



Modeling Stock Market Returns under Self-exciting Threshold Autoregressive Model: Evidence from West Africa

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ABSTRACT

The study seeks to investigate whether non-linear patterns are present in the returns of two indices on the stock markets in Ghana and Nigeria between the period of 2011 and 2015. The results of applying four linearity tests on the returns concluded that the null of linearity is rejected on all four tests for the Ghanaian index but mixed for the Nigerian index. We modelled the indices under the non-linear self-exciting threshold autoregressive (SETAR) model. We compared the modelling performance of the non-linear SETAR model with that of the standard AR (1) and AR (2) by analyzing Akaike information criterion values of the respective models. Our results show that the SETAR model fits the data well. Hence, modelling stock market returns from Ghana and Nigeria using linear models might lead to spurious conclusions.

Keywords: Threshold Models, Linearity Tests, Self-exciting Threshold Autoregressive Model

JEL Classifications: C12, C13, C24

1. INTRODUCTION

The efficient market hypothesis (EMH) posits that information on a market is correctly and instantaneously incorporated in setting current asset prices. This assertion means there is a linear relationship between information flow to market participants and how prices are set on the market.

Thus modelling and forecasting of the returns are done using linear models. However, some researchers have raised objection to the EMH. Researchers such as Hinich and Paterson (1985), Cochrane (1998), Fama and French (1988), Lo and McKinlay (1988), Hsieh (1991), Ryden et al. (1998), Garcia and Genay (2000) have questioned whether it is appropriate for linear models to be used in analyzing complex models that come about as a result of how prices are determined and the market negotiation process. These researchers believe that market participants do not have an even trading field. It is believed that information flow on the market is not simultaneously relayed to all agents on the market therefore a non-linear model is appropriate for capturing the dynamics on the market. Non-linearity on the market might be due to agents having different objectives and targets for trading on the market.

Also agents vary in their negotiation times and how agents view risk and the need to diversify their portfolio.

These sources of non-linearity on the market has increased the interest in analyzing stock returns with non-linear methods. Though it is reported in the extant literature of the non-easiness of non-linear models because they can sometimes create spurious fits; Granger and Terasvirta (1993), we employ a discrete transition regime switching model; the self-exciting threshold autoregressive (SETAR) model because of its variety and flexibility. The SETAR model is robust to heteroscedasticity in the data.

The aim of this study is to determine if there exists non-linear patterns on the composite index of Ghana. We first apply four linearity tests on the returns of the series; Keenan (1985) test, Tsay (1986) test, BDS (1987) test and the delay vector variance (DVV) test developed by Gautama et al. (2004). If non-linearity exists, we model the returns of the series with the SETAR model and compare the results of the SETAR model with the results of the standard AR (1) and AR (2) models to see which model fits the data well by analyzing Akaike information criterion (AIC) values. The study is organised as follows:

Section two describes the data and the methodology employed. Section three presents the empirical results and discusses the findings observed. Section four concludes the study.

2. DATA AND METHODOLOGY

2.1. Data

We used the daily closing values from the Nigeria all share index (NIGALSH) and the Ghana composite index (GSEALSH) for the period between January 04, 2011 to August 09, 2015. The data was obtained from DataStream. The daily closing prices were transformed into returns which were calculated as: $Y_t = P_t - P_{t-1}$ where P_t and P_{t-1} are the daily closing prices of the index on two consecutive trading days.

2.2. Methodology

We employ Keenan test, Tsay test, the BDS test and the DVV test to confirm the existence or otherwise of non-linear patterns in the data. If non-linearity is present, we model the returns using the non-linear SETAR.

2.2.1. Linearity tests

We first test if the return series exhibit non-linear patterns before we proceed with modelling with non-linear models. We present four linearity tests, namely Keenan (1985) test and Tsay (1986) test, BDS (1987) test and the DVV (2004) test.

2.2.2. Keenan test

We present the Keenan test (1985) as in Cryer and Chan (2008). This test is based on a second-order Volterra type expansion similar to the Taylor expansion. The Volterra expansion is used for non-linear modelling and it is able to capture memory effects.

