2024, 9 (2), 1-21

Akademik İzdüşüm Dergisi (AİD)

Journal of Academic Projection (JAP)

https://dergipark.org.tr/tr/pub/beuiibfaid

Geliş/Received: 06/02/2024
Kabul/Accepted: 29/06/2024
Araştırma Makalesi / Research Article

ISSN: 2547-9725

FINANCIAL MARKETS: CLASSICAL AND BAYESIAN APPROACH TO UNIVARIATE VOLATILITY MODELS

Tuğçe ACAR KARA*

Abstract

There are two different approaches to the development of statistics. These are the "Classical" and the "Bayesian" approaches. We encounter the concept of "objectivity", which in the classical approach refers to ignoring prior information about the process being measured. However, in the presence of prior information about the process under consideration, there is a loss of information because the existing information is ignored. Since the parameters are not random in the classical approach, no probability statements can be made about the parameters. The Bayesian approach takes into account prior information about the process and takes a more disciplined approach to uncertainty. It is therefore an approach derived from Bayes' theorem. The Bayesian approach treats parameters as probabilistic and random variables. There are no assumptions to be made as in the classical approach. Given this information, the aim is to evaluate the univariate volatility models under the Classical and Bayesian approaches. Volatility, which corresponds to uncertainty in the financial markets, also represents the risk of the financial asset. Therefore, it is expected that it will be beneficial to evaluate the effect of both approaches on the analysis of volatility models.

Keywords: *Financial Markets, Volatility Models, Classical Approach, Bayesian Approach.*

JEL Codes: *C01, C11, E44*

Finansal Piyasalar: Tek Değişkenli Volatilite Modellerine Klasik ve Bayesyen Yaklaşım

Öz

İstatistiğin gelişiminde iki farklı yaklaşım vardır. Bunlar "Klasik" ve "Bayesci" yaklaşımlardır. Klasik yaklaşımda, ölçülen süreçle ilgili ön bilgilerin göz ardı

* Dr., İstanbul Aydın Üniversitesi, Teknoloji Geliştirme Merkezi (TEKMER), tugcekara@aydin.edu.tr, ORCID: 0000-0001-9223-0089

edilmesini ifade eden "objektiflik" kavramı karşımıza çıkar. Ancak, söz konusu süreçle ilgili ön bilgilerin varlığında, mevcut bilgiler göz ardı edildiği için bir bilgi kaybı söz konusudur. Parametreler rastgele olmadığından, parametreler hakkında olasılık ifadeleri yapılamaz. Bayesci yaklaşım, süreçle ilgili ön bilgileri dikkate alır ve belirsizliğe daha disiplinli bir yaklaşım getirir. Bu nedenle Bayes teoreminden türetilmiş bir yaklaşımdır. Bayesci yaklaşım, parametreleri olasılıksal ve rastgele değişkenler olarak ele alır. Klasik yaklaşımda olduğu gibi sağlanması gereken varsayımlar yoktur. Bu bilgiler ışığında amaç, tek değişkenli volatilite modellerini Klasik ve Bayesyen yaklaşımlar altında değerlendirmektir. Finansal piyasalarda belirsizliğe karşılık gelen volatilite, aynı zamanda finansal varlığın riskini de temsil etmektedir. Dolayısıyla her iki yaklaşımın volatilite modellerinin analizine etkisi açısından değerlendirilmesinin faydalı olacağı beklenmektedir.

Anahtar Kelimeler: *Finansal Piyasalar, Volatilite Modelleri, Klasik Yaklaşım, Bayes Yaklaşımı.*

JEL Kodu: *C01, C11, E44*

1. INTRODUCTION

Financial asset movements in the financial markets include upward and downward changes. Although the financial series are stationary on average, they are not stationary in variance. Here we come across with the concept of volatility. The concept we call volatility means that variance changes over time. Considering that this concept meets uncertainty, volatility is also expressed as the total risk of any financial asset. The high volatility of the financial asset tells us how risky the asset is. High volatility means that the index is risky and that the return index is spread over a wide range of values. This means that the index price will change significantly in a short period of time (Karolyi, 2001:2). In general, returns on financial assets have three main characteristics. These are the leptokurtic feature, the volatility cluster and the leverage effect. Leptokurtic means that financial series are thick in the tails and pointed at the end. While the thickness in the tails corresponds to the points where there are excessive movements in the financial series, excessive pointedness means that the periods are more likely to have extreme situations, i.e. highly volatile movements. Mandelbrot (1963) found that large changes in the returns of financial assets are followed by large changes and small changes are followed by small changes (Mendelbrot, 1963: 394). This situation is known as a volatility cluster in the financial markets. On the other hand, the high rate of depreciation of financial assets over time creates greater volatility than the same level of appreciation and increases the risk level of the financial asset. In addition, market participants react differently to positive and negative market news. Accordingly, news with a negative impact on financial markets creates more volatility than news with a positive impact. Therefore, the direction of the price change has an asymmetric effect on volatility. We can say that the asymmetric effect must be present for the leverage effect to be present. One of the characteristic features of financial time series is that the conditional variance changes over time. Models that take into account the concept of heteroskedasticity are called "autoregressive conditional heteroskedasticity models". The models are divided into univariate and multivariate volatility models. While univariate volatility models ensure that the volatility of a particular financial asset is reported independently of the returns of other financial assets, multivariate volatility models also take into account the time dependence between the financial market and the assets. ARCH and GARCH models, which can be used to model the volatility of a single financial asset, are inadequate for multivariate structures. In particular, the fact that the financial markets of many countries are interconnected and interdependent means that the financial assets traded in the financial markets are also interconnected and interdependent.

