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New Ratio Estimators for Estimating Population Mean in Simple Random Sampling using a Coefficient of Variation, Correlation Coefficient and a Regression Coefficient

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Abstract

We propose new ratio estimators for estimating population mean using known auxiliary variables in this paper. To estimate the population mean of the study variable we use a known population coefficient of variation of the auxiliary variable, correlation coefficient between an interest variable and an auxiliary variable and also the sample regression coefficient of interest variable of an auxiliary variable. The expressions for the bias and mean square error (MSE) of the proposed estimators up to the first order of approximation have been obtained. The performance of proposed estimators are compared with existing estimators using both theoretical and empirical data.

1. INTRODUCTION

The use of auxiliary information in sample survey that is highly correlated with the study variable can help to improve precision in estimating the population mean estimator. One of the well-known estimators for estimating population mean using auxiliary variables is known as the ratio estimator. The ratio estimator is recommended for use when the correlation between study variable Y and auxiliary variable X is positive. Conversely; a ratio product type estimator will be used when the correlation between the study variable and auxiliary variable is negative. Cochran [2] was first to propose using the classical ratio estimator to improve precision in estimating population mean. The classical ratio estimator is given by

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X}, \tag{1}$$

where $\bar{y}$ and $\bar{x}$ denote the sample mean of the study and auxiliary variables respectively. It is assumed that the population mean $\bar{X}$ of the auxiliary variable X is known. Up to the first degree of approximation, the mean squared error (MSE) of the classical ratio estimator is given by

$$MSE\left(\hat{\bar{Y}}_R\right) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right), \tag{2}$$

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where $f = \frac{n}{N}$, n is the sample size, N is the population size, C_y and C_x denote the population coefficient of variation of the study and auxiliary variables respectively. ρ is the correlation coefficient between the study variable and auxiliary variable.

Later, Sisodia and Dwivedi [10] proposed to improve the classical ratio estimator by using the population coefficient of variation of auxiliary variable C_x. Sisodia and Dwivedi's [10] ratio estimator is given as

$$\hat{\bar{Y}}_{SD} = \bar{y}\left(\frac{\bar{X} + C_x}{\bar{x} + C_x}\right). \tag{3}$$

The MSE of the estimator $\hat{\bar{Y}}_{SD}$ is given as

$$MSE\left(\hat{\bar{Y}}_{SD}\right) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2 + C_x^2\theta_1\left(\theta_1 - 2K\right)\right), \tag{4}$$

where $\theta_1 = \frac{\bar{X}}{\bar{X} + C_x}$ and $K = \frac{\rho C_y}{C_x}$.

Using the population correlation coefficient between the study variable and auxiliary variable (ρ) can also improve the efficiency of the population mean estimator. Singh and Tailor [9] suggested a new ratio estimator for estimating the population mean by using ρ. The suggested estimator is given as

$$\hat{\bar{Y}}_{ST} = \bar{y}\left(\frac{\bar{X} + \rho}{\bar{x} + \rho}\right). \tag{5}$$

The MSE of the estimator $\hat{\bar{Y}}_{ST}$ is given as

$$MSE\left(\hat{\bar{Y}}_{ST}\right) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2 + C_x^2\theta_2\left(\theta_2 - 2K\right)\right), \tag{6}$$

where $\theta_2 = \frac{\bar{X}}{\bar{X} + \rho}$.

Later, many researchers have improved and developed the ratio type estimator to estimate the population mean by using other population values of auxiliary variables such as the coefficient of skewness, coefficient of kurtosis, median and quartile function have all been proposed (see, e.g., Singh [8], Kadilar and Cingi [4], Kadilar and Cingi [5], Alomari et al. [1], Yan and Tian [12], Subramani and Kumarapandiyan [11], Irfan et al. [3])

Some researchers proposed using an estimation of the auxiliary variable in the sample instead of assuming a known value for the population auxiliary variable. Nangsue [7] suggested the ratio and regression type estimator by using the sample correlation coefficient of Y on X in the case of missing data. Nangsue [7] estimator is given as

$$\hat{\bar{Y}}_N = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{b_1}. \tag{7}$$

The mean square error of the estimator $\hat{\bar{Y}}_N$ is given as

$$MSE\left(\hat{\bar{Y}}_N\right) = \bar{Y}^2 \frac{1-f}{n}\left[C_y^2 - K^2 C_x^2\right]. \tag{8}$$

