



Pricing Ability of Four Factor Model using Quantile Regression: Evidences from India

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ABSTRACT

With the assumption that the returns are normally distributed with no fat tails, most of the existing studies have used ordinary least square (OLS) method to test the pricing ability of asset pricing models. These assumptions are not valid in numerous cases. Thus, to overcome such problem, the present study tests the pricing ability of Carhart (1997) four factor model using quantile regression which provides superior fitting of pricing factors than the traditional OLS model. The study uses daily data of Indian firms for period from December 1993 to March 2016. The results of the study reveal that the quantile regression model is having superior fitting across all percentile levels than OLS as it fails to fit these four factors across all percentile levels.

Keywords: Asset Pricing, Fama-French Factor Model, Quantile Regression, Carhart's Momentum

JEL Classifications: C30, G11, G12

1. INTRODUCTION

The asset pricing models focus to spot and gauge both systematic and unsystematic risk attached to a particular security and assess the fair returns for a security, which is very decisive factor for corporations, individual investors and policy makers. Over past few decades, the development of a narrow but well-performing asset pricing model has been one of the major tasks in financial economics. Prior to Fama and French (1992) three factor model, single factor capital asset pricing model (CAPM) of Sharpe (1964), Treynor (1961) and Lintner (1965) was best model to determine the fair price of security. The study conducted by Fama and French (1992) in context of NYSE, AMEX and NASDAQ for period of 1963-1990 was having high empirical validity and ease of application as it was focused to see the joint impact of market beta, size factor (SMB¹) and value factor (HML²) on cross-section of stock returns. The findings of Fama and French (1992) indicated that the cross-sectional pricing ability of size and value factor holds significantly and it was concluded that stocks pricing should be

rational and adjusted with multidimensional risk factors. This study was further extended by the two and termed as Fama and French (1993) where they made two key changes. The first change was to use time-series regression in place of cross-sectional regression for not only stocks but for bond market also. In the second change, Fama and French (1993) added two more factors term spread and default spread to existing three factor model to test the pricing ability and documented that the new model is able to capture much variation of the cross section of average US stock returns.

Further, Carhart (1997) extended the work and added one more pricing factor as 'momentum' (WML³) in existing three factor model and the new model was titled as Carhart's four factor model. Jegadeesh and Titman (1993) initiated the discussion on momentum factor and reported various investment strategies which have generated significant positive abnormal returns equal to approximately 1% per month for the upcoming years by having long position in well performing stocks and short position in poor performing stocks on the basis of their past 3-12 months performance. The positive correlation between the past and future returns was tested by numerous other studies also. For

1 SMB is the return difference between the returns on the small and big size portfolios.

2 HML is the return difference between the returns on the high and low book-to-market-ratio portfolios.

3 WML is the return difference between the returns on the high and low prior return portfolios.

US context, Fama and French (1996) and Jegadeesh and Titman (2001) supported these findings. Similarly, for Asian market, Rouwenhorst (1998), Chui et al. (2000), for other emerging markets Rouwenhorst (1999) and Grundy and Martin (2001) confirmed the role of momentum factor and recommended to use momentum as fourth factor in existing three factor model. The existence of SMB, HML and WML was also confirmed by Nartea et al. (2009) in New Zealand stock market. The author reported significant effects of HML and WML factor while the role of SMB was relatively weaker. The explanatory power of four factor model was also significantly higher than CAPM single factor model.

Most of these existing models have assumed that returns of individual securities or portfolios are normally distributed with no fat tails and also having linear relationship with these pricing factors. As result of same, the existing studies have used ordinary least square method (OLS) to model the means of distribution covariates and test the pricing ability of these models. The success of these models is questioned by studies conducted by Black (1993), Kothari and Shanken (1995), Levhari and Levy (1977), Officer (1972), Knez and Ready (1997) and Horowitz et al. (2000). Officer (1972) documented that the distribution of returns have fat tails as compared to normal distribution. Supporting this, Levhari and Levy (1977) indicated that the stock returns carries fat tails and the beta estimates using monthly data are not same as the beta estimated using yearly data. Black (1993) emphasized that the pricing ability of Fama and French factors is significant due to data mining while Horowitz et al. (2000) indicated that the results of size effect do not hold robust across different sample periods and become disappear since 1982. In order to overcome this issue, Chan and Lakonishok (1992) suggested the usage of some more robust methods of testing the cross-sectional pricing ability of these factors. In line with this, the studies conducted by Taylor (2000), Basset and Chen (2001), Barnes and Hughes (2002) and Ma and Pohlman (2008) have used the quantile regression suggested by Koenker and Bassett (1978) to test the relationship in their study model. Taylor (2000) has tried to forecast the exchange rates fat tails using the quantile regression while Basset and Chen (2001) have used the same for portfolio analysis. Barnes and Hughes (2002) tested the cross-sectional pricing ability of CAPM model using quantile regression while Ma and Pohlman (2008) tested the similar relationship for different asset pricing factors. Prior to Allen et al. (2011), the testing of Fama and French factor model was not done using quantile regression. Allen et al. (2011) have tested the pricing ability of Fama and French three factor model using quantile regression This study tests the role size, value and market factor in explaining the returns of 30 Dow Jones Industrial Average Stocks using quantile regression for the period of global financial crisis starting from January 2005 to December 2008. The scope of this study is limited to model the individual stock returns for the financial crisis period and not the portfolio returns. The three factor model of Fama and French (1992; 1993) and Carhart (1997) four factor model advocated to model the portfolio returns not stock returns.

In the light of above background, the present study has been designed to study the pricing ability of Carhart (1997) four factor model using quantile regression in Indian context. The study

contributes in the existing body of literature; firstly by testing the pricing ability of four factor model using quantile regression and comparing these results with OLS results to see which fits better with Indian data; and secondly by enriching the asset pricing domain with in-depth analysis of testing of asset pricing models in Indian context which is not having much of literature available in this context. The asset pricing studies in India are limited to testing of CAPM (Manjunatha and Mallikarjunappa, 2011; Manjunatha and Mallikarjunappa, 2009; Nair et al., 2009; Varma, 1988; Yalwar, 1988; Srinivasan, 1988; Choudhary and Choudhary, 2010; Gupta and Seghal, 1993; Vaidyanathan, 1995; Madhusoodanan, 1997; Sehgal, 1997; Ansari, 2000; Rao, 2004; Mallikarjunappa et al., 2006; Nair et al., 2009; and Basu and Chawla, 2010;) testing of Fama and French three factor model (Connor and Sehgal, 2003; Bahl, 2006; Taneja, 2010) using OLS and investigating the role of idiosyncratic volatility in asset pricing models using Newey-West estimators (Sharma and Kumar, 2016).

