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An investigation of energetic particles in the magnetically confined fusion plasma

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ABSTRACT

The investigation of energetic particles in magnetically confined plasma is a critical area of research in the field of controlled thermonuclear fusion, with implications for the development of fusion reactors such as tokamaks and stellarators. This study focuses on understanding the behaviour of energetic particles, such as alpha particles produced in fusion reactions, and their interactions with the magnetic fields and plasma waves within the confinement devices. The main objectives are to analyze the mechanisms of particle heating, confinement, and the potential instabilities induced by the energetic particles.The study also explores the implications of these interactions for the stability of magnetically confined plasma.It further examines the conditions under which energetic particles can excite instabilities, such as Alfvén eigenmodes, leading to enhanced particle losses and potentially compromising the efficiency of fusion reactors.

1. Introduction

In the enduring quest to unlock the secrets of controlled nuclear fusion—a process that promises an almost inexhaustible source of clean energy—scientists and engineers have relentlessly pursued various methods to achieve and sustain nuclear fusion reactions under terrestrial conditions[1].Among the various approaches to achieve controlled thermonuclear fusion, the confinement of plasma using magnetic fields has emerged as one of the most promising paths[2].In view of this, the current research paper delves into an in-depth investigation of energetic particles within magnetically confined fusion plasma, a key component in the quest to harness fusion energy for practical use. Magnetically confined fusion devices, such as tokamaks and stellarators, aim to replicate the conditions necessary for fusion reactions by heating and confining plasma—a hot, ionized state of matter—in magnetic fields[3].Within this plasma, the fusion of light nuclei, such as hydrogen isotopes, produces not only vast amounts of energy but also energetic particles, including neutrons and alpha particles[4]. These energetic particles are integral to the fusion process, not only as by-products but also as key agents in sustaining the plasma temperature through their collisions with the plasma ions and electrons, effectively driving the plasma heating process and maintaining the conditions necessary for ongoing fusion reactions[5].However, the presence of energetic particles in the plasma introduces a suite of complex phenomena that can both benefit and challenge the stability and efficiency of the fusion process.Their interactions with the magnetic fields and plasma waves can lead to a variety of effects, including the transfer of energy to the bulk plasma, the excitation of instabilities, and potentially the loss of confinement of these particles from the plasma core[6].Such dynamics are critical to understand for the optimization of fusion reactor designs, as they directly influence the reactor's performance, safety, and longevity[7]. In view of this the current study aims to analyse the multifaceted roles and behaviours of energetic particles within magnetically confined fusion plasma. The behaviour of energetic particles is complex and multifaceted, encompassing issues such as heating mechanisms, energy transfer processes, resonance phenomena, and the excitation of instabilities like Alfvén eigenmodes[8].These interactions are crucial for understanding how to optimize energy confinement and mitigate potential losses that could impede the efficiency of energy production in fusion devices[9]. The focus on understanding the interaction between energetic particles and plasma instabilities inside magnetic fields and plasma waves offers a novel approach to studying fusion plasma dynamics.While previous studies have examined individual aspects of plasma behaviour, such as energetic particle effects or instability mechanisms, this research integrates these factors into a comprehensive analysis, providing a more holistic understanding of plasma behaviour[10]. This research hopes to integrate various factors that influence plasma behaviour, including energetic particle dynamics, magnetic field effects, and plasma wave interactions. By considering these complex interactions simultaneously, the research offers a

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comprehensive understanding of plasma instabilities and dynamics that goes beyond traditional single-factor analyses.The study likely adopts a multiscale approach, examining phenomena across different spatial and temporal scales. This allows for the exploration of how processes at the microscale, such as individual particle motions, interact with macroscopic plasma behaviour, leading to a more nuanced understanding of plasma dynamics.