Multivariate structures enable more rational decisions in areas such as asset pricing, portfolio selection, option pricing and risk management. While univariate ARCH-GARCH models include conditional variances, multivariate structures include conditional variances and covariances include dynamic relationships. The differentiation of the models by name is related to the parameterization techniques used. Furthermore, these models have two characteristics as symmetric and asymmetric autoregressive heteroskedasticity models. In symmetric conditional variance models, the effect of positive and negative news on volatility is treated as equal. In asymmetric models, this effect occurs at a different level. Accordingly, Engle (1982) firstly analyzed the UK inflation rate data and showed that the variance of the error term is not fixed (Engle, 1982: 987). In the study in question, it was determined that the estimation errors vary depending on the prediction errors of the previous period. He proposed autoregressive conditional variance model (ARCH Model) to calculate conditional volatility. On the other hand, there are two different approaches to the development of statistics. These are the "Classical" and the "Bayesian" approaches. In particular, the importance of being able to obtain information about the distribution of variables makes the Bayesian approach attractive. Accordingly, the study aims to examine the univariate volatility models within the framework of the classical and Bayesian approaches. The study will also contribute to the literature by guiding the selection of a better model that takes into account future fluctuations with applications in financial markets. It is believed that it would be beneficial to make an assessment in terms of the differences and advantages over the classical approach to the subject, rather than the methodological differences in the Bayesian approach. Volatility, which corresponds to uncertainty in the financial markets, also represents the risk of the financial asset. Therefore, it is believed that it will be useful to show the effect of both approaches on the analysis of volatility models. In the study, following the introduction, univariate symmetric and asymmetric autoregressive heteroskedasticity models are analysed, and then how these models are handled in the Bayesian approach and the similarities and differences between them are examined.

2. UNIVARIATE SYMMETRIC AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTİCİTY MODELS

Models commonly used in research areas such as financial markets, risk management, equities and exchange rates model the volatility of financial assets. They are symmetric in that the effects of positive and negative shocks are treated equally. The model structures are ARCH, ARCH-M, GARCH and GARCH-M, as described below.

2.1. ARCH and ARCH-M Models

The ARCH model is based on the basic logic of explaining the model with the previous periods of the square of the residuals. Therefore, the prediction of variance in the next period depends on the information available in the previous periods [9]. B_{t-1}, is a linear function of past returns and information in (t-1) time. In order to model volatility correctly, it is important to use the information of previous periods in the calculation of the conditional average and variance of financial returns. The conditional mean and variance of a financial return (r_t) obtained by using past period information are as follows;

$$\mu_t = E\,(r_t \mid B_{t-1}) \tag{1}$$

$$\sigma_t^2 = Var\,(r_t \mid B_{t-1}) = E\,[\,(rt - \mu_t)^2 \mid B_{t-1}] \tag{2}$$

Here, the ARCH (p) model with the most general expression for the return of a financial asset is as follows;

$$\text{ARCH (p)}: \sigma_t^2 = \alpha_0 + \alpha_1\,\varepsilon_{t-1}^2 + \alpha_2\,\varepsilon_{t-2}^2 + \ldots\ldots\ldots + \alpha_p\varepsilon_{t-p}^2 \tag{3}$$

Errors are derived from an average model. Considering that the mean model is a simple AR(1) process in the form of $Y_t = \theta Y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N\,(0, \sigma_t^2)$ the error terms must be Gaussian.

If we make inference from ARCH (1) model,

$$\text{ARCH(1)}: \sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 \tag{4}$$

In the ARCH model, parameter estimates are made using the maximum likelihood method. The α_1 parameter is the ARCH(1) parameter. It should be $0 < \alpha_1 < 1$. The fact that this parameter is close to 1 indicates that volatility is continuous (with a permanent effect), in other words, the presence of the volatility cluster. The fact that the parameter is less than 1 indicates that the ARCH model satisfies the stationary condition.

Engle (1982) suggested first testing for the ARCH effect, i.e. the presence of conditional variance in variance modelling. If there is no ARCH effect in the residual, then the use of the ARCH model would be incorrect.

The implementation of the ARCH test is as follows:

A model is established with the square of the residues obtained $Y_t = \theta Y_{t-1} + \varepsilon_t$ over the appropriate average equation of AR (1).

With the help of the R^2 value obtained from the $\varepsilon_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \varepsilon_{t-2}^2 + \dots\dots\dots + \beta_p \varepsilon_{t-p}^2 + u_t$ model, the test statistic is calculated as nR^2. Here, the test statistic has a p degrees of freedom $\aleph^2$distribution. The null hypothesis is that there is no ARCH effect present.