The Keenan test can be written as:

$$y_t = \mu + \sum_{i=-\infty}^{\infty} \theta_i \varepsilon_{t-i} + \sum_{i,j=-\infty}^{\infty} \theta_{ij} \varepsilon_{t-i} \varepsilon_{t-j} + \sum_{i,j,k=-\infty}^{\infty} \theta_{ijk} \varepsilon_{t-i} \varepsilon_{t-j} \varepsilon_{t-k} + \dots \quad (1)$$

Here $\{\varepsilon_t, -\infty < t < \infty\}$ are a sequence of independent and identically distributed (IID) random variables with mean zero while y_1, \dots, y_n are the observations. The process $\{y_t\}$ is linear if the double sum of the right hand side of the equation disappears. Thus, testing the non-linearity of a series y_t consists practically in testing whether the double sum is zero or not.

Alternatively, as proven by Cryer and Chan (2008), Keenan test can also be heuristically derived as follows:

$$y_t = \theta_0 + \varnothing_1 Y_{t-1} + \dots + \varnothing_m Y_{t-m} + \exp \left\{ \eta \left(\sum_{j=1}^m \varnothing_j Y_{t-j} \right)^2 \right\} + \varepsilon_t \quad (2)$$

Where, $\{\varepsilon_t\}$ are independent and normally distributed with zero mean and finite variance. If the regression coefficient $\eta = 0$ then the exponential term becomes 1. Equation (2) becomes an AR model with order m . However, if the regression coefficient η is different from zero, then Equation (2) is non-linear. Using the expansion

$\exp(x) \approx 1+x$, which holds for x of small magnitude, we can see that for small η , Y_t follows approximately a quadratic AR model:

$$Y_t = \theta_0 + 1 + \varnothing_1 Y_{t-1} + \dots + \varnothing_m Y_{t-m} + \eta \left\{ \left(\sum_{j=1}^m \varnothing_j Y_{t-j} \right)^2 \right\} + \varepsilon_t \quad (3)$$

The test statistic $\hat{F} = \frac{\eta^2 (n-2m-2)}{RSS - \eta^2}$ is approximately distributed as an F-distribution with degrees of freedom 1 and $n-2m-2$.

2.2.3. Tsay test

Tsay (1986) extended the Keenan test due to its some limitations. As shown in Keenan (1985), though Keenan test is robust in detecting non-linearity in the form of the square of the approximating linear conditional mean function, the strength of the test is sometimes low. This limitation brought about Tsay (1986) test. We again present the Tsay test as in Cryer and Chan (2008).

Tsay replaced the term $\eta \left\{ \left(\sum_{j=1}^m \varnothing_j Y_{t-j} \right)^2 \right\}$ of Equation (2) by:

$$\exp \left(\begin{matrix} \varsigma_{1,1} Y_{t-1}^2 + \varsigma_{1,2} Y_{t-1} Y_{t-2} + \dots + \varsigma_{1,m} Y_{t-1} Y_{t-m} \\ + \varsigma_{2,2} Y_{t-2}^2 + \varsigma_{2,3} Y_{t-2} Y_{t-3} + \dots + \varsigma_{2,m} Y_{t-2} Y_{t-m} \\ + \dots + \varsigma_{m-1,m-1} Y_{t-m}^2 \\ + \varsigma_{m-1,m} Y_{t-m+1} Y_{t-m} + \varsigma_{m,m} Y_{t-m}^2 + \varepsilon_t \end{matrix} \right) \quad (4)$$

From this approximation in Equation (4), we can observe that the non-linear model is approximately a quadratic AR model but the coefficient of the quadratic terms are unconstrained. Therefore, the Tsay test considers the following quadratic regression model:

$$Y_t = \theta_0 + \varnothing_1 Y_{t-1} + \dots + \varnothing_m Y_{t-m} + \varsigma_{1,2} Y_{t-1}^2 + \varsigma_{1,2} Y_{t-1} Y_{t-2} + \dots + \varsigma_{1,m} Y_{t-1} Y_{t-m} + \varsigma_{2,2} Y_{t-2}^2 + \varsigma_{2,3} Y_{t-2} Y_{t-3} + \dots + \varsigma_{2,m} Y_{t-2} Y_{t-m} + \dots + \varsigma_{m-1,m} Y_{t-m+1}^2 + \varsigma_{m-1,m} Y_{t-m+1} Y_{t-m} + \varsigma_{m,m} Y_{t-m}^2 + \varepsilon_t \quad (5)$$

And tests whether all $m(m+1)/2$ coefficients $\varsigma_{i,j} = 0$.

