RISK ANALYSIS
IN
PROJECT EVALUATION*

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I. CONCEPTUAL BASES OF RISK ANALYSIS
IN PROJECT APPRAISAL

In general, a project is conceived of as an investment proposal with a planned series of capital expenditures, i.e. cash outflows, undertaken in expectation of their generating a larger series of cash inflows at various times in the future. Thus, the basic aim of project appraisal procedure is simply to compare the future cash outflows with future cash inflows associated with the investment proposal so as to evaluate in advance the desirability of undertaking it.[1] However, future cash flows are uncertain. That is, the future cash flows associated with a project can hardly be estimated in advance, if not impossible, due to various socio-economic changes that the future brings about.

For the purpose of this paper regarding the uncertainty relevant to a project we should distinguish between two types of uncertainty. The first type refers to a situation in which neither the outcomes of the project nor their likelihoods are known. This is a complete uncertain situation where a different medhodology is required for project appraisal and it is

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[1] Sariaslan, Halil. "Project Evaluation Under Inflationary Conditions", A Paper Presented at the Training Program on Project Appraisal, Organized by State Investment Bank of Turkey and Islamic Development Bank, June 8-23, 1987, Ankara, p. 1.

out of the scope of this paper. The second type of uncertainty refers to a situation in which the likelihoods of the outcomes of a project can be specified. That is, rather than a best point estimate in time a range of possible values for the outcomes of a project can be specified.[2] In other words, risk refers to the situation in which a probability distribution of future returns can be established for the project and that the riskiness of a project can be defined as the variability or dispersion of its possible future returns.[3] Similarly, risk is the discrepancy between what is expected and what is happened. Therefore, the future returns of a risky project can only be estimated through a probability distribution since it reflects the variability or dispersion of possible returns.

As such, from the view point of project appraisal or evaluation, risk analysis is a methodology that takes into account the recognized fact that variables used in determining the profitability of a project depend on future events whose occurrence cannot be estimated with certainty. Its purpose is to determine risks associated with a project and to provide a means by which the various project outcomes can be converted to a form from which a judgmental decision regarding the possible range of future returns as well as the likelihood of each value within this range can be made.[4]

Despite the discernible context and purpose of risk analysis in project appraisal process it sometimes is confused with the term risk management by some novices. It should be indicated clearly that although the terms risk analysis and risk management are related to each other, risk management pertains to the development and implementation stage of a project's life cycle and it is an administrative function for a project manager, whereas risk analysis as defined above is related to the planning stage of the project's life cycle and is an economic and financial function of a project analyst. The term risk management; which indicates that hazards are present in advanced projects or systems and that they must be identified, analyzed, evaluated and controlled or rationally accepted, requires all foreseeable alternatives to remedy a hazardous situation and putting them together in an adequate manner for management decisions.[5] Thus, risk

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[2] Sarıaslan, Halil. "Risk Management', in Project Management (ed.) State Investment Bank, (Ankara: State Investment Bank, 1986), p. 183.

[3] Clifton, D.S. Jr. and Fyffe, D.E. Project Feasibility Analysis (New York: John Wiley and Sons, Inc., 1977), p. 160.

[4] Ibid, p. 160-161.

[5] Coutinho, John de S., Advanced Systems Development Management, (New York: John Wiley and Sons, Inc., 1977), p. 317.

management necessitates a different methodology than risk analysis in project appraisal process.[6]

II. METHODS OF RISK DETERMINATION: SENSITIVITY AND PROBABILITY ANALYSES

Analyzing and measuring risk inherent in a project is not an easy task since it involves determining the uncertainty prevailed over the life cycle of the project. However, a number methods ranging from informal judgements to complex statistical anlyses involving large scale computer models are available. Since riskiness of a project is the variability of its possible returns and risk anlysis is a methodology dealing with the problem of the variability of the possible range of each variable and the likelihood of each value within this range, the starting point for risk analysis involves determining the risk inherent in a project.[7] For this purpose, sensivity analysis is a more commonly employed method since it involves an investigation of how and to what extent the individual variables (factors or elements) and parameters involved in a project are likely to influence the returns of a project.[8]

Sensitivity Analysis:

The purpose of sensitivity analysis is to specify the possible range for a variable (e.g. price, quantity sold, variable unit cost) and to calculate the effect of changes in this variable on the project's profitability. Through such a procedure the analyst can determine the relative importance of each of the variables to profitability and then identify the variables that most affect the outcome of the project, i.e., the most criticial variables to which the profitability of the project is very sensitive.[9] In other words, sensitivity analysis finds out exactly how much the profitibility of a project will be changed by a given change in an input variable under the "ceteris paribus" condition.

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[6] For further explanation concerning risk management, see, Ibid. 317-324. Sarıaslan, Halil, "Risk Management"..., pp. 181-186.

[7] Pouliquen, Louis Y. Risk Analysis in Project Appraisal, World Bank Staff Occasional Papers, No: 11, (Washington, D.C.: IBRD, 1970), p. 2.
Brigham, Eugene F., Fundamentals of Financial Management, Third Edition, (New York: The Dryden Press, 1983), p. 336.