In India, Connor and Sehgal (2003) has tested the FF model using CRISIL 500 list for period of June 1989 to March 1999 and documented that cross-section of returns are explained by the mix of these three factors, not just with market factors. Subsequently Bahl (2006) also empirically studied the FF model using BSE-100 Index data for period of June 2001 to June 2006 and confirms the empirical validity of FF model in Indian context. Further, Taneja (2010) has used the S and P CNX 500 and showed that the role of FF factors can't be ignored. Since the factor i.e., size or value are highly correlated so any of them is sufficient to improve the efficiency of model. There is dearth of literature for examining the empirical validity of four factor model in Indian context. Also, all these studies are testing the empirical validity of FF three factor model using OLS method to estimate the model parameters which suffers from the problem of modeling using the conditional mean of the distribution.

Further the study has been structured in following sections. Section 2 presents the data and asset pricing factors construction methodology. Section 3 provides the empirical methodology of testing the pricing ability of four factor model using quantile regression. Section 4 discussed the results while section 5 provides the findings and conclusions.

2. DATA

The study employs daily data of pricing factors ERM, SMB, HML and WML along with test portfolios SL, SM, SH, BL, BM and BH of Indian firms for period starting from 1st December 1993 to 31st March 2016. The data are collected from Agarwalla et al. (2013) working paper which has developed the asset pricing factors using standard Fama and French (1992) and Carhart (1997) methodology. As per this methodology, all the stocks are divided into two groups (Small or "S" and Big or "B") on the basis of market capitalization while the same is again divided into three groups (High or "H," Medium or "M" and Low or "L") on the basis of book equity to market equity ratio. A total of 6 portfolios (SL, SM, SH, BL, BM, BH) are calculated by the intersection of these 2×3 portfolios. The calculation of factor SMB and HML are shown in Table 1. Further, all the stocks are sorted into two more

Table 1: Description of pricing factors and test portfolios

Factor/portfolios	Description
ERM	Excess return on market index is calculated by subtracting the risk free rate of returns from market benchmark indices returns
SMB	Size factor is the daily difference of average returns on three small portfolios (S/L, S/M, and S/H) and three big portfolios (B/L, B/M, and B/H)
HML	Value factor is the daily difference of average returns on two high book to market equity portfolios (S/H and B/H) and two low book to market equity portfolios (S/L and B/L)
WML	The momentum factor is the daily difference of returns of portfolio (WS-LS) and portfolio (WB-LB)
SL	Firms which are small in market capitalization and having low book to market equity ratio
SM	Firms which are small in market capitalization and having medium book to market equity ratio
SH	Firms which are small in market capitalization and having high book to market equity ratio
BL	Firms which are big in market capitalization and having low book to market equity ratio
BM	Firms which are big in market capitalization and having medium book to market equity ratio
BH	Firms which are big in market capitalization and having high book to market equity ratio

categories (Winner or 'W' and 'Loser' or L) as per their momentum level and with the intersection of these two groups (W and L) and other two (S and B) shorted using market capitalization, a total of 4 portfolios (WS, WB, LS, LB) are created. The calculation function of factor WML is given in Table 1. The brief description and calculation procedure of asset pricing factors and sample portfolios are given in following Table 1.

3. EMPIRICAL METHODOLOGY

The standard four factor model proposed by Carhart (1997) is used to investigate the pricing ability of these factors in Indian context. The pooled cross-sectional regression expressed in equation 1 is pressed into service using OLS and quantile regression (section 3.1 for details about the quantile regression).

$$R_{p,t} = \alpha_p + \beta_{p1,t}(ERM_t) + \beta_{p2,t}(SMB_t) + \beta_{p3,t}(HML_t) + \beta_{p4,t}(WML_t) + \varepsilon_{p,t} \quad (1)$$

Where $R_{p,t}$ are the returns on test portfolios, ERM_t (Market factor) is the excess return on market index at time t, SMB_t (size factor) is the return of a portfolio on the size factor, HML_t (value factor) is the return on portfolio on value factor at time t, WML_t (momentum factor) is the return on portfolio on previous performance of stocks at time t. The coefficients β_{p1} , β_{p2} , β_{p3} and β_{p4} are coefficients of ERM_t , SMB_t , HML_t and WML_t factors respectively. $\varepsilon_{p,t}$ is the error of regression analysis.

3.1. Quantile Regression Methodology by Koenker and Bassett (1978)

The limitation of linear regression model is that the dependent variable is always shown as the linear function/combination of the one or more independent variables which is subject to random errors or disturbances. The linear regression model tries to estimate the average or mean value of the dependent variable with the help of a set of predefined independent variables. So the linear regression or OLS models are useful to the condition where the researcher is interested to estimate or predict the mean value of the dependent variable. The question arises on the usability of OLS models when the researcher is interested to estimate other forms of central tendencies and also would like to see the resulting regression relationship in the forms of quartiles and quantiles. Thus in these cases, the OLS loses its validity and effectiveness. The

existing asset pricing studies have used OLS to model the average portfolio returns with respect to ERM, SMB, HML and WML factors and reported significance of these factors in asset pricing but will these results hold true for situation where the test portfolio have given extraordinary returns or performed very poor?.

In order to answer such problem, Koenker and Bassett (1978) have introduced another estimation model which is popularly known as the "Quantile regression." This model is basically an extension of OLS model to estimate the conditional quantile means of dependent variables as function of covariates of observed variables.

The values for different quantiles can be acquired by minimizing a sum of asymmetrically weighted absolute residuals where the different weights are assigned to positive and negative residuals. To achieve beta coefficient (β_τ) of τ^{th} sample quantile, the following equation 2 can be solved.

$$\beta_\tau = \min_{\xi \in R} \sum_{i=1}^n \rho_\tau(Y_i - X_i \beta) \quad (2)$$

Where β_τ is the coefficient of τ^{th} quantile.

In the past decade, many researchers have used this approach in the field of applied econometrics and other areas. The study conducted by Buchinsky and Phillip (1997) have used quantile regression to test the wage structure and earning mobility while Engle and Manganelli (1999) and Morillo (2000) have applied the same to solve the problems of Value at Risk and option pricing respectively. Similarly Barnes and Hughes (2002) have applied quantile regression to study pricing ability of CAPM using cross section of stock market returns. The study conducted by Allen et al. (2011) has used the same for testing the pricing ability of Fama French model.