In general, the ARCH effect in ARCH (1) does not require long delays to be considered. For this reason, in practice, it is generally applied via ARCH (1).

Since the conditional variance (σ_t^2) or square root is included in the mean equation, the model is called ARCH in Mean.

Average equation is as follows;

$$Y_t = \theta Y_{t-1} + \varphi\sigma_t^2 + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_t^2) \quad (5)$$

Model is as follows;

$$\text{ARCH-M(p)} = \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots\dots\dots + \alpha_p \varepsilon_{t-p}^2 \quad (6)$$

Also, the parameter φ in the average equation is the risk premium coefficient that changes over time.

Although the ARCH model has a parametric structure in estimating volatility and has produced some empirical results on financial asset returns, it also has some weaknesses. These weaknesses are as follows:

i. The ARCH model is only used to determine the behaviour of the conditional variance. It is ineffective in explaining the changes in the financial series (Degiannakis and Xekalaki, 2015: 272).

ii. Since the shocks of the previous period are included in the model with their squares, positive and negative shocks are assumed to have the same effect on volatility. However, it is well known that the prices of financial assets in financial markets can react differently to these shocks.

iii. Since the ARCH model tends to react slowly to large shocks to financial returns, it can predict volatility more than it does.

iv. The excess of variables that are significant in the delay number of the error term square in the ARCH model increases the number of parameters to be estimated (Çil, 2015: 449).

2.2. GARCH and GARCH-M Models

The Generalised ARCH (GARCH) model was developed by Bollerslev (1986) to overcome the implementation difficulties of the ARCH model. The main limitation of the ARCH model is that it requires many lags to capture the effect of past returns on today's volatility. Bollerslev (1986) developed the GARCH model by including the ARCH model's own lag in the volatility equation.

For the GARCH model, as in the ARCH model, r_t denotes the logarithmic return and $\varepsilon_t = r_t - \mu$, shock at time t. The GARCH (p,q) model used to capture the volatility in the financial return series with $\varepsilon_t = z_t . \sigma_t$ is as follows:

$$\text{GARCH(p,q)}: \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i {\varepsilon^2}_{t-i} + \sum_{i=1}^{q} \beta_i {\sigma^2}_{t-i} \qquad (7)$$

The random variable array with $z_t \sim N(0,1)$expresses the degree of the parameter GARCH, that is, the degree of the conditional variance (σ_t^2), while the parameter p expresses the degree of the ARCH and the past period of the residues.

$$\text{GARCH(1,1)}: \sigma_t^2 = \alpha_0 + \alpha_1 {\varepsilon^2}_{t-1} + \beta_1 {\sigma^2}_{t-1} \qquad (8)$$

In the GARCH (1,1) model, the parameter α_0 indicates the long-run volatility, α_1 indicates the size of the shocks arriving in the series, and the parameter β_1 indicates the effect of past volatility on today's volatility. In the equation, the response of the series to shocks is equal to the value of the parameter α_1. The size of the parameter β_1, which indicates the volatility lag, means that the shocks in the series last for a long time.

For example, if α_1= 0.30 and β_1= 0.60, one unit of unexpected return changes cause the volatility to increase by 0.30 units, when the volatility of the previous period increases by one unit, the volatility of the next period increases by 0.60.

For the significance of the GARCH model, α_0, α_1 and $\beta_1 > 0$ must be 0.

Also, α_1 + β_1<1 condition must be fulfilled for covariance stationarity. (Covariance Stationary: It means that the mean and variance of the stochastic process does not change over time. Besides, it requires that the correlation, which is the indicator of the relationship between x (t) and x (t + h), depends on the h parameter, that is, the distance between processes and not on time.)

The GARCH model also uses the maksimum likelihood method for parameter estimations.

3. UNIVARIATE ASYMMETRIC AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY MODELS

3.1. Exponential GARCH (EGARCH) and Threshold GARCH (TARCH) Model

The main disadvantage of ARCH and GARCH models is that the variance effect is assumed to be constant (Çil,2015:461). However, the reaction to negative news in financial markets is greater than the reaction to positive news. In other words, the fall in stock prices causes more volatility in financial markets than the rise in prices. The reason for this is that the company is seen as more risky because of the increase in the ratio of debt to equity, known as the leverage effect. The leverage effect was first introduced by Black (1976) (Black, 1976:177-181).

The EGARCH model developed by Nelson (1991) to capture this asymmetry is as follows (Nelson, 1991:347-370):

$$\ln({\sigma_t}^2) = \alpha_0 + \sum_{i=1}^{p} \beta_i ln({\sigma^2}_{t-i}) + \sum_{i=}^{p} \alpha_i \left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| + \sum_{i=1}^{p} \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \tag{9}$$

As seen from the equation, the conditional variance of a time series in the EGARCH model is a nonlinear function of its past values, the lagged values and sign of the residues. The term $\frac{\varepsilon_{t-i}}{\sigma_{t-i}}$ in the equation is standardized error terms.