[8] Reutlinger, Shlomo., Techniques for Project Appraisal Under Uncertainty, World Bank Staff Occasional Papers, No: 10, (Washington, D.C.: IBRD, 1970), p. 67.

[9] Clifton and Fyffe, Project Feasibility Analysis..., p. 153.

For such an analysis any mathematical model that specifies and relates the effects of variables of a project to its profitability can be used. A most common and the simplest type of sensitivity analysis is that of break-even analysis where an analyst can examine how changes in four basic variables, namely, production volume (Q), price (P), fixed costs (F), and variable costs (V) affect the profitability of a project through a formula defined as

$$QP = F + VQ$$

Perhaps in another more detailed case, an analyst may want to determine the effect of changes in variables on net present value through the following model.

$$NPV = \sum_{t=1}^{n} \frac{(I_t - O_t)(1 - T) + F_t T}{(1+k)^t} - C_o + \frac{S}{(1+k)^n}$$

where:

NPV = the net present value of the project at time 0

n   = project's expected life

I_t = expected value of cash inflows in period t

O_t = expected value of cash outflows in period t

T   = corporate tax rate

F_t = depreciation charges on the assets in period t

C_o = value of initial investment at time 0

k   = cost of capital or required rate of return.

S   = salvage value of the project

The sensitivity of NPV to changes in the variables defined above can be determined on the basis of the most likely or base case values. For example, assume that for a project a NPV of $ 12.000 is calculated on the basis of a 30% cost of capital and a 48% corporate tax rate. The analyst then asks "what if" questions such as: "what if the cost of capital is 5% below or above the 30%?" or, "what if the corporate tax rate increases to 52% or decreases to 45%?" Based on each assumption a NPV can be calculated and the resultant changes can be determined, and also might be represented in graphical forms like the ones given below.
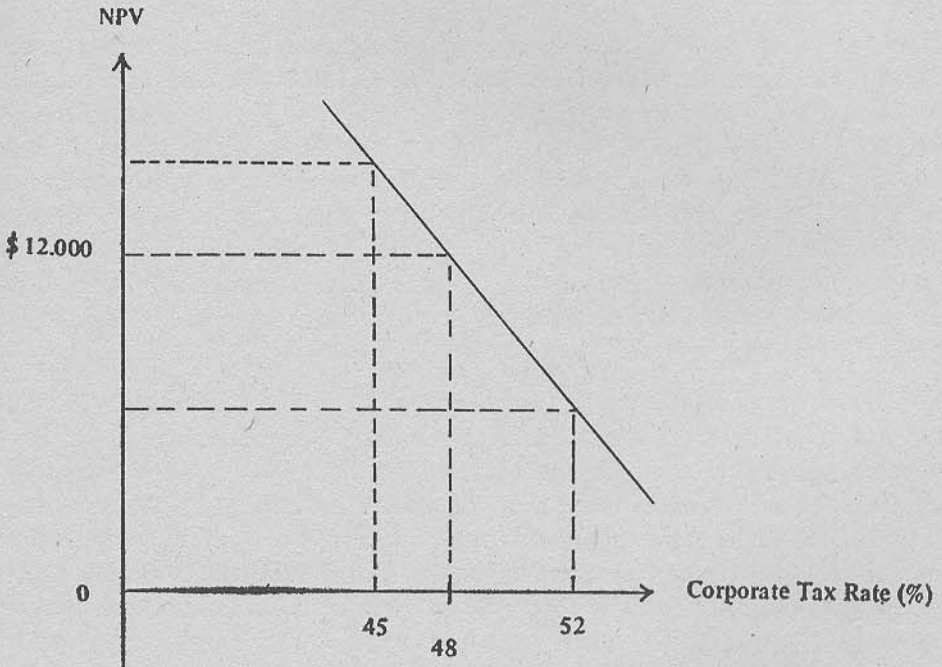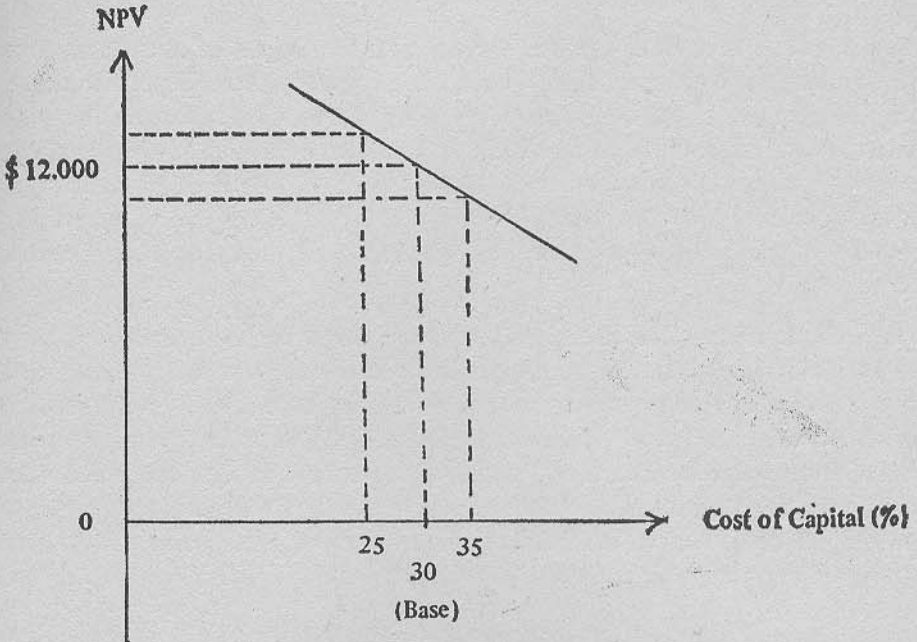
As seen, in a sensitivity analysis each variable is changed by specific percentages above and below the base case value and new NPVs are calculated (other variables being constant). The greater the resultant changes in NPV the more sensitive the NPV is to the changes in the

