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On The Non-Newtonian Padovan and Non-Newtonian Perrin Numbers

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Research Article

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Abstract
In this work, we introduce a novel version of Padovan and Perrin numbers which we refer to as non-Newtonian Padovan and non-Newtonian Perrin numbers. Furthermore, we examine about a number of their properties. Additionally, we provide a variety of identities and formulas involving these new kinds, including the Binet-like formulas, the generating functions, the partial sum formulas, and the binomial sum formulas.

Keywords: Non-Newtonian calculus, Padovan numbers, Binet formula, generating functions, binomial sum

Newtonian Olmayan Padovan ve Newtonian Olmayan Perrin Sayıları Hakkında

Mersin University, Faculty of Science, Department of Mathematics, Mersin, Türkiye

Öz
Bu çalışmada Newtonian olmayan Padovan ve Newtonian olmayan Perrin sayıları olarak adlandırdığımız Padovan ve Perrin sayılarının yeni bir versiyonunu tanıtıyoruz. Ayrıca bunların bazı özelliklerini de inceliyoruz. Ek olarak, Binet benzeri formüller, üreteç fonksiyonları, kısmi toplam formülleri ve binom toplam formülleri de dahil olmak üzere bu yeni türleri içeren çeşitli özdeşlikler ve formüller sağlıyoruz.

Anahtar Kelimeler: Newtonian olmayan analiz, Padovan sayıları, Binet formülü, üreteç fonksiyonları, binom toplamı

Introduction

Numerous calculi with various features have been developed since Newton and Leibnitz established modern calculus. As an alternative to Newton and Leibniz's standard calculus, Grossman and Katz created a new family of calculi known as non-Newtonian calculus, and, defined modern forms of integrals and derivatives that converted addition and subtraction operations into multiplication and division operations [1]. In classical calculus, every property finds an analogue in non-Newtonian calculus. Non-Newtonian calculus offers a distinct approach to problems traditionally tackled by calculus. For certain scenarios, such as those involving wage rates (in dollars, euros, etc.), proponents suggest employing bigeometric calculus, a type of non-Newtonian calculus, instead of the conventional Newtonian approach [1–3].

Duyar and Sağır aimed to generalize the traditional Lebesgue measure on real numbers to the context of non-Newtonian real numbers. To achieve this, they introduced the concept of Lebesgue measure for both open and closed sets in the non-Newtonian framework and explored its fundamental properties [4].
Erdoğan and Duyar introduced non-Newtonian improper integrals and investigated their convergence conditions. Additionally, key theorems such as the second mean value theorem and the intermediate value theorem were proved within the non-Newtonian framework to provide convergence tests [5].
Değirmen and Duyar introduced one of these gains by introducing non-Newtonian Fibonacci and non-Newtonian Lucas numbers. Additionally, they provide some formulas and identities such as the Binet formula, d'Ocagne identity, Cassini identity, and Gelin-Cesaro identity, and find the functions that generate these numbers [6].
Yağmur introduced a new type of Pell and Pell-Lucas numbers in terms of non-Newtonian calculus, and studied some significant identities and formulas for classical Pell and Pell-Lucas numbers [7].
In this research endeavor, we commence by providing essential insights into non-Newtonian calculus. Subsequently, we present the definition and certain properties of Padovan numbers, establishing their connection with non-Newtonian calculus. We then delve into an examination of various identities associated with these numbers. An injective function is called a generator, and its codomain is a subset of $\mathbb{R}$, and whose domain is $\mathbb{R}$. There is precisely one arithmetic produced by each generator, and there is exactly one generator that produces each arithmetic. We select the exponential function from $\mathbb{R}$ to the set $\mathbb{R}^+$ as a generator, in other words,

$$\alpha : \mathbb{R} \to \mathbb{R}^+, \quad \lambda \rightarrowtail \alpha(\lambda) = e^\lambda = \mu,$$
$$\alpha^{-1} : \mathbb{R}^+ \to \mathbb{R}, \quad \lambda \rightarrowtail \alpha^{-1}(\lambda) = \ln \mu = \lambda.$$

I is referred to as the unit function, whose inverse is oneself, if $I(\lambda) = \lambda$ for all $\lambda \in \mathbb{R}$. The definition of the non-Newtonian real number set $\mathbb{R}(\mathcal{N})$ is as follows:

$$\mathbb{R}(\mathcal{N}) := \{\alpha(\lambda) : \lambda \in \mathbb{R}\}.$$

It is important to recognize that every concept in classical arithmetic has an inherent equivalent in $\alpha-$arithmetic. As an example, the $\alpha-$integers shown to be as follows:

$$..., \alpha(-3), \alpha(-2), \alpha(-1), \alpha(0), \alpha(1), \alpha(2), \alpha(3), \ldots.$$

For every integer m, we set $\dot{m} = \alpha(m)$. Naturally, if $\dot{m}$ is an $\alpha-$positive integer, then

$$\dot{m} = \underbrace{\dot{1}\dot{+}\ldots\dot{+}\dot{1}}_{m \quad \text{terms}}.$$

For a generator α with range $A = \mathbb{R}(\mathcal{N})$, which is a subset of the real numbers, $\alpha-$arithmetic denotes arithmetic operations defined on the domain A as follows: For any generator α and $\lambda, \mu \in \mathbb{R}(\mathcal{N})$,

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$$\begin{aligned}
\alpha-addition &\rightarrow \lambda\dot{+}\mu=\alpha\{\alpha^{-1}(\lambda)+\alpha^{-1}(\mu)\},\\
\alpha-subtraction &\rightarrow \lambda\dot{-}\mu=\alpha\{\alpha^{-1}(\lambda)-\alpha^{-1}(\mu)\},\\
\alpha-multiplication &\rightarrow \lambda\dot{\times}\mu=\alpha\{\alpha^{-1}(\lambda)\times\alpha^{-1}(\mu)\},\\
\alpha-division &\rightarrow \lambda\dot{/}\mu=\alpha\{\alpha^{-1}(\lambda)\div\alpha^{-1}(\mu)\}=\frac{\lambda}{\mu}\mathcal{N},\\
\alpha-order &\rightarrow \lambda\dot{<}\mu=\alpha\{\alpha^{-1}(\lambda)<\alpha^{-1}(\mu)\}.
\end{aligned}$$

