



## FORECASTING WATER DEMAND BY USING MONTE CARLO SIMULATION

Yard.Doç.Dr.Selim Tüzüntürk<sup>1</sup>

Arş.Gör.Arzu Eren Şenaras<sup>2</sup>

Prof.Dr. H. Kemal Sezen<sup>3</sup>

### Abstract

Numerous forecasting methods such as qualitative methods, naïve approach, time series methods, judgmental methods, artificial intelligence methods, regression analysis, simulation methods and etc. are available for practitioners in the forecasting literature. The main goal of this paper is to present how Monte Carlo Simulation Method is used for forecasting the demand practically and for forecasting the future demands that would help managerial decisions. For this purpose, the data that is comprised of the demand of the dispenser size (19 liters) water bottles of a water company in Bursa was gathered and Monte Carlo Simulation monthly and seasonal forecasts were obtained. Results show that the monthly and seasonal actual values and estimated values were close to each other. By decreasing uncertainty about future through Monte Carlo forecasting method, company managers had the advantage of making healthy future decisions about controlling their inventory, purchasing new equipment, hiring a new worker and other sources.

**Key Words:** Monte Carlo Simulation, Demand Forecasting, Dispenser Size Water Bottles.

## MONTE CARLO BENZETİMİ KULLANARAK SU TALEBİNİN KESTİRİMİ

### Öz

Kestirim literatüründe kalitatif yöntemler, naïve yaklaşım, zaman serileri yöntemleri, muhakeme yöntemleri, yapay zeka yöntemleri, regresyon analizi, simülasyon yöntemleri ve benzeri yöntemler uygulamacıların kullanımına sunulmuştur. Bu makalede, asıl amaç Monte Carlo Benzetim Yönteminin talep kestiriminde uygulamalı olarak nasıl kullanıldığının gösterilmesi ve yönetici kararlarına yardımcı olacak gelecekteki taleplerin kestirilmesidir. Bu amaçla, damacana su (19 litre) talebini içeren veriler Bursa'daki bir su şirketinden elde edilmiştir ve aylık ve mevsimlik Monte Carlo kestirimleri elde edilmiştir. Sonuçlar aylık ve mevsimsel gerçek değerler ile tahminlerin birbirine yakın olduğunu göstermektedir. Gelecekteki belirsizliğin bilinmesi ile firma yöneticileri envanterlerini kontrol etme, yeni teçhizat satın alma, yeni bir elemanı işe alma ve diğer kaynaklar hakkında gelecekte sağlıklı kararlar alma avantajına sahip olmuştur.

**Anahtar Kelimeler:** Monte Carlo Benzetimi, Talep Kestirimi, Damacana Su.

### 1. INTRODUCTION

Forecasting is the art and science of predicting future events (Heizer and Render, 2000: 38). Suppose that you have an observed time series variable that you are concerned about and the observations of this variable are sequencing such as  $x_1, x_2, x_3, \dots, x_n$ . You might be interested in the future values of this variable for planning or decision making situations. When this is the case, the basic problem is to estimate future values such as  $x_{n+k}$  (Chatfield, 1996:66). Forecasting methods resolve this difficulty. In the related literature,

<sup>1</sup> Uludağ Üniversitesi, İktisadi ve İdari Bilimler Fakültesi, Ekonometri Bölümü, [selimtuzunturk@uludag.edu.tr](mailto:selimtuzunturk@uludag.edu.tr)

<sup>2</sup> Uludağ Üniversitesi, İktisadi ve İdari Bilimler Fakültesi, Ekonometri Bölümü, [arzueren@uludag.edu.tr](mailto:arzueren@uludag.edu.tr)

<sup>3</sup> Uludağ Üniversitesi, İktisadi ve İdari Bilimler Fakültesi, Ekonometri Bölümü, [kemal@uludag.edu.tr](mailto:kemal@uludag.edu.tr)



several methods are used such as qualitative methods which are based on the opinions of experts, naïve approach, time series methods, judgmental methods (e.g. Delphi method), artificial intelligence methods (e.g. neural networks, support vector machines and machine learning), regression analysis, simulation methods and so on. In forecasting processes following steps are taken into consideration (Hanke and Wichern, 2005: 5):

1. Problem formulation and data collection
2. Data manipulation and cleaning
3. Model building and evaluation
4. Model implementation
5. Forecast evaluation.