variables, or if shown on graph the steeper the slopes of the lines. Thus, in appraising and comparing projects, the projects with steeper sensitivity lines, i.e., greater changes in the NPV, should be regarded as riskier, since this situation indicates that a relatively small error in estimating variables would bring about a large error in the estimated profitability of a project. As such, sensitivity analysis provides useful insights into the relative riskiness of projects and is an indispensable method for risk determination.[10]

Needless to say that the possible range for a variable in a sensitivity analysis is as important to be determined as to conduct the analysis since the estimates of changes in variables are expected values taken from probability distributions.[11] Therefore, determining probability distributions render significance to sensitivity analysis as a risk determination method. That is to say "a project's risk depends on both (1) its sensitivity to changes in key variables and (2) the range of likely values of these variables - the variables probability distributions".[12]

Probability Analysis:

As explained in the preceding pages, sensitivity analysis aims at an investigation of how and to what extent the variables related to a project are likely to affect the profitability of a project. However, sensitivity analysis does not indicate the possible range of each individual variable and the probability of each value within this range. These are the purpose of probability analysis. Stated in different words, probability analysis specifies the possible range of each variable and the probability of occurrence of each value within this range.[13] Thus ,the core of probability analysis is to assign to each variable a probability distribution which is obtained in various ways.

Constructing a probability distribution for each individual variable involved in a project can perhaps be exhaustive in nature and time consuming, and yet may be redundant. For this reason, only variables that have been identified as being important or critical in determining the profitability of a project (most likely through a sensitivity analysis) need be involved in the probability analysis and that the project analyst should then collect the necessary information and data that will enable

[10] Brigham, Fundamentals of ..., p. 337.

[11] Ibid. p. 136.

[12] Ibid., p. 338.

[13] UNIDO., (Manual of Industrial Project Appraisal), Endüstri Projelerini Değerlendirme El Kitabı, (Ankara: Birleşmiş Milletler Sınai Kalkınma Teşkilatı, 1977), p. 135.

him to construct probability distributions relevant to the variables in consideration. As can be concluded, sensitivity and probability analyses are complementary to each other rather than being alternatives for each other, in that each renders significance to the other.

Derivation of a probability distribution for a sensitive or critical variable can be in various ways. Although some studies suggest to construct probability distributions through dialogue between the analyst and decision maker,[14] team evaluation, and Bayesian approaches,[15] the most commonly practiced two ways of deriving a probability distribution are the portrait or the standard statistical distributions and step-rectangular distribution approaches.[16]

The portrait approach resembles the portrait method used to identify suspects. Based on the limited information obtained a distribution is chosen among the standard statistical distributions, such as normal, beta, triangle, chi-square distributions, with the judgement that it fits the case best.

The step-rectangular approach starts with asking an informant the highest and the lowest expected values for a variable. Then the range specified is divided into two intervals and the informant is asked to assign a probability to each interval. The procedure goes on until the subdivision of intervals reaches a point where the informant is indecisive about assigning probabilities to subintervals any further. For example, a sales-manager may develop a step-rectangular probability distribution for the price of a product through the following steps.