4. RESULTS AND DISCUSSION

The present section presents the results of regression analysis (using OLS and quantile regression) conducted to test the pricing ability of Carhart (1997) four factor model in Indian context.

4.1. Descriptive Statistics

The Table 2 presents the descriptive statistics of six test portfolios (SL, SM, SH, BL, BM and BH), and the four asset pricing factors (ERM, SMB, HML and WML). In our sample period, the size factor has generated negative average daily returns of -0.010 while the other three factors i.e. ERM, HML and WML have generated average daily returns of 0.015 , 0.021 and 0.075 respectively. Similarly, the average daily returns given by six test portfolios i.e., SL, SM, SH, BL, BM, BH are 0.008 , 0.018 , 0.022 , 0.015 , 0.031 and 0.015 respectively (Figure 1). All the variables are normally distributed as the Jarque-Bera test statistics is statistically significant for all the variables ($P < 0.05$).

4.2. The Correlation Coefficients amongst Pricing Factors

The Table 3 represents the correlation coefficients amongst the size, value, momentum and the market factor. The ERM factor is having very low correlation coefficients value of 0.023 , -0.360 and -0.107 with HML, SMB and WML factors respectively. Similarly, the factor HML is also having low correlation coefficient value of 0.155 and -0.075 with SMB and WML factors respectively. The size factor has documented very low negative correlation of -0.005 with WML. Thus all the pricing factors are having very low correlation with each other and provide indication of no problem of multicollinearity problem in the sample.

4.3. Pricing Ability of Carhart (1997) Four Factor Model

The present section provides the results of regression analysis conducted to test the pricing ability of all four factors using OLS and quantile regression. The results for all six test portfolios are as follows.

4.3.1. Test portfolio SL returns

The results of pooled cross-sectional regression analysis conducted to test the pricing ability of Carhart (1997) asset pricing model on test portfolio SL using OLS and quantile regression are presented in Table 4. The panel 1 of the table shows the OLS results (coefficients followed by standard error) while panels 2-6 display the results of quantile regression at 5th, 25th, 50th, 75th, and 95th percentile levels respectively. From panel 1, the regression coefficients of all four asset pricing factors ERM, SMB, HML and WML found significant at 1% level of significance with coefficient value of 0.907 , 0.543 , 0.158 and -0.032 respectively. These four factors explain 90.3% variation in the dependent variable (SL) thus in this context, it can be concluded that the four factor model holds significant in India.

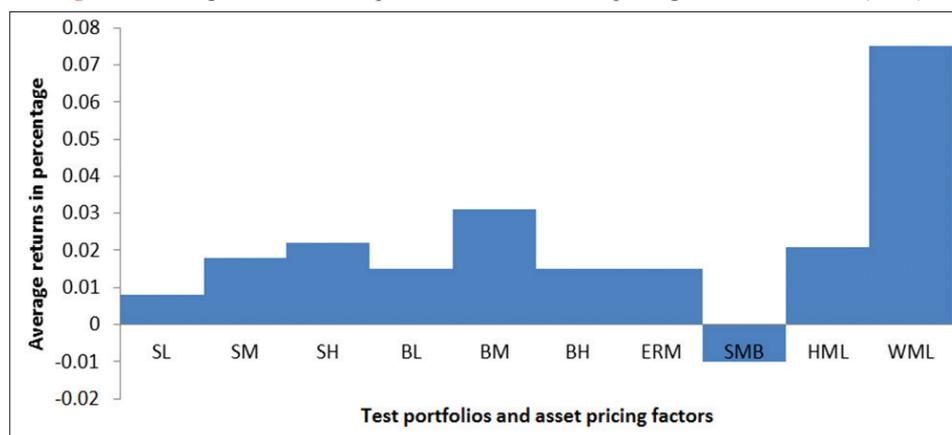
The significance of these pricing factors in explaining the returns of test portfolio SL holds robust at all 5 percentile levels except the ERM factor, which is not best fit beyond 85% percentile and

Table 2: Descriptive statistics of the test portfolios and pricing factors

Statistics	SL	SM	SH	BL	BM	BH	ERM	SMB	HML	WML
Mean	0.008	0.018	0.022	0.015	0.031	0.015	0.015	-0.010	0.021	0.075
Median	0.053	0.046	0.030	0.048	0.056	-0.072	0.073	0.026	-0.031	0.127
Maximum	7.316	9.372	14.564	15.816	15.281	24.723	14.922	6.040	9.625	7.774
Minimum	-10.262	-10.555	-10.392	-11.537	-14.413	-22.174	-11.478	-9.656	-8.252	-7.832
Standard deviation	1.363	1.497	1.669	1.517	1.890	2.861	1.503	0.956	1.072	1.148
Skewness	-0.485	-0.351	0.082	-0.160	0.034	0.519	-0.234	-0.331	0.577	-0.370
Kurtosis	7.079	7.036	7.088	9.380	8.047	11.125	8.920	7.359	8.663	8.597
Jarque-Bera	3672.047	3504.571	3496.260	8522.688	5319.892	14010.260	7364.391	4058.869	6975.529	6656.067
P	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum	38.103	88.349	107.910	77.472	157.819	74.127	76.793	-48.313	105.497	374.894
Sum squared deviations	9304.335	11227.170	13955.490	11532.990	17905.530	41018.750	11312.660	4575.205	5753.231	6598.734

The sample is from 1st December 1993 to 31st March 2016. The factors SL, SM, SH, BL, BM and BH are the test portfolios and ERM, SMB, HML and WML are pricing factors of Carhart (1997) four factor model. OLS: Ordinary least square

Figure 1: Average returns on test portfolios and four asset pricing factors of Carhart (1997)



HML factor is also not good fit in between 75% to 85% percentile levels (Figure 2).

4.3.2. Test portfolio SM returns

In continuation to previous analysis, the Table 5 presents the results of pooled cross-sectional regression analysis conducted to test

Table 3: Correlation coefficient amongst pricing factors

	ERM	HML	SMB	WML
ERM	1	0.023762	-0.36035	-0.10709
HML	0.023762	1	0.155357	-0.07542
SMB	-0.36035	0.155357	1	-0.00545
WML	-0.10709	-0.07542	-0.00545	1

The sample is from 1st December 1993 to 31st March 2016. The factors ERM, SMB, HML and WML are pricing factors of Carhart (1997) four factor model. OLS: Ordinary least square

the pricing ability of Carhart (1997) asset pricing model on test portfolio SM using OLS and quantile regression. The portfolio SM is average daily reruns on those variables which are having small market capitalization and medium book to market equity ratio. From panel 1, the regression coefficients of all four asset pricing factors ERM, SMB, HML and WML found significant at 1% level of significance with coefficient value of 0.957, 0.593, 0.348, and -0.041 respectively. The goodness of fit for this model is 0.924 indicating that these four factors explain 92.4% variation in the dependent variable (SM).