In the present study, a theoretical framework was developed to predict the thresholds and growth rates of plasma instabilities based on energetic particle interaction analysis.Furthermore the current study also investigates how energetic particle convection, driven by instabilities or non-axisymmetric magnetic fields, affects plasma dynamics which involves quantifying changes in particle confinement times or studying the redistribution of energy within the plasma due to convection.This analysis could uniquely contribute to the future studies by employing novel simulation techniques or diagnostics to measure particle transport with high accuracy and exploring non-linear effects in the coupling between particles, fields, and waves. In a wider perspective, the current study is structured to provide an analysis of the interaction of energetic particles in magnetically confined fusion plasma with magnetic fields and plasma waves that contribute to various plasma instabilities and the effects of energetic particle convection on plasma dynamics which is crucial in shaping fusion plasma behaviour, influencing energy transport, plasma stability, mode structures, and diagnostic signatures.In summary, investigating the interaction of energetic particles and their convection in magnetically confined fusion plasma is unique and distinct research because the approach involves developing entirely novel theoretical frameworks to describe the interplay between energetic particles, magnetic fields, and plasma waves. Finally, deriving novel analytical predictions for instability growth rates, thresholds, and characteristics based on energetic particle interactions is another avenue for unique research.Therefore investigating the interaction and convection of energetic particles in magnetically confined fusion plasma is a unique and distinct research endeavour with far-reaching implications for fusion energy research. By unravelling the complex dynamics of energetic particles, the current research aims to unlock the full potential of fusion energy and address the global energy challenges of the future.

2. Governing Equations

The investigation of energetic particles in magnetically confined fusion plasmas involves understanding their behaviour within the complex electromagnetic environment of the plasma[11]. The theoretical framework for this research involves several key components, including kinetic theory, magnetohydrodynamics (MHD), and the interaction between particles and electromagnetic fields. The distribution and dynamics of energetic particles within the plasma are described by kinetic theory[12]. The distribution function $f(\vec{r},\vec{v},t)$ represents the density of particles in phase space, where $\vec{r}$ denotes spatial coordinates, $\vec{v}$ represents velocity, and t is time. The evolution of f is governed by the Vlasov equation[13]:

$$\frac{\partial f}{\partial t}+\vec{v}.\frac{\partial f}{\partial \vec{r}}+\frac{q}{m}\left(\vec{E}+\vec{v}\times\vec{B}\right).\frac{\partial f}{\partial \vec{v}}=0 \tag{1}$$

This equation describes how the distribution function changes as particles move in response to electromagnetic forces, where q is the particle charge, m is the particle mass, $\vec{E}$ is the electric field and $\vec{B}$ is the magnetic field. The behaviour of the overall plasma, including its collective motion and stability, is described by MHD equations. These equations govern the evolution of plasma density, velocity, and magnetic field. One of the fundamental equations is the MHD momentum equation[14]:

$$\rho\left(\frac{\partial \vec{v}}{\partial t}+\vec{v}.\vec{\nabla}\vec{v}\right)=-\vec{\nabla}p+\frac{1}{\mu_0}\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B} \tag{2}$$

where ρ is the plasma density, $\vec{v}$ is the plasma velocity, p is the plasma pressure and μ_0 is the permeability of free space. This equation describes how the plasma responds to electromagnetic forces, including magnetic confinement.Energetic particles interact with the magnetic and electric fields present in the plasma[15].These interactions are crucial for understanding particle confinement, heating, and transport.For instance, particles may undergo drift motion due to the gradient of the magnetic field, or they may resonate with specific electromagnetic waves, leading to instabilities[16].

3. Analysis of the Instabilities

The equations describing these interactions depend on the specific geometry of the magnetic field and the properties of the particles involved[17].Alfvén instabilities are a class of magnetohydrodynamic (MHD) waves that arise in magnetically confined fusion plasmas due to the interaction between plasma particles and the confining magnetic fields[18].These instabilities can have detrimental effects on plasma performance, including particle and energy transport, plasma heating, and overall confinement[19].In the context of energetic particle dynamics, Alfvén instabilities play a crucial role as they can lead to enhanced particle losses and reduced plasma stability. The dynamics of Alfvén instabilities in fusion plasmas can be described by the magnetohydrodynamic equations, which govern the evolution of plasma density, velocity and magnetic field. In the presence of energetic particles, additional terms arise in these equations due to particle kinetic effects[20]. One of the key expressions used to analyse Alfvén instabilities is the linearized MHD wave equation:

$$\frac{\partial \vec{B}}{\partial t}=\vec{\nabla}\times\left(\vec{C}_A\times\vec{B}\right)-\frac{1}{\mu_0\rho}\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) \tag{3}$$