The autoregressive order m must be specified using the AIC to test the null of linearity.

2.2.4. BDS test

The BDS test was developed in 1987 by Brock, Dechert and Scheinkman and not Brock, Dechert. We employed the BDS test to detect non-linearity because it is used in detecting serial dependence in time series. The null of IID hypothesis is tested against an unspecified alternative. We present the procedure below in computing the BDS test:

Let $\{y_t\}$ be a time series with N observations. Thus $\{y_t\}$ is the first difference of the natural logarithms of raw data in time series. Thus:

$$\{y_t\} = [y_1, y_2, y_3, \dots, y_N] \quad (6)$$

An embedding dimension, m is selected to embed the time series into m -dimensional vectors, by taking each m successive points in the series. The series of scalars is as thus converted into a series of vectors with overlapping entries.

$$y_1^m = (y_1, y_2, \dots, y_m)$$

$$y_2^m = (y_2, y_3, \dots, y_{m+1})$$

$$y_{N-m}^m = (y_{n-m}, y_{N-m+1}, \dots, y_N)$$

We compute the correlation integral, which is a measure of the spatial correlation among the points, by adding the number of pairs of points (t, j) where:

$1 \leq t \leq N$ and $1 \leq j \leq N$, in the m -dimensional space which are “close” in the sense that the points are within a radius or tolerance ε of each other.

$$C_{\varepsilon, m} = \frac{1}{N_m(N_m - 1)} \sum_{t \neq j} I_{t, j; \varepsilon} \tag{7}$$

$$\text{Where, } I_{t, j; \varepsilon} = \begin{cases} 1 & \text{if } \|y_t^m - y_j^m\| \leq \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

BDS (1987) proved that if the time series is IID.

$$C_{\varepsilon, m} \approx [C_{\varepsilon, 1}]^m \tag{8}$$

According to Lin (1997), if the ratio $\frac{N}{m}$ is >200 , the values of $\frac{\varepsilon}{\sigma}$ range from 0.5 to 2 and the values of m are between 2 and 5 (Brock et al., 1996).

The quantity $[C_{\varepsilon, m} - (C_{\varepsilon, 1})^m]$ has an asymptotic normal distribution with zero mean and a variance $V_{\varepsilon, m}$ defined as:

$$V_{\varepsilon, m} = 4 \left[K^m + 2 \sum_{j=1}^{m-1} K^{m-j} C_{\varepsilon}^{2j} + (m-1)^2 C_{\varepsilon}^{2m} - m^2 K C_{\varepsilon}^{2m-2} \right] \tag{9}$$

$$\text{Where, } K = K_{\varepsilon} = \frac{6}{N_m(N_m - 1)(N_m - 2)} \sum_{t < j < N} h_{t, j, N; \varepsilon}; h_{t, j, N; \varepsilon} = \frac{[I_{t, j; \varepsilon} I_{j, N; \varepsilon} + I_{t, N; \varepsilon} I_{N, j; \varepsilon} + I_{j, t; \varepsilon} I_{t, N; \varepsilon}]}{3}$$

The BDS test statistic is thus stated as:

$$BDS_{\varepsilon, m} = \frac{\sqrt{N}[C_{\varepsilon, m} - (C_{\varepsilon, 1})^m]}{\sqrt{V_{\varepsilon, m}}} \tag{10}$$

The null of an IID is rejected at the 5% significance level if $|BDS_{\varepsilon, m}| > 1.96$.

2.3. DVV

The DVV was developed by Gautama et al. (2004a) for signal characterization. Characterizing signal non-linearities have been adopted in predicting survival in heart failure cases, practical engineering applications (Ho et al., 1997; Chambers and Mandic, 2001) and in economic time series (Caraiani, 2015; Addo et al., 2013a; 2013b; 2013c).

DVV is based on surrogate data and it is more suitable for signal processing application because the data generating process can

be described by a linear or non-linear equations theoretically. The combination with the concept of surrogate data gives an additional account of the non-linear behavior of the time series.