If we Figure 3, the OLS model is not best fit as the SMB factor is not able to explain the dependent variable beyond 50th percentile level. Similarly, the HML factor is also not best fit in between 40th and 60th percentiles. The factor WML is also not showing good

Table 4: Regression results four factor model using returns of SL portfolio as dependent variable

Independent Variables	Dependent variable: Test portfolio (SL) returns					
	OLS		Quantile regression			
	1	2	3	4	5	6
ERM	0.907*** (0.0040)	0.897*** (0.0110)	0.900*** (0.0050)	0.901*** (0.0040)	0.904*** (0.0040)	0.926*** (0.0130)
SMB	0.543*** (0.0070)	0.556*** (0.0160)	0.543*** (0.0070)	0.536*** (0.0060)	0.533*** (0.0080)	0.544*** (0.0210)
HML	0.158*** (0.0050)	0.145*** (0.0130)	0.159*** (0.0060)	0.159*** (0.0050)	0.171*** (0.0060)	0.154*** (0.0180)
WML	-0.032*** (0.0050)	-0.038*** (0.0140)	-0.028*** (0.0060)	-0.023*** (0.0050)	-0.030*** (0.0050)	-0.038*** (0.0170)
Constant	0.0040 (0.0060)	-0.656*** (0.0160)	-0.235*** (0.0060)	-0.010 (0.0060)	0.226*** (0.0070)	0.667*** (0.0190)
Observations	5528	5528	5528	5528	5528	5528
R ²	0.9030					
Adjusted R ²	0.9030					
Residual standard error	0.418 (df=5523)					
F Statistic	12859.760*** (df=5523)					

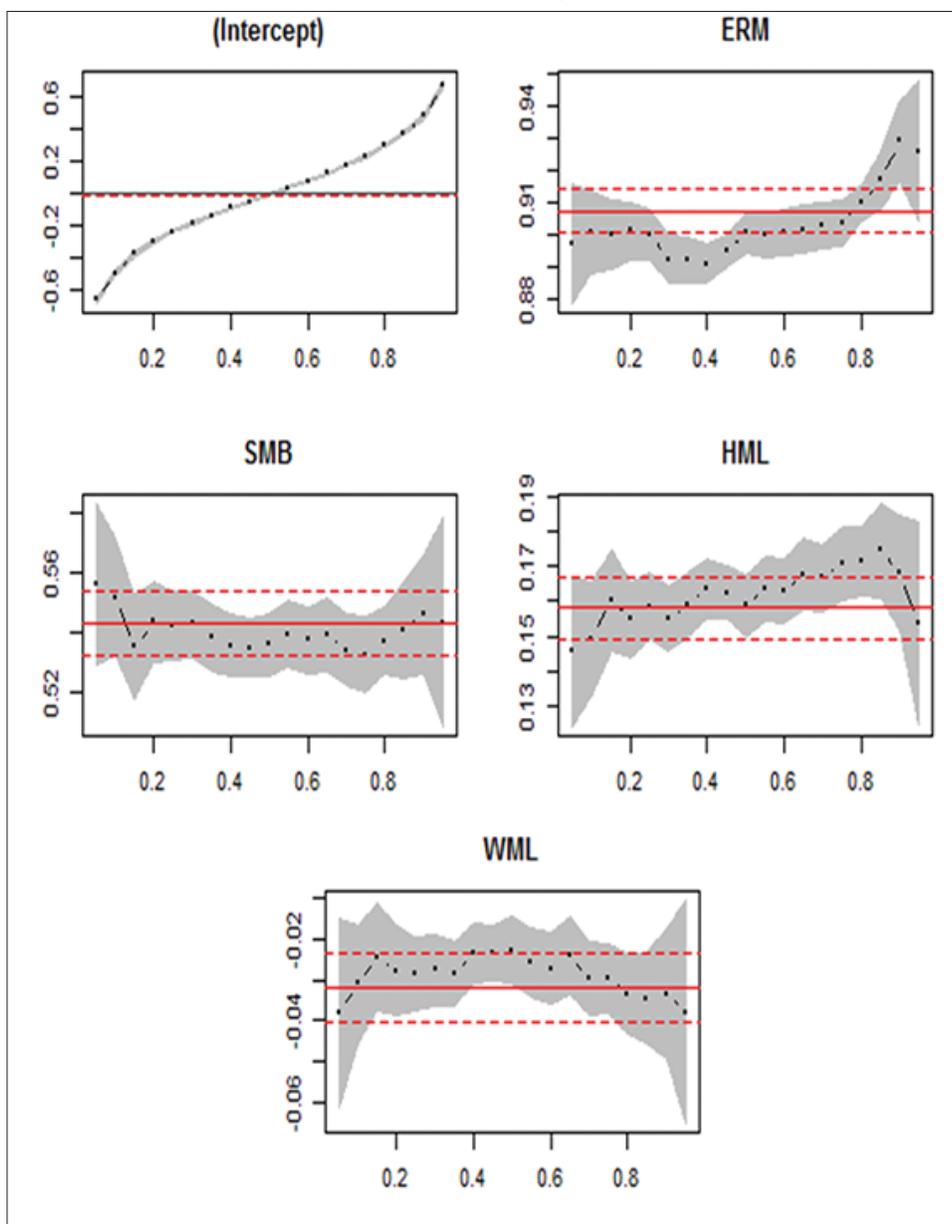
P<0.05; *P<0.01. The sample is from 1st December 1993 to 31st March 2016. The table represents the results of OLS (in panel 1) and quantile regression (in panel 2 to 6 for 5th, 25th, 50th, 75th and 95th percentile respectively. The portfolio returns of SL are taken as dependent variable. The coefficients are reported opposite to the variable (Right side, below the respective panel) followed by standard error. OLS: Ordinary least square

Table 5: Regression results four factor model using returns of SM portfolio as dependent variable