Specifically, selecting $\alpha-$generator I, which is the identity function $\alpha(t)=t$ for all $t\in\mathbb{R}\Rightarrow\alpha^{-1}(t)=t$, makes it evident that $\alpha-$arithmetic is indeed classical arithmetic. Assuming that the $\alpha-$generator is *exp* and that $\alpha(k)=e^k$ for $k\in\mathbb{R}$, $\alpha^{-1}(k)=\ln k$, $\alpha-$arithmetic transforms into geometric arithmetic as follows:

$$\begin{aligned}
Geometric\quad addition \rightarrow\ & \lambda\dot{+}\mu=\alpha\{\alpha^{-1}(\lambda)+\alpha^{-1}(\mu)\}=e^{\{\ln\lambda+\ln\mu\}}=\lambda\cdot\mu,\\
Geometric\quad subtraction \rightarrow\ & \lambda\dot{-}\mu=\alpha\{\alpha^{-1}(\lambda)-\alpha^{-1}(\mu)\}=e^{\{\ln\lambda-\ln\mu\}}=\frac{\lambda}{\mu}\quad(\mu\neq 0),\\
Geometric\quad multiplication \rightarrow\ & \lambda\dot{\times}\mu=\alpha\{\alpha^{-1}(\lambda)\times\alpha^{-1}(\mu)\}=e^{\{\ln\lambda\times\ln\mu\}}=\lambda^{\ln\mu}=\mu^{\ln\lambda},\\
Geometric\quad division \rightarrow\ & \lambda\dot{/}\mu=\alpha\{\alpha^{-1}(\lambda)\div\alpha^{-1}(\mu)\}=e^{\{\ln\lambda\div\ln\mu\}}=\lambda^{\{1\div\ln\mu\}}\quad(\mu\neq 1).
\end{aligned}$$

These operations define α-arithmetic, where α is typically the exponential function in this context.
Now, we create the non-Newtonian real field $\mathbb{R}(\mathcal{N})$ and provide some associated characteristics.
Definition of the dual operations addition and multiplication for the set of non-Newtonian real numbers $\mathbb{R}(\mathcal{N})$ as follows, respectively:

$$\begin{aligned}
\dot{+}:&\mathbb{R}(\mathcal{N})\times\mathbb{R}(\mathcal{N})\rightarrow\mathbb{R}(\mathcal{N})\\
&(\lambda,\mu)\longmapsto\lambda\dot{+}\mu=\alpha\{\alpha^{-1}(\lambda)+\alpha^{-1}(\mu)\},\\
\dot{\times}:&\mathbb{R}(\mathcal{N})\times\mathbb{R}(\mathcal{N})\rightarrow\mathbb{R}(\mathcal{N})\\
&(\lambda,\mu)\longmapsto\lambda\dot{\times}\mu=\alpha\{\alpha^{-1}(\lambda)\times\alpha^{-1}(\mu)\}.
\end{aligned}$$

It is demonstrable that $\left(\mathbb{R}(\mathcal{N}),\dot{+},\dot{\times}\right)$ is an complete field by normal checking, so we omit out the specifics. Let $A\subset\mathbb{R}(\mathcal{N})$ be a set of integers in the extended real numbers [3].
The $v-$th $\alpha-$power of $\lambda\in\mathbb{R}(\mathcal{N})$ for a given integer v is represented by

$$\lambda^{v_{\mathcal{N}}}=\underbrace{\lambda\dot{\times}\lambda\dot{\times}\ldots\dot{\times}\lambda}_{v\quad\text{terms}}.$$

The α-factorial of a given positive integer v is represented by

$$\dot{v}!_{\mathcal{N}}=\dot{v}\dot{\times}(\dot{v}\dot{-}\dot{1})\dot{\times}(\dot{v}\dot{-}\dot{2})\dot{\times}\ldots\dot{\times}\dot{2}\dot{\times}\dot{1}.$$

The α-binomial of a given positive integer u and v is represented by

$$\binom{\dot{u}}{\dot{v}}_{\mathcal{N}} = \frac{\dot{u}!_{\mathcal{N}}}{\dot{v}!_{\mathcal{N}} \dot{\times} (\dot{u} \dot{-} \dot{v})!_{\mathcal{N}}} \mathcal{N}.$$

The α-binomial sum of a given integer λ, μ and u is represented by

$$(\dot{\lambda} \dot{+} \dot{\mu})^{u_{\mathcal{N}}} =_{\mathcal{N}} \sum_{v=0}^{u} \binom{\dot{u}}{\dot{v}}_{\mathcal{N}} \dot{\times} \dot{\lambda}^{v_{\mathcal{N}}} \dot{\times} \dot{\mu}^{(u-v)_{\mathcal{N}}}. \tag{1}$$

The notation $\sqrt{\lambda}^{\mathcal{N}}$ will be utilized for $s = \alpha\{\sqrt{\alpha^{-1}(\lambda)}\}$ where every s is $\alpha-$nonnegative integer. This indicates that $s^{2_{\mathcal{N}}} = \lambda$ is the only $\alpha-$nonnegative integer whose $\alpha-$square equals λ. Additionally, $\alpha(-\lambda) = \alpha\{-\alpha^{-1}(\dot{\lambda})\} = \dot{-}\dot{\lambda}$ all $\lambda \in \mathbb{R}$.

