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Research Article

Imaginary latent variables: Empirical testing for detecting deficiency in reflective measures

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Abstract: Imaginary latent variables are variables with negative variances and have been used to implement constraints in measurement models. This article aimed to advance this practice and rationalize the imaginary latent variables as a method to detect possible latent deficiencies in measurement models. This rationale is based on the theory of complex numbers used in the measurement process of common factor model–based structural equation modeling. Modeling an imaginary latent variable produces a potential deficiency within its relative reflective measures through a considerable reduction in common variance indicating the most affected indicator(s).

1. INTRODUCTION

Rindskopf (1984, p. 38) first defined imaginary latent variables as: "… variables with negative variances, or, equivalently, variables with positive variance but whose influence on other variables is represented by an imaginary rather than a real number.'' These variables are of no interest themselves, but only exist to implement the constraints." Considering the first situation, in which an imaginary latent variable has a negative variance, what might it mean in applied psychological and/or educational measurement? Above and beyond of implementing constraints? Might it be useful for detecting potential latent variable deficiency?

Rindskopf (1984) described the use of imaginary latent variables by recalling Bentler and Lee's (1983) work where imaginary latent variables were used by fixing the variances to –1 to permit a measurement model having factors with the same variance as 1: a computational detracting strategy to allow the covariance matrix being able to run the correlational structure. However, this empirical exercise did not reveal the usefulness of the imaginary latent variable unless it was used as a constraint to produce equality restrictions in linear structural models. In my view, constraining a latent variable to be imaginary is not limited to a computational way to implement constraints in measurement models; however, it has potential conceptual

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implications in the underlying measures that, as will be explained in the next section, have ground in the field of the imaginary complex numbers.

Essentially, and to be as reasonable as possible, whenever a latent variable is considered as imaginary, with its negative variance, a researcher postulates a sort of "what if" scenario concerning a potential deficiency of that latent concept in a specific context. That is to say, this imaginary interrogation may want to test what could happen to a latent concept if it has been affected by some causes that have triggered its absence. Consequently, this deficiency will be spread throughout those observed measures that are a reflection, manifestation, and an effect of that latent concept. These observed measures (i.e., the well-known reflective indicators of a latent variable) under imaginary interrogation can determine which aspects of that latent concept are more affected by this potential deficiency/absence.

In this respect, the aim of this article was to propose a simple empirical test based on constraining latent variables to become imaginary and thus verifying what could happen to their reflective measures if they are affected by a potential deficiency in a measurement model and hence within a context of application.

The remainder of this paper is structured as follows. The next section presents the conceptual foundations of this imaginary latent process in a measurement model. Successively, the following section presents a computational demonstration on Schwartz' (1992) human values taxonomy applied to an Italian sample. Finally, a short discussion with limitations and future perspectives concludes this work.

1.1. The Parallelism Between The Measurement Process of Common Factor Models and The Rationale of Imaginary Complex Numbers

The logic and rationale of the classic measurement process, taken from the classic measurement process based on the classical test theory (Lord & Novick, 1968) of true and error scores, postulates that any measure x_i, even the one obtained with the most sophisticated procedures, is affected by a measurement error e_i (a nonsystematic but normally distributed with zero mean and nonzero variance); therefore, this measure is functional/dependent on the true measure t_i (which may be latent in nature and thereby unknown) and the measurement error itself:

$$x_i = t_i + e_i \quad (1)$$

As a logical computational consequence, the true measure is indeed the expected value of the initial measures and is not related to the measurement error:

$$E\,(x_i) = t_i \quad (2)$$

$$Cov\,(t_i, e_i) = 0 \quad (3)$$

According to Equations 1 and 3, a researcher may have a set of observed measures x_i with variances σ^2_{xi} that can be decomposed of another set of true measures with latent true error–free variable variances σ^2_{ti} and a set of measurement errors with variances σ^2_{ei}:

$$\sigma^2_{xi} = \sigma^2_{ti} + \sigma^2_{ei} \quad (4)$$

$$\rho = \sigma^2_{ti}/\sigma^2_{xi} \quad (5)$$

Equation 4 depicts the famous definition of reliability[†] ρ (5) of the classic measurement process where a true value is a value free of measurement error. This true value is indeed a value that is still unknown and requires a set of observed measures to be revealed as precisely as possible by partial-out measurement errors from the common values.