Independent Variables	Dependent variable: Test portfolio (SM) returns					
	OLS		Quantile regression			
	1	2	3	4	5	6
ERM	0.965*** (0.0040)	0.957*** (0.0110)	0.968*** (0.0050)	0.970*** (0.0030)	0.971*** (0.0040)	0.954*** (0.0110)
SMB	0.591*** (0.0060)	0.593*** (0.0160)	0.584*** (0.0070)	0.598*** (0.0050)	0.608*** (0.0070)	0.609*** (0.0180)
HML	0.364*** (0.0050)	0.348*** (0.0120)	0.348*** (0.0050)	0.364*** (0.0040)	0.381*** (0.0060)	0.387*** (0.0150)
WML	-0.065*** (0.0050)	-0.041*** (0.0120)	-0.060*** (0.0060)	-0.063*** (0.0050)	-0.066*** (0.0060)	-0.093*** (0.0130)
Constant	0.0000 (0.0060)	-0.644*** (0.0160)	-0.224*** (0.0070)	0.0002 (0.0050)	0.227*** (0.0070)	0.644*** (0.0150)
Observations	5528	5528	5528	5528	5528	5528
R ²	0.9240					
Adjusted R ²	0.9240					
Residual standard error	0.409 (df=5523)					
F Statistic	16763.450*** (df=4; 5523)					

** shows significant at 5% level of significance (p<0.05) while *** indicates significant at 1% level of significance (p<0.01). The sample is from 1st December 1993 to 31st March 2016. The table represents the results of OLS (in panel 1) and quantile regression (in panel 2 to 6 for 5th, 25th, 50th, 75th and 95th percentile respectively. The portfolio returns of SM are taken as dependent variable. The coefficients are reported opposite to the variable (Right side, below the respective panel) followed by standard error. OLS: Ordinary least square

Figure 2: The fitting lines of ordinary least square and quantile regression for dependent variable SL and independent factors (ERM, SMB, HML and WML)



fit for top percentile SM portfolio returns (beyond 80th percentile) and bottom percentile SM portfolio returns (below 20th percentile).

4.3.3. Test portfolio SH returns

Further, the similar analysis was conducted after taking SH portfolio returns as dependent variable and the results are reported in Table 6. The results of OLS from panel 1 of Table 6 show the regression coefficients of all four asset pricing factors ERM, SMB, HML and WML as 1.015, 0.736, 0.668, and 0.0010 respectively. These coefficients are significant at 1% level of significance except the coefficient of WML factor. These four factors explain 96.8% variation in the dependent variable (SH). In consistent to previous analysis conducted for SL and SM dependent variables, the WML factor has become insignificant now for dependent variable SH. The WML factor is significant only at 25th percentile level. If we see the fitting lines of both OLS and quantile regression (Figure 4),

the SMB factor is not appearing good fit with OLS fitting line as the straight line is not modeling it completely. Similarly, the factor HML is not able to model the dependent variable (SH) returns for below 60th percentile and above 80th percentile levels. The WML is also not best fit below 50th percentile levels.

4.3.4. Test portfolio BL returns

The Table 7 reports the results of the similar analysis conducted after taking the return of BL portfolio returns. The panel 1 of Table 7 shows the regression coefficients of all four asset pricing factors ERM, SMB, HML and WML as 0.987, -0.048, -0.102 and 0.008 respectively. These coefficients are significant at 1% level of significance except the coefficient of WML factor which loses the significance beyond 75th percentile level (panel 5 and 6 of table). These four factors explain 98.2% variation in the dependent variable (BL).

Figure 3: The fitting lines of ordinary least square and quantile regression for dependent variable SM and independent factors (ERM, SMB, HML and WML)

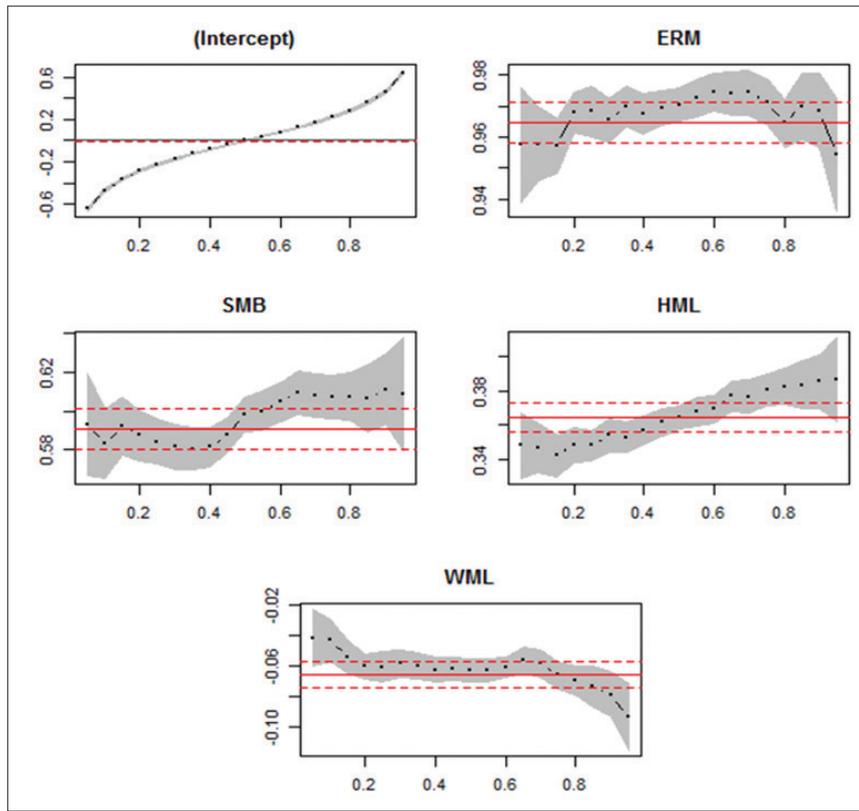


Figure 4: The fitting lines of ordinary least square and quantile regression for dependent variable SH and independent factors (ERM, SMB, HML and WML)