where $\vec{B}$ is the magnetic field, $\vec{C}_A$ is the Alfvén velocity,

μ_0 is the permeability of free space, and ρ is the plasma density. This equation describes the propagation of Alfvén waves in a magnetized plasma and provides insights into the stability properties of the system.In the presence of energetic particles, the dispersion relation for Alfvén waves is modified due to the coupling between the wave fields and the energetic particle distribution function.The presence of resonant particles can lead to the excitation of Alfvén eigenmodes, which are localized oscillations in the plasma that can cause enhanced particle transport and reduced plasma confinement.Alfvén eigenmodes (AEs) are collective oscillations of the plasma magnetic field and plasma particles that can occur in magnetically confined fusion plasmas. Alfvén eigenmodes are generated through resonant interactions between energetic particles and the plasma's magnetic field. These interactions can occur through various mechanisms, including wave-particle resonances. When the frequency of an Alfvén eigenmode matches the gyrofrequency of an energetic particle, resonance occurs, leading to the excitation of the eigenmode.The Fishbone instability occurs when the precession frequency of the energetic particles matches the frequency of certain plasma modes, typically Alfvén eigenmodes. These modes are oscillations in the plasma that involve the magnetic field lines. When the frequencies match, there is a resonant interaction between the energetic particles and the plasma modes.The presence of energetic particles, such as fusion-born alpha particles or neutral beam injection ions, can drive the instability of these modes through their interaction with the plasma's magnetic field.Alfvén eigenmodes exhibit characteristic features depending on the plasma parameters and the properties of the energetic particles.These modes typically involve oscillations of the plasma's magnetic field lines and are characterized by a certain mode structure determined by the mode number n. The mode structure describes how the magnetic field and plasma variables vary spatially along the magnetic field lines.Additionally, Alfvén eigenmodes can have different polarization properties, such as toroidal or poloidal polarization, depending on the magnetic geometry of the fusion device.The presence of Alfvén eigenmodes driven by energetic particles can have both beneficial and detrimental effects on plasma stability and confinement.On one hand, these modes can enhance plasma heating and improve plasma confinement by redistributing energetic particles within the plasma while on the other, if the mode amplitudes grow excessively, they can lead to particle losses and degrade plasma performance.The dispersion relation for Alfvén eigenmodes describes the relationship between the mode frequency ω and the mode number n. In a magnetically confined plasma, this can be derived from the MHD equations. One common approach is to linearize the MHD equations around an equilibrium state and solve for small perturbations.The resulting dispersion relation typically takes the form:

$$\omega^2 = k_{||}^2 C_A^2 \qquad (4)$$

where $k_{\parallel}$ is the wavevector parallel to the magnetic field and $\vec{C}_A$ is the Alfvén velocity, given by $|\vec{C}_A| = |\vec{B}|/\sqrt{\mu_0\rho}$, where $|\vec{B}|$ is the magnetic field strength and ρ is the plasma density.The interaction between energetic particles and Alfvén eigenmodes arises from resonant conditions. When the frequency of an eigenmode matches the gyrofrequency Ω of an energetic particle, resonance occurs.The resonant condition is given by $\omega=\Omega$ leading to resonance islands in phase space.The stability of Alfvén eigenmodes is determined by the growth rate γ or damping rate γ_d.These rates can be obtained by solving the linearized kinetic equation, considering the resonant particle distribution and accounting for other damping mechanisms such as collisional effects. Stability analysis involves comparing the growth rate of the eigenmode to other damping mechanisms present in the plasma. If the growth rate exceeds the damping rates, the eigenmode becomes unstable, leading to its excitation.Conversely, if the damping rates dominate, the eigenmode is damped, and its amplitude decreases over time. The Alfvén wave is often known as one of the three basic MHD solutions.Essentially, any MHD solution may be expressed as a linear combination of the basic solutions.In case of magnetically confined fusion plasma,the plasma oscillations may be entailed into an equation for the plasma displacement vector $(\vec{\xi})$ in a single fluid MHD theory of perfectly conducting homogenous plasma:

$$\frac{\partial^2}{\partial t^2}(\vec{\xi}) = C_A^2 \vec{\nabla}(\vec{\nabla}.\vec{\xi}) + C_A^2 \vec{\nabla}_{\perp}(\vec{\nabla}.\vec{\xi}_{\perp}) + C_A^2 \frac{\partial^2}{\partial z^2}(\vec{\xi}_{\perp}) \quad (5)$$

Three different wave types can be obtained from equation (5) and each one corresponds to a term on the R.H.S of the equation.The above equation represents 'Alfvén ion-cyclotron waves' which are characterized by shear motions parallel to the magnetic field direction in a plasma. They are called 'slow' because they propagate slower than the fast magnetoacoustic waves and involve predominantly shear motions of the magnetic field provided the conditions $(\vec{\nabla}.\vec{\xi}_{\perp}) = 0$ and $\vec{\xi}_{||} = 0$ meet, essentially having the form:

$$\frac{\partial^2}{\partial t^2}(\vec{\xi}_{\perp}) = C_A^2 \frac{\partial^2}{\partial z^2}(\vec{\xi}_{\perp}), \text{ or } \omega^2 = \omega_A^2 = C_A^2 k_{||}^2,$$