The use of standardised errors instead of past values of the error terms in the EGARCH model provides information on the size and persistence of the shock. If $\sum_{i=1}^{p} \beta_i$< 1, the process is covariance stationary. Since the conditional variance is modeled linearly in the EGARCH model, there are no non-negative constraints imposed on the parameters for the GARCH model to be positive, so the model does not restrict the parameters α_i and β_i.

Let's examine the parameters on the EGARCH (1,1) model ,for the term $\frac{\varepsilon_{t-i}}{\sigma_{t-i}}$;

$$\ln(\sigma_t{}^2) = \alpha_0 + \beta_1 ln(\sigma^2{}_{t-1}) + \alpha_1 \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{10}$$

If this term is positive, the effect of shocks on conditional variance is equal to(α_1 + γ_1). If the ratio is negative, the effect of shocks on conditional variance will be (α_1-γ_1) (Enders, 2009: 156). The most important concept that distinguishes this model from other models is the parameter γ_1. The statistically significant γ_1 parameter indicates the presence of asymmetric volatility. While $\gamma_1 < 0$ indicates the presence of leverage effect, $\gamma_1 = 0$ means that the positive shock ($\varepsilon_{t-1} > 0$) and negative shock ($\varepsilon_{t-1} < 0$) have the same effect on volatility.

The main feature of this model is that a dummy variable expressing the threshold is added to the conditional variance model based on a threshold for the residual of the mean model. The conditional variance equation of the TARCH model is as follows:

$$\sigma_t{}^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma^2{}_{t-i} + \sum_{j=1}^{q} (\alpha_i \varepsilon^2{}_{t-j} + \gamma_j D_{j,t-j} \varepsilon^2{}_{t-j}) \tag{11}$$

$$\text{TARCH}(1,1)\text{: } \sigma_t{}^2 = \alpha_0 + \beta_1 \sigma^2{}_{t-1} + \alpha_1 \varepsilon^2{}_{t-1} + \gamma_1 D_{t-1} \varepsilon^2{}_{t-1} \tag{12}$$

In here, $\varepsilon^2{}_{t-1}$ is the residues obtained from the average equation.

$$D_{t-1} = \begin{cases} 1 & \varepsilon_{t-1} < 0 \\ 0 & \varepsilon_{t-1} \geq 0 \end{cases}$$

Also, the statistically significant γ_1 parameter indicates the presence of asymmetric effect.

4. BAYESIAN ARCH and GARCH VOLATILITY MODELS

Classical ARCH and GARCH models have been mentioned in the previous section. These are models that can be treated using a Bayesian approach. The Bayesian approach takes into account prior information about the process and takes a more disciplined approach to uncertainty. It provides an advantage by utilising prior knowledge of parameters and better performance in model comparison. The approach involves prior knowledge and posterior distribution. The posterior probabilities are calculated using the first and likelihood probabilities. This process includes the steps of the Bayesian approach. The process is followed in the Bayesian forecasting of ARCH-GARCH models. Therefore, in this section, we will discuss Bayesian ARCH and GARCH models, and the ARCH model will be mentioned primarily within the Bayesian approach, as it forms the basis of the GARCH model.

4.1. Bayesian ARCH Models

Bayesian extraction of ARCH models was first developed by Geweke (1986a, 1986b) (Geweke, 1988: 73). In Bayesian prediction, it is easier to achieve constraints such as stationary assumption in classical ARCH models. It's well known that in the Bayesian method, inference is made via the similarity function. The likelihood function is a concept that plays a role in the calculation of the prior distribution in the Bayesian approach and is used in the calculation of the probability depending on the parameters. It is also critical for algorithm performance (Thornton 2007: 598). Markov Chain Monte Carlo (MCMC) provides flexibility in finding the last probability distribution of both model parameters and functions, since the similarity function is not linear with respect to the parameters, optimising of the similarity function is difficult. MCMC is a class of simulations in Bayesian statistics and is a powerful tool for calculating integrals in high-dimensional problems for which no analytical solution exists (Chernozhukov and Hong, 2003: 294).

Accordingly, let's determine the similarity function for the ARCH model in the Bayesian Approach.

Let's assume that the mean model is the simple AR (1) model in the form of $Y_t = \theta Y_{t-1} + \varepsilon_t$. In this case, it is expressed as ARCH (p): $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots\ldots\ldots + \alpha_p \varepsilon_{t-p}^2$. Here, conditional similarity function of Y_t with the variation $\varepsilon_t \sim N(0, 1)$,it is expressed as $l(\theta; y, \Im_0) = \prod_{t=1}^{T} f(y_t \mid \Im_{t-1}, \theta)$ [12]. $\theta' = (\alpha_0, \alpha_1, \ldots\ldots \alpha_p)$, $y = (y_1, y_2, \ldots\ldots y_T)$, $\Im_0 = (y_0, y_{-1}, \ldots\ldots y_{1-p})$ are the vector of the initial conditions. $f(y_t \mid \Im_{t-1}, \theta)$ is the conditional probability density function. Each conditional probability density function has a normal distribution with an average of zero and variance of σ_t^2. Similarity function when initial observations are taken as initial conditions;

$$\ell(\theta;y,\Im_0)\propto \prod_{t=1}^{T}(\sigma_t{}^2)^{-1/2}\exp\left(-\frac{1}{2}\sum_{t=1}^{T}\frac{y_t{}^2}{\sigma^2}\right) \quad (13)$$