Step 1: The lowest and the highest prices are specified as $50 and $80, respectively. The range of variation is divided into two intervals: 50 to 70 and 70 to 80 with assigned probabilities of 65 and 35 percents, respectively.
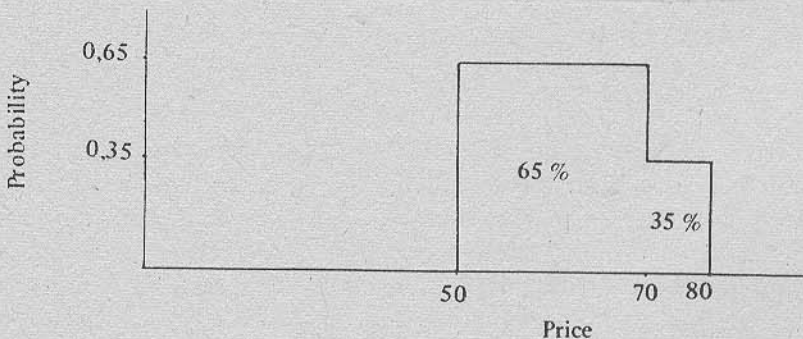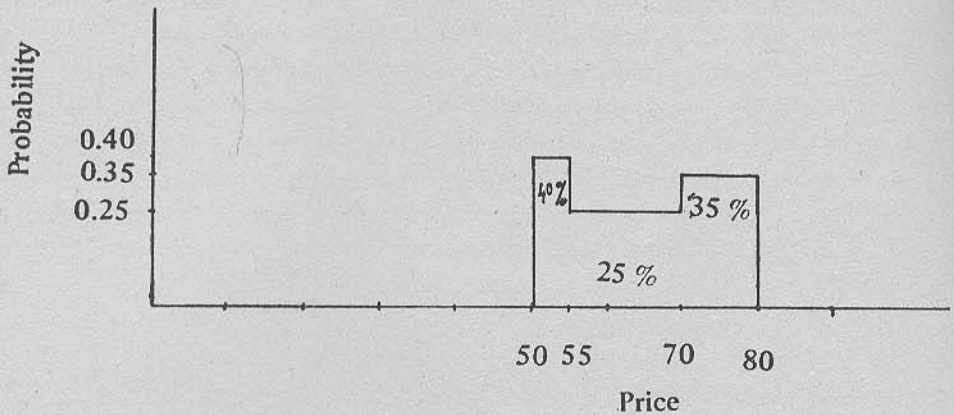
[14] Raiffa, Howard., Decision Analysis, (Reading, Mass.: Addison-Wesley, 1968), pp. 161-168.
[15] Winkler, Robert L., "The Consensus of Subjective Probability Distributions", Management Science, 15 - 2 (October 1968).
[16] Pouliquen, Risk Analysis in ..., pp. 52-61.

Step 2: The 50 - 70 range is divided into intervals of 50 to 55 and 55 to 70 with assigned probabilities of 40 and 25 respectively.

Step 3: The 70 - 80 range is divided into 70 to 75 and 75 to 80 intervals with probabilities 20 and 15 percents, respectively.
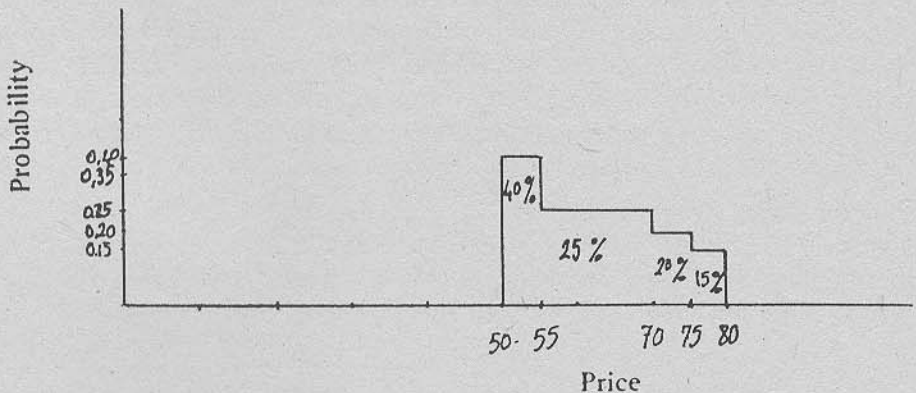
Practical experience have proven that the step rectangular approach is better than the portrait approach.[17] In case of discrete variables the step-rectangular approach can also be used. The only difference is that the probabilities can be assigned to specific values rather than to intervals. In summary, whatever approach is used in deriving a probability distribution, a probability analysis gives insight into the variability or dispersion of a variable in particular and into the risk of a project in general. As such, it is an essential stage of a risk analysis.[18]

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[17] Ibid. pp. 52-62.
[18] Ibid. p. 13.

III. STATISTICAL TECHNIQUES FOR RISK MEASUREMENT

Notwithstanding the fact that sensitivity and probability analyses indicate the presence of risks associated with a variable in particular and a project in general; however, such analyses do not provide a project analyst with any objective measure of risk. To evaluate or measure the risk associated with a probability distribution the following statistics need to be calculated from the distribution.

1. **Expected Value:** The expected value of a probability distribution can be calculated by the formula

$$E(A) = \sum_{i=1}^{m} A_i P_i$$

For example, assume that the net cash flows associated with a project indicate the following distribution.

Net Cash Flow (A_i)	Probabilities (P_i)
$ 5000	0.25
6500	0.60
8500	0.15

The expected value of cash flows it then:

$E(A) = 5000 \ (0.25) + 6500 \ (0.60) + 8500 \ (0.15)$

$E(A) = 6425$ dollars.

The analyst has now a better perception of the risk associated with the cash flow since the expected value of $6425 has a range of probable outcomes from $5000 to $8500 in contrast to the best estimate of $6500. Expected value can be considered to be a type of weighed average or a long-run average.

2. **Standard Deviation:** A more formalized statistics to measure risk is the standard deviation of a probability distribution which measures the variability or dispersion of expected values.

For the net cash flow example given above, the standard deviation is.[19]

$$S = \sqrt{\sum_{i=1}^{m} (A_i - E(A))^2 P_i}$$

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[19] Here we assume independence among the values of the distribution. This subject will be dealt in the last section of this paper.

$$S = \sqrt{(5000-6425)^2 \ (0.25) + (6500-6425)^2 \ (060) + (8500-6425) \ (0.15)}$$

$$S = 1075.6 \text{ dollars.}$$

As a measure of risk, the smaller the standard deviation the lower the risk associated with a variable or a project, whereas the higher the expected value the lower the risk is. This indicates that if one is to chose between two mutually exclusive projects, he or she should choose the project with the higher expected value. Moreover, if their standard deviations are different and the decision-maker is risk-averse the project with the low standard deviation should be selected.