(from equation (5) and (4)) (6)

A superposition of waves travelling down the z-axis at the Alfvén velocity $(\vec{C}_A)$ can be used to describe any solution to equation (6) . By dropping the terms associated with velocity of sound in equation (6), we get

$$\frac{\partial^2}{\partial t^2}(\vec{\nabla}.\vec{\xi}_{\perp}) = C_A^2(\vec{\nabla}.\vec{\xi}_{\perp}) \text{ and } \omega^2 = \omega_{CA}^2 = C_A^2 k^2 \quad (7)$$

The above equation represents 'fast magnetoacoustic waves' which are characterized by compressional motions perpendicular to the magnetic field direction in a plasma. They are called 'fast' because they propagate faster than ordinary Alfvén waves and involve both magnetic and plasma compressions. Meanwhile, nonzero parallel displacement $\vec{\xi}_{||} \neq 0$ and

$(\vec{\nabla}.\vec{\xi}_{\perp}) \neq 0$ characterise another branch that meets :

$$\frac{\partial^2}{\partial t^2}(|\vec{\xi}_{||}|) = C_A^2 \frac{\partial^2}{\partial z^2}(|\vec{\xi}_{||}|) \text{ and } \omega^2 = \omega_a^2 = C_a^2 k_{||}^2 \quad (8)$$

The above equation represents the acoustic branch or ion acoustic wave, is a type of longitudinal wave that propagates through the plasma.When plasma beta approaches zero (β→0) i.e. when the magnetic pressure dominates over the plasma pressure, the plasma is characterised as strongly magnetized and it is well-confined by the magnetic fields , in such case the displacement of the plasma is along the magnetic field lines i.e. along the z-direction .Therefore one can outline these branches as (fig.1) :

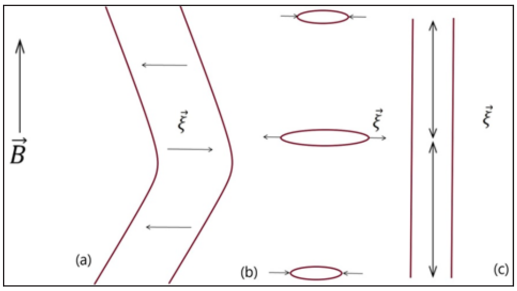

Figure 1. Three essential MHD wave branches are highlighted with plasma displacement ($\vec{\xi}$); a: Alfvén ion-cyclotron waves ; b: Fast magnetoacoustic waves and c: Acoustic branch or ion acoustic wave.

Table 1. Branches and their properties in magnetically confined homogenous plasma.

Branches	Properties
Alfvén ion-cyclotron waves	Shear motions parallel to the magnetic field direction in a plasma, also called 'slow' waves.
Fast magnetoacoustic waves	Compressional motions perpendicular to the magnetic field direction in a plasma.
Acoustic branch or ion acoustic wave	Oscillations in the electron density of a plasma and propagate longitudinally along the direction of the magnetic field.