The Padovan and Perrin sequences are defined as follows. The Padovan sequence $\{P_v\}_{v\geq 0}$ and Perrin sequence $\{R_v\}_{v\geq 0}$ are defined by the third order recurrences, respectively,

$$P_{v+3} = P_{v+1} + P_v, \tag{2}$$

$$R_{v+3} = R_{v+1} + R_v, \tag{3}$$

with the initial conditions $P_0 = P_1 = P_2 = 1$ and $R_0 = 3$, $R_1 = 0$, $R_2 = 2$, respectively [8, 9]. The Padovan and Perrin sequences appear as sequences A000931 and A001608 on the On-Line Encyclopedia of Integer Sequences (OEIS), respectively [10]. The first few values of these sequences are $1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37$ and $3, 0, 2, 3, 2, 5, 5, 7, 10, 12, 17, 22$, respectively. The unique real root of the characteristic equation of these sequences is known as the plastic number:

$$t^3 - t - 1 = 0$$

with a value of

$$\sqrt[3]{\frac{1}{2} + \frac{1}{6}\sqrt{\frac{23}{3}}} + \sqrt[3]{\frac{1}{2} - \frac{1}{6}\sqrt{\frac{23}{3}}} \approx 1.324718.$$

By designating its roots as δ, η, and γ, the subsequent equalities can be obtained

$$\delta + \eta + \gamma = 0,$$

$$\delta\eta + \delta\gamma + \eta\gamma = -1,$$

$$\delta\eta\gamma = 1.$$

In addition, the Binet-like formulas for the Padovan and Perrin sequences are as follows:

$$P_v = a\delta^v + b\eta^v + c\gamma^v, \tag{4}$$

$$R_v = \delta^v + \eta^v + \gamma^v, \tag{5}$$

respectively, where

$$a = \frac{(\eta-1)(\gamma-1)}{(\delta-\eta)(\delta-\gamma)}, \quad b = \frac{(\delta-1)(\gamma-1)}{(\eta-\delta)(\eta-\gamma)}, \quad c = \frac{(\delta-1)(\eta-1)}{(\gamma-\delta)(\gamma-\eta)}.$$

It is commonly known that the following identities are recognized based on [11, 12]:

$$P_{-v-7} = P_v^2 - P_{v+1}P_{v-1}, \tag{6}$$

$$P_v = P_{v-1} + P_{v-5}, \tag{7}$$

$$R_v = P_{v-10} + P_{v+1}, \tag{8}$$

$$P_{v-5}P_{-v-5} + P_{v-3}P_{-v-4} + P_{v-4}P_{-v-3} = 1, \tag{9}$$

$$P_v = P_{u-1}P_{v-u-4} + P_{u+1}P_{v-u-3} + P_uP_{v-u-2}, \tag{10}$$

$$\sum_{v=1}^{u} P_v = P_{u+5} - 3, \tag{11}$$

$$\sum_{v=1}^{u} R_v = R_{u+5} - 5, \tag{12}$$

$$\sum_{v=0}^{\infty} P_v x^v = \frac{1+x}{1-x^2-x^3}, \tag{13}$$

$$\sum_{v=0}^{\infty} R_v x^v = \frac{3-x^2}{1-x^2-x^3}, \tag{14}$$

$$\sum_{v=0}^{\infty} \frac{P_v}{v!} x^v = ae^{\delta x} + be^{\mu x} + ce^{\gamma x}, \tag{15}$$

$$\sum_{v=0}^{\infty} \frac{R_v}{v!} x^v = e^{\delta x} + e^{\mu x} + e^{\gamma x}. \tag{16}$$

In recent years, numerous studies on Padovan numbers have been published. For further information, please refer to the works cited in [13–19].

Some Properties of the Non-Newtonian Padovan and Non-Newtonian Perrin Numbers

This section provides a novel interpretation of the definitions of a non-Newtonian Padovan number and a non-Newtonian Perrin number from a different perspective. Along with dealing with the non-Newtonian versions of various formulas and identities, we also demonstrate their links by comparing them in an analogy with several well-known identities and formulas for their classical equivalents.

Definition 1. The non-Newtonian Padovan and non-Newtonian Perrin numbers are defined by

$$\mathcal{N}P_v = \dot{P}_v = \alpha(P_v) \quad \text{and} \quad \mathcal{N}R_v = \dot{R}_v = \alpha(R_v),$$

respectively, where the $v-$th Padovan and Perrin numbers are P_v and R_v, respectively. The $\mathcal{N}P$ and $\mathcal{N}R$ represent the set of the non-Newtonian Padovan and non-Newtonian Perrin numbers, respectively. That is,

$$\mathcal{N}P = \{\mathcal{N}P_v : v \in \mathbb{N}\} = \{\dot{1}, \dot{1}, \dot{1}, \dot{2}, \dot{2}, \dot{3}, \dot{4}, \dot{5}, \dot{7}, \dot{9}, \dot{12}, \dot{16}, ..., \dot{P}_v, ...\}$$

and

$$\mathcal{N}R = \{\mathcal{N}R_v : n \in \mathbb{N}\} = \{\dot{3}, \dot{0}, \dot{2}, \dot{3}, \dot{2}, \dot{5}, \dot{5}, \dot{7}, \dot{10}, \dot{12}, \dot{17}, \dot{22}, ..., \dot{R}_v, ...\}$$

Regarding classical arithmetic, we obtain Padovan and Perrin numbers if we utilize the generator I specified by $\alpha(x) = x$ for any $x \in \mathbb{R}$.
Additionally, by selecting the generator *exp*, which is defined as $\alpha(x) = e^x$ for any $x \in \mathbb{R}$, we may derive the following Padovan and Perrin numbers in terms of geometric arithmetic:

$$\begin{aligned}\mathcal{N}GP &= \{\alpha(P_v) : v \in \mathbb{N}\} \\ &= \{e^{P_v} : v \in \mathbb{N}\} \\ &= \{e^1, e^1, e^1, e^2, e^2, e^3, e^4, e^5, e^7, e^9, e^{12}, e^{16}, ..., e^{P_v}, ...\}\end{aligned}$$

and

$$\begin{aligned}\mathcal{N}GR &= \{\alpha(R_v) : v \in \mathbb{N}\} \\ &= \{e^{R_v} : v \in \mathbb{N}\} \\ &= \{e^3, e^0, e^2, e^3, e^2, e^5, e^5, e^7, e^{10}, e^{12}, e^{17}, e^{22}, ..., e^{R_v}, ...\}.\end{aligned}$$

Now, we emphasize on a few relations concerning non-Newtonian Padovan and non-Newtonian Perrin numbers, as well as their respective relationships.