However, in cases of evaluating a probability distribution per se a knowledge of the probability distribution is required to evaluate the risk of the distribution. For instance, if the distribution is assumed to be a normal one, the dispersion can be standardized and the areas under the normal curve and their associated probability can be found. Since this subject is explained in any statistical text book it is not dealt with in this paper.

3. **Coefficient of Variation:** In comparing the risks of two mutually exclusive projects with largely varying sizes, standard deviation will be misleading since a project of a large size would have a large standard deviation due just to having a large expected value. The coefficient of variation can overcome this problem because it is a standardized risk measure which is computed by dividing the standard deviation of a probability distribution by its expected value. The larger the coefficient of variation of a project the riskier it will be.

For instance, the coefficient of variation (V) for our net cash flow distribution given earlier, where expected value E(A) was $6425 and standard deviation S was $ 1075,6 is:

$$V = \frac{S}{E(A)}$$

$$V = \frac{1075.6}{6425}$$

$$V = 0.167$$

If we compare it, for example, with a distribution whose standard deviation is $5300 and expected value is $42400 it can be said that the

former distribution is riskier since it has a larger coefficient of variation than the latter whose coefficient of variation is

$$V_2 = \frac{5300}{42400} = 0.125$$

As seen, the advantage of the coefficient of variation over the standard deviation is that it can be used to compare distributions with varying expected values. As above, the standard deviations indicate that the second distribution is riskier than the first since it has a larger standard deviation while the coefficient of variations indicates just the opposite. Thus, coefficient of variation is a good measure to evaluate competing projects whose investments and/or cash flow probability distributions differ substantially in dollar size.

4. **Skewness and Kurtosis:** In cases of equal coefficients of variation the shape of distributions determined by their measures of skewness and kurtosis can give insight into the riskiness of distributions. Measures of skewness and kurtosis can be calculated in various ways, e.g. Pearson coefficients. However, among them the measures obtained from the third and the fourth moments about the expected value in the sense of the arithmetic mean is as follows:

$$\text{Third moment } (M_3) = \sum_{i=1}^{m} (A_i - E(A))^3 P_i$$

$$\text{Fourth moment } (M_4) = \sum_{i=1}^{m} (A_i - E(A))^4 P_i$$

Thus, the measure of skewness G_1 for a probability distribution is obtained by dividing the third moment M_3 by the third power (cube) of standard deviation S^3,

$$G_1 = \frac{M_3}{S^3}$$

If $G_1 = 0$, it is said that the distribution is symetrical. However, if the distribution has a long tail to the right, the distribution is positively skewed and that G_1 is positive. Conversely, if the distribution has a long tail to the left, G_1 is negative and the distribution is negatively skewed as shown below.
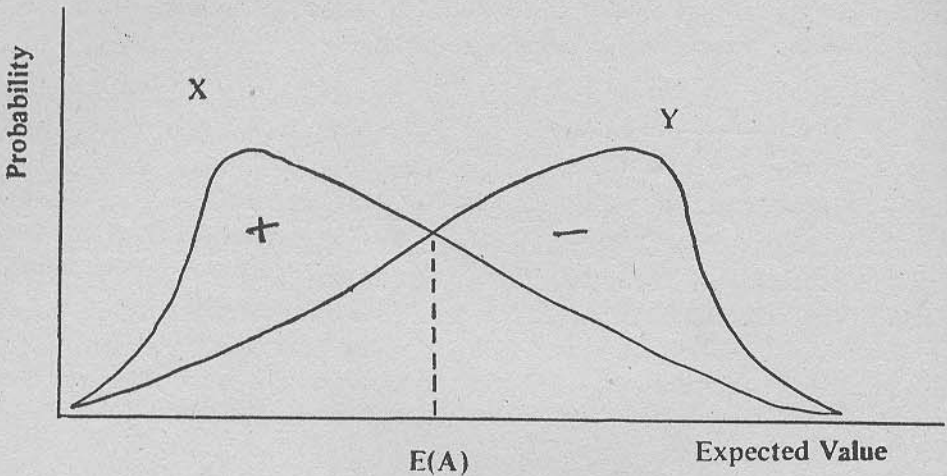
A positively skewed distribution indicates less risk since the sum of cubes of deviations above the expected value is greather than those below the expected value in contrast to a negatively skewed distribution with the same coefficient of variation. Consequently, for the distributions with equal coefficients of variation the one with the greatest measure skewness indicates less risk.