Each of these has bearing on the instabilities that EPs stimulate in magnetically confined fusion plasma. The damping of MHD oscillations in the presence of finite conductivity or viscous effects occurs due to the conversion of the oscillation's energy into thermal energy. In the case of finite conductivity, the dissipation is primarily through Joule heating, while in the presence of viscous effects, it is through the transfer of kinetic energy into heat due to fluid friction.These damping mechanisms are essential considerations in understanding the behaviour and stability of magnetized plasmas.In several experiments these branches are found to be dominant while involving little coupling from others, for instance, beta-induced shear Alfvén–acoustic eigenmodes (BAEs) which arise due to the interaction between kinetic Alfvén waves (KAWs), acoustic waves, and the plasma flow, particularly in plasmas with finite plasma beta (the ratio of plasma pressure to magnetic pressure). The term "shear Alfvén" refers to Alfvén waves that involve shear motions perpendicular to the magnetic field direction, while "acoustic" refers to compressional motions parallel to the magnetic field. BAEs are characterized by the coupling between these shear Alfvén and acoustic waves, resulting in hybrid modes that exhibit both shear and compressional components.The "beta-induced" aspect of BAEs highlights the influence of plasma beta on their excitation and properties. In plasmas with finite beta, the pressure gradients associated with the plasma can influence the mode structure and stability of MHD modes. BAEs are particularly affected by the plasma beta, and their excitation is enhanced in regions of high beta.Considering the shear Alfvén wave as one of the branches. Compared to the thermal ion speed, its phase velocity is much higher , i.e. $C_A » C_i$, which is an obvious outcome of low plasma beta, i.e. $\beta_i « 1$. Now to be able to heat the plasma, energetic particles are introduced, and this enables resonances between the oscillations and energetic particles , i.e. $E_f » T_i$ and $C_f \sim C_A » C_i$. In plasma medium, ions and electrons have a Maxwellian distribution of velocities, which describes the distribution of particle velocities at a given temperature.The Maxwellian distribution is characterized by a peak around the most probable velocity (corresponding to the thermal speed) and tails that extend to lower and higher velocities. When considering the interaction between thermal ions and acoustic eigenmodes, it is important to understand that the coupling between ions and the acoustic mode occurs through the particle velocity.The amplitude of the interaction depends on the relative velocities of the ions and the phase velocity of the acoustic mode, which implies that ions with velocities corresponding to the tails of the Maxwellian distribution are most likely to resonate with the frequency of the acoustic eigenmode.This resonance occurs when the velocity of the ions matches the phase velocity of the acoustic mode, allowing for efficient energy transfer between the ions and the mode. In contrast, ions with velocities close to the peak of the Maxwellian distribution are less likely to interact strongly with the acoustic eigenmodes because their velocities may not match the phase velocity of the mode as effectively. Therefore, the interaction between thermal ions and acoustic eigenmodes is predominantly observed in the tails of their Maxwellian distribution, where ions have velocities that can resonate with the mode frequency.It has been observed that the plasma oscillations resembling the plasma eigenmodes bear distinctive frequency relationships resulting from their dispersions properties i.e. $\omega_{cA} » \omega_{sA} » \omega_a$. The impacts of these modes on EPs are largely spatial transfer from sound and shear Alfvén waves and energy diffusion from compressional Alfvén waves dependent on their frequencies.Unless they are triggered externally or relevant eigenmodes are stimulated by fast ions, these waves are not present in the plasma.The low frequency oscillations, which cause radial particle redistribution, can interact with EPs and pose an obstacle to radial confinement, as revealed from the shear Alfvén branch.The relevant shear Alfvén eigenmodes can be expressed in the form of solutions to a more generalised set of coupled equations for plasma oscillations as :

$$\sum_{m,n} \hat{L}_{m,n} \Phi_{m,n} = 0 \quad (9)$$

The solutions to the system of coupled equations provide insights into the behaviour and stability of shear Alfvén eigenmodes in magnetized plasmas. These modes can influence phenomena such as energy transport, wave-particle interactions, and plasma confinement and therefore understanding these solutions is essential for predicting and analysing the behaviour of shear Alfvén eigenmodes and their impact on plasma dynamics.

4. Energetic Particle Convection

Energetic particle convection refers to the transport of energetic particles over large spatial scales in a plasma driven by global plasma dynamics. These energetic particles can include ions, electrons, or other charged particles that have gained significant kinetic energy through processes such as plasma heating, acceleration, or interaction with plasma waves and instabilities.Energetic particle convection occurs due to various physical mechanisms within the plasma. These mechanisms can include:

i. Advection by plasma flows: Energetic particles can be carried along with the bulk plasma flow, which may be driven by gradients in pressure, temperature, or magnetic field strength.
ii. Drift motion in magnetic fields: Energetic particles can experience drift motions in magnetic fields, such as magnetic gradient drift, curvature drift, and drift due to magnetic field asymmetries. These drift motions can lead to particle transport across magnetic field lines.
iii. Interaction with plasma waves: Energetic particles can interact with plasma waves, such as Alfvén waves, magnetosonic waves, or kinetic Alfvén waves, leading to wave-particle interactions that can drive particle convection.