To increase the efficiency of the population mean estimator, in this study, we propose two new ratio estimators derived from estimators by Sisodia and Dwivedi [10], Singh and Tailor [9] and Nangsue [7] considered under simple random sampling without replacement (SRSWOR) using auxiliary information. The bias and mean square error of the proposed estimators have been obtained up to the first order of approximation. The performance of the proposed estimators are compared with all four existing estimators; the classical ratio estimator, Sisodia and Dwivedi's [10] estimator, Singh and Tailor's [9] estimator and Nangsue's [7] estimator using empirical data with percent relative efficiencies (PRE) as a criterion.

The proposed ratio estimators are presented in Section 2 and the efficiency comparison of the proposed estimators and the existing ratio estimators are given in Section 3. The properties of these estimators are studied empirically in Section 4. Concluding comments are given in Section 5.

2. PROPOSED ESTIMATOR

We proposed new ratio estimators under SRSWOR following Sisodia and Dwivedi [10], Singh and Tailor [9] and Nangsue [7]. The proposed estimators are given as

$$\hat{\bar{Y}}_{NSD} = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)^{b_1}. \tag{9}$$

$$\hat{\bar{Y}}_{NST} = \bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)^{b_1}. \tag{10}$$

To obtain the bias and MSE of the estimator $\hat{\bar{Y}}_{NSD}$ in equation (9), we define

$\bar{y} = \bar{Y} + e_0$ and $\bar{x} = \bar{X} + e_1$ such that, $E(e_0) = E(e_1) = 0$, and $E\left(e_0^2\right) = \frac{1-f}{n}C_y^2$, $E\left(e_1^2\right) = \frac{1-f}{n}C_x^2$, $E\left(e_0 e_1\right) = \frac{1-f}{n}C_{xy} = \frac{1-f}{n}\rho C_y C_x$.

Expressing (9) in term of e's, we have

$$\hat{\hat{Y}}_{NSD} = \bar{Y} - \bar{Y}b_1\theta_1 e_1 + \bar{Y}\frac{b_1\left(b_1+1\right)}{2}\theta_1^2 e_1^2 + \bar{Y}e_0 - \bar{Y}b_1\theta_1 e_0 e_1 + \bar{Y}\frac{b_1\left(b_1+1\right)}{2}\theta_1^2 e_0 e_1^2. \quad (11)$$

To the first degree of approximation, the bias and MSE of the estimator $\hat{\hat{Y}}_{NSD}$ are respectively obtained as

$$Bias\left(\hat{\hat{Y}}_{NSD}\right) = \bar{Y}\left(\frac{1-f}{n}\right)\left(\frac{b_1\left(b_1+1\right)}{2}\theta_1^2 C_x^2 - b_1\theta_1\rho C_y C_x\right), \quad (12)$$

and

$$MSE\left(\hat{\hat{Y}}_{NSD}\right) = \bar{Y}^2\frac{1-f}{n}\left(C_y^2 + b_1^2\theta_1^2 C_x^2 - 2b_1\theta_1\rho C_y C_x\right). \quad (13)$$

The MSE of estimator $\hat{\hat{Y}}_{NSD}$ in (13) is minimized for

$$b_1 = \frac{K}{\theta_1} = b_{1opt} \quad \text{, when } K = \rho\frac{C_y}{C_x}. \quad (14)$$

Substitution of (14) in (9) yields the optimum of estimator $\hat{\hat{Y}}_{NSD}$ as

$$\hat{\hat{Y}}_{NSD}^{(opt)} = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)^{b_{1opt}}. \quad (15)$$

When we substitute (14) in (12) and (13), we obtain the bias and MSE of estimator $\hat{\hat{Y}}_{NSD}^{(opt)}$ respectively. Therefore, the bias and the minimum MSE of the proposed ratio estimator $\hat{\hat{Y}}_{NSD}^{(opt)}$ are given as

$$Bias\left(\hat{\hat{Y}}_{NSD}^{(opt)}\right) = \bar{Y}\left(\frac{1-f}{n}\right)\left(\frac{1}{2}K\left(\frac{K}{\theta_1}+1\right)\theta_1 C_x^2 - K\rho C_y C_x\right), \quad (16)$$

and

$$MSE_{\min}\left(\hat{\hat{Y}}_{NSD}^{(opt)}\right) = \bar{Y}^2\frac{1-f}{n}\left[C_y^2\left(1-\rho^2\right)\right]. \quad (17)$$