Engle (1982) proposed a linear decreasing structure for the α_i coefficients of the ARCH (p) model (from (p+1) to 2) in order to reduce the number of parameters. In this case,

ARCH(p) : $\sigma_t^2 = \alpha_0 + 2\alpha \sum_{j=1}^{p}\frac{p+1-j}{p(p+1)}y^2{}_{t-j}$ equation replace ARCH(p): $\sigma_t^2 = \alpha_0 + \alpha_1\, \varepsilon_{t-1}^2 + \alpha_2\, \varepsilon_{t-2}^2 + \ldots\ldots\ldots + \alpha_p\, \varepsilon_{t-p}^2$ equation.

Since it is $\sum_{j=1}^{p}\frac{p+1-j}{p(p+1)}y^2{}_{t-j}$ =0.5, $\frac{p+1-j}{p(p+1)} > 0$, $\forall j \le p$, the stationary condition is similarly provided with $\alpha < 1$.

The inequality constraints required to provide the positivity constraint and the stationary condition, expressed as $0 \le \alpha < 1$ when estimating the model with the maximum likelihood method, make the solution very difficult. In the Bayesian approach, values that don't confirm the inequalities are rejected by simulating the final probability distribution. In this way, the condition of inequality is satisfied. However, if the rejected values are too many, problems will arise because the process will not be stationary (Bauwens and Lubrano, 1999:208).

In the general ARCH (p) model, if p is greater than 1, the Monte Carlo method can be used. Since numerical integration is used, any initial probability density function can be chosen.

4.2. Bayesian GARCH Models

Bayesian GARCH models were developed by Kleibergen and Van Dijk (1993) and Bauwens and Lubrano (1998) (Bauwens and Lubrano, 1988: 23). For the Bayesian estimation of the GARCH model, the likelihood function, the pre-distributions of the parameters and the final distribution calculated within the framework of the Bayes rule should be specified. At this point, it is necessary to make a distribution assumption for the return errors, and the expression "mixture of normal distributions" is generally used for the GARCH model.

The mixture of normal distributions is a concept proposed for thick tails and asymmetry. The mixture of normals is determined by changing either the variance or the mean of a normally distributed random variable. If only the normal variance is changed, the distribution obtained refers to the scale mixture of normals, and if both the mean and variance are changed, it is expressed as the position scale mixture of normals. In this respect, the Student t distribution better captures the nature of the distribution of the data when it comes to returns. So let's first look at the similarity function for the GARCH model. The similarity function of the ARCH model was expressed as;

$$\text{ɥ}(\theta;y,\Im_0)\propto \prod_{t=1}^{T}({\sigma_t}^2)^{-1/2}\exp\left(-\frac{1}{2}\sum_{t=1}^{T}\frac{{y_t}^2}{{\sigma_t}^2}\right) \tag{14}$$

In this notation, under the conditional normality hypothesis, the similarity function can be written for GARCH (1,1) process by adding changes in ${\sigma_t}^2$.

If $\varepsilon_t \sim t\,(0,\ 1,1/(v\text{-}2),v)$ is similarity function;

$$\text{ɥ}(\theta;y,\Im_0)\propto \prod_{t=1}^{T}[\,\Gamma\,(\frac{(v+1)}{2}/\ \Gamma(\frac{v}{2})]\left((v-2){\sigma_t}^2\right)^2[1+\frac{{y_t}^2}{(v-2){\sigma_t}^2}]^{-(v+1)/2} \tag{15}$$

If $\varepsilon_t \sim t\,(0,\ 1,1/v,v)$ is similarity function;

$$\text{ɥ}(\theta;y,\Im_0)\propto \prod_{t=1}^{T}[\,\Gamma\,(\frac{(v+1)}{2}/\ \Gamma(\frac{v}{2})]\left((v){\sigma_t}^2\right)^2[1+\frac{{y_t}^2}{v{\sigma_t}^2}]^{-(v+1)/2} \tag{16}$$

In many applications of Bayesian GARCH models, the numerical computational methods of the models are more important than the determination of the first distribution for the parameters, because there are limitations in the choice of the first probability distributions and the computations are done numerically (Rachev et.al., 2008: 203).

Bollerslev et al. (1992) model GARCH (1,1) with Student-t residues (Student-GARCH (1,1)),

$y_t = \varepsilon_t.\sqrt{{\sigma_t}^2}$ $\quad\varepsilon_t\ /\ \Im_{t-1} \sim Student\,(0,1,v)$, Variance can be stated as (Bollerslev et.al., 1992: 5);

$${\sigma_t}^2=\alpha_0+\alpha_1{\varepsilon^2}_{t-1}+\beta_1{\sigma^2}_{t-1} \tag{17}$$

Here t = 1, 2, ...is T. As mentioned earlier, the Student t distribution reflects the leptrophicity in the financial series better than the normal distribution due to its thick tails.