Finally, the measure of kurtosis can also provide some information concerning the riskiness of the distributions with equal coefficients of variations and skewnes in cases of distributions having more than one peak.. In such cases, the measure of kurtosis together with the measure of skewness will shed light on the riskiness of the distributions. Kurtosis (G_2) as the measure of peakedness or flatness of a distribution is calculated by dividing the fourth moment M_4 by the fourth power of standard deviation S^4 (i.e., the square of variance) as given below.[20]

$$G_2 = \frac{M^4}{S^4} - 3$$

That is,

$$G_2 = \frac{\sum\limits_{i=1}^{m} (A_i - E(A))^4 P_i}{\sum\limits_{i=1}^{m} (A_i - E(A))^2 P_i} - 3$$

[20] Ferguson, George A. Statistical Analysis, (New York: McGraw-Hill, 1971) p. 68.

where, E(A) is the expected value of all probable A_i values of a distribution and P_i denotes the probabilities associated with A_i values.

The measure of kurtosis equals to zero for a symetrical (normal) distribution. The values above and below zero shows whether a distribution is more or less peaked than a normal distribution. Moreover, for the distributions with equal coefficients of variation, it can be said that the one with the greatest measure of kurtosis is the least risky one. For example in the below given figure representing distributions of cash flows X is riskier than Y.
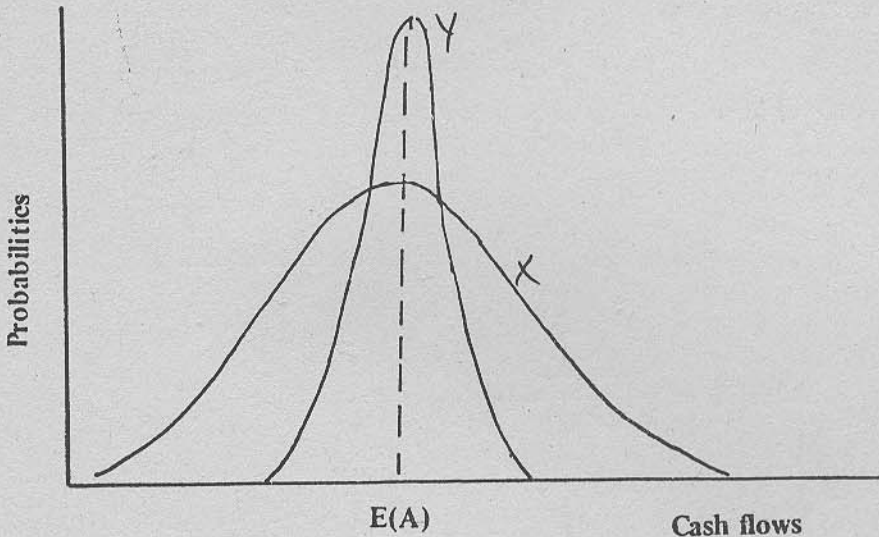
IV. EVALUATING RISKY PROJECTS

Up to this point we have explained how the risk associated with a variable in particular or a project in general can be determined and measured through various methods. In this section of the paper we will discuss how risky projects can be evaluated or appraised on the basis of the explainations provided in the foregoing pages.

In appraising risky projects the following methods which will be explained separately are used.

1. **Expected Net Present Value Method:** One method of appraising a risky project is to calculate its expected net present value (i.e. the expected value of the net present value) based on the probability distributions of annual cash flows, preferably obtained through sensitivity and probability analyses. To illustrate the calculations involved assume that a

project costing 9000 dollars at time zero is likely to generate the net cash flows during its three-year life, further assuming that it has no salvage value.

Year 1		Year 2		Year 3	
Net Cash Flows (A$_i$)	Probability (P$_i$)	Net Cash Flows (A$_i$)	Probability (P$_i$)	Net Cash Flows (A$_i$)	Probability (P$_i$)
$ 4000	0.20	$ 3000	0.25	$ 6000	0.15
5000	0.30	4000	0.40	7000	0.25
6000	0.40	5000	0.20	8000	0.45
8000	0.10	6000	0.15	9000	0.15

The expected net present value, E(NPV), of the project is obtained as the discounted sum of the expected annual net cash flows, $E_t(A)$, in each year as given below.

$$E_t(A) = \sum_{i=1}^{m} A_{it} P_{it} \qquad \text{(for all t's, } t = 1,2,\ldots\ldots,n)$$

and

$$E(NPV) = \sum_{t=0}^{n} \frac{E_t(A)}{(1+R)^t} - C_o$$

where:

$E_t(A)$ = expected net cash flow in year t

A_{it} = net cash flow for the (i) th probability in year t

P_{it} = probability of A_{it}

m = number of probable net cash flows in each year

n = final time period in which cash flows are obtained

R = risk - free discount rate. That is, the cost of capital that does not embody a risk preium. Otherwise, double counting would result with respect to this method.

Co = Cost of project at time 0

Thus,

$E_1(A) = 4000\ (0.20) + 5000\ (0.30) + 6000\ (0.40) + 8000\ (0.10)$
$E_1(A) = 5500$ dollars.
$E_2(A) = 3000\ (0.25) + 4000\ (0.40) + 5000\ (0.20) + 6000\ (0.15)$
$E_2(A) = 4250$ dollars.

$E_2(A) = 6000 \ (0.15) + 7000 \ (0.25) + 8000 \ (0.45) + 9000 \ (0.15)$

$E_3(A) = 7600$ dollars.