Energetic particle convection typically occurs over large spatial scales, spanning regions of the plasma from localized regions to entire plasma volumes. The spatial extent of convection depends on the strength and scale of plasma flows, magnetic field structures, and wave phenomena.Temporally, convection can occur over a range of timescales, from rapid convective bursts associated with transient plasma events to slower, steady-state convection driven by persistent plasma dynamics.The presence of energetic particles and their convection can influence the stability of the plasma. Energetic particles can drive various plasma instabilities through their interaction with plasma waves and collective plasma modes. For instance,energetic particle convection can trigger Alfvén instabilities, kinetic instabilities, or drift waves, leading to enhanced turbulence and transport processes in the plasma. These instabilities can have significant effects on plasma confinement, energy transport, and overall plasma performance.Energetic particles can drive Alfvén instabilities through wave-particle interactions. In magnetized plasmas, energetic particles can resonate with Alfvén waves, leading to the transfer of energy and momentum between the particles and the waves. This resonance can destabilize Alfvén waves, triggering instabilities such as Alfvén eigenmodes, shear Alfvén waves, or kinetic Alfvén waves and fishbone instability(*as discussed in Section: 3*) .The resonant interaction between the energetic particles and the plasma modes can lead to the growth of the fishbone instability. Energy and momentum can be exchanged between the particles and the modes, leading to an amplification of the mode amplitude.If the resonant interaction between the energetic particles and the plasma modes results in a positive feedback mechanism, it can lead to the growth of the instability. The mode amplitude increases over time, potentially leading to disruptive effects on the plasma confinement and stability.The impairment of the confinement of alpha particles and other rapid ions, which would prevent a self-sustained fusion reaction, is one of the primary issues regarding Alfvénic instabilities in fusion devices, which is generally not the case for each Alfvénic mode.The justification for this can be since only a limited portion of the particle phase space can be occupied by the wave-particle resonances corresponding to a single low-amplitude mode, in order to trigger global diffusion, resonance overlap across a substantial large region of phase space must be achieved by several modes and within the framework of quasilinear theory, particle diffusion emerges over an ensemble of overlapping resonances.In principle, this diffusion undermines the motion constants ($E; P_{\phi};\mu$) that describe the particle orbits of unperturbed particles.Nonetheless, in the context of Alfvén modes, the particle magnetic moment μ maintains to function as a reliable motion constant.Furthermore, if the wave frequency is lower than the multiples of the toroidal and poloidal particle frequencies under the resonance situation, the particle energy is almost constant :

$$\omega - n\omega_{\varphi}(E; P_{\varphi}; \mu) - l\omega_{\theta}(E; P_{\varphi}; \mu) = 0 \quad (10)$$

The Fishbone instability occurs when the precession frequency of the energetic particles matches the frequency of the plasma mode under resonant condition.Mathematically, this can be expressed as:

$$\omega(\vec{k}) = \Omega(\vec{B}, \vec{E}, m)$$

The cause of the Fishbone instability lies in the resonant interaction between energetic particles and certain plasma modes, which can drive the growth of unstable oscillations in the plasma.In the realm of magnetically confined fusion plasmas,the (canonical) toroidal angular momentum relates to the rotational motion of energetic particles within the plasma.In the context of magnetically confined fusion plasma, the canonical toroidal angular momentum refers to the rotational momentum associated with the toroidal motion of charged particles within the plasma. It can be defined in terms of the canonical momentum (P_ϕ) which accounts for both the linear and angular momentum of a particle within a magnetic field. Mathematically, the canonical toroidal angular momentum (L_ϕ) can be given by :

$$L_\varphi = m v_\varphi R = P_\varphi R$$

where *m* can be considered as reduced mass of the energetic particles , υ_φ is the toroidal component of the particle's velocity (velocity in the toroidal direction) and *R* is the major radius of the torus (the distance from the centre of the toroidal device to the centre of the plasma).In a magnetically confined fusion plasma, the particles' motion is influenced by the magnetic field geometry and the plasma dynamics.It affects the plasma's rotational profile, which in turn influences the toroidal angular momentum or, in other words, the radial site of the particle orbit in the poloidal cross-section of the tokamak is therefore primarily affected by the waves.In view of this, the resonance width δP_φ can be given by :

$$\delta P_\varphi \frac{\partial}{\partial P_\varphi}\left[\omega - n\omega_\varphi(E;P_\varphi;\mu) - l\omega_\theta(E;P_\varphi;\mu)\right] = \omega_b \quad (11)$$

where a resonant particle's nonlinear bounce frequency in the wave field is represented by ω_b . The form of the 1-D quasilinear diffusion equation in P_φ is as follows [21] :

$$\frac{\partial f}{\partial t} - \frac{\partial}{\partial P_\varphi} D \frac{\partial f}{\partial P_\varphi} = -v(f - f_0) \quad (12)$$

where the particle source and relaxation process of the Krook type which tend to generate a classical equilibrium distribution function, f_0 are taken into consideration by the R.H.S of the equation.The growth rate (γ) of the instability, which describes how fast the mode amplitude increases, is determined by the resonant interaction between the energetic particles and the plasma mode.The waves are driven by instability due to the gradient of this distribution in P_φ, which grows at a linear pace, $\gamma_0 = a\frac{\partial f_0}{\partial P_\varphi}$; ; where the factor *a* relies upon the details of the modes that resonate with a particular value of P_φ.The coefficient of diffusion, *D* related to the intensity of the wave can be written as :

$$\frac{\partial D}{\partial t} = 2\left[a\frac{\partial f}{\partial P_\varphi} - \gamma_b\right] D \quad (13)$$