Depending on the past information ($\Im_{t-1}$), Student t is distributed with y_t , 0 average $\sigma_t{}^2 v/(v-2)$ variance (v>2).

The parameters of the variance equation are constrained to $\alpha_0 \geq 0$, $\alpha_1 > 0$ and $\beta_1 \geq 0$ to ensure that $\sigma_t{}^2$ is positive. Last probability function with T observation number,

Shown as $\varphi(\theta/y) \propto \varphi(\theta)\mathrm{I}$ (θ/y) (Bauwens and Lubrano,1988: 25).

Here $\varphi(\theta)$ denotes the marginal probability density, and I (θ/y)denotes the similarity function. θ (α_0, α_1, β_1, v) shows the parameter vector. $\varphi(\theta)$, the first probability density function must satisfy the positivity constraint on parameters and the condition $\beta_1 < 1$. Besides, $\beta_1 < 1$ condition is necessary for weak and strong stasis.

An important feature of Bayesian inference is that the final probability density function can be integrated. Here we encounter the integral because of the concept of the probability density function, because here are the values corresponding to each point. This leads us to the concept of area and thus to the expression integral. If you use an integrable first probability density function, or a particular first probability density function, and if the similarity function also makes sense for some parameter values, then the last probability function can also be integrated. Failure to integrate the last probability function may be due to the inability to integrate the first probability function. Therefore, sufficient initial information is required for the degrees of freedom of v to ensure that the final probability function converges sufficiently to zero in the tail region and that the integral can be obtained.

In the Bayesian ARCH - GARCH model estimation, the Metropolis - Hastings algorithm, Importance Sampling and Griddy - Gibbs Sampler are used. The M-H algorithm is one of the MCMC algorithms. Due to the repetitive structure of the variance equation in models, the conjugate process between the likelihood function and the predistribution cannot be performed. Sampling is done from the parameter vector, which contains all parameters together (Rachev et.al., 2008: 208). Simulation is then performed by adding additional input parameters such as the v (degrees of freedom) parameter.

Griddy Gibbs sampling is a special form of Gibbs sampling and developed by Geman and Geman (1984) (Geman and Geman, 1984: 721). The basic idea in this sampling is that if it is possible to express conditionally every coefficient to the others, it is possible to achieve the desired accurate common distribution by looping these conditional expressions. Griddy Gibbs sampling is carried out by evaluating the univariate conditional final density in the system to which the parameter value belongs. Therefore, in order to apply the Gibbs sampler, all of the last probability density functions must be analytically known. This sampling is a combination of the known Gibbs sampling and a standard numerical computation and was first used by Bauwens and Lubrano to estimate the parameters of the GARCH models. The Gibbs sampler can be applied by applying the dimensionless deterministic integral rule to each coordinate of the last probability density function. The approach should be followed up by creating all the conditional densities in order to obtain random samples from the common final probability distribution. Significance sampling is a version of rejection sampling with greater emphasis on "major regions". In this case, points that do not reflect the target distribution are not discarded, instead they are given less weight. With the significance sampling, it is aimed to increase the accuracy of the estimator by reducing the variance and by giving more weight to the simulations that are important. In the significance sampling, the last probability density function is brought closer to the importance function to obtain random shots. Bayesian calculation requires the obtaining of $E[g(\theta)] = \frac{\int g(\theta)\varphi(\theta)d\theta}{\int \varphi(\theta|y)d\theta}$.

Here φ (θ / y) is the kernel of the last probability density function and g (.) Is an integrable function. The integrals in the statement above are based on the expected values according to the significance function (I (θ)) can be stated as;

$\int \varphi(\theta|y)d\theta = \int \frac{\varphi(\theta|y)}{I(\theta)} I(\theta)d(\theta) = E_I \{\frac{\varphi(\theta|y)}{I(\theta)}\}$. Sample mean can be stated as;

$$E_I\left\{\frac{\varphi(\theta|y)}{I(\theta)}\right\} \approx \frac{1}{N}\sum_{i=1}^{N}\frac{\varphi(\theta_i|y)}{I(\theta_i)} \tag{18}$$

The convergence of the estimate is obtained by limiting the weighting function φ (θ/y)/I(θ). The small coefficient of variation of this ratio increases the sensitivity. It is very difficult to find a good importance function for this method. The choice is often based on the Student t-density, which is easy to simulate. Student's density can determine the tail thickness of the degrees of freedom parameter, so it is more flexible. If there is no specific information about the tails of the final probability distribution function, it is determined by trying the v selection. Since GARCH models need to be truncated to ensure the positivity constraint of the student t function, significance sampling is not suitable for these models (Saçıldı Saçaklı, 2011: 96).