Forecasting has several applications. Some leading forecasting topics that were performed by using Monte Carlo Simulation Method are as follows: wind energy (Desrochers, Blanchard and Sud, 1986), weather forecasting (Cubasch and others, 1994; Barnett, 1995), travel demand forecasting (Kitamura and others, 2000), sales forecasting (Demirdögen, 1998; Patır and Yıldız, 2003), transportation forecasting (Bruno and Clarke, 2003), pharmaceutical industry demand forecasting (Kiely, 2004), electricity demand forecasting (McQueen, Hyland and Watson, 2004), political forecasting (Udina and Delicado, 2005), budgeting forecasting (Aygören and İlem, 2006), health forecasting (Yu and others, 2009), hotel arrivals forecasting (Zakhary and others, 2009), automotive demand forecasting (Dobrican, 2013), private investment forecasting (Tadeu and Silva, 2013), supply chain forecasting (Križanová and others, 2013), financial forecasting (Hujala and Hilmola, 2009; Shahbandarzadeh, Slimifard and Moghdani, 2013; Park and Tomek, 2014), water demand forecasting (Almutaz and others, 2013; Hague and others, 2014) and etc.

As one can see, Monte Carlo Simulation Methods can be used in many cases. Especially, in such cases when it is considered in business world that managers make decisions without knowing what will happen in future. The usage of Monte Carlo Simulation Method enables them to estimate critical questions such as future sales, future demands and so on. In this way, they can control their inventory by being informed about the scientific estimated values of what sales and demands will be in the future. Thus, they can purchase enough new equipment, hire workers and can make use of their sources properly. A good forecast, then, becomes critical in all aspects of a business world. Heizer and Render (2000) explains that the forecast is the only estimate of demand until actual demand becomes known.

In the context explained above, the main aim of this study is to present how Monte Carlo Simulation Method is used for forecasting the demand practically. For this purpose, the data that is comprised of the demand of the dispenser size (19 liters) water bottles of a water company was used in the application section. Rest of the paper organized as follows: next section describes the Monte Carlo Simulation Method. Section three presents the application. And the last Section covers the conclusions of this study.

## 2. MONTE CARLO SIMULATION

A model is a representation of a system or process (Carson, 2004: 9). Simulation is experimentation with models (Korn, 2007: 1). Simulation is a powerful tool for the analysis of new system designs, retrofits to existing systems and proposed changes to operating rules (Carson, 2004: 9). A simulation model of a complex system can only be an approximation to the actual system (Law, 2005: 24). Simulation models may be further classified as follows:



- Static or Dynamic
- Deterministic or Stochastic
- Discrete or Continuous

A static simulation model represents a system at a particular point in time (Banks et al., 2005: 13). A dynamic system model relates model-system states to earlier states (Korn, 2007: 1). If a system behavior can be represented fully, it is deterministic; otherwise, it is stochastic (Sezen and Günel, 2009:21). A discrete model is one that changes only at discrete points in time, and not continuously (Carson, 2004: 9).

In the above classification, Monte Carlo simulation is classified as a static simulation model (Banks et al., 2005: 13). This method of simulation is very closely related to random experiments, experiments for which the specific result is not known in advance. Monte Carlo simulation can be considered as a methodical way of doing so-called what-if analysis (Raychaudhuri, 2008: 91). Although we are primarily interested in the use of the Monte Carlo technique for simulating probabilistic events, it can also be used in certain completely deterministic problems that cannot be solved analytically (Pegden and Shannon, 1990: 12). The name “Monte Carlo” simulation or method originated during World War II, when this approach was applied to problems related to the development of the atomic bomb (Law, 2007: 73). The Monte Carlo method has its roots in the work of American mathematician, Stanislaw Ulam (Travers and Gray, 1981: 327). It was invented by Ulam in the late 1940s and it was named by Nicholas Metropolis.

Monte Carlo simulation builds a model of possible outcomes by substituting random values from a specified distribution of all possible values for each input variable. The model continues to recalculate outcomes by using a different set of random values from the probability distribution of each variable defined by the researcher (Valle and Norvell, 2013: 35). Methods of simulation are based on the production of random variables that are distributed according to a distribution that is not necessarily explicitly known (Robert and Casella, 2004: 35). Usually, random numbers that are generated from uniform distribution on the interval [0, 1], which provides basic probabilistic representation of randomness and also other distributions, require a sequence of uniform values to be simulated (Robert and Casella, 2004: 35).