The above equation posits a background damping-rate, $\gamma_b<\gamma_0$ and defines an instability threshold.In view of this , the distribution function *f* in absence of waves can be given by :

$$f \cong f_0 \cong \frac{\gamma_0}{a} P_\varphi \quad (14)$$

However when waves are present,the equilibrium solutions of equations (12) and (13) limit *f* to $f \cong \frac{\gamma_b}{a} P_\varphi < f_0$. Using this, the diffusion coefficient , *D* can be given by:

$$D \cong \frac{a}{\gamma_b} v f_0 P_\varphi \cong v \frac{\gamma_0}{\gamma_b} P_\varphi^2 \quad (15)$$

The resonance overlap limit on the steady regime can be established by considering an ensemble of rarely overlapped resonances corresponding to the correlation time , $1/\omega_b$ and diffusion coefficient ,

$$D \approx \omega_b(\delta P_\varphi)^2 \cong \omega_b^3 \times \frac{1}{\left\{\frac{\partial}{\partial P_\varphi}\left[\omega - n\omega_\varphi(E;P_\varphi;\mu) - l\omega_\theta(E;P_\varphi;\mu)\right]\right\}^2} \quad (16)$$

The above equation can further be simplified as :

$$D \approx (\delta P_\varphi)^3 \times \frac{\partial}{\partial P_\varphi}\left[\omega - n\omega_\varphi(E;P_\varphi;\mu) - l\omega_\theta(E;P_\varphi;\mu)\right] \quad (17)$$

The overlap criterion has the following form if *N* resonant modes are present over the whole range of P_ϕ:

$$D > \left(\frac{P_\varphi}{N}\right)^3 \times \frac{\partial}{\partial P_\varphi}\left[\omega - n\omega_\varphi(E;P_\varphi;\mu) - l\omega_\theta(E;P_\varphi;\mu)\right] \quad (18)$$

From equations (15) and (18) , the overlap criterion for resonance condition necessitate the source to be adequately strong ,

$$v\frac{\gamma_0}{\gamma_b} > \frac{P_\varphi}{N^3} \times \frac{\partial}{\partial P_\varphi}\left[\omega - n\omega_\varphi(E;P_\varphi;\mu) - l\omega_\theta(E;P_\varphi;\mu)\right] \quad (19)$$

The energetic particle convection either interrupts or becomes discontinuous if the source is weaker than the value indicated by equation (19) based on whether, at certain moments during their nonlinear growth, each individual mode could fulfil the overlap condition.The KAM surfaces that divide the resonances behave as transport obstacles for fast particles in the lack of overlap.In particular, compared to equation (19), taking into account the saturated modes independently results in a far more restricted overlap condition as :

$$\gamma_0\left(1 + \frac{v}{\gamma_b}\right) > \frac{1}{N} \times P_\varphi \times \frac{\partial}{\partial P_\varphi}\left[\omega - n\omega_\varphi(E;P_\varphi;\mu) - l\omega_\theta(E;P_\varphi;\mu)\right] \quad (20)$$

This requirement is immediately satisfied by equation (11) and the expected nonlinear bounce frequency is found as :

$$\omega_b \cong \gamma_0\left(1 + \frac{v}{\gamma_b}\right) \quad (21)$$

For large values of *N*, the considerable difference between equations (19) and (20) suggests that, in contrast to isolated modes, the energy release per mode is greatly enhanced by the overlap of several resonances.Therefore, it is probable that even if two closely spaced resonances are linearly stable, their overlap might set off an avalanche-type relaxation phenomenon that involves nearby modes.In such a situation , rapid quasilinear diffusion can lower the energetic particle concentration to a hypercritical amount underneath the threshold of linear instability as shown in (fig.2) :

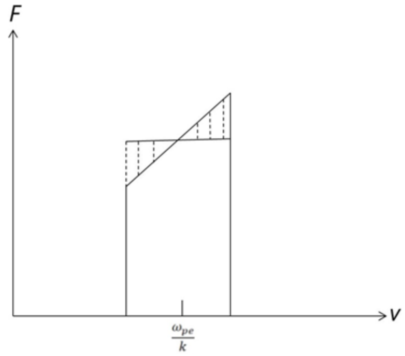

Figure 2. Figure illustrating rapid quasilinear diffusion can lower the energetic particle concentration to a hypercritical amount underneath the threshold of linear instability, where the dashed lines represent the boundaries in velocity space.