4. CONCLUSION

Methods that have been used for a long time on issues such as financial assets, risk management, exchange rates, where parameters are assumed to be fixed and inferences are made based on data, belong to the classical approach. On the other hand, the Bayesian approach is the approach in which the parameters are considered random, uses prior knowledge and makes probabilistic inferences about the parameters. These approaches provide useful tools for different needs in the financial field and take volatility into account when determining the risk of financial assets. Therefore, determining the difference between the two approaches plays an important role in determining what purpose the Bayesian approach will serve, such as whether it works well in small samples. Therefore, identifying the similarities and differences between both approaches will contribute to the literature in guiding the selection of a better model that takes into account future fluctuations with applications in financial markets. It is thought that it would be beneficial to make an assessment in terms of the differences and advantages over the classical approach to the subject, rather than the methodological differences in the Bayesian approach. The theory shows that expected utility maximization provides the basis for rational decision making, and Bayes' theorem explores ways of combining beliefs in the light of changing evidence. The aim is to create a set of rules and procedures through a disciplined approach to uncertainty. Maximization of the likelihood function used in ARCH - GARCH model estimation should be ensured by a constrained optimization technique. The model parameters must be positive to provide a positive conditional variance and the covariance must be stationary. The optimization process is subject to some inequality constraints which make the process difficult. The optimization becomes difficult to converge when the actual parameter values are not close to the bounds of the parameter space, or when the process is closer to a non-stationary state. Also, since it is difficult to obtain the optimal covariance matrix, some approaches should be taken to obtain reliable results. In classical applications of ARCH-GARCH models, the focus is not directly on the parameters of the model, but on the possible complex non-linear functions of the parameters. When the models are treated by the Bayesian approach, these difficulties disappear. First, the constraints considered appropriate for the model parameters can be incorporated into the model with appropriate predistributions. Secondly, Monte-Chain Monte-Carlo (MCMC) can be used to study the combined final distributions of the model parameters. This approach avoids the local maxima encountered in maximum likelihood estimation of the regime-modified GARCH model. Nonlinear distributions of model parameters can be easily achieved by simulating from the combined final distribution. Therefore, while constraints such as the constancy constraint are difficult assumptions to achieve according to the classical approach, this difficulty disappears in the Bayesian approach and solutions are provided for many assumptions and constraints made outside the framework of the stationary state.

Etik Beyanı: *Bu çalışmanın tüm hazırlanma süreçlerinde etik kurallara uyulduğunu yazar beyan eder. Aksi bir durumun tespiti halinde Akademik İzdüşüm Dergisinin hiçbir sorumluluğu olmayıp, tüm sorumluluk çalışmanın yazarına aittir.*

Destek ve Teşekkür: *Bu araştırmanın hazırlanmasında herhangi bir kurumdan destek alınmamıştır.*

Katkı Oranı Beyanı: *Araştırmanın tüm süreci makalenin beyan edilen tek yazarı tarafından gerçekleştirilmiştir.*

Çatışma Beyanı: *Araştırmada herhangi bir çıkar çatışma beyanı bulunmamaktadır.*

REFERENCES

Bauwens, L. L. (1999). Bayesyen inference in dynamic econometric models, New York: Oxford University Press.

Bauwens, L., Lubrano, M. (1998). Bayesian inference on GARCH models using the Gibbs Sample, *Econometrics Journal*, 1, 23-46.

Black, F. (1976). Studies of stock prices volatility changes, American Statistical Association, 177-181.

Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH modelling in finance: A review of the theory and empirical evidence, *Journal of Economerics*, 52, 5-59.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 31(3), 307-327.

Bolstad, William M. (2004). Introduction to bayesian statistics, John Wiley&Sons.

Carter, C.K., R. Kohn. (1994). On gibbs sampling for state space models, *Biometrika,* 81, 541-553.

Chernozhukov, V., Hong, H. (2003). An MCMC approach to classical estimation. *Journal of Econometrics*, 115(2), 293-346.

Cowles, M.K., B.P.Carlin. (1996). Markov chain monte carlo convergence diagnostics: A comparative review, *Journal of the American Statistical Association*, 91, 883-905.

Çil, N. (2015). Finansal Ekonometri, Der Yayınları.

Degiannakis, S., Xekalaki, E. (2004). Autoregressive conditional heteroscedasticity (ARCH) models: A review, *Quality Technology & Quantitative Management*, 1(2), 271-324.

Ekici, O. (2009). İstatistikte bayesyen ve klasik yaklaşımın kavramsal farklılıkları, *Balıkesir Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, 12(21), 89-101.

Enders, W. (2009). Applied econometrics time series (3b.), Wiley Series.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, *Econometrica*, 50(4), 987-1007.

Geman, S., Geman, D. (1984). Stochastic relaxation gibbs disributions and the bayesian restoration of images, IEEE Transactions on Pattern Analysis and Machine Intelligence, 6, 721-741,

Geyer, C.J. (1992). Practical markov chain monte carlo, *Statistical Science*, 473-483.

Geweke, J., (1986a). Exact inference in the inequality constrained normal linear regression model, *Journal of Applied Econometrics*, 1, 127-141.

Geweke, J., (1986b). Bayesian inference in econometric models using Monte Carlo integration, Manuscript (Duke Lhriversity, Durham, NC).

Geweke, John. (1988). Antithetic acceleration of Monte Carlo integration in Bayesian inference, *Journal of Econometrics*, 38(1), 73-89.