In connection therewith, we know the common factor model theory of Thurstone (1947), which

[†] "Reliability is the ratio of true score's variance to the observed variable's variance" (Bollen, 1989, p.208).

constitutes the key to factor analysis, that each set of observed variables may be written, or better decomposed of, as a linear function of that part of common shared variance and that part that is unique in each observed itself. These two concepts of common shared variance and unique variance represent what have been above formalized with the expression (4) where σ^2_{ti} is the common shared variance needed to reflect the manifestation of a common latent factor (i.e., the true value to be sought), whereas σ^2_{ei} is the unique variance that embodies the following: (a) the part of the observed variance that each observed variable does not share with the observed variances of the other observed variables and thus not useful to manifest the true value and (b) the random error owing to the measurement process.

Hence, by combining the classical test theory of measurement process with a typical confirmatory factor analysis (CFA) model (Bollen, 1989; Jöreskog, 1966), a type[‡] of common factor model where the relations between measures and factors are a priori specified, Equation 1 can be explicated in a system of simple linear regression equations as follows:

$$x_i = \tau_i + \lambda_i \xi + \delta_I \quad (6)$$

where x_i is a set of observed variables (i = 1, …, n), ξ is a hypothetical common latent factor, λ_i is the factor loading or regression slope, τ_i is the intercept, and δ_i is the measurement error. The difference between Equation 6 and a typical regression equation is that the independent variable is the latent factor ξ and the criterion is constituted by multiple observed variables x_i. Therefore, it does mean that the latent concept ξ is trying to explain, and summarize, all those observed variables x_i, and the magnitude of how much the latent factor can do that is owing to the regression slopes or factor loadings λ_i associated with each x_i . The magnitude of what was not captured by the latent factor is δ_i, which is an error in this sort of interpolation process. This error has an expected value E (δ_i) = 0 and Cov ($\xi;\delta_i$) = 0.

Equation 6 estimates parameters τ_i, λ_i, and δ_i using all the information of the observed measures x_i that constitute all the sources of covariation of x_i: the variances and covariances of each involved x_i. This leads to the fundamentals of the structural equation model applied to measured variable and latent variables path analysis (Bollen, 1989): decomposition of observed variances and covariances (i.e., the matrix Σ_{xx}) into the model-implied parameters (i.e., the model-implied matrix $\Sigma(\theta)$):

$$\Sigma = \Sigma[\theta] \quad (7)$$

If a researcher can write the system of Equation 7 he/she can list all the necessary parameters of the model (6).

For an example with two-latent factors ξ_1 and ξ_2 and four measures (x_1, x_2, x_3, x_4) as depicted in Figure 1[§], it is possible to rewrite the covariance matrix of the four measures following the system of Equations 6, as shown in Table 1.

[‡]The other type of common factor model is the famous explorative factor analysis (EFA) where the relations between measures and factors are not a priori specified. EFA and CFA can partial out common variance from unique variance. However, the former assumes measurement error at random; hence, it cannot be modeled while the latter may assume measurement error at random, or not, and thus it can be modeled (Brown, 2006; Fabricar et al., 1999).

[§]The model in Figure 1 is not identified, and it requires to fix one of the λ_i to 1 for each latent factor. As soon as this identification is done, relative decomposition Table 1 will be simplified accordingly, and the imaginary process will involve only the other not fixed λ_i. However, for a better understanding of the process, I did not indicate either in Figure 1 or Table 1 that the λ_i needs to be equal to 1 to trigger the idea that all the λ_i must be involved into the imaginary process alternatively as described in the results section.