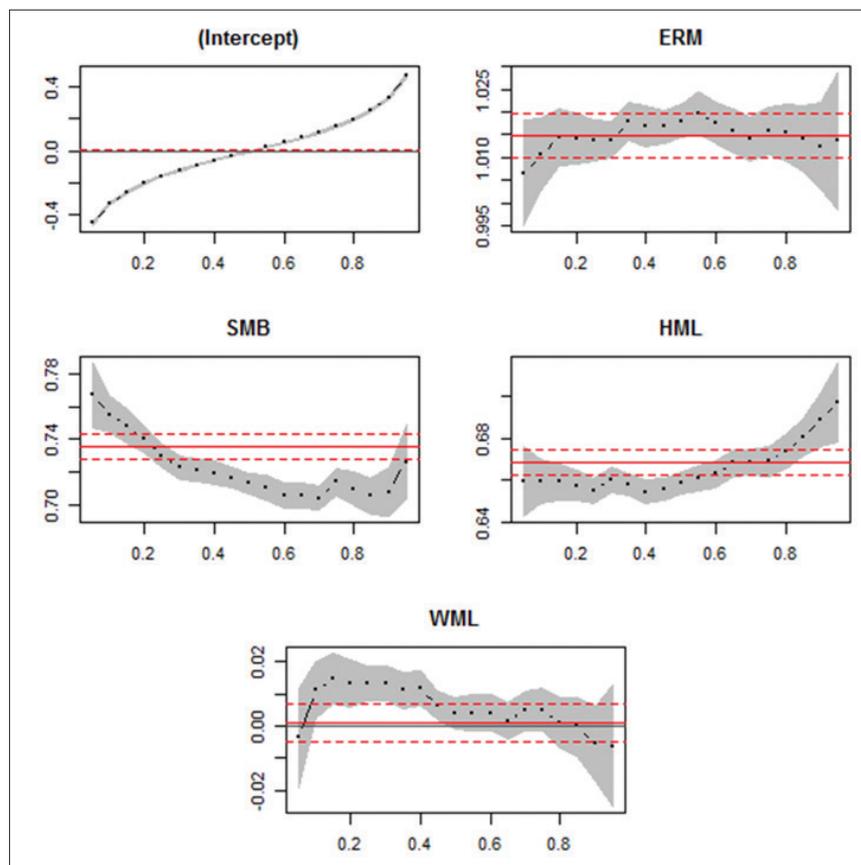


Table 6: Regression results four factor model using returns of SH portfolio as dependent variable

Independent Variables	Dependent variable: Test portfolio (SH) returns					
	OLS	Quantile regression				
	1	2	3	4	5	6
ERM	1.015*** (0.0030)	1.007*** (0.0070)	1.014*** (0.0030)	1.018*** (0.0020)	1.016*** (0.0030)	1.014*** (0.0090)
SMB	0.736*** (0.0050)	0.767*** (0.0120)	0.730*** (0.0040)	0.713*** (0.0040)	0.714*** (0.0050)	0.726*** (0.0140)
HML	0.668*** (0.0040)	0.659*** (0.0100)	0.655*** (0.0040)	0.658*** (0.0030)	0.669*** (0.0040)	0.697*** (0.0110)
WML	0.0010 (0.0040)	0.0040 (0.0090)	0.013*** (0.0030)	0.0040 (0.0030)	0.0050 (0.0040)	0.0060 (0.0120)
Constant	0.0010 (0.0040)	-0.453*** (0.0110)	-0.158*** (0.0040)	-0.007 (0.0040)	0.153*** (0.0050)	0.469*** (0.0140)
Observations	5528	5528	5528	5528	5528	5528
R ²	0.9680					
Adjusted R ²	0.9680					
Residual standard error	0.295 (df=5523)					
F Statistic	42178.870*** (df=4; 5523)					

** shows significant at 5% level of significance (p<0.05) while *** indicates significant at 1% level of significance (p<0.01). The sample is from 1st December 1993 to 31st March 2016. The table represents the results of OLS (in panel 1) and quantile regression (in panel 2 to 6 for 5th, 25th, 50th, 75th and 95th percentile respectively). The portfolio returns of SH are taken as dependent variable. The coefficients are reported opposite to the variable (Right side, below the respective panel) followed by standard error. OLS: Ordinary least square

Table 7: Regression results four factor model using returns of BL portfolio as dependent variable

Independent Variables	Dependent Variable: Test portfolio (BL) returns					
	OLS	Quantile regression				
	1	2	3	4	5	6
ERM	0.987*** (0.0020)	0.986*** (0.0040)	0.993*** (0.0020)	0.993*** (0.0020)	0.990*** (0.0020)	0.990*** (0.0060)
SMB	-0.048*** (0.0030)	-0.039*** (0.0090)	-0.037*** (0.0030)	-0.041*** (0.0020)	-0.050*** (0.0030)	-0.069*** (0.0100)
HML	-0.102*** (0.0030)	-0.126*** (0.0060)	-0.103*** (0.0020)	-0.097*** (0.0020)	-0.096*** (0.0020)	-0.097*** (0.0080)
WML	0.008*** (0.0020)	0.014** (0.0070)	0.006*** (0.0020)	0.006*** (0.0020)	0.0030 (0.0020)	0.0090 (0.0080)
Constant	0.0020 (0.0030)	-0.300*** (0.0090)	-0.086*** (0.0030)	0.004 (0.0020)	0.094*** (0.0030)	0.293*** (0.0090)
Observations	5528	5528	5528	5528	5528	5528
R ²	0.9820					
Adjusted R ²	0.9820					
Residual standard error	0.202 (df=5523)					
F Statistic	73370.340*** (df=4; 5523)					

** shows significant at 5% level of significance (p<0.05) while *** indicates significant at 1% level of significance (p<0.01). The sample is from 1st December 1993 to 31st March 2016. The table represents the results of OLS (in panel 1) and quantile regression (in panel 2 to 6 for 5th, 25th, 50th, 75th and 95th percentile respectively). The portfolio returns of BL are taken as dependent variable. The coefficients are reported opposite to the variable (Right side, below the respective panel) followed by standard error. OLS: Ordinary least square

From Figure 5, the OLS fitting line is not sufficient enough to gauge the impact of ERM factor on dependent variable BL below 70th percentile level. It is only fitting well above 70th percentile level. The SMB factor is also not showing good fit with OLS line except for 40-75th percentile. The factor HML is also not fitted well below 20th percentile level.

4.3.5. Test portfolio BM returns

In consistent with existing analysis conducted for four variables (SL, SM, SH and BL) the OLS results for BM (Table 8) are also not best fit across all quantile levels. From Figure 6, it is evident that SMB factor is not best fit below 25th percentile level and above 60th percentile level. The factor WML is also not best fit for below 20th percentile level and HML for 80th percentile level.

4.3.6. Test portfolio BH return

The OLS results reported in Table 9 are having low R² value of 0.259 which indicates that the model is not best fit and only explains 25.9% variation in the dependent variable (BH portfolio returns) (Figure 7). The OLS line is also not able to fit SMB and HML across all quantile levels and factor ERM is not fitted for below 20th percentile level.