Theorem 1. For $v, u \geq 0$, the following identities valid:

i) $\mathcal{N}P_v \dot{+} \mathcal{N}P_{v+1} = \mathcal{N}P_{v+3}$,

ii) $\mathcal{N}P_{v-1}\dot{+}\mathcal{N}P_{v-5}=\mathcal{N}P_v$,

iii) $\mathcal{N}R_v\dot{+}\mathcal{N}R_{v+1}=\mathcal{N}R_{v+3}$,

iv) $\mathcal{N}P_{v-10}\dot{+}\mathcal{N}P_{v+1}=\mathcal{N}R_v$,

v) $\mathcal{N}P_v^2\dot{-}\mathcal{N}P_{v+1}\dot{\times}\mathcal{N}P_{v+1}=\mathcal{N}P_{-v-7}$,

vi) $\mathcal{N}P_{v-5}\dot{\times}\mathcal{N}P_{-v-5}\dot{+}\mathcal{N}P_{v-3}\dot{\times}\mathcal{N}P_{-v-4}\dot{+}\mathcal{N}P_{v-4}\dot{\times}\mathcal{N}P_{-v-3}=1$,

vii) $\mathcal{N}P_{v-1}\dot{\times}\mathcal{N}P_{u-v}\dot{+}\mathcal{N}P_{v+1}\dot{\times}\mathcal{N}P_{u-v+1}\dot{+}\mathcal{N}P_v\dot{\times}\mathcal{N}P_{u-v+2}=\mathcal{N}P_u$.

Proof. The proofs of i, ii, iii, iv, v and vi are obvious based on the addition and subtraction properties of non-Newtonian real numbers by using expressions (2), (7), (3), (8), (6) and (9), respectively. The proof of equality vii is shown by using expression (10) as follows:

$$\begin{aligned}
&\mathcal{N}P_{v-1}\dot{\times}\mathcal{N}P_{u-v}\dot{+}\mathcal{N}P_{v+1}\dot{\times}\mathcal{N}P_{u-v+1}\dot{+}\mathcal{N}P_v\dot{\times}\mathcal{N}P_{u-v+2}\\
&=\alpha(P_{v-1})\dot{\times}\alpha(P_{u-v})\dot{+}\alpha(P_{v+1})\dot{\times}\alpha(P_{u-v+1})\dot{+}\alpha(P_v)\dot{\times}\alpha(P_{u-v+2})\\
&=\alpha\{\alpha^{-1}\alpha(P_{v-1})\times\alpha^{-1}\alpha(P_{u-v})\}\dot{+}\alpha\{\alpha^{-1}\alpha(P_{v+1})\times\alpha^{-1}\alpha(P_{u-v+1})\}\\
&\dot{+}\alpha\{\alpha^{-1}\alpha(P_v)\times\alpha^{-1}\alpha(P_{u-v+2})\}\\
&=\alpha\{\alpha^{-1}\alpha\{\alpha^{-1}\alpha(P_{v-1})\times\alpha^{-1}\alpha(P_{u-v})\}+\alpha^{-1}\alpha\{\alpha^{-1}\alpha(P_{v+1})\times\alpha^{-1}\alpha(P_{u-v+1})\}\}\\
&\dot{+}\alpha\{\alpha^{-1}\alpha(P_v)\times\alpha^{-1}\alpha(P_{u-v+2})\}\\
&=\alpha\{\alpha^{-1}\alpha\{\alpha^{-1}\alpha\{\alpha^{-1}\alpha(P_{v-1})\times\alpha^{-1}\alpha(P_{u-v})\}+\alpha^{-1}\alpha\{\alpha^{-1}\alpha(P_{v+1})\times\alpha^{-1}\alpha(P_{u-v+1})\}\}\\
&+\alpha^{-1}\alpha\{\alpha^{-1}\alpha(P_v)\times\alpha^{-1}\alpha(P_{u-v+2})\}\}\\
&=\alpha(P_{v-1}P_{u-v}+P_{v+1}P_{u-v+1}+P_vP_{u-v+2})\\
&=\alpha(P_u)\\
&=\mathcal{N}P_u.
\end{aligned}$$

Remark 1. The non-Newtonian characteristic equation of the recurrence relation in the i-th item of the above theorem is as follows:

$$\dot{t}^{3_{\mathcal{N}}}=\dot{t}\dot{+}\dot{1}. \tag{17}$$

If $\mathcal{N}P_v=\dot{t}^{v_{\mathcal{N}}}$ is taken in the recurrence relation in the i-th

$$\dot{t}^{v+3_{\mathcal{N}}}=\dot{t}^{v+1_{\mathcal{N}}}\dot{+}\dot{t}^{v_{\mathcal{N}}}$$

is obtained.
Thus, we get the result by simplifying the $\dot{t}^{v_{\mathcal{N}}}$'s.

Theorem 2. For all $v\geq 0$, the Binet-like formulas for $\mathcal{N}P_v$ and $\mathcal{N}R_v$ are given by

$$\mathcal{N}P_v=\dot{a}\dot{\times}\dot{\delta}^{v_N}\dot{+}\dot{b}\dot{\times}\dot{\eta}^{v_N}\dot{+}\dot{c}\dot{\times}\dot{\gamma}^{v_N} \tag{18}$$

and

$$\mathcal{N}R_v = \dot{\delta}^{v_N} \dot{+} \dot{\eta}^{v_N} \dot{+} \dot{\gamma}^{v_N}, \tag{19}$$

respectively, where

$$\dot{a} = \frac{(\dot{\eta} \dot{-} \dot{1}) \dot{\times} (\dot{\gamma} \dot{-} \dot{1})}{(\dot{\delta} \dot{-} \dot{\eta}) \dot{\times} (\dot{\delta} \dot{-} \dot{\gamma})}\mathcal{N}, \quad \dot{b} = \frac{(\dot{\delta} \dot{-} \dot{1}) \dot{\times} (\dot{\gamma} \dot{-} \dot{1})}{(\dot{\eta} \dot{-} \dot{\delta}) \dot{\times} (\dot{\eta} \dot{-} \dot{\gamma})}\mathcal{N} \quad \text{and} \quad \dot{c} = \frac{(\dot{\delta} \dot{-} \dot{1}) \dot{\times} (\dot{\eta} \dot{-} \dot{1})}{(\dot{\gamma} \dot{-} \dot{\delta}) \dot{\times} (\dot{\gamma} \dot{-} \dot{\eta})}\mathcal{N}.$$

Also, $\dot{\delta}$, $\dot{\eta}$ and $\dot{\gamma}$ are roots of the characteristic equation $\dot{t}^{3_N} \dot{-} \dot{t} \dot{-} \dot{1} = \dot{0}$.