The DVV analysis calculates the target variance, σ^{*2} which is an inverse measure of the predictability of a time series. We present the summarized algorithm of the DVV analysis below:

1. For an optimal embedding dimension m which are obtained via a differential entropy based method using wavelet-based surrogates and time lag τ , generate DV: $y(k)=[y_{k-\tau}, \dots, y_{k-m\tau}]$ and corresponding target y_k
2. The mean μ_d and standard deviation, σ_d are computed over all pairwise distances between DVs, $\|y(t) - y(j)\|$ for $t \neq j$
3. The sets Ω_k are generated such that $\Omega_k = \{y(t) \mid \|y(t) - y(k)\| \leq \varrho_d\}$, i.e., sets which consist of all DVs that lie closer to $x(k)$ than a certain ϱ_d taken from the interval $[\min\{0, \mu_d - n_d \sigma_d\}; \mu_d + n_d \sigma_d]$, e.g., uniformly spaced, where n_d is a parameter controlling the span over which to perform the DVV analysis
4. For every set Ω_k the variance of the corresponding targets σ_k^2 is computed. The average over all sets Ω_k , normalized by the variance of the time series, σ_x^2 yields the target variance σ^{*2} , as:

$$\sigma^{*2}(\varrho_d) = \frac{\frac{1}{N} \sum_{k=1}^N \sigma_k^2(\varrho_d)}{\sigma_x^2} \tag{11}$$

Where, N denotes the total number of sets $\Omega_k(\varrho_d)$.

The DVV analysis can be represented by graphically plotting of $\sigma^{*2}(\varrho_d)$ as a function of the standardized distance (ϱ_d) . The minimum target variance, $\sigma_{min}^{*2} = \min(\varrho_d) [\sigma^{*2}(\varrho_d)]$. A measure of the amount of noise present corresponds to the lowest point of the curve.

A DVV analysis where the surrogate and the original time series provide similar plots, the series is said to be linear else non-linear. Also, because of the standardization of the distance axis, the plots can be combined within a scatter diagram, where the horizontal axis corresponds to the DVV plot of the original time series and the vertical axis corresponds to the surrogate time series. If the DVV scatter diagram deviates from the bisector line, the series is said to be non-linear.

2.4. SETAR Model

The SETAR model is the simplest form of threshold autoregressive models (TAR). Tong (1978) and Tong and Lim (1980) proposed TAR models where the regime was determined by the value of an observable variable relative to a threshold value. The SETAR model can account for conditional heteroscedasticity in the data because the error variance may be different in the regimes.

We employ a first-order SETAR model for our analysis. The SETAR model is presented in summary from Cryer and Chan (2008) as below:

$$y_t = \begin{cases} \mu_{1,0} + \rho_{1,1}y_{t-1} + \sigma_1 e_t & \text{if } y_{t-1} < \theta \\ \mu_{2,0} + \rho_{2,1}y_{t-1} + \sigma_2 e_t & \text{if } \theta < y_{t-1} \end{cases} \tag{12}$$

Where, ρ are the autoregressive parameters, σ are noise standard deviations, θ is the threshold parameter and $\{e_t\}$ is a sequence of IID random variables with mean 0 and variance 1.

Table 1: Summary statistics for returns

Index	Mean	Standard deviation	Skewness	Kurtosis	ADF	Jarque-Bera
GSEALSH	0.0006	0.0063	-0.2349	46.4947	-7.0935	95704
NIGALSH	0.0001	0.0093	0.4674	7.6474	-11.029	3002.4

GSEALSH: Ghana composite index, NIGALSH: Nigeria all share index, ADF: Augmented Dickey-Fuller

Therefore, if the lag 1 value of y_t is not greater than the threshold, then the conditional distribution of y_t is similar to the first AR (1) process and we are in the lower regime, else the second AR (1) model is operational and we will be in the upper regime. This means the process switches between two linear models depending on the position of the lag 1 value.

3. EMPIRICAL RESULTS

The results of the summary statistics as shown in Table 1 indicates that the return series is non-normal with high values of kurtosis and highly skewed. The Jarque-Bera test statistic null hypothesis is rejected at the 1% level of significance. The augmented Dickey-Fuller test result shows return series is stationary.

We use AIC to select autoregressive order m in testing for linearity using the Keenan, Tsay, DVV and BDS tests. The autoregressive order m was given as 10 for GSEALSH and 1 for NIGALSH.