A simulation calculates numerous scenarios of a model by repeatedly picking values from a user-predefined probability distribution for the uncertain variables and using those values for the model (Mun, 2006: 74). In the Monte Carlo technique, artificial data are generated through the use of a random number generator and cumulative distribution of interest. Although a number of methods have been used in the past to generate random numbers, today we use algorithms executed on computers. Because using an algorithm generates the random numbers, they are in fact not truly random, and so they are called pseudo-random-numbers-, meaning that what is produced is in fact reproducible and hence not random (Pegden and Shannon, 1990: 12). The technique breaks down into five steps (Heizer and Render, 2000: 852-855):

**Step 1. Establishing Probability Distributions** (setting up a probability distribution for important variables): One common way to establish a probability distribution for a given variable is to examine



historical outcomes. We can find the probability, or relative frequency, for each possible outcome of a variable by dividing the frequency of observation by the total number of observations.

**Step 2. Building a Cumulative Probability Distribution for each Variable:** The cumulative probability distribution for each level of demand is the sum of the number in the probability column added to the previous cumulative probability.

**Step 3. Setting Random Number Intervals** (establishing an interval of random numbers for each variable): Once we have established a cumulative probability distribution for each variable in the simulation, we must assign a set of numbers to represent each possible value or outcome. These are referred to as random number intervals (For example; intervals of random numbers: 0,1 through 0,15, 0,16 through 0,45, 0,46 through 0,70, and 0,71 through 1,0).

**Step 4. Generating Random Numbers:** Random numbers may be generated for simulation in two ways. If the problem is large and the process under study involves many simulation trials, computer programs are available to generate the needed random numbers. If the simulation is being done by hand, the numbers may be selected from a table of random digits.

**Step 5. Simulating the Experiment** (actually simulating a series of trials): We may simulate outcomes of an experiment by random numbers. For example if the random number chosen is 12 and the interval 01 through 15 represents a daily demand for first demand value.

Simulation models are increasingly being used in problem solving and to aid in decision-making (Sargent, 2004: 17). Monte Carlo simulation can help predict income and demand for services (Kalar, 2013: 43). Valle and Norvell (2013) explain that organizations have discovered the benefits of using simulation. Researchers underline this discovery by giving some real life examples:

*“For example, **General Motors, Procter and Gamble, and Eli Lilly** use simulation to estimate both the average return and riskiness of new products. This helps determine which products come to market and which products need further development. **Sears** uses simulation to determine how many units of each product line to order from suppliers and **Lilly** uses it to determine the optimal plant capacity that should be built for each drug. Marketing departments use simulation for such activities as improving sales forecasts and modeling consumer behavior on the internet.”*



### 3. MONTE CARLO SIMULATION APPLICATION

The main aim of this study is to present how Monte Carlo Simulation Method is used for forecasting the demand practically. For this purpose, the data that is comprised of the demand of the dispenser size (19 liters) water bottles of a water company was used in the application section. Especially, we focused on forecasting the middle term<sup>4</sup> demand of the dispenser size (19 liters) water bottles by using Monte Carlo Simulation Method. To do so, data was gathered from a water company that runs for home delivery of this product. This water company was established in 2007 and located in Nilüfer district Bursa City. Service zones include İhsaniye, Esentepe, Karaman, Ataevler, Cumhuriyet, and Barış quarters.

The data that is analyzed in this study consists of the number of dispenser size water bottles that were demanded by the actual members and potential members (new customers) of this company. The frequency of the data is monthly. The period of this data is composed of 39 months that begins by January 2011 till March 2014. All data obtained from the computer archives of the company by the owners' permissions.

The analysis part has three stages:

**1. First Stage:** In the first stage, by using the actual data between January 2011 and March 2013 Period (27 Months) Monte Carlo Simulation Method was performed. Hence, the monthly data between April 2013 and March 2014 were forecasted. Then, these forecasted monthly data enabled us to compare them with the actual data between April 2013 and March 2014.

**2. Second Stage:** In the second stage, by using the actual data between January 2011 and March 2014 Period (39 Months) Monte Carlo Simulation Method was performed. Hence, the monthly data between April 2014 and March 2015 were forecasted. Thereby, the forecasted values that present the future monthly middle term demands of the dispenser size water bottles for this water company were obtained.

**3. Third Stage:** In the third stage, data between January 2011 and March 2013 were divided into three seasons. First season is composed of January, February and March. Second season is composed of April, May, June, July, and August. Third season is composed of September, October, November and December. Hence, seasonal data between 2013 and 2014 were forecasted. Then, these forecasted seasonal data enabled us to compare them with the actual seasonal data between 2013 and 2014.