Proof. By using the equation (4) for Padovan numbers, we obtain

$$\begin{aligned}
&\dot{a} \dot{\times} \dot{\delta}^{v_N} \dot{+} \dot{b} \dot{\times} \dot{\eta}^{v_N} \dot{+} \dot{c} \dot{\times} \dot{\gamma}^{v_N} \\
&= \alpha\{\alpha^{-1}(\dot{a}) \times \alpha^{-1}(\alpha\{(\alpha^{-1}(\dot{\delta}))^v\})\} \dot{+} \alpha\{\alpha^{-1}(\dot{b}) \times \alpha^{-1}(\alpha\{(\alpha^{-1}(\dot{\eta}))^v\})\} \\
&\dot{+} \alpha\{\alpha^{-1}(\dot{c}) \times \alpha^{-1}(\alpha\{(\alpha^{-1}(\dot{\gamma}))^v\})\} \\
&= \alpha\{\alpha^{-1}(\alpha\{\alpha^{-1}(\dot{a}) \times \alpha^{-1}(\alpha\{(\alpha^{-1}(\dot{\delta}))^v\})\}) + \alpha^{-1}(\alpha\{\alpha^{-1}(\dot{b}) \times \alpha^{-1}(\alpha\{(\alpha^{-1}(\dot{\eta}))^v\})\}) \\
&+ \alpha^{-1}(\alpha\{\alpha^{-1}(\dot{c}) \times \alpha^{-1}(\alpha\{(\alpha^{-1}(\dot{\gamma}))^v\})\})\} \\
&= \alpha\{\alpha^{-1}(\dot{a}) \times (\alpha^{-1}(\dot{\delta}))^v + \alpha^{-1}(\dot{b}) \times (\alpha^{-1}(\dot{\eta}))^v + \alpha^{-1}(\dot{c}) \times (\alpha^{-1}(\dot{\gamma}))^v\} \\
&= \alpha\{a\delta^v + b\eta^v + c\gamma^v\} \\
&= \alpha\{P_v\} = \mathcal{N}P_v.
\end{aligned}$$

On the other hand, by performing the equation (5) for the Perrin numbers, one can easily observe that

$$\begin{aligned}
\dot{\delta}^{v_N} \dot{+} \dot{\eta}^{v_N} \dot{+} \dot{\gamma}^{v_N} &= \alpha\{(\alpha^{-1}(\dot{\delta}))^v\} \dot{+} \alpha\{(\alpha^{-1}(\dot{\eta}))^v\} \dot{+} \alpha\{(\alpha^{-1}(\dot{\gamma}))^v\} \\
&= \alpha\{\alpha^{-1}(\alpha\{\alpha^{-1}(\alpha\{(\alpha^{-1}(\dot{\delta}))^v\}) + \alpha^{-1}(\alpha\{(\alpha^{-1}(\dot{\eta}))^v\})\}) + \alpha^{-1}(\alpha\{(\alpha^{-1}(\dot{\gamma}))^v\})\} \\
&= \alpha\{\delta^v + \eta^v + \gamma^v\} \\
&= \alpha\{R_v\} = \mathcal{N}R_v.
\end{aligned}$$

Theorem 3. The generating functions for $\mathcal{N}P_v$ and $\mathcal{N}R_v$ are as follows, respectively,

i.

$$_{\mathcal{N}}\sum_{v=0}^{\infty}(\mathcal{N}P_v \dot{\times} t^{v_{\mathcal{N}}}) = \frac{\dot{1} \dot{+} t}{\dot{1} \dot{-} t^{2_{\mathcal{N}}} \dot{-} t^{3_{\mathcal{N}}}}\mathcal{N},$$

ii.

$$_{\mathcal{N}}\sum_{v=0}^{\infty}(\mathcal{N}R_v \dot{\times} t^{v_{\mathcal{N}}}) = \frac{\dot{3} \dot{-} t^{2_{\mathcal{N}}}}{\dot{1} \dot{-} t^{2_{\mathcal{N}}} \dot{-} t^{3_{\mathcal{N}}}}\mathcal{N},$$

where the notation $_{\mathcal{N}}\sum_{v=0}^{\infty}$ denotes the non-Newtonian real number series as elaborated in [20].

Proof. $\quad i.$ Let $g_{\mathcal{N}P}(t) =_{\mathcal{N}} \sum_{v=0}^{\infty}(\mathcal{N}P_v \dot{\times} t^{v_{\mathcal{N}}})$. We have

$$\begin{aligned} g_{\mathcal{N}P}(t) &= \dot{1}\dot{+}t\dot{+}t^{2_{\mathcal{N}}}\dot{+}_{\mathcal{N}} \sum_{v=3}^{\infty}(\mathcal{N}P_v \dot{\times} t^{v_{\mathcal{N}}}) \\ &= \dot{1}\dot{+}t\dot{+}t^{2_{\mathcal{N}}}\dot{+}_{\mathcal{N}} \sum_{v=3}^{\infty}(\mathcal{N}P_{v-2} \dot{\times} t^{v_{\mathcal{N}}})\dot{+}_{\mathcal{N}} \sum_{v=3}^{\infty}(\mathcal{N}P_{v-3} \dot{\times} t^{v_{\mathcal{N}}}) \end{aligned}$$