Even though the Keenan test at lags 2 and 6 fail to reject the null of linearity for GSEALSH in Table 2 and the Keenan and Tsay tests fail to reject the null of linearity for NIGALSH in Tables 3-5; the Tsay test in Table 2 for GSEALSH rejects the null of linearity, the BDS tests results in Tables 6 and 7 give an indication of the presence of non-linearity in the returns of the series because the $P < 0.05$.

Also the DVV analysis with iterative amplitude adjusted Fourier Transform surrogates performed on the returns of GSEALSH using $m = 10$ and on NIGALSH using $m = 1$ via the differential entropy-based method (Gautama et al., 2003) shows that there is a clear deviation from the bisector line on the DVV scatter plots in Figures 1 and 2. The DVV plots in Figures 1 and 2 also show that non-linear patterns exist in the series.

Since non-linearity exists in the data, we proceed to model the data under the SETAR model with 2-regimes, low and high. The 2-regime SETAR was chosen after observing the data and concluding that there are no derivatives on the GSEALSH and the NIGALSH. The maximum autoregressive order for the low and high regimes was chosen to be 2. The findings are presented in Tables 8 and 9.

3.1. SETAR model - GSEALSH

- Non-linear autoregressive model
- SETAR model (2 regimes)
- Proportion of points
 - a. Low regime: 15.43%
 - b. High regime: 84.57%
- Fit: Residuals variance = $3.532e-05$, AIC = -12431, MAPE = 257%.

Table 2: Keenan test results-GSEALSH

m	1	2	3	4	5	6	7	8	9	10
F-test	108.50	2.05	5.76	5.46	3.91	2.12	4.99	5.67	6.75	7.54
P value	0.0002	0.15	0.02	0.02	0.05	0.15	0.03	0.02	0.01	0.01

GSEALSH: Ghana composite index

Table 3: Keenan test results-NIGALSH

m	1
F-test	0.07
P	0.79

NIGALSH: Nigeria all share index

3.2. SETAR model - NIGALSH

- Non-linear autoregressive model
- SETAR model (2 regimes)
- Proportion of points:
 - a. Low regime: 84.82%
 - b. High regime: 15.18%.
- Fit: Residuals variance = $7.58e-05$, AIC = -11504, MAPE = 639.7%.

The results show that SETAR for both GSEALSH and NIGALSH is stationary because the coefficients in the low and high regimes are < 1 and also the product of the coefficients in the low and high regimes are < 1 . Thus the necessary and sufficient condition as in Chan et al. (1985) is satisfied.

The SETAR model results for GSEALSH shows that the number of observations in the high regime (84.57%) is more than that in the low regime (15.43%). Growth in returns are decreasing in the low regime with negative coefficients hence agents are found more in the high regime because of the high opportunities. The positive coefficients show an increasing rate of returns in the high regime.

However, results for NIGALSH are opposite that of GSEALSH. The number of observations is higher in the low regime (84.82%) than in the low regime (15.18%). Returns are increasing in the low regime because coefficients are positive making it risky for agents to in the high regime.

3.3. Modeling Performance

We compared the modelling performance of the SETAR model with the standard AR (1) and AR (2) models to confirm whether it is appropriate to model stock market returns from Ghana and Nigeria with the non-linear SETAR model. Comparing AIC values as shown in Tables 10 and 11 for GSEALSH and NIGALSH, we conclude that the SETAR model performs and fit the data well than the standard AR (1) and AR (2) models.

Figure 1: Delay vector variance plot and delay vector variance scatter plot for Ghana composite index