**4. Fourth Stage:** In the fourth stage, data between January 2011 and March 2014 were divided into three seasons. First season is composed

<sup>4</sup> Time horizons can be chosen subjectively that is directly related to the researchers interest. Time horizons generally can be divided into immediate term (less than one month), short term (one to three months), medium term (three months to two years) and long term (two years or more) (Makridakis and Wheelwright, 1989: 27).



of January, February and March. Second season is composed of April, May, June, July, and August. Third season is composed of September, October, November and December. Hence, seasonal data between 2014 and 2015 were forecasted. Thereby, the forecasted values that present the future seasonal middle term demands of the dispenser size water bottles for this water company were obtained.

### 3.1. First Stage

The initial step of the Monte Carlo Simulation is sorting the data in the ascending order as follows:

**Table 1. Ascending Order of the Monthly Data (January 2011-March 2013 Period)**

No	Data	No	Data	No	Data
1	5313	10	7083	19	8005
2	5958	11	7244	20	8031
3	6582	12	7267	21	8166
4	6733	13	7320	22	8396
5	6745	14	7398	23	8405
6	6827	15	7621	24	8458
7	6905	16	7894	25	8622
8	7030	17	7943	26	8628
9	7077	18	7955	27	9374

In the second step, the difference between the maximum and minimum value is computed ( $9374-5313=4061$ ). Then, the numbers of intervals is calculated with the following formula:  $\sqrt{n} = \sqrt{27} = 5,19$ . Namely, the number of intervals found 5. After then, 4061 is divided by 5 and found 812,2. This last number shows the margin between upper and lower bound for each one of the 5 intervals. The calculated interval values are found as is in the following table:

**Table 2: Interval Values for January 2011-March 2013 Period**

Interval No.	Interval Values
1	5313-6125
2	6126-6937
3	6938-7749
4	7750-8561
5	8562-9374

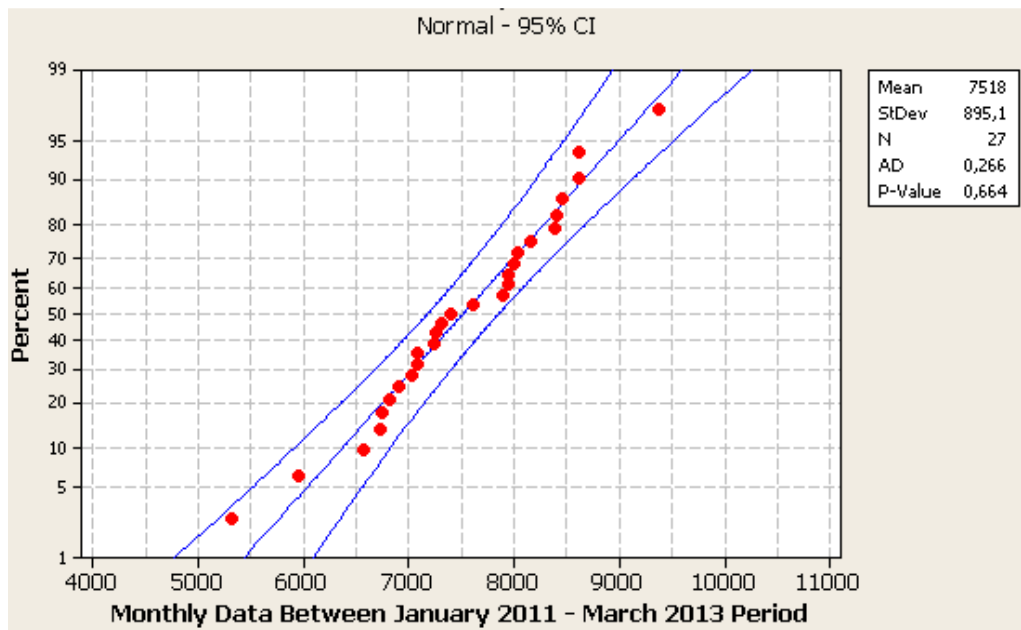
Then, by using the table above frequency distribution is computed and the values of the frequency distribution are converted into the probability values and then cumulative probability distribution is computed as is seen in the Table below.



**Table 3: Probability Distribution of Dispenser Size Water Bottles Demand**

Interval No.	Interval Values	Frequency	Probability of Occurrence	Cumulative Probability
1	5313-6125	2	0,07	0,07
2	6126-6937	5	0,19	0,26
3	6938-7749	8	0,30	0,56
4	7750-8561	9	0,33	0,89
5	8562-9374	3	0,11	1,00

Then, the distribution of the monthly data that was between January 2011 and March 2013 Period is tested.