Similarly, the bias and MSE of the proposed estimator $\hat{\hat{Y}}_{NST}$ have been obtained as,

$$Bias\left(\hat{\hat{Y}}_{NST}\right) = \bar{Y}\left(\frac{1-f}{n}\right)\left(\frac{b_1\left(b_1+1\right)}{2}\theta_2^2 C_x^2 - b_1\theta_2\rho C_y C_x\right), \quad (18)$$

and

$$MSE\left(\hat{\hat{Y}}_{NST}\right)=\overline{Y}^2\frac{1-f}{n}\left(C_y^2+b_1^2\theta_2^2C_x^2-2b_1\theta_2\rho C_yC_x\right). \tag{19}$$

The MSE of estimator $\hat{\hat{Y}}_{NST}$ in (19) is minimized for

$$b_1=\frac{K}{\theta_2}=b_{1opt}. \tag{20}$$

The optimum of estimator $\hat{\hat{Y}}_{NST}$, is given as

$$\hat{\hat{Y}}_{NST}^{(opt)}=\overline{y}\left(\frac{\overline{X}+\rho}{\overline{x}+\rho}\right)^{b_1opt}. \tag{21}$$

Therefore, the bias and the minimum MSE of the proposed estimator $\hat{\hat{Y}}_{NST}^{(opt)}$ are given by

$$Bias\left(\hat{\hat{Y}}_{NST}^{(opt)}\right)=\overline{Y}\left(\frac{1-f}{n}\right)\left(\frac{1}{2}K\left(\frac{K}{\theta_2}+1\right)\theta_2C_x^2-K\rho C_yC_x\right), \tag{22}$$

and

$$MSE_{\min}\left(\hat{\hat{Y}}_{NST}^{(opt)}\right)=\overline{Y}^2\frac{1-f}{n}\left[C_y^2\left(1-\rho^2\right)\right]. \tag{23}$$

3. EFFICIENCY COMPARISONS

In this section, the performance of the proposed ratio estimators are compared with four existing ratio estimators; the classical ratio estimator, Sisodia and Dwivedi's [10] estimator, Singh and Tailor's [9] estimator and Nangsue's [7] estimator. The expressions of MSE of the proposed estimators up to the first order of approximation are required in order to be able to compare them with the noted existing estimators.

3.1.Efficiency Comparison of $\hat{\hat{Y}}_{NSD}$ with existing estimators

We compare MSE and the minimum MSE of the proposed ratio estimator $\hat{\hat{Y}}_{NSD}$ from (13) and (17) with the MSE of the four existing ratio estimators from (2), (4), (6) and (8) respectively. The proposed estimator $\hat{\hat{Y}}_{NSD}$ is more efficient than all four existing estimators if the conditions below are satisfied. The details are as follows:

3.1.1 The classical ratio estimator

$$MSE\left(\hat{\hat{Y}}_{NSD}\right) < MSE\left(\hat{\hat{Y}}_{R}\right),$$

$$\left[C_x^2-b_1^2\theta_1^2C_x^2+2b_1\theta_1\rho C_yC_x-2\rho C_xC_y\right]>0,$$

$$\rho < \frac{C_x - b_1^2\theta_1^2 C_x}{2C_y - 2b_1\theta_1 C_y}, \tag{24}$$

and

$$MSE_{\min}\left(\hat{\bar{Y}}_{NSD}\right) < MSE\left(\hat{\bar{Y}}_{R}\right),$$

$$\bar{Y}^2\frac{1-f}{n}\left[C_y^2 - \rho^2 C_y^2\right] < \frac{1-f}{n}\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right),$$

$$\rho < \frac{C_x^2}{2C_y C_x - \rho C_y^2}. \tag{25}$$

3.1.2 Sisodia and Dwivedi's [10] estimator

$$MSE\left(\hat{\bar{Y}}_{NSD}\right) < MSE\left(\hat{\bar{Y}}_{SD}\right),$$

$$\left[C_x^2\theta_1^2 - b_1^2\theta_1^2 C_x^2 + 2b_1\theta_1\rho C_y C_x - 2\theta_1\rho C_y C_x\right] > 0,$$