Figure 1. *Path diagram of two common factors with four measures.*

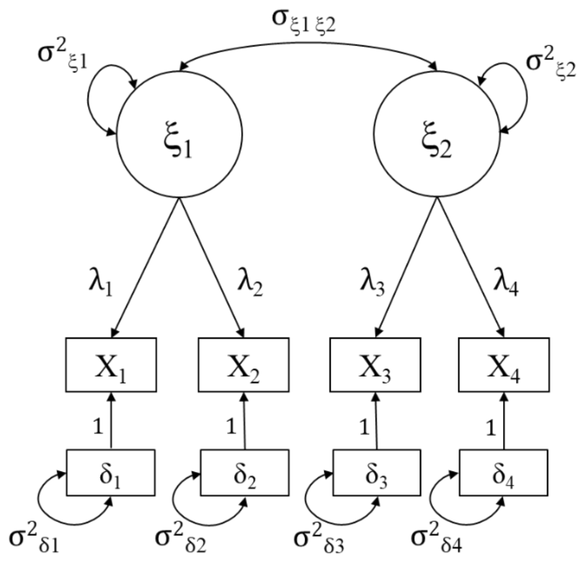


Table 1. *Decomposition table of structural parameters of two common factor models with four measures (adapted from Hancock et al., 2009).*

		Unknown parameters										
info	decomposition	λ_1	λ_2	λ_3	λ_4	$\sigma^2_{\xi1}$	$\sigma_{\xi1\xi2}$	$\sigma^2_{\xi2}$	$\sigma^2_{\delta1}$	$\sigma^2_{\delta2}$	$\sigma^2_{\delta3}$	$\sigma^2_{\delta4}$
σ^2_{x1}	$\lambda_1^2\sigma^2_{\xi1} + \sigma^2_{\delta1}$	√				√			√			
σ^2_{x2}	$\lambda_2^2\sigma^2_{\xi1} + \sigma^2_{\delta2}$		√			√				√		
σ^2_{x3}	$\lambda_3^2\sigma^2_{\xi2} + \sigma^2_{\delta3}$			√				√			√	
σ^2_{x4}	$\lambda_4^2\sigma^2_{\xi2} + \sigma^2_{\delta4}$				√			√				√
σ_{x1x2}	$\lambda_1\lambda_2\sigma^2_{\xi1}$	√	√			√						
σ_{x1x3}	$\lambda_1\lambda_3\sigma_{\xi1\xi2}$	√		√			√					
σ_{x1x4}	$\lambda_1\lambda_4\sigma_{\xi1\xi2}$	√			√		√					
σ_{x2x3}	$\lambda_2\lambda_3\sigma_{\xi1\xi2}$		√	√			√					
σ_{x2x4}	$\lambda_2\lambda_4\sigma_{\xi1\xi2}$		√		√		√					
σ_{x3x4}	$\lambda_3\lambda_4\sigma^2_{\xi2}$			√	√			√				

Reading the table horizontally indicates how many and which piece of information we need to estimate the unknown parameters (Hancock et al., 2009). On the contrary, by reading the table vertically, we are aware of which decomposition expression is directly involved in the estimation of that particular parameter (Hancock et al., 2009). The checkmarks indicate the combinations. It is noteworthy that to estimate the latent variances $\sigma^2_{\xi1}$ and $\sigma^2_{\xi2}$, we require all the information available in the observed measures as expected. Furthermore, the latent variances are functions of all other parameters because they are involved in almost all the decomposition expressions, although unevenly.

Considering the abovementioned, and recalling the theory of imaginary and complex numbers, we acknowledge that an imaginary number is $i^2 = -1$ (or $i = \sqrt{-1}$), and thus a complex number is the sum of a real number x with an imaginary part i (i.e., x + i when the weight of i is 1); on the contrary, a latent variable (LV) is imaginary if its variance (var) is negative (i.e., var (LV) = −1), and thus looking again at decomposition Table 1 for the latent variances, the following expression for the measure x_1 (the same for the other three left) can be written as

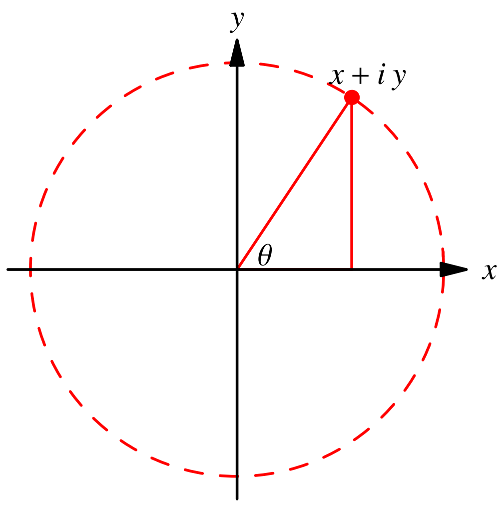
$$\sigma^2_{x1} = \lambda^2_1(-1) + \sigma^2_{\delta 1} \tag{8}$$