$$t^{2_{\mathcal{N}}} \dot{\times} g_{\mathcal{N}P}(t) =_{\mathcal{N}} \sum_{v=0}^{\infty}(\mathcal{N}P_v \dot{\times} t^{(v\dot{+}2)_{\mathcal{N}}}) = t^{2_{\mathcal{N}}}\dot{+}_{\mathcal{N}} \sum_{v=3}^{\infty}(\mathcal{N}P_{v-2} \dot{\times} t^{v_{\mathcal{N}}})$$

$$t^{3_{\mathcal{N}}} \dot{\times} g_{\mathcal{N}P}(t) =_{\mathcal{N}} \sum_{v=0}^{\infty}(\mathcal{N}P_v \dot{\times} t^{(v\dot{+}3)_{\mathcal{N}}}) =_{\mathcal{N}} \sum_{v=3}^{\infty}(\mathcal{N}P_{v-3} \dot{\times} t^{v_{\mathcal{N}}})$$

Therefore, it follows that

$$\begin{aligned} (\dot{1}\dot{-}t^{2_{\mathcal{N}}}\dot{-}t^{3_{\mathcal{N}}})\dot{\times} g_{\mathcal{N}P}(t) &= g_{\mathcal{N}P}(t)\dot{-}t^{2_{\mathcal{N}}}\dot{\times} g_{\mathcal{N}P}(t)\dot{-}t^{3_{\mathcal{N}}}\dot{\times} g_{\mathcal{N}P}(t) \\ &= \dot{1}\dot{+}t. \end{aligned}$$

Thus, we obtain the function $g_{\mathcal{N}P}(t) = \frac{\dot{1}\dot{+}t}{\dot{1}\dot{-}t^{2_{\mathcal{N}}}\dot{-}t^{3_{\mathcal{N}}}}\mathcal{N}$ as the intended outcome.

$ii.$ The proof can be obtained in a manner similar to the proof above.

Theorem 4. The exponential generating functions for $\mathcal{N}P_v$ and $\mathcal{N}R_v$ are, respectively,

i.

$$_{\mathcal{N}}\sum_{v=0}^{\infty}(\mathcal{N}P_v \dot{\times} \dot{x}^{v_{\mathcal{N}}})\dot{/}\dot{v}!^{\mathcal{N}} = \dot{a}\dot{\times}\dot{e}^{\dot{\delta}\dot{\times}\dot{x}_{\mathcal{N}}}\dot{+}\dot{b}\dot{\times}\dot{e}^{\dot{\eta}\dot{\times}\dot{x}_{\mathcal{N}}}\dot{+}\dot{c}\dot{\times}\dot{e}^{\dot{\gamma}\dot{\times}\dot{x}_{\mathcal{N}}},$$

ii.

$$_{\mathcal{N}}\sum_{v=0}^{\infty}(\mathcal{N}R_v \dot{\times} x^{v_{\mathcal{N}}})\dot{/}\dot{v}!^{\mathcal{N}} = \dot{e}^{\dot{\delta}\dot{\times}\dot{x}_{\mathcal{N}}}\dot{+}\dot{e}^{\dot{\eta}\dot{\times}\dot{x}_{\mathcal{N}}}\dot{+}\dot{e}^{\dot{\gamma}\dot{\times}\dot{x}_{\mathcal{N}}},$$

where the notation $_{\mathcal{N}}\sum_{v=0}^{\infty}$ denotes the non-Newtonian real number series as elaborated.

Proof. $\quad i.$ Using the identity (15), we get

$$\begin{aligned} &_{\mathcal{N}}\sum_{v=0}^{\infty}(\mathcal{N}P_v \dot{\times} \dot{x}^{v_{\mathcal{N}}})\dot{/}\dot{v}! \\ &=_{\mathcal{N}} \sum_{v=0}^{\infty}(\alpha(P_v)\dot{\times}\alpha\{(\alpha^{-1}(\dot{x}))^v\})\dot{/}\dot{v}! \end{aligned}$$

$$
\begin{aligned}
&= \alpha\{\alpha^{-1}(\alpha\{\alpha^{-1}(\alpha\{\alpha^{-1}(\alpha(P_0)) \times \alpha^{-1}(\alpha\{(\alpha^{-1}(\alpha(x)))^0\})\}) \div \alpha^{-1}(\alpha(0!))\}) \\
&+ \alpha^{-1}(\alpha\{\alpha^{-1}(\alpha\{\alpha^{-1}(\alpha(P_1)) \times \alpha^{-1}(\alpha\{(\alpha^{-1}(\alpha(x)))^1\})\}) \div \alpha^{-1}(\alpha(1!))\}) \\
&+ \alpha^{-1}(\alpha\{\alpha^{-1}(\alpha\{\alpha^{-1}(\alpha(P_2)) \times \alpha^{-1}(\alpha\{(\alpha^{-1}(\alpha(x)))^2\})\}) \div \alpha^{-1}(\alpha(2!))\}) + \dots\} \\
&= \alpha\{(P_0 \times x^0) + (P_1 \times x^1) + (P_2 \times x^2) \div 2 + \dots\} \\
&= \alpha\{\sum_{v=0}^{\infty}(P_v x^v) \div v!\} \\
&= \alpha\{ae^{\delta x} + be^{\eta x} + ce^{\gamma x}\} \\
&= \alpha\{\alpha\{\alpha^{-1}(\alpha(a)) \times \alpha\{(\alpha^{-1}(\alpha(e)))^{\alpha\{\alpha^{-1}(\alpha(\delta))\times\alpha^{-1}(\alpha(x))\}}\}\} \\
&+ \alpha\{\alpha^{-1}(\alpha(b)) \times \alpha\{(\alpha^{-1}(\alpha(e)))^{\alpha\{\alpha^{-1}(\alpha(\eta))\times\alpha^{-1}(\alpha(x))\}}\}\} \\
&+ \alpha\{\alpha^{-1}(\alpha(c)) \times \alpha\{(\alpha^{-1}(\alpha(e)))^{\alpha\{\alpha^{-1}(\alpha(\gamma))\times\alpha^{-1}(\alpha(x))\}}\}\}\} \\
&= \alpha\{\alpha^{-1}(\dot{a}) \times \alpha\{(\alpha^{-1}(\dot{e}))^{\alpha\{\alpha^{-1}(\dot{\delta})\times\alpha^{-1}(\dot{x})\}}\}\} \\
&\dot{+}\alpha\{\alpha^{-1}(\dot{b}) \times \alpha\{(\alpha^{-1}(\dot{e}))^{\alpha\{\alpha^{-1}(\dot{\eta})\times\alpha^{-1}(\dot{x})\}}\}\} \\
&\dot{+}\alpha\{\alpha^{-1}(\dot{c}) \times \alpha\{(\alpha^{-1}(\dot{e}))^{\alpha\{\alpha^{-1}(\dot{\gamma})\times\alpha^{-1}(\dot{x})\}}\}\} \\
&= \dot{a}\dot{\times}\dot{e}^{\dot{\delta}\dot{\times}\dot{x}_{\mathcal{N}}}\dot{+}\dot{b}\dot{\times}\dot{e}^{\dot{\eta}\dot{\times}\dot{x}_{\mathcal{N}}}\dot{+}\dot{c}\dot{\times}\dot{e}^{\dot{\gamma}\dot{\times}\dot{x}_{\mathcal{N}}}.
\end{aligned}
$$