$$\rho < \frac{C_x\theta_1 - b_1^2\theta_1 C_x}{2C_y - 2b_1 C_y}, \tag{26}$$

and

$$MSE_{\min}\left(\hat{\bar{Y}}_{NSD}\right) < MSE\left(\hat{\bar{Y}}_{SD}\right),$$

$$\bar{Y}^2\frac{1-f}{n}\left(C_y^2 - \rho^2 C_y^2\right) < \frac{1-f}{n}\bar{Y}^2\left(C_y^2 + C_x^2\theta_1^2 - 2\theta_1\rho C_y C_x\right),$$

$$\rho < \frac{C_x^2\theta_1^2}{2\theta_1 C_y C_x - \rho C_y^2}. \tag{27}$$

3.1.3 Singh and Tailor's [9] estimator

$$MSE\left(\hat{\bar{Y}}_{NSD}\right) < MSE\left(\hat{\bar{Y}}_{ST}\right),$$

$$\left[C_x^2\theta_2^2 - b_1^2\theta_1^2 C_x^2 + 2b_1\theta_1\rho C_y C_x - 2\theta_2\rho C_y C_x\right] > 0,$$

$$\rho > \frac{C_x\left[b_1^2\theta_1^2 - \theta_2^2\right]}{2\left[b_1\theta_1 C_y - \theta_2 C_y\right]}, \tag{28}$$

and

$$MSE_{\min}\left(\hat{\bar{Y}}_{NSD}\right) < MSE\left(\hat{\bar{Y}}_{ST}\right),$$

$$\left[C_x^2\theta_2^2+\rho^2C_y^2-2\theta_2\rho C_yC_x\right]>0,$$

$$\left(C_x\theta_2-C_y\rho\right)^2>0,$$

$$\rho>\frac{C_x^2\theta_2^2}{2\theta_2C_yC_x-\rho C_y^2}. \tag{29}$$

3.1.4 Nangsue's [7] estimator

$$MSE\left(\hat{\bar{Y}}_{NSD}\right) < MSE\left(\hat{\bar{Y}}_N\right),$$

$$\left[\rho^2C_y^2-2b_1\rho C_yC_x+b_1^2C_x^2\right]>0,$$

$$\left(C_xb_1-C_y\rho\right)^2>0,$$

$$\rho<\frac{b_1C_x-b_1\theta_1^2C_x}{2C_y-2\theta_1C_y}. \tag{30}$$

and

$$MSE_{\min}\left(\bar{y}_{NSD}\right)<MSE\left(\bar{y}_N\right),$$

$$\bar{Y}^2\frac{1-f}{n}\left(C_y^2-\rho^2C_y^2\right)<\frac{1-f}{n}\bar{Y}^2\left(C_y^2+C_x^2b_1^2-2b_1\rho C_yC_x\right),$$

$$\rho<\frac{C_x^2b_1^2}{2b_1C_yC_x-\rho C_y^2}. \tag{31}$$

3.2 Efficiency Comparison of $\hat{\bar{Y}}_{NST}$ with existing estimators

We compare the MSE and the minimum MSE of the proposed ratio estimator $\hat{\bar{Y}}_{NST}$ from (19) and (23) against the MSE of four existing ratio estimators from (2), (4), (6) and (8) respectively. The proposed estimator $\hat{\bar{Y}}_{NST}$ is more efficient than all four existing estimators if the conditions below are satisfied. The details are as follows.

3.2.1 The classical ratio estimator

$$MSE\left(\hat{\bar{Y}}_{NST}\right)<MSE\left(\hat{\bar{Y}}_R\right),$$

$$\left[C_x^2-b_1^2\theta_2^2C_x^2+2b_1\theta_2\rho C_yC_x-2\rho C_xC_y\right]>0,$$

$$\rho < \frac{C_x - b_1^2\theta_2^2 C_x}{2C_y - 2b_1\theta_2 C_y}, \tag{32}$$

and

$$MSE_{\min}\left(\hat{\bar{Y}}_{NST}\right) < MSE\left(\hat{\bar{Y}}_R\right)$$

$$\bar{Y}^2\frac{1-f}{n}\left[C_y^2 - \rho^2 C_y^2\right] < \frac{1-f}{n}\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right)$$

