the hedge ratios are typically low, suggesting that hedging effectiveness involving commodity and stock

markets is quite good, which is consistent with the view that the incorporation of commodities in a diversified portfolio of stocks increases its risk-adjusted performance.

Optimal weights in the hedged portfolios vary substantially across stock and commodity markets, but they are only slightly different across the used class models. This results corroborate with those obtained by similar recent empirical studies such as Arouri et al. (2011b). Values of w_t range between 0.34 for stock-wheat in VAR-BEKK-GARCH model and 0.85 for stock-WTI in the same class model. When introduced structural breaks in volatility, average values drop off slightly and remains somewhat close across models.

In the main, we observe that, to maximize the risk-adjusted return of the same one-dollar stock-commodity portfolio, international investor should hold, on average, fewer financial assets (i.e., stock) with $w_t = 80\%$. When hedging with foods assets, he is supposed to overweight financial assets ($w_t = 34\%$ for wheat and 37% for barley). This finding suggests that the food assets are substantially riskier or do not provide higher benefits comparing to the holding of oil or precious metal assets. We state that the three class models point out the same findings with and relatively smallest values for VAR-DCC-GARCH followed by VAR-BEKK-GARCH model. The obtained results confirm those obtained by Arouri et al. (2011b).

As for hedge ratios, we find that they are varying over markets but slightly over VAR-GARCH class models. Average values range between 0.03 for stock-gold using VAR-CCC-GARCH and 0.16 for stock-wheat using VAR-DCC-GARCH model. Greatest values of β_t were found for stock-wheat (range between 0.13 and 0.16) and stock barley (range between 0.08 and 0.12).

Interpretation of these records makes it possible to infer that VAR-GARCH class models overweight food assets than the others to minimize or hedge International Portfolio Risk. Finally, we see that gold displays the smallest hedge ratios. We state that the present evidence in not unexpected in so far as holding gold is a successful strategy for speculation and hedging against economic and financial risks. Previous recent studies (we cite, for instance, Baur and Lucey, 2010; Baur and McDermott, 2010; Ciner et al., 2013; Chkili et al., 2014) declare that international investors

Table 10: Summary statistics for optimal portfolio weights and hedge ratios

Portfolio	Weights and hedge ratios	VAR-CCC-GARCH		VAR-DCC-GARCH		VAR-BEKK-GARCH	
		Without SB	With SB	Without SB	With SB	Without SB	With SB
SP 500/Brent	ω_t	0.8004	0.7998	0.7984	0.7971	0.8020	0.8023
	β_t	0.0605	0.0611	0.0813	0.0816	0.0767	0.0768
SP 500/WTI	ω_t	0.8404	0.8402	0.8483	0.7888	0.8473	0.7474
	β_t	0.0835	0.0837	0.1046	0.0833	0.0984	0.0983
SP 500/Wheat	ω_t	0.3452	0.3452	0.3446	0.3444	0.3444	0.3438
	β_t	0.1432	0.1427	0.1646	0.1650	0.1391	0.1413
SP 500/Barley	ω_t	0.3763	0.3758	0.3769	0.3766	0.3638	0.3625
	β_t	0.0866	0.0865	0.0787	0.0785	0.1246	0.1240
SP 500/Gold	ω_t	0.5232	0.5217	0.5234	0.5219	0.5240	0.5217
	β_t	0.0300	0.0328	0.0430	0.0435	0.0352	0.0381
SP 500/Silver	ω_t	0.7386	0.7383	0.7536	0.75365	0.7642	0.7646
	β_t	0.0912	0.0907	0.0643	0.0641	0.0647	0.0635

The table reports summary statistics for average values of optimal weights (w_t) and hedge ratios (β_t) for stock-commodity portfolio using conditional variance and covariance estimated from three competitive return and volatility spillover models in a bivariate specification. GARCH: Generalized autoregressive conditional heteroskedasticity, VAR: Vector autoregression, BEKK: Baba, Engle, Kroner, Kraft, CCC: Constant conditional correlation, DCC: Dynamic conditional correlation

Table 11a: Diversification and hedging effectiveness (PF1)