$ii.$ The proof can be obtained in a manner similar to the proof above by using the identity (16).

Theorem 5. The partial sum formulas for $\mathcal{N}P_v$ and $\mathcal{N}R_v$ are, respectively,

i.

$$
{}_{\mathcal{N}}\sum_{v=1}^{u}\mathcal{N}P_v = \mathcal{N}P_{u+5}\dot{-}\dot{3},
$$

ii.

$$
{}_{\mathcal{N}}\sum_{v=1}^{u}\mathcal{N}R_v = \mathcal{N}R_{u+5}\dot{-}\dot{5},
$$

where the notation ${}_{\mathcal{N}}\sum_{v=1}^{u}$ represents a finite sum based on $\alpha-$arithmetic, and u is a non-negative integer.

Proof. $i.$ Using the formula (11), we find

$$
\begin{aligned}
{}_{\mathcal{N}}\sum_{v=1}^{u}\mathcal{N}P_v &= {}_{\mathcal{N}}\sum_{v=1}^{u}\alpha(P_v) \\
&= \alpha\{\alpha^{-1}(\alpha(P_1)) + \alpha^{-1}(\alpha(P_2)) + \cdots + \alpha^{-1}(\alpha(P_u))\} \\
&= \alpha\{P_1 + P_2 + \cdots + P_u\} \\
&= \alpha\{\sum_{v=1}^{u}P_v\} \\
&= \alpha\{P_{u+5} - 3\}
\end{aligned}
$$

$$= \alpha\{\alpha^{-1}(\alpha(P_{u+5})) - \alpha^{-1}(\alpha(3))\}$$
$$= \mathcal{N}P_{u+5}\dot{-}\dot{3}.$$

$ii.$ The proof can be obtained in a manner similar to the proof above by using the formula (12).

Theorem 6. The binomial sum formulas for $\mathcal{N}P_v$ and $\mathcal{N}R_v$ are, respectively,

i.
$$_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\mathcal{N}P_v = \mathcal{N}P_{3u},$$

ii.
$$_{\mathcal{N}}\sum_{v=0}^{m}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\mathcal{N}R_v = \mathcal{N}R_{3u},$$

where the notation $_{\mathcal{N}}\sum_{v=0}^{u}$ represents a finite sum based on $\alpha-$arithmetic, and v is a non-negative integer.

Proof. $i.$ Using the equalities (18), (17) and (1), we obtain

$$_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\mathcal{N}P_v$$
$$=_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}(\dot{a}\dot{\times}\dot{\delta}^{v_N}\dot{+}\dot{b}\dot{\times}\dot{\eta}^{v_N}\dot{+}\dot{c}\dot{\times}\dot{\gamma}^{v_N})$$
$$= \dot{a}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\dot{\delta}^{v_N}\dot{+}\dot{b}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\dot{\eta}^{v_N}\dot{+}\dot{c}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\dot{\gamma}^{v_N}$$
$$= \dot{a}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{(\alpha^{-1}(\dot{\delta}))^v\}\dot{+}\dot{b}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{(\alpha^{-1}(\dot{\eta}))^v\}$$
$$\dot{+}\dot{c}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{(\alpha^{-1}(\dot{\gamma}))^v\}$$
$$= \dot{a}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{\delta^v\}\dot{+}\dot{b}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{\eta^v\}\dot{+}\dot{c}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{\gamma^v\}$$
$$= \dot{a}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{\delta^v\dot{\times}1^{u-v}\}\dot{+}\dot{b}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{\eta^v\dot{\times}1^{u-v}\}$$
$$\dot{+}\dot{c}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{\gamma^v\dot{\times}1^{u-v}\}$$
$$= \dot{a}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{(\alpha^{-1}(\dot{\delta}))^v\times(\alpha^{-1}(\dot{1}))^{u-v}\}$$
$$\dot{+}\dot{b}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{m}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\alpha\{(\alpha^{-1}(\dot{\eta}))^v\times(\alpha^{-1}(\dot{1}))^{u-v}\}$$