$$\rho < \frac{C_x^2}{2C_y C_x - \rho C_y^2} \tag{33}$$

3.2.2 Sisodia and Dwivedi's [10] estimator

$$MSE\left(\hat{\bar{Y}}_{NST}\right) < MSE\left(\hat{\bar{Y}}_{SD}\right),$$

$$\left[C_x^2\theta_1^2 - b_1^2\theta_2^2 C_x^2 + 2b_1\theta_2\rho C_y C_x - 2\theta_1\rho C_y C_x\right] > 0,$$

$$\rho < \frac{C_x\theta_2 - b_1^2\theta_2 C_x}{2C_y - 2b_1 C_y}, \tag{34}$$

and

$$MSE_{\min}\left(\hat{\bar{Y}}_{NST}\right) < MSE\left(\hat{\bar{Y}}_{SD}\right),$$

$$\left(C_x\theta_1 - C_y\rho\right)^2 > 0,$$

$$\rho > \frac{C_x^2\theta_1^2}{2\theta_1 C_y C_x - \rho C_y^2}. \tag{35}$$

3.2.3 Singh and Tailor's [9] estimator

$$MSE\left(\hat{\bar{Y}}_{NST}\right) < MSE\left(\hat{\bar{Y}}_{ST}\right),$$

$$\left[C_x^2\theta_2^2 - b_1^2\theta_2^2 C_x^2 + 2b_1\theta_2\rho C_y C_x - 2\theta_2\rho C_y C_x\right] > 0,$$

$$\rho > \frac{\theta_2 C_x\left[b_1^2 - 1\right]}{\left[2b_1 C_y - C_x\right]}, \tag{36}$$

and

$$MSE_{\min}\left(\hat{\bar{Y}}_{NST}\right) < MSE\left(\hat{\bar{Y}}_{ST}\right),$$

$$\overline{Y}^2 \frac{1-f}{n}\left(C_y^2 - \rho^2 C_y^2\right) < \frac{1-f}{n}\overline{Y}^2\left(C_y^2 + C_x^2\theta_2^2 - 2\theta_2\rho C_y C_x\right),$$

$$\rho < \frac{C_x^2\theta_2^2}{2\theta_2 C_y C_x - \rho C_y^2}. \tag{37}$$

3.2.4 Nangsue's [7] estimator

$$MSE\left(\hat{\overline{Y}}_{NST}\right) < MSE\left(\hat{\overline{Y}}_N\right),$$

$$\left[b_1^2\theta_2^2 C_x^2 - 2b_1\theta_2\rho C_y C_x + \rho^2 C_y^2\right] > 0,$$

$$\rho < \frac{b_1 C_x - b_1\theta_2^2 C_x}{2C_y - 2\theta_2 C_y}. \tag{38}$$

and

$$MSE_{\min}\left(\hat{\overline{Y}}_{NST}\right) < MSE\left(\hat{\overline{Y}}_N\right),$$

$$\overline{Y}^2 \frac{1-f}{n}\left(C_y^2 - \rho^2 C_y^2\right) < \frac{1-f}{n}\overline{Y}^2\left(C_y^2 + C_x^2 b_1^2 - 2b_1\rho C_y C_x\right),$$

$$\rho < \frac{C_x^2 b_1^2}{2b_1 C_y C_x - \rho C_y^2}. \tag{39}$$

4. EMPIRICAL STUDY

To illustrate the performance of the proposed estimators with the existing estimators we have taken data from Mehta and Sharma [6] which belongs to the population census 2001 published by the Government of India for the Rajasthan state. This study assumed that the working population of all districts in the Rajasthan State (comprised of thousands) was given as study variable Y and the total population of all districts in the Rajasthan State (comprised of thousands) was given as an auxiliary variable X. The descriptions of the population parameter under their study were given as follows:

$N = 32$, $n = 17$, $\overline{Y} = 742.71$, $\overline{X} = 1765$, $\rho = 0.97$, $C_y = 0.45$, $C_x = 0.5$, $\theta_1 = 0.9997$,