The expected net present value can be calculated by assuming a 20 percent riskfree cost of capital

$$E(NPV) = \frac{5500}{(1+0.20)} + \frac{4250}{(1+0.20)^2} + \frac{7600}{(1+0.20)^3} - 9000$$

$E(NPV) = 11933 - 9000$

$E(NPV) = 2933$ dollars.

To have some information concerning the risk associated with it, we need to calculate the standard deviation about $E(NPV)$. If we assume serial independence of net cash flows for future periods, the standard deviation for the probability distribution of the expected net present value is calculated by the following formula.[21]

$$S = \sqrt{\sum_{t=0}^{n} \frac{S_t^2}{(1+R)^{2t}}} \qquad \text{(If } t_0 \text{ is certain then } t=1,2,\ldots\ldots, n)$$

Here, S_t denotes the standard deviation of the probability distribution of probable net cash flows in year t, and is calculated by

$$S_t = \sqrt{\sum_{i=1}^{m} (A_{it} - E_t(A))^2 \ P_{it}} \qquad \text{(for all t's, } t=1,2,\ldots\ldots,n)$$

Where, all the symbols are as defined before.

For our example, the standard deviation of net cash flows for year 1 is:

$S_1 = \sqrt{(4000-5500)^2 \ (0.20) + (5000-5500)^2 \ (0.30) + (6000-5500)^2 \ (0.40)}$
$\qquad \overline{\qquad\qquad\qquad\qquad + (8000-5500)^2 \ (0.10)}$

$S_1 = \sqrt{1250000}$

$S_1 = 1118$ dollars.

Additionally, for the years 2 and 3, respectively

$S_2 = 994$ dollars.

$S_3 = 916$ dollars.

[21] Van Horne, James., Financial Management and Policy, Fifth Edition ,(London: Prentice-Hall International, Inc., 1980), p. 152.

Thus, with the assumption of serial independence, the standard deviation about the expected net present value is

$$S = \sqrt{\frac{1118^2}{(1+0.20)^2} + \frac{994^2}{(1+0.20)^4} + \frac{916^2}{(1+0.20)^6}}$$

S = 1275 dollars.

The coefficient of variation for the distribution of the expected net present value is

$$V = \frac{S}{E(NPV)} = \frac{1275}{2933}$$

V = 0.43

Consequently, based on E(NPV), S, and V the risk associated with this project can be compared with other competing alternative projects as explained in the previous section, However, to evaluate the riskiness of this project per se, the dispersion of the distribution of E(NPV) should be standardized. For instance, if it is assumed that the distribution is normal, through E(NPV) and S we can calculate the probability of having a net present value less or more than a specified amount of X by standardizing the difference of X and E(NPV) as

$$Z = \frac{X - E(NPV)}{S}$$

Then, the probability is found from the standardized normal curve distribution by determining the area under the curve to the left or to the right of E(NPV) on the basis of the calculated Z, as explained in all statistics text books. However, if the distribution is not normal probability statements concerning the risk of the project may be made by Tchebycheff's inequality.[22]

At this point, it has to be pointed out that in calculating the standard deviation for cash flows where the assumption of serial independence is not valid, standard deviation cannot be calculated by the formula given earlier. In this case, if the cash flows are perfectly correlated, standard deviation is significantly larger than that in the case of independence, and thus the formula becomes.[23]

[22] For this technique, see Stephen L. Buzby, "Extending the Applicability of Pro-
babilistic Management and Control Models", Accounting Review, 49 (January
1974), 42-49.
[23] Van Horne, James. Financial Management and Policy, p. 158.

$$S = \sum_{t=0}^{n} \frac{S_t}{(1+R)^t}$$

Therefore, for our given example with the assumption of perfect correlation

$$S = \frac{1118}{(1+0.20)} + \frac{994}{(1+0.20)^2} + \frac{916}{(1+0.20)^3}$$

$S = 2152$ dollars.

Finally, the standard deviation for less than perfectly correlated cash flows is determined to be somewhere between these two extremes.[24] Additionally, for the projects where the cash flows are moderate; i.e., neither approximately independent nor perfectly correlated, the problem of moderate correlation should be dealt with by a method requiring derivation of conditional and/or joint probabilities. That is, the probabilities of moderately correlated cash flows should first of all be specified in terms of conditional and/or joint probabilities. Nonetheless, calculation of the expected net present value in this case is the same as before, but computing the standard deviation for complex situations becomes cumbersome and a computer simulation approach becomes necessary.[25]

2. Simulation Approach: Computer simulation reconciling sensitivity probability analyses is presently a very promising approach in dealing with appraising risky projects and quantifying their risks. The approach is usually named as Monte Carlo simulation. Although building a simulation model and designing experiments as well as handling the statistical problems associated with it is not an easy job to explain succinctly here[26] the approach may simply be summarized by several consecutive steps as follows.

1. The first step in a computer simulation is to determine a probability distribution for each of the critical key variables in a project.

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[24] For a formula to calculate the standard deviation for such cash flows, see Freedrick S. Hiller, "The Deviation of Probabilistic Information for the Evaluation of Risky Investments", Management Science, 9 (April 1963), pp. 443-457.

[25] For example, see Ibid. pp.
Bonini, Charles p., "Comment on Formulating Correlated Cash Flow Streams", Engineering Economist, 20 (Spring 1975), 210-214.