$$\dot{+}\dot{c}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{u}{v}_{\mathcal{N}}\dot{\times}\alpha\{(\alpha^{-1}(\dot{\gamma}))^{v}\times(\alpha^{-1}(\dot{1}))^{u-v}\}$$
$$=\dot{a}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\dot{\delta}^{v_N}\dot{\times}\dot{1}^{u-v_N}\dot{+}\dot{b}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\dot{\eta}^{v_N}\dot{\times}\dot{1}^{u-v_N}$$
$$\dot{+}\dot{c}\dot{\times}_{\mathcal{N}}\sum_{v=0}^{u}\binom{\dot{u}}{\dot{v}}_{\mathcal{N}}\dot{\times}\dot{\gamma}^{v_N}\dot{\times}\dot{1}^{u-v_N}$$
$$=\dot{a}\dot{\times}_{\mathcal{N}}(\dot{\delta}\dot{+}\dot{1})^{u_{\mathcal{N}}}\dot{+}\dot{b}\dot{\times}_{\mathcal{N}}(\dot{\eta}\dot{+}\dot{1})^{u_{\mathcal{N}}}\dot{+}\dot{c}\dot{\times}_{\mathcal{N}}(\dot{\gamma}\dot{+}\dot{1})^{u_{\mathcal{N}}}$$
$$=\dot{a}\dot{\times}_{\mathcal{N}}\dot{\delta}^{3u_{\mathcal{N}}}\dot{+}\dot{b}\dot{\times}_{\mathcal{N}}\dot{\eta}^{3u_{\mathcal{N}}}\dot{+}\dot{c}\dot{\times}_{\mathcal{N}}\dot{\gamma}^{3u_{\mathcal{N}}}$$
$$=\mathcal{N}P_{3u}$$

$ii.$ The proof can be obtained in a manner similar to the proof above by using the the equalities (19), (17) and (1).

Conclusions

In this work, we have introduced a novel perspective on Padovan and Perrin numbers, termed as non-Newtonian Padovan and non-Newtonian Perrin numbers. We have explored various properties of these new numbers and provided a range of identities and formulas. These include Binet-like formulas, generating functions, partial sum formulas, and binomial sum formulas.

Important conclusions from our research include:

Recurrence relations and recursive formulations for non-Newtonian Padovan and non-Newtonian Perrin numbers. Derivation and proof of several formulas applicable in the analysis of these new numbers. In-depth investigation into the analytic and combinatorial properties of these numbers. In conclusion, this study aims to enhance understanding of the mathematical structures of non-Newtonian Padovan and non-Newtonian Perrin numbers, potentially opening new avenues for advanced mathematical research and applications. Future work could explore further applications of these new numbers in different fields and investigate extended properties.

By contributing an innovative perspective to the classical theories of Padovan and Perrin numbers, this work encourages further exploration and utilization of these new types within those working in this field.

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Authors Contribution -

References

[1] Grossman, M. & Katz, R. (1972). *Non-Newtonian calculus*, Lee Press: Pigeon Cove, MA, USA.

[2] Grossman, M. (1979). An introduction to non-Newtonian calculus. *International Journal of Mathematical Educational in Science and Technology 10*(4), 525–528. https://doi.org/10.1080/0020739790100406

[3] Çakmak, A. F. & Başar, F. (2012). Some new results on sequence spaces with respect to non-Newtonian calculus. *Journal of Inequalities and Applications*, 1–17. https://doi.org/10.1186/1029-242X-2012-228

[4] Duyar, C., & Sağır, B. (2017). Non-Newtonian Comment of Lebesgue Measure in Real Numbers. *Journal of Mathematics 2017*(1), 6507013. https://doi.org/10.1155/2017/6507013

[5] Erdogan, M., & Duyar, C. (2018). Non-Newtonian improper integrals. *Journal of Science and Arts 18*(1), 49–74.

[6] Degirmen, N. & Duyar, C. (2023). A new perspective on Fibonacci and Lucas numbers. *Filomat*, *37*(28), 9561–9574. https://doi.org/10.2298/FIL2328561D

[7] Yağmur, T. (2024). Non-Newtonian Pell and Pell-Lucas numbers. *Journal of New Results in Science*, *13*(1), 22–35. https://doi.org/10.54187/jnrs.1447678

[8] Shannon, A. G., Horadam, A. F., & Anderson, P. R. (2006). The Auxiliary Equation Associated with the Plastic Numbers, *Notes Number Theory Discrete Mathematics*, *12*(1), 1–12.

[9] Shannon, A. G., Anderson, P. R. & Horadam, A. F. (2006). Properties of Cordonnier, Perrin and Van der Laan numbers, *International Journal of Mathematical Education in Science and Technology*, *37*(7), 825–831. https://doi.org/10.1080/00207390600712554

[10] Sloane, N. J. A. (1973). A Handbook of Integer Sequences, Academic Press, NY.

[11] Dişkaya, O. (2023). Padovan sayılarının genellemeleri ve uygulamaları . (Tez no. 826990) [Doktora Tezi, Mersin Üniversitesi].

[12] Adegoke, K. (2022). Summation identities involving Padovan and Perrin numbers.*Palestine Journal of Mathematics*, *11*(1), 633—650.

[13] Yilmaz, N. & Taskara, N. (2013). Matrix sequences in terms of Padovan and Perrin numbers. *Journal of Applied Mathematics*, *2013*(1), 941673. https://doi.org/10.1155/2013/941673

[14] Dişkaya, O. & Menken, H. (2021). Some properties of the plastic constant. *Journal of Science and Arts 21*(4), 883–894. https://doi.org/10.46939/J.Sci.Arts-21.4-a01

[15] Diskaya, O., & Menken, H. (2020). On the Padovan Triangle. *Journal of Contemporary Applied Mathematics 10*(2), 77–83.

[16] Diskaya, O., & Menken, H. (2024). On the pulsating Padovan sequence. *Notes on Number Theory and Discrete Mathematics 30*(1), 1–7. https://doi.org/10.7546/nntdm.2024.30.1.1-7

E-ISSN: 2564-7873
[17] Diskaya, O., & Menken, H. (2023). Padovan Polynomials Matrix. *Bulletin of The International Mathematical Virtual Institute 13*(3), 499–509. https://doi.org/10.7251/BIMVI2303499D

[18] Diskaya, O., & Menken, H. (2020). Some Identities of Gadovan Numbers. *Journal of Science and Arts*, *20*(2), 317–322.

[19] Deveci, O., & Karaduman, E. (2017). On the Padovan p-numbers. *Hacettepe Journal of Mathematics and Statistics 46*(4), 579–592.

[20] Duyar, C. & Erdogan, M. (2016). On non-Newtonian real number series. *IOSR Journal of Mathematics*, *12*(6), 34–48.