Subsequently, the waves will diminish over a linear damping period and the system will ‘ keep off ’ for the particle sources to replenish the energetic particle concentration, causing instability and resulting in the subsequent avalanche. One inherent characteristic of this type of discontinuous diffusion is that the triggering activity causes the bursts of various modes to synchronise.The concentration of energetic particles is prompted to remain near the marginally stable level by the bursts of several modes.This means that the energetic particle distribution remains close to the threshold of stability, where the growth rates of plasma instabilities are balanced by damping mechanisms. The synchronized bursts of modes trigger enhanced particle transport, leading to a quasi-stable state where the energetic particle population hovers around the marginally stable level. In conclusion, energetic particle convection in magnetically confined fusion plasma is a critical phenomenon that significantly impacts plasma heating, stability, and confinement in fusion devices. Through their interaction with plasma waves and instabilities, energetic particles contribute to the overall dynamics of the plasma and play a crucial role in sustaining fusion reactions.

5. Conclusion

The investigation into energetic particles in magnetically confined fusion plasma has underscored their significant influence on plasma stability. Addressing the instabilities they induce is crucial for maintaining efficient and sustained fusion reactions. These instabilities arise from the interaction between energetic particles and plasma waves, leading to enhanced turbulence and transport phenomena in the plasma. Energetic particles play a central role in driving plasma instabilities through resonant interactions with plasma waves. By providing a source of free energy, energetic particles contribute to the excitation and amplification of instabilities, leading to enhanced particle transport and modification of plasma profiles. The presence of energetic particle-driven instabilities has significant implications for plasma confinement and stability. These instabilities can lead to increased particle transport across magnetic field lines, resulting in particle losses and reduced plasma confinement times. The current analysis has highlighted the importance of energetic particle convection in shaping plasma dynamics. Energetic particle convection leads to spatial redistribution of particles, enhanced particle diffusion, and modification of plasma profiles.This phenomenon significantly influences plasma heating, stability, and overall fusion performance.Existing literature provides insights into the role of energetic particles in driving plasma instabilities, such as Alfven instabilities and fishbone modes[22].The current analysis reveals previously unrecognized mechanisms through which energetic particle convection amplifies certain instabilities while suppressing others.This nuanced understanding enhances predictive capabilities and provide an indication of targeted mitigation strategies. Prior studies have elucidated the contribution of energetic particles to plasma heating through fusion reactions and collisional processes[23].The current analysis uncovers the intricate interplay between energetic particle convection, magnetic field topology, and plasma heating mechanisms.This reveals non-linear effects that govern plasma temperature profiles, offering insights into optimizing heating efficiency. Previous literature has established the importance of energetic particle transport in plasma confinement and energy balance[24].The current analysis quantifies the role of energetic particle convection in modifying plasma transport properties, including particle diffusion and energy loss mechanisms.These findings challenge conventional models of plasma transport and highlight the need for a more refined theoretical framework.In summary, analysing the interaction and convection of energetic particles in magnetically confined fusion plasmas yields results that build upon existing knowledge in the literature while also offering new insights and advancing the broader goals to the ongoing pursuit of achieving controlled nuclear fusion energy.

6. Significance

Energetic particles, a by-product of fusion reactions, can be both beneficial (heating the plasma) and detrimental (driving instabilities).Understanding how these particles interact with the magnetic field and plasma waves, and how they convect within the plasma, is crucial for achieving stable and efficient fusion. Instabilities triggered by energetic particle interactions and convection can disrupt the plasma, hindering sustained fusion reactions. Plasma instabilities, such as Alfven instabilities and fishbone modes, pose significant challenges for achieving and maintaining stable fusion reactions.This research helps identify the mechanisms behind these instabilities, paving the way for developing methods to control them and achieve a stable plasma.Insights gained from this research can apprise the experimental design and operation,guiding the development of diagnostic techniques and experimental protocols to study energetic particle behaviour more effectively.

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