$$\sigma^2_{x1} - \sigma^2_{\delta 1} = \lambda^2_1(-1) \tag{9}$$

From Equation 9, we know that when an imaginary latent is postulated, the relative common variance λ^2_i is negative, which can occur when the unique variances are high. From decomposition Table 1 and Equation 9, it is straightforward noticing how this process involves all the four measures.

This intuition becomes a deduction while referring to the properties of imaginary numbers and thus to the well-known complex number geometrical representation of the Argand diagram (Weisstein, 2023), as shown in Figure 2, where the imaginary part *iy* is on the vertical axis, whereas the real numbers *x* are on the horizontal axis.

Figure 2. *The Argand diagram (Weisstein, 2023).*



The logic of the circle is as follows: The more the real number *x* increases, the more the imaginary part *iy* decreases. By translating this rationale to the case of imaginary LVs, the same logic can be applied to its reflective measures. To measure x_1 in Equations 8 and 9, the more the unique variance $\sigma^2_{\delta 1}$ (i.e., the real number x in Figure 2) increases, the more the common variance $i\lambda^2_1$ decreases (i.e., the imaginary part *iy* in Figure 2): This explains the deficiency in items while posing var (LV) to −1, to let it imaginary.

Therefore, it seems reasonable to assume that an imaginary LV is not a proper variable that does not exist because its variance is not zero but equal to a number, although imaginary. Hence, constraining a latent factor to have a negative variance seems to hypothesize *what* could happen *if*, for some reason, there was a deficiency in that factor within its measurement model. Consequently, this deficiency spreads out within its reflective measures, most precisely affecting the common variances (i.e., factor loadings). This can pragmatically indicate which items might be more affected by a potential latent deficiency and suggest which latent aspects (i.e., measures) a specific sample of respondents may be deficient in. The estimation process of the system (7) for the two-latent model in Figure 1 with the imaginary testing with Equation 8 (i.e., by constraining the latent variance $\sigma^2_{\xi 1}$ to –1) will yield to new factor loadings values affected by the imaginary constraint. Furthermore, in the decomposition properties in Table 1, even the estimated latent covariance $\sigma_{\xi 1 \xi 2}$ will be affected by the factor loading modifications, and thus the deficiency in the latent ξ_1 will possibly modify the relation with the other latent ξ_2 as well.

2. METHOD AND METHODS: An example of imaginary latent process

An empirical example of the proposed imaginary latent process will be conducted from the European Social Survey (ESS) (ESS Round 10: European Social Survey, 2022) Italian data of the latest round 10 (ESS Round 10: European Social Survey Round 10 Data, 2020). The ESS is a biennial cross-national survey organized by the European Research Infrastructure Consortium to collect data on the attitudes, values, beliefs, and many behavioral patterns of European countries citizens.

The Schwartz human values section H of the ESS questionnaire (ESS Round 10: European Social Survey, 2022) will be used to select items relative to the two domains of Universalism and Benevolence.

Universalism

(1) He thinks it is important that every person in the world should be treated equally. He believes that everyone should have equal opportunities in life (i.e., item C in ESS questionnaire named ipeqopt).

(2) It is important to him to listen to people who are different from him. Even when he disagrees with them, he still wants to understand them (i.e., item H in ESS questionnaire named ipudrst).

(3) He strongly believes that people should care for nature. Looking after the environment is important to him (i.e., item S in ESS questionnaire named impenv).

Benevolence[**]

(1) It is essential for him to help the people around him. He wants to take care for their well-being (i.e., item L in ESS questionnaire named iphlppl).

(2) It is important for him to be loyal to his friends. He wants to devote himself to people close to him (i.e., item R in ESS questionnaire named iplylfr).

The ESS uses the Schwartz's Portrait Value Questionnaire (Schwartz, 2004; Schwartz et al., 2001) with the unipolar 6-point Likert scale (i.e., from 1 = very like me to 6 = not like me at all) to measure the aforementioned items.