Portfolio	Ignoring structural breaks				Incorporating structural breaks			
	Mean	Variance	Risk-adjusted return ($\times 100$)	Hedging effectiveness	Mean	Variance	Risk-adjusted return	Hedging effectiveness
SP 500/Brent								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0344	3.5232	1.8313	-1.0561	0.0344	3.5198	1.8311	-1.054
PF2. VAR-DCC-GARCH	0.0343	3.4179	1.8556	-0.9947	0.0343	3.4088	1.8557	-0.989
PF2. VAR-BEKK-GARCH	0.0344	3.4378	1.8569	-1.0063	0.0098	1.4745	0.8083	0.139
PF3.	0.0412	5.2258	1.8023	-	0.0412	5.228	1.8020	-
SP 500/WTI								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0331	4.4872	1.5603	-1.6188	0.0332	4.4857	1.5682	-1.618
PF2. VAR-DCC-GARCH	0.0333	4.4418	1.5799	-1.5922	0.0316	3.8808	1.6046	-1.265
PF2. VAR-BEKK-GARCH	0.0333	4.4096	1.5842	-1.5735	0.0303	3.4893	1.6231	-1.036
PF3.	0.0380	6.0856	1.5404	-	0.0382	6.0860	1.5485	-
SP 500/Wheat								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0136	0.8647	1.4581	0.495	0.0134	0.8646	1.4433	0.495
PF2. VAR-DCC-GARCH	0.0135	0.8251	1.4914	0.518	0.0134	0.8254	1.4755	0.518
PF2. VAR-BEKK-GARCH	0.0135	0.8253	1.4908	0.518	0.0134	0.8234	1.4761	0.519
PF3.	0.0260	0.7230	3.0588	-	0.0256	0.7230	3.0118	-
SP 500/Barley								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0145	0.8330	1.5915	0.514	0.0143	0.8337	1.5692	0.513
PF2. VAR-DCC-GARCH	0.0145	0.7989	1.6265	0.534	0.0143	0.7991	1.6046	0.534
PF2. VAR-BEKK-GARCH	0.0143	0.8354	1.5619	0.512	0.0141	0.8376	1.5372	0.511
PF3.	0.0270	0.9230	2.8110	-	0.0265	0.923	2.7590	-
SP 500/Gold								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0258	0.7956	2.8965	0.536	0.0256	0.7976	2.8692	0.535
PF2. VAR-DCC-GARCH	0.0258	0.7808	2.9245	0.544	0.0256	0.7751	2.9115	0.548
PF2. VAR-BEKK-GARCH	0.0259	0.7706	2.9464	0.550	0.0256	0.7719	2.9167	0.550
PF3.	0.0430	1.401	3.6324	-	0.0427	1.401	3.6070	-
SP 500/Silver								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0305	2.7079	1.8527	-0.580	0.0305	2.7054	1.8530	-0.579
PF2. VAR-DCC-GARCH	0.0310	2.6566	1.8998	-0.550	0.0310	2.6899	1.8880	-0.570
PF2. VAR-BEKK-GARCH	0.0313	2.7177	1.8988	-0.586	0.0313	2.7195	1.8989	-0.587
PF3.	0.0388	4.4790	1.8334	-	0.0388	4.479	1.8334	-

GARCH: Generalized autoregressive conditional heteroskedasticity, VAR: Vector autoregression, BEKK: Baba, Engle, Kroner, Kraft, CCC: Constant conditional correlation, DCC: Dynamic conditional correlation

implement a more defensive allocation strategy by investing in “refuge” or safe haven assets.

5.3. Diversification and Hedging Effectiveness

We actually manage and simulate global portfolio diversification with the estimated optimal parameters (weights and hedge ratios), cited here before, to learn about the hedging effectiveness. We use the estimates of three VAR-GARCH class models to conceive three portfolios: A first full-stocks portfolio (PF1); a second weighted stock-commodity portfolio (PF2) and a third full-commodity portfolio (PF3). We point out that some previous studies have been limited for only (PF1) and (PF2). We attempt to test the contribution of a weighted stock-commodity portfolio to two unhedged portfolios: A full unhedged stock portfolio (PF1) and a full unhedged commodity portfolio (PF3).