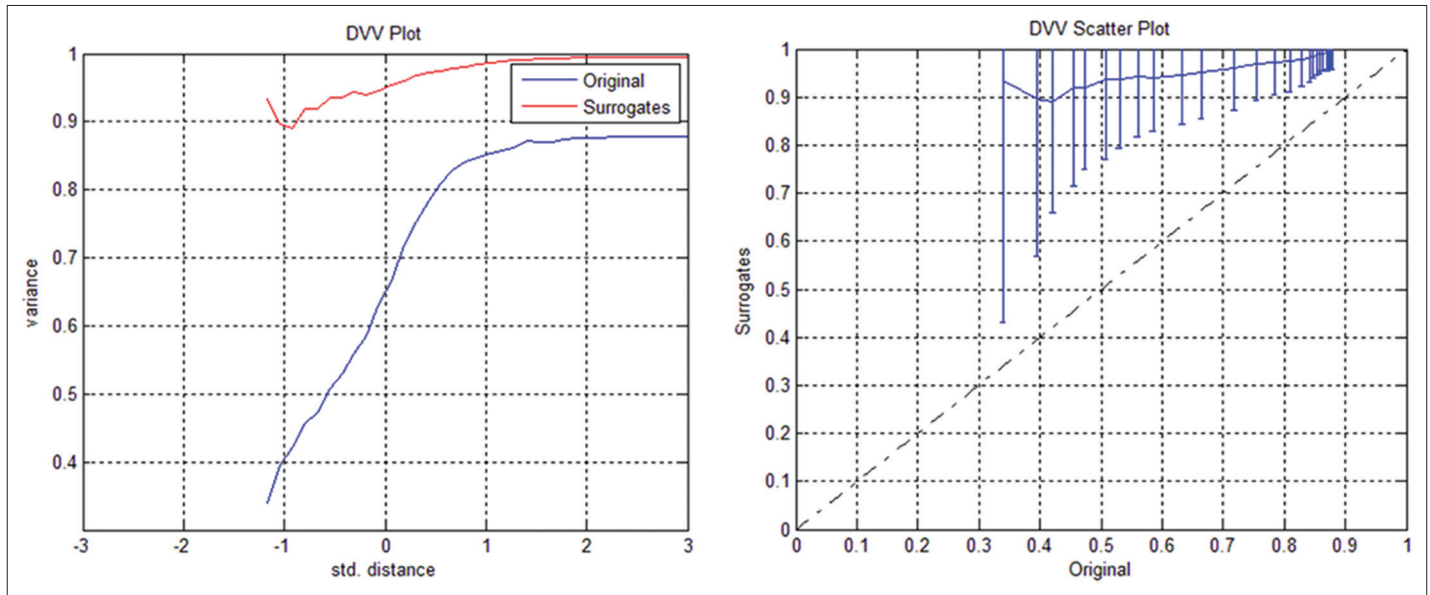


Figure 2: Delay vector variance plot and delay vector variance scatter plot for Nigeria all share index