The structural equation modeling (SEM) analyses will be conducted using LISREL v.9.30 (Jöreskog & Sörbom, 2017).

3. RESULTS

The general SEM model's fit was assessed using the classical goodness-of-fit indexes: the maximum likelihood ratio chi-square test, the goodness-of-fit index (GFI), and standardized root-mean-square residual (SRMR) as absolute goodness-of-fit indexes; the root-mean-square error of approximation (RMSEA) as parsimonious fit index; and the comparative fit index (CFI) and the non-normed fit index (NNFI) as incremental fit indices. Most of the SEM scientific community (Fan et al., 2016; Hu & Bentler, 1999; Kline, 2011; Schermelleh-Engel et al., 2003) suggests cutoff values of the aforementioned fit indexes: (a) low and not significant chi-square values are symptoms of good fit even though they are often found significant owing to the well-known limitations of this index, which is sensible to sample size. However, the chi-square magnitude is always reported as the first indication of discrepancy between the data and the hypothesized model; (b) values of RMSEA equal to or less than 0.05 are a good fit, in the range between 0.05 and 0.08 marginal, and greater than 0.10 is a poor fit; (c) GFI is similar to the coefficient of determination used in linear regression but applied to the entire model, and it reveals the amount of variance and covariance explained by the model (Bollen, 1989); (d)

[**] For simplicity's sake only two domains of the Schwartz' taxonomy have been selected, but the analyses can be expanded to the complete taxonomy or considering other domains of interest. It does not jeopardize the imaginary latent process.

SRMR values below 0.09 are considered good data-model fit; and (e) values greater than 0.90 for CFI and NNFI are considered adequate for a good model fit, although values approaching and over 0.95 are preferred.

Tables 2 and 3 present the CFA results of the Universalism and Benevolence latent Schwartz domains tested for the imaginary process with the maximum likelihood (ML) method of estimation[††] and the bootstrapping analysis[‡‡] on the constrained covariation matrix for testing the estimation stability caused by the sampling fluctuation. The first columns of both tables show the CFA solutions with no restrictions unless the first item is fixed to 1 to measure the respective latent as scaling indicators to identify the model (Bollen, 1989). This initial model with an effective sample size of 2546 respondents and 4 degrees of freedom performed fairly well regarding factor loadings (all over .5 and statistically significant from 0) and fit indices (i.e., chi square = 67.10 (p < .000); GFI = .99; RMSEA = .079 with 90% confidence interval [.063–.096]; CFI = .98; NNFI = .96; SRMR = .02). This CFA model is the one to be tested for an imaginary process. Starting from Table 2, the Universalism is investigated as an imaginary value domain first with constraining its latent variance to –1[§§]. This process was repeated by selecting each item as scaling indicator alternatively to test for each item deficiency[***]. Therefore, the item coded impenv (i.e., He strongly believes that people should care for nature. Looking after the environment is important to him.) seems to be the only one found to be more resilient (i.e., factor loadings are greater) than the other two in the presence of a potential deficiency of the Universalism domain in Italy concerning the ESS sample. Practically, this means that for these citizens, a deficiency in Universalism will more likely affect their relationships with other people than their concern for preserving the environment. Passing to the Benevolence domain from Table 3 is straightforward, indicating that the most resilient item at a potential deficiency seems is the iplylfr (i.e., It is important to him to be loyal to his friends. He wants to devote himself to people close to him.) even though the bootstrapping solution did not confirm owing to the sampling fluctuation. However, these two items require further attention and investigation because they seem to preserve their own purposes, whereas Universalism and Benevolence concepts are more and more tenuous. Attention may regard, for instance, the context from which the items were surveyed, the research questions of the study, the characteristics of the sample, and so forth.

[††] Robust Maximum Likelihood (RML) and Robust Diagonally Weighted Least Square (DWLS) methods of estimation have been performed for considering also the potential ordinal nature of the variables (Finney & DiStefano, 2013), but here I just reported the ML solutions because they did not substantially differ from the other two strategies. All the RML and DWLS solutions are not reported, but they can be requested to the author.

[‡‡] The number of bootstrap samples was of 1000 (Hair et al., 2018) with 100% resampling of the raw data.