5. FINDINGS AND CONCLUSIONS

The results reported in section 4.3 have shown the pricing ability of Carhart (1997) four factor model using six test portfolios as dependent variables. The results have indicated that the four factor model explain significant variation in the

Figure 5: The fitting lines of ordinary least square and quantile regression for dependent variable BL and independent factors (ERM, SMB, HML and WML)

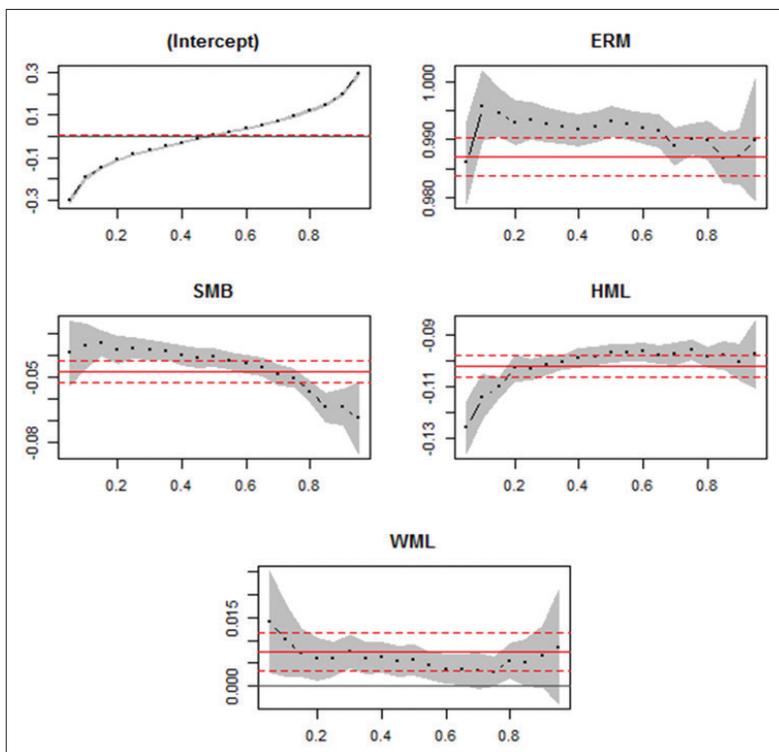


Figure 6: The fitting lines of ordinary least square and quantile regression for dependent variable BM and independent factors (ERM, SMB, HML and WML)

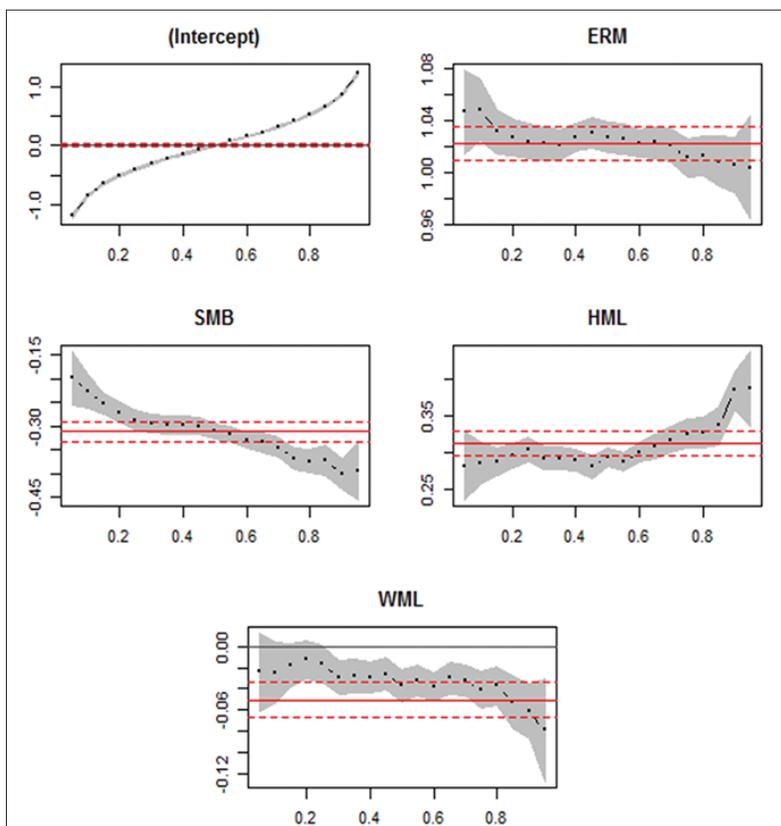


Table 8: Regression results four factor model using returns of BM portfolio as dependent variable

Independent Variables	Dependent variable: Test portfolio (BM) returns					
	OLS	Quantile regression				
	1	2	3	4	5	6
ERM	1.022*** (0.0080)	1.046*** (0.0200)	1.024*** (0.0080)	1.027*** (0.0070)	1.011*** (0.0090)	1.004*** (0.0240)
SMB	-0.311*** (0.0120)	-0.199*** (0.0350)	-0.287*** (0.0130)	-0.310*** (0.0100)	-0.367*** (0.0140)	-0.394*** (0.0370)
HML	0.312*** (0.0100)	0.281*** (0.0290)	0.304*** (0.0110)	0.293*** (0.0080)	0.325*** (0.0120)	0.386*** (0.0310)
WML	-0.050*** (0.0100)	0.0240 (0.0220)	0.0170 (0.0100)	-0.037*** (0.0090)	-0.041*** (0.0110)	-0.079*** (0.0300)
Constant	0.0100 (0.0110)	-1.192*** (0.0320)	-0.399*** (0.0120)	0.0060 (0.0100)	0.414*** (0.0120)	1.237*** (0.0330)
Observations	5528	5528	5528	5528	5528	5528
R ²	0.8140					
Adjusted R ²	0.8140					
Residual standard error	0.798 (df=5523)					
F Statistic	6060.242*** (df=4; 5523)					

** shows significant at 5% level of significance (p<0.05) while *** indicates significant at 1% level of significance (p<0.01). The sample is from 1st December 1993 to 31st March 2016. The table represents the results of OLS (in panel 1) and quantile regression (in panel 2 to 6 for 5th, 25th, 50th, 75th and 95th percentile respectively). The portfolio returns of BM are taken as dependent variable. The coefficients are reported opposite to the variable (Right side, below the respective panel) followed by standard error. OLS: Ordinary least square

Table 9: Regression results four factor model using returns of BH portfolio as dependent variable