Ghosh, K.J., Delampady, M., Samanta, T. (2006). An introduction to bayesian analysis- theory and methods, Springer.

Güriş, S., Saçıldı Saçaklı, İ. (2011). İstanbul menkul kıymetler borsasında hisse senedi getiri volatilitesinin klasik ve bayesyen GARCH modelleri ile analizi, Trakya *Üniversitesi Sosyal Bilimler Dergisi*, 13(2), 153-172.

Heckman, J., Leamer, E. (2001). Markov chain monte carlo methods: computation and inference", S. Chib içinde, Handbook of Econometrics, 5, 3571-3642.

Jacquier, E., N.G. Polson, P.E.Rossi, (1994). Bayesian analysis of stochastic volatility models, *Journal of Business and Economic Statistics*, 371-389.

Jose M. Bernardo, Adrian F.M.Smith. (1994). Bayesian Theory, John Wiley&Sons.

Karolyi, A.,G. (2001). Why stock return volatility really matters, *Institutional Investor Journals Series*, 614, 1-16.

Leonard, T. (1975). A Bayesian approach to the linear model with unequal variances. *Technometrics*, 84: 95-102.

Mendelbrot, B, (1963). The variation of certain speculative prices, *The Journal of Business*, 394-419.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A New Approach, *Econometrica*, 347-370.

Rachev, Svetlozar, T., Hsu, J.S.J., Bagasheva, B.S., Fabozzi, F.J. (2008). Bayesian methods in finance, Canada: John Wiley & Sons.

Saçıldı Saçaklı, İ. (2011). Hisse senedi getiri volatilitelerinin bayesyen yaklaşım ile analizi, İstanbul: Marmara Üniversitesi.

Songül, H. (2010). Otoregresif koşullu değişen varyans modelleri: Döviz kurları üzerine uygulama. Ankara: Türkiye Cumhuriyeti Merkez Bankası Araştırma ve Para Politikası Genel Müdürlüğü.

Wakefield, J. (2013). Bayesian and frequentist regression methods, Wahington, USA: Springer.

Zellner, A. (1971). An introduction to bayesian inference in econometrics, New York, John Wiley & Sons.

Zeytinoğlu, Ç. F. (2015). Bayesyen istatistik, İstanbul: Yalın Yayıncılık.

CLASSICAL AND BAYESIAN APPROACH TO UNIVARIATE VOLATILITY MODELS

Extended Summary

Aim:

Global developments, including economic, political and social developments in recent years, directly affect exchange rates and other financial instruments in financial markets and cause significant volatility in their return changes. At this point, the concept of volatility becomes important. Volatility, which corresponds to uncertainty in financial markets, also represents the risk of the financial asset. Volatility estimates are used extensively in asset management, portfolio management and derivative product pricing and are very important. The constraints in the applications of classical ARCH - GARCH models are eliminated in the Bayesian approach, and solutions are provided to many assumptions and restrictions outside the scope of the stationarity condition. In this regard, the aim of the manuscript is to compare the Classical and Bayesian volatility models theoretically and to clarify the approach by discussing the advantages of the Bayesian approach, since the necessity of choosing the right econometric forecasting method directly affects the reliability of the results to be obtained.

Research Questions:

The research questions that this manuscript aims to answer are; - What are the basic characteristics of financial asset returns? What does the concept of volatility in financial markets mean and why is it important? What are the characteristics of univariate symmetric and asymmetric autoregressive conditional variance models in the classical approach and volatility models in the Bayesian approach? What are the disadvantages of the classical approach and the advantages of the Bayesian approach over the classical approach?

Method(s):

In this manuscript, classical and Bayesian approaches to volatility models are discussed, taking into account that the

choice of appropriate econometric method is crucial for the reliability of the results in model estimations.

Findings and Discussion:

The maximisation of the likelihood function used in ARCH - GARCH model estimation is achieved by a constrained optimisation technique. To ensure a positive conditional variance, the model parameters must be positive and the covariance must be stationary. The optimisation process is subject to some inequality constraints, which complicates the process. If the actual parameter values are not close to the boundaries of the parameter space or if the process is closer to the non-stationary state, the convergence of the optimisation becomes difficult. Moreover, since it is difficult to obtain the optimal covariance matrix, some approximations must be adopted for reliable results. In classical applications of ARCH - GARCH models, the focus is not directly on the parameters of the model, but on the possible complex nonlinear functions of the parameters. These difficulties are eliminated when the models are handled with a Bayesian approach. Firstly, constraints that are deemed appropriate for the model parameters can be included in the model with appropriate prior distributions. Furthermore, the Monte Chain Monte Carlo (MCMC) process can be used to investigate the joint final distributions of the model parameters. With this approach, local maxima encountered in the Maximum Likelihood estimation of the regime-switched GARCH model can be avoided. Nonlinear distributions of the model parameters can be easily obtained by simulation from the final distribution. Therefore, while constraints such as the stationarity constraint are difficult assumptions to obtain according to the classical approach, this difficulty is eliminated in the Bayesian approach, and many assumptions and constraints made outside the scope of the stationarity condition are solved.