[§§]The SIMPLIS syntax, a program language that works under LISREL (Jöreskog & Sörbom, 2017) ambient, has been reported in the Appendix.

[***]Goodness-of-fit indices of the constrained model obviously got worse, even for bootstrapping, than the unconstrained solution because imposing a latent variance to be –1 computationally sounds improper (the worst example of fit indices found: chi square = 2462.02 (p < .000); GFI = .73; RMSEA = .439 with 90% confidence interval (.425–.454); CFI = .40; SRMR = .33; the reader can easily run the CFAs reported in Table 2 with the SIMPLIS syntax provided in the Appendix). All that was expected and the goodness-of-fit indices here are not very informative because the purpose was not to find a good adaptation of original data matrix to the model-implied matrix but to look at the modifications of the indicators' common variances (i.e., factor loadings) while imposing an imaginary constraint.

Table 2. *Unstandardized (Std) factor loadings, latent variances, and covariances for Universalism as imaginary latent (*not significant at the 95% confidence level). Fixed values are indicated in bold. Bootstrapping results are indicated in italics.*

UNIVERSALISM				
		Latent Variance		
	0.41 (1.00)	**-1.00**	**-1.00**	**-1.00**
ipeqopt	**1.00** (.64)	**1.00** (.96) *1.00 (1.02)*	-.07 (-.07) *-.12 (-.12)*	-.01* (-.01) *-.02 (-.02)*
ipudrst	.97 (.66)	-.05 (-.05) *-.09 (-.10)*	**1.00** (1.03) *1.00 (1.10)*	.01* (.01) *.01*(.01)*
impenv	1.06 (.73)	.09 (.09) *.08 (.08)*	.08 (.09) *.08 (.08)*	**1.00** (1.07) *1.00 (1.08)*
BENEVOLENCE				
		Latent Variance		
	.43 (1.00)	.43 (1.00) *.43 (1.00)*	.46 (1.00) *.43 (1.00)*	.38 (1.00) *.36 (1.00)*
iphlppl	**1.00** (.70)	**1.00** (.71) *1.00 (.71)*	**1.00** (.73) *1.00 (.71)*	**1.00** (.67) *1.00 (.66)*
iplylfr	1.00 (.73)	.98 (.72) *.98 (.73)*	.92 (.70) *.98 (.73)*	1.12 (.77) *1.15 (.79)*
UNIVERSALISM-BENEVOLENCE				
		Latent Covariance		
	.42 (1.00)	.45 (.68) *.38 (.58)*	.43 (.64) *.35 (.54)*	.43 (.70) *.41 (.68)*

Table 3. *Unstandardized (Std) factor loadings, latent variances, and covariances for Benevolence as imaginary latent. (*not significant at the 95% confidence level). Fixed values are indicated in bold. Bootstrapping results are indicated in italics*

BENEVOLENCE			
		Latent Variance	
	0.41 (1.00)	**-1.00**	**-1.00**
iphlppl	**1.00** (.70)	**1.00** (.1.06) *1.00 (1.09)*	.01* (.01) *-.02 (-.02)*
iplylfr	1.00 (.73)	.02 (.02) *-.00* (-.00)*	**1.00** (1.11) *1.00 (1.16)*
UNIVERSALISM			
		Latent Variance	
	.43 (1.00)	.44 (1.00) *.43 (1.00)*	.40 (1.00) *.39 (1.00)*
ipeqopt	**1.00** (.64)	**1.00** (.66) *1.00 (.67)*	**1.00** (.64) *1.00 (.64)*
ipudrst	.97 (.66)	.98 (.68) *.94 (.68)*	.95 (.64) *.94 (.64)*
impenv	1.06 (.73)	.97 (.69) *.98 (.69)*	1.10 (.75) *1.14 (.76)*
BENEVOLENCE - UNIVERSALISM			
		Latent Covariance	
	.42 (1.00)	.45 (.69) *.41 (.63)*	.43 (.67) *.39 (.63)*