Independent Variables	Dependent variable: Test portfolio (BH) returns					
	OLS	Quantile regression				
	1	2	3	4	5	6
ERM	254.724*** (11.7470)	14.6150 (12.5560)	267.300*** (23.7080)	296.050*** (11.0170)	284.281*** (7.3020)	249.323*** (14.0210)
SMB	-316.823*** (18.4800)	23.8050 (20.5050)	-468.784*** (41.9850)	-406.866*** (14.0510)	-274.188*** (10.4460)	-110.895*** (16.5280)
HML	421.709*** (15.3350)	60.857*** (16.4750)	536.521*** (34.9690)	488.503*** (11.1270)	444.728*** (7.9090)	245.623*** (13.7880)
WML	-65.987*** (14.5230)	9.0250 (15.2240)	-50.582 (30.3340)	-89.911*** (14.6310)	-58.387*** (8.6690)	-33.103 (17.2190)
Constant	2493.897*** (16.0110)	279.242*** (19.7520)	1721.934*** (37.9370)	2713.807*** (17.7800)	3328.085*** (16.5260)	4162.914*** (23.0260)
Observations	5528	5528	5528	5528	5528	5528
R ²	0.259					
Adjusted R ²	0.259					
Residual standard error	1186.173 (df=5523)					
F statistic	482.927*** (df=4; 5523)					

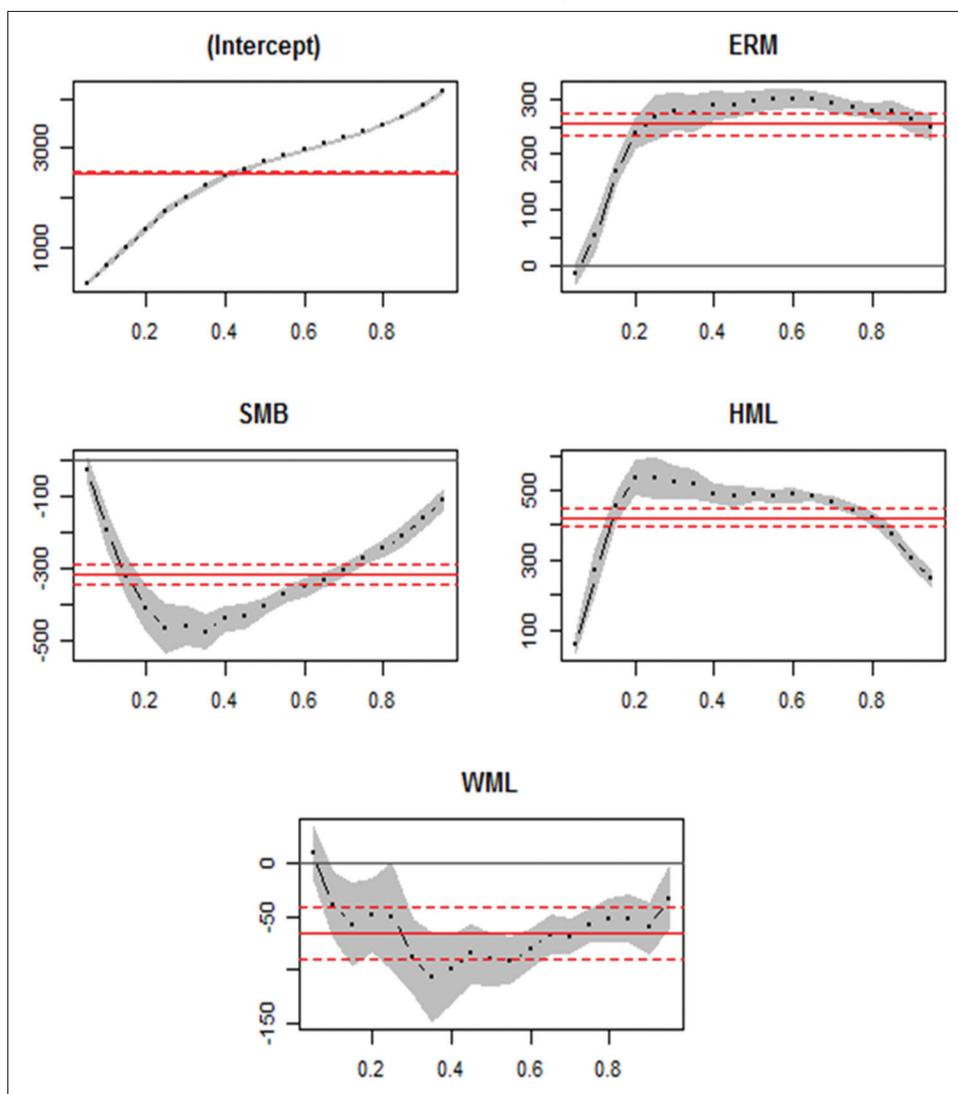
** shows significant at 5% level of significance (p<0.05) while *** indicates significant at 1% level of significance (p<0.01). The sample is from 1st December 1993 to 31st March 2016. The table represents the results of OLS (in panel 1) and quantile regression (in panel 2 to 6 for 5th, 25th, 50th, 75th and 95th percentile respectively). The portfolio returns of BH are taken as dependent variable. The coefficients are reported opposite to the variable (Right side, below the respective panel) followed by standard error. OLS: Ordinary least square

dependent variables. The results of all three factors ERM, SMB and HML are robust for all sample portfolios but same is not the case with WML factor as it loses its pricing ability for dependent variable SH, BL, BM and BH. The study also compares the OLS results with quantile regression to see whether the OLS fitting line is best for all the explanatory variables across all the quantiles. The results show that all four factors are not best fit using OLS as these factors are not fitting well at different percentile/quantile levels. The size factor (SMB) is not fitting well with OLS for test portfolios SH, SM (beyond 50th percentile), BL, BM (less than 25th percentile and above 60th percentile) and BH. Similarly, the HML factor is not good fit for SH and BH test portfolios. The ERM factor is fitting well across all quantiles with OLS except BL test portfolio. The factor WML is also not best fit with OLS for

SM (below 20th percentile and above 80th percentile) and SH (below 50th percentile) dependent variables.

In light of the results reported above, we can conclude that the pricing ability of four factor model using daily data holds significant for data of Indian firms. The results of OLS are robust and falls in line with the results of quantile regression conducted for 5th, 25th, 50th, 75th, and 95th percentile levels. In contrast to this, the fitting lines of OLS model fail to model the pricing ability of independent variables for all the quantile levels. Thus we can conclude that the returns are not distributed in perfect linear form and the OLS fails to model those returns series which are having fat tails. Thus based on the findings of this study, it is suggested to use quantile regression to model the asset pricing models as it holds its superiority for modeling the data across all quantile period. The

Figure 7: The fitting lines of ordinary least square and quantile regression for dependent variable BH and independent factors (ERM, SMB, HML and WML)



results of the study fall in consistent with the study conducted by Allen et al. (2011) which documented the superiority of quantile regression over OLS while testing Fama and French (1993) three factor asset pricing model.

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