As decision rule, we assess the effectiveness of the diversification strategy by comparing the realized risk and return. We make use of the realized hedging errors of Ku et al. (2007) which is presented as follows:

$$HE = \frac{var^u - var^h}{var^u} \quad (15)$$

Where, var^u and var^h denote variances of the unhedged and hedged portfolios, respectively. We interpret a higher value of HE ratio as representative of a better hedging effectiveness in terms of the portfolio’s variance reduction, and consider the associated hedging investment strategy as successful.

Table 11 displays summary statistics of hedging effectiveness ratios in presence and absence of break dates effect. We consider two non-diversified portfolios. A first one (Table 11a) with 100% stocks and we incorporate commodity assets to implement diversification strategy and a second one (Table 11b) with 100% commodities and we incorporate stocks. We assess for each portfolio the reward-to-risk and the hedging effectiveness.

Results, in Table 11a and b, prove that adding assets to the diversified portfolios improves the risk-adjusted return ratios. The improvement ranges between 3 and 6 times. More

Table 11b: Diversification and hedging effectiveness (PF3)

Portfolio	Ignoring structural breaks				Incorporating structural breaks			
	Mean	Variance	Risk-adjusted return (×100)	Hedging effectiveness	Mean	Variance	Risk-adjusted return (×100)	Hedging effectiveness
SP500/Brent								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0344	3.5232	1.8313	0.3258	0.0344	3.5198	1.8311	0.327
PF2. VAR-DCC-GARCH	0.0343	3.4179	1.8556	0.3460	0.0343	3.4088	1.8557	0.348
PF2. VAR-BEKK-GARCH	0.0344	3.4378	1.8569	0.3422	0.0098	1.4745	0.8083	0.718
PF3.	0.0412	5.2258	1.8023	-	0.0412	5.228	1.8020	-
SP500/WTI								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0331	4.4872	1.5603	0.2627	0.0332	4.4857	1.5682	0.263
PF2. VAR-DCC-GARCH	0.0333	4.4418	1.5799	0.2701	0.0316	3.8808	1.6046	0.362
PF2. VAR-BEKK-GARCH	0.0333	4.4096	1.5842	0.2754	0.0303	3.4893	1.6231	0.427
PF3.	0.0380	6.0856	1.5404	-	0.0382	6.0860	1.5485	-
SP500/Wheat								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0136	0.8647	1.4581	-0.197	0.0134	0.8646	1.4433	-0.197
PF2. VAR-DCC-GARCH	0.0135	0.8251	1.4914	-0.142	0.0134	0.8254	1.4755	-0.142
PF2. VAR-BEKK-GARCH	0.0135	0.8253	1.4908	-0.142	0.0134	0.8234	1.4761	-0.140
PF3.	0.0260	0.7230	3.0588	-	0.0256	0.7231	3.0118	-
SP500/Barley								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0145	0.8330	1.5915	0.097	0.0143	0.8337	1.5692	0.096
PF2. VAR-DCC-GARCH	0.0145	0.7989	1.6265	0.134	0.0143	0.7991	1.6046	0.134
PF2. VAR-BEKK-GARCH	0.0143	0.8354	1.5619	0.094	0.0141	0.8376	1.5372	0.092
PF3.	0.0270	0.9230	2.8110	-	0.0265	0.9232	2.7590	-
SP500/Gold								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0258	0.7956	2.8965	0.432	0.0256	0.7976	2.8692	0.431
PF2. VAR-DCC-GARCH	0.0258	0.7808	2.9245	0.443	0.0256	0.7751	2.9115	0.447
PF2. VAR-BEKK-GARCH	0.0259	0.7706	2.9464	0.450	0.0256	0.7719	2.9167	0.449
PF3.	0.0430	1.4011	3.6324	-	0.0427	1.4012	3.6070	-
SP500/Silver								
PF1.	0.0070	1.7135	0.5348	-	0.0070	1.7135	0.5348	-
PF2. VAR-CCC-GARCH	0.0305	2.7079	1.8527	0.395	0.0305	2.7054	1.8530	0.396
PF2. VAR-DCC-GARCH	0.0310	2.6566	1.8998	0.407	0.0310	2.6899	1.8880	0.399
PF2. VAR-BEKK-GARCH	0.0313	2.7177	1.8988	0.393	0.0313	2.7195	1.8989	0.393
PF3.	0.0388	4.4791	1.8334	-	0.0388	4.4794	1.8334	-

GARCH: Generalized autoregressive conditional heteroskedasticity, VAR: Vector autoregression, BEKK: Baba, Engle, Kroner, Kraft, CCC: Constant conditional correlation, DCC: Dynamic conditional correlation

importantly, this evidence holds for all considered models and for all portfolios.