$\theta_2 = 0.9995$

Table 1. *PREs of proposed estimators with respect to existing estimators.*

Proposed Estimator	Existing Estimator			
	$\hat{\overline{Y}}_R$	$\hat{\overline{Y}}_{SD}$	$\hat{\overline{Y}}_{ST}$	$\hat{\overline{Y}}_N$
$\hat{\overline{Y}}_{NSD}$	358.52	358.31	358.11	100.01
$\hat{\overline{Y}}_{NST}$	358.54	358.34	358.14	100.02

From Table 1 we can see that the proposed estimator $\hat{\bar{Y}}_{NST}$ performed the best when compared to other existing estimators. The PRE of the proposed estimators $\hat{\bar{Y}}_{NSD}$ and $\hat{\bar{Y}}_{NST}$ are much larger than all existing estimators except $\hat{\bar{Y}}_{N}$ which is a bit larger. Thus, the proposed estimators $\hat{\bar{Y}}_{NSD}$ and $\hat{\bar{Y}}_{NST}$ can be recommended for practical use.

5. SIMULATION STUDY

In this section, a simulation study is also used to compare the performance of the proposed estimators against the existing estimators. We generate (X,Y) from the bivariate normal distribution with a mean equal to 10 and 18 and the variance equal to 49 and 16 respectively with the population size being N = 1,000. Sample size n = 50, 150 and 400, selected using simple random sampling without replacement. The correlation between interest variable and auxiliary variable is 0.4 and 0.7. Mean square error and percentage relative efficiency is used to compare the performance of the proposed estimators with the classical ratio estimator. The results are presented in Table 2 below.

***Table 2**. PREs of proposed estimators with respect to the classical ratio estimator.*

ρ	**Estimator**	**n=50**		**n=150**		**n=400**	
		MSE	**PRE**	**MSE**	**PRE**	**MSE**	**PRE**
0.4	$\hat{\bar{Y}}_{R}$	2.597	100.00	0.745	100	0.194	100
	$\hat{\bar{Y}}_{SD}$	2.253	115.28	0.651	114.58	0.170	114.31
	$\hat{\bar{Y}}_{ST}$	2.388	108.74	0.688	108.34	0.179	108.19
	$\hat{\bar{Y}}_{N}$	0.308	843.11	0.085	872.94	0.022	867.58
	$\hat{\bar{Y}}_{NSD}$	**0.298**	**872.31**	**0.083**	**900.36**	**0.021**	**923.81**
	$\hat{\bar{Y}}_{NST}$	0.302	860.76	0.084	889.56	0.022	882.27
0.9	$\hat{\bar{Y}}_{R}$	1.601	100.00	0.465	100.00	0.121	100.00
	$\hat{\bar{Y}}_{SD}$	1.335	120.51	0.388	119.73	0.101	119.46
	$\hat{\bar{Y}}_{ST}$	1.260	127.73	0.367	126.67	0.096	126.30
	$\hat{\bar{Y}}_{N}$	0.219	733.97	0.063	740.17	0.016	745.48
	$\hat{\bar{Y}}_{NSD}$	0.177	910.29	0.051	917.62	0.013	922.05
	$\hat{\bar{Y}}_{NST}$	**0.166**	**972.27**	**0.047**	**980.27**	**0.012**	**984.24**

From Table 2 we can see clearly that the proposed estimators $\hat{\bar{Y}}_{NSD}$ and $\hat{\bar{Y}}_{NST}$ performed a lot better than all other existing estimators because they gave smaller MSE and bigger PRE when compared to other estimators. When $\rho = 0.4$, $\hat{\hat{Y}}_{NSD}$ performed the best and gave a very slightly lower MSE than $\hat{\hat{Y}}_{NST}$. When ρ is increased to 0.9, $\hat{\hat{Y}}_{NST}$ seemed to perform better than $\hat{\hat{Y}}_{NSD}$ and other estimators.

6. CONCLUSION

The efficiency of the population mean estimator can be improved by using known auxiliary variables for sample survey. We proposed the improved ratio estimators following estimators by Sisodia and Dwivedi [10], Singh and Tailor [9] and Nangsue [7] considered under simple random sampling without replacement. The expressions for the bias and mean square error (MSE) of the proposed estimators up to the first order of approximation have been studied. We can see from the theoretical and empirical study that the proposed estimators performed better than the existing estimators in term of percent relative bias under some satisfied conditions. Therefore, the proposed estimators are functional for practical use.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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