4. DISCUSSION and CONCLUSION

Recalling Rindskopf (1984, p.38), the imaginary LVs should be variables useful to implement specific constraints in measurement models. Above and beyond this initial definition and based on the empirical test provided in this manuscript, it can be reasonable to propose that imaginary LVs are variables useful for testing a latent deficiency within a specific context of the application. Explicitly, the imaginary LVs while postulating variances equal to −1 reflect this negative effect within their observed indicators that turn into complex numbers. Consequently, the measurement equations of confirmatory factor models with imaginary LVs turn into measurement equations with complex numbers. However, on one hand, solving these new complex equations with the usual SEM techniques yields expected unacceptable fit indices; on the contrary, it still provides significant structural parameters and thus potential indications on which indicator, loading the imaginary latent, is less (or more) affected by this latent deficiency. That is to say, because a negative latent variance is a variability that is absent in a latent concept, this sort of latent lacking will be reflected in the indicators, and thus, it can sensibly give signals on what would happen if that latent concept is flawed: which latent aspect (measured by each indicator) will be more affected by, and which is more resilient to, this potential deficiency. These potential indications need to be more investigated and/or validated by other SEM-based strategies (like measurement invariance across groups for instance), but I strongly suggest that it is something not to be ignored. This empirical test can also add further potential information on the selection of scaling indicators while a deficiency scenario in the LVs is hypothesized and therefore contributes to expanding the list of criteria for this selection (Bollen et al., 2022).

Furthermore, and perhaps most importantly, this empirical test of the imaginary latent interrogation opens new possibilities regarding the promising usefulness of the complex numbers in measurement models with latent variables that, to my knowledge, are still unexplored and so are the subsequent estimation methods of these types of SEM models. While using the well-known methods of estimation (e.g., ML, RML, DWLS), a researcher obtains bad fit indices because you are running models with offending constraints like fixing latent variances to –1. Consequently, new methods, possibly even completely different from the usual ones, that include the math process of imaginary and complex numbers in the estimation process are eagerly necessary, although it goes beyond the purpose of this work that remains essentially pioneering. However, the two-factor model tested in this initial experiment yielded promising results that warrant further investigation, particularly involving multifactor structures with additional reflective items to be tested across different respondent groups.

Finally, the evident limitations of this approach need to be considered. The first was just partially mentioned above and regards the methodological way how to model an imaginary latent. In this experiment, the LISREL computational system was pragmatically forced to converge to a solution by fixing the variance of a latent variable to be equal to –1. Other statistical software like M-Plus (Muthen & Muthen, 1998-2017) and lavaan (Rosseel, 2012) under R (R core Team, 2021) can be tried, but I am more than certain that other methods of estimation are needed. A second limitation is that only reflective indicators have been tested for potential deficiency in a latent variable. However, what happens when formative causal indicators are included? They are typical predictors of a latent variance such as the multiple indicators multiple causes (MIMIC) model (Jöreskog & Goldberger, 1975) (Bollen & Diamantopoulos, 2015). Whenever an imaginary interrogation is requested for latent variable models with formative indicators, it would mean that they predict a negative latent variance by estimating possible causes behind the deficiency found in the relative reflective indicators. This sounds like another extremely challenging perspective to be explored in the future.

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Declaration of Conflicting Interests and Ethics

The author declares no conflict of interest. This research study complies with research publishing ethics. The scientific and legal responsibility for manuscripts published in IJATE belongs to the author.

Authorship Contribution Statement

Marco Vassallo: Conceived the presented idea, conducted the formal analyses, and wrote the article.

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APPENDIX

SIMPLIS syntax to run the maximum likelihood analysis (no bootstrapping) of the path model in Table 2 and 3 within the main text; “!” stands for comments. The reader can alternatively set the variance of U (Universalism) or B (Benevolence) to -1 to check the U and B items, respectively):

```
Observed variables ipeqopt ipudrst impenv iphlppl iplylfr
Covariance Matrix
0.997
0.444 0.896
0.427 0.401 0.870
0.427 0.431 0.431 0.860
0.402 0.384 0.485 0.428 0.804
Latent variables U B
Sample Size = 2546
Relationships
ipeqopt=1*U
ipudrst=U
impenv=U
iphlppl=1*B
iplylfr=B
Set Variance of U to -1 ! Set Variance of B to -1
Path Diagram
Print Residuals
Admissibility check = off
End of Problem
```