Moreover, different VAR-GARCH class models display analogous results. The VAR-BEKK-GARCH model provides the best risk-adjusted return ratios in 7 out of 12 pairs of stock-commodity portfolios, followed closely by the VAR-DCC-GARCH with 5 out of 12 cases which confirm that interdependencies and markets linkages are far from to be constant over time. We infer that foods and gold assets help improve and hedge full stock portfolios (PF1) while oil, precious metals and barley help improve and hedge full commodity designed portfolios (PF3). The role of the gold and food assets appear to be more fundamental for financial asset made portfolios as safe haven or accomplishment factors.

The results show that hedging strategies involving stock and commodity assets allow reducing considerably portfolio risk (variance). Indeed, the variance reduction ranges from 49.50% (wheat) to 55% (gold) for full stock made portfolio and from 9.20% (barley) to 71.80% (Brent) for full commodity made portfolio.

The variance reduction is then significantly different across sectors, but remains relatively stable across the three VAR-GARCH class models. The portfolio variance is reduced, or the hedging effectiveness is greater, when the BEKK-GARCH and DCC-GARCH models are used. However, we state here that the BEKK-GARCH is the best one. Chang et al. (2011), Arouri et al. (2011) get to the same finding as regards the superior ability of bivariate diagonal BEKK-GARCH over the DCC-GARCH and CCC-GARCH when examining the optimal hedging effectiveness between crude oil spot and futures markets and between oil prices and stock sector returns respectively. We state here that the current findings are plausible and economically interpretable and provide practical usefulness for portfolio management as well as for sectors' governance.

6. CONCLUSION

It is interesting for investors and policy makers to make known of asset pricing process and markets dynamics in the presence

of time-varying volatility. Similarly, for prediction and hedging purposes, markets linkage and volatility spillovers remain a renewable topic.

In that framework, volatility has a key meaning. It involves instability but also investment opportunities given market efficiency hypothesis and investments horizon. Accordingly, volatility persistence and breaking events are prime factors for international portfolio management.

The major aim of this paper is to highlight the stock-commodity market linkages through both conditional return dynamics and volatility spillovers and demonstrate how to get out lessons for portfolio management purpose.

The used empirical methodology is a VAR-GARCH approach of Ling and McAleer (2003). We try to perform the analysis using three competing specifications; namely VAR-CCC-GARCH, VAR-DCC-GARCH and VAR-BEKK-GARCH while interesting in the effect of structural breaks in volatility. Given that the estimated CCC were very weak and not statistically significant, which means that the cross-markets constant correlation of conditional shocks were absent. Conversely, we find for the DCC model, that the estimates of the conditional correlations were for all time significant, which is far from supporting empirically the assumption of CCCs. This evidence highlights the dynamic conditional correlations. Accordingly, we try to run three competitive specifications to learn about implications on portfolio diversification. The implementation of break-dates helps answer to raised questions about information content of those breaking events.

Our findings show the evidence of factual effect of past on current daily behavior within each market, on the one hand, and a mutual interference between stock and commodity markets, on the other hand, which help forecast markets performance. The one lagged stock market daily return affects current commodity-market returns which draw attention to the presence of behavioral effects on daily pricing process. Conversely, only wheat and gold daily past returns help predict today's stock market return. The current conditional volatility of stock market is significantly affected by both own past volatility and the 1 day lagged volatility of commodity markets in absence and presence of dummy volatility break dates except for wheat and precious metals. Reciprocally, past stock market's volatility help significantly predict current commodity markets GARCH terms except for Brent, barley, gold and silver. The information content of breaking events helps forecast present conditional volatility of both stock and commodity markets.

The results corroborate recent previous findings (Arouri et al., 2011; Arouri et al., 2012; Ewing and Malik, 2013; Mensi et al., 2013; 2014) on volatility persistence and the effect of sudden changes. Results have then been operated to present their managerial usefulness for portfolio investment. Indeed, optimal weights, hedge ratios and hedging effectiveness have been estimated and discussed. The results show the importance of adding stocks as well as commodities to an international unhedged portfolio investment.

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