[26] For a detailed discussion of simulation modeling and Analysis, see Halil Sarıaslan, (Simulation Techniques in Queuing Systems) Sıra Bekleme Sistemlerinde Simulasyon Tekniği, (Ankara: Siyasal Bilgiler Fakültesi, 1986).

2. A random number generating mechanism associated with the probability distributions is used to produce variates (i.e., simulation values) for a variable.

3. When for all critical values a random variate is generated, a set of cash flows is calculated.

4. Using this set of cash flows a net present value of the project is calculated.

5. This process of the computer run is repeated many times. At the end of each computer run a net present value is colculated and stored. For these present values, perhaps 400 of them, a frequency distribution and thereby a probability distribution can be derived for the net present value.

6. Finally, on the basis of the distribution the expected net present value, standard deviation, coefficient of variation, and perhaps the measures of skewness and kurtosis can be calculated to judge the riskiness of the project; as explained in the preceding pages.

At this point it should be added that after the first introduction of this approach by Hertz,[27] the computer simulation in project appraisal has made significant progress and today there have been many well-developed computer simulation packages.[28]

3. **Risk- Adjusted Discount Rate:** A relatively simple yet more widely practiced method for appraising risky projects is the risk-adjusted discount rate method.[29] This method indicates that when discounting cash flows of risky projects a risk adjustment factor should be added to the discount rate, that is the discount rate should be increased.

For example, if the risk-free discount factors is $(1+i)^t$, to account for the risk involved a risk premium (e) is added to it. Thus, the discount factor becomes $(1+i+e)$. More specifically if the risk-free discount factor is $(1+0.20)^t$ and the risk premium is assumed to be 10% then the discount factor is taken as $(1+0.20+0.10)^t = (1+0.30)^t$.

Determination of the risk premium is based on the judgment regarding the variability of the cash flows over the life of the project. However,

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[27] Hertz, David B., "Risk Analysis in Capital Investments", Harvard Business Review, (Jan.-Fab. 1964), pp. 95-106.

[28] A program package called Interactive Financial Planning System (IFPS) is developed at the University of Florida and is ready for commercial use.

[29] Petry reports that among the USA corporations he surveyed 71% of them accounted for risk in project appraisal and that 30% of that 71% used the risk - adjusted discount rate method, see Petry, Glenn H. "Effective Use of Capital budgeting Pools", Business Horizons, 19, No: 5 (Oct. 1975), pp. 57-65.

such a practice can lead to erroneous results by underestimating the value of a project since in practice the discount rate usually embodies a risk premium and adding another risk premium would be double counting. Secondly, keeping the risk premium constant throughout the life of the project would undervalue the worth of project if risks associated with a project vary during the development stages of the project as in the case of research and development projects. In such cases the project analyst should certainly differentiate between the risks attributable to each stage of project development. Thirdly, the selection of the risk premium is frequently arbitrary even though it seldom is based on the coefficient of variation of cash flows, i.e. the higher the coefficient of variation the greater the risk premium will be.

4. **Certainty Equivalent Method:** An alternative method to the risk-adjusted discount rate method is the one called the certainty equivalent method which requires to first determine the certainty equivalent values of cash flows and then to discount the certainty equivalent values to present time by using a risk-free discount rate.

In order to determine the certainty equivalent value of each cash flow in period t, each cash flow is multiplied by its corresponding coefficient of certainty equivalent, α_t, which is obtained by dividing the certain amount of a cash flow in period t by the risky (uncertain) amount of the cash flow, A_t. Then the certainty equivalents of cash flows are discounted by using a risk-free discount rate to calculate the net present value. That is,

$$\alpha_t = \frac{A^*_t}{A_t} = \frac{\text{Certain Cash Flow}}{\text{Risky Cash Flow}}$$

$$NBD = \sum_{t=0}^{n} \frac{\alpha_t A_t}{(1+i)^t}$$

The certain amount of a risky cash flow is perceived as the amount to which the decision maker is indifferent with respect to a risky or uncertain cash flow. For instance, a decision-maker may be indifferent between a certain amount of $15000 and a risky amount of $25000 in the year 2. Then, the certainty equivalent coefficient is

$$\alpha_2 = \frac{15000}{25000} = 0.6$$

No doubt that certainty equivalent coefficients α_t takes values bet-

ween 1 and 0, and that the larger the degree of risk associated with a cash flow the closer α_t is to 0.

V. CONCLUSION

Risk analysis is a useful approach to dissipate uncertainties associated with a project in the future and to focus our attention on the critical points of a project in practice or implementation stage, since it enables us to attack problems that we would otherwise avoid and to make decisions we would not otherwise feel competent to make. As pointed out by Pouliquen risk analysis provides the following advantages.[30]

a)   Risk analysis provides a complete picture of the project from which a decision can be made more quickly.

b)   Risk analysis permits more people to make contributions to project appraisals through a highly efficient channel of communication.

c)   It provides convenience for people to express their judgment for it is easy to express judgment in probabilitistic terms than in terms of a best estimate.

d)   It enables analysts to use a great deal of information which might otherwise be lost in the conventional method.

[30] Pouliquen, pp. 76-79.