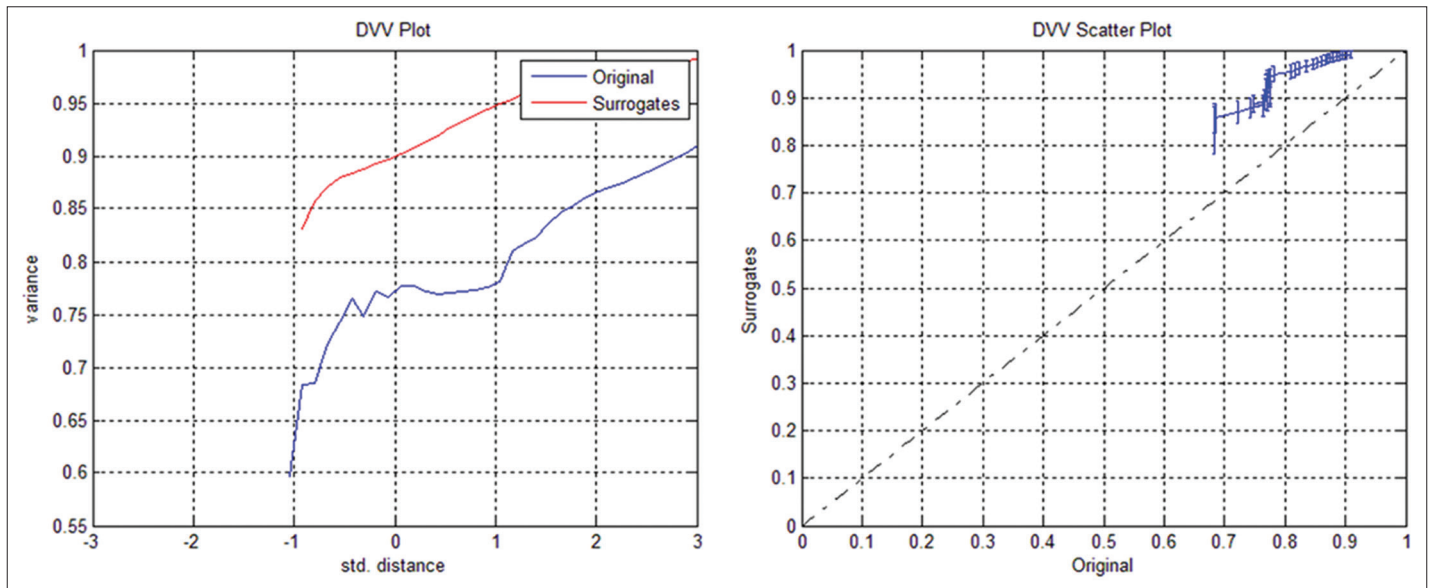


Table 4: Tsay test results-GSEALSH

<i>m</i>	1	2	3	4	5	6	7	8	9	10
F-test	99.9	59.3	28.9	18.7	13.7	9.8	7.9	6.3	5.7	5.1
P	1.17e-22	9.89e-36	1.65e-32	3.63e-32	1.55e-32	1.42e-29	8.49e-29	1.82e-26	2.21e-27	2.89e-27

NIGALSH: Nigeria all share index

Table 5: Tsay test results-NIGALSH

<i>m</i>	1
F-test	0.05
P	0.83

NIGALSH: Nigeria all share index

Table 6: BDS test results-GSEALSH

<i>m</i>	2	3	4	5	6	7	8	9	10
P	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

GSEALSH: Ghana composite index

4. CONCLUSION

We have shown that non-linear patterns exist in the GSEALSH and NIGALSH on the Ghanaian and Nigerian stock markets by employing four linearity tests; Keenan (1985), Tsay (1986), BDS (1987) and DVV (2004) tests. The BDS and DVV tests consistently detected non-linearities in both series unlike the Tsay test which failed to reject the null of linearity in NIGALSH. The series was therefore modelled using the SETAR model. The AIC results of the SETAR model was compared with that of the standard AR (1) and

Table 7: BDS test results-NIGALSH

<i>m</i>	2	3	4	5	6	7	8	9	10
P	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

NIGALSH: Nigeria all share index

Table 8: SETAR model results-GSEALSH

Coefficient	Estimate	Standard error	t-value	Pr(> t)
const.L	-0.00511	0.00066	-7.7588	1.815E-14***
phiL.1	-0.70410	0.06421	-10.9653	2.2E-16***
phiL.2	0.30660	0.08313	3.6882	0.0002359***
const.H	0.00025	0.00021	1.1957	0.232067
phiH.1	0.17359	0.03827	4.5363	6.296e-6
phiH.2	0.10713	0.02881	3.7184	0.0002096

Significant codes: 0 *** 0.001 ** 0.01 * 0.05. 0.1 1, Threshold variable: $Z(t)=+(1)X(t)+(0)X(t-1)$. Value: -0.003436. GSEALSH: Ghana composite index

Table 9: SETAR model-NIGALSH

Coefficient	Estimate	Standard error	t-value	Pr(> t)
const.L	0.00030	0.00029	1.0488	0.29450
phiL.1	0.36066	0.04154	8.6815	2.2e-16***
phiL.2	0.00755	0.03266	0.2312	0.81718
const.H	-0.00276	0.00132	-2.1003	0.03591
phiH.1	0.44584	0.08498	5.2466	1.829e-7
phiH.2	0.03383	0.06048	0.5593	0.57604

Significant codes: 0 *** 0.001 ** 0.01 * 0.05. 0.1 1. Threshold variable: $Z(t)=+(1)X(t)+(0)X(t-1)$. Value: 0.007295. NIGALSH: Nigeria all share index, SETAR: Self-exciting threshold autoregressive

Table 10: AIC values comparison-GSEALSH

Model	AR (1)	AR (2)	SETAR
AIC	-8844.56	-8856.39	-12431

GSEALSH: Ghana composite index, AIC: Akaike information criterion

Table 11: AIC values comparison-NIGALSH

Model	AR (1)	AR (2)	SETAR
AIC	-8045.03	-8043.55	-11504

AIC: Akaike information criterion, NIGALSH: Nigeria all share index

AR (2) models. We observed that, the SETAR model fits the data well than the AR (1) and AR (2) models. Therefore, the GSEALSH and the NIGALSH are best modelled with non-linear models like the SETAR model else results might be spurious.

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