**Figure 1. Probability Plot and Anderson Darling Test Values**

In the probability plot, all the points are observed between the lower and upper bounds which means that the data follows a normal probability distribution. Also, as a formal test Anderson Darling test result shows that null hypothesis cannot be rejected, which means that the data follows a normal probability distribution ( $p\text{-value}=0,664 \geq \alpha=0,05$ ). For this reason, fifty random numbers which follow normal probability distribution were generated for twelve months (April 2013 to March 2014) by using Minitab. The generated random numbers are seen in the table below:

**Table 4: Generated Random Numbers for Twelve Months (50 Trials)**

Random Numbers	RN1	RN2	.....	RN50
April 2013	0,2728	0,6090	.....	0,3710
May 2013	0,1862	0,2760	.....	0,2978
June 2013	0,5231	0,6966	.....	0,1238
July 2013	0,7325	0,1558	.....	0,1077



August 2013	0,0209	0,0287	.....	0,3390
September 2013	0,7796	0,9640	.....	0,3850
October 2013	0,0391	0,5194	.....	0,4946
November 2013	0,5973	0,4807	.....	0,1597
December 2013	0,3962	0,0727	.....	0,1034
January 2014	0,8767	0,4463	.....	0,0064
February 2014	0,1224	0,0703	.....	0,1810
March 2014	0,5923	0,3956	.....	0,2176

Then cumulative probability distribution that was computed in the table above is used for creating the probability intervals (the upper and lower intervals) probabilities as is seen in the table below:

**Table 5: Probability Intervals**

Interval No.	Probability Intervals
1	0-0,07399
2	0,074-0,2589
3	0,259-0,5559
4	0,556-0,8889
5	0,889-1,0

Averages of upper and lower bound values (interval values) are computed for each one of the interval that the random number was classified. The following table presents the average values for each classification.

**Table 6: Average Values for Each Classification**

	RN1	RN2	.....	RN50	Row Average
April 2013	7344	8156	.....	7344	7473
May 2013	6531	7344	.....	7344	7344
June 2013	7344	8156	.....	6531	7555
July 2013	8156	6531	.....	6531	7327
August 2013	5719	5719	.....	7344	7051
September 2013	8156	8968	.....	7344	7701
October 2013	5719	7344	.....	7344	7100
November 2013	8156	7344	.....	6531	7408
December 2013	7344	5719	.....	6531	7425
January 2014	8156	7344	.....	5719	7473
February 2014	6531	5719	.....	6531	7360
March 2014	8156	7344	.....	6531	7555





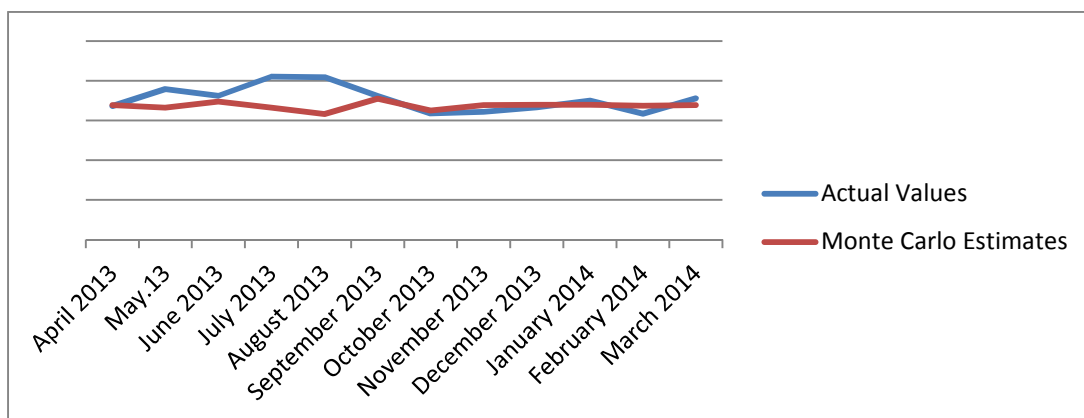
At the end the averages of rows for each month is computed. These computed values are treated as the estimates of Monte Carlo Simulation for the months between April 2013 and March 2014 Period. These values can be compared with the actual values as it is shown in the table below:

**Table 7: Comparisons of Actual Values and Monte Carlo Simulation Estimates**

	<b>Actual Values</b>	<b>Monte Carlo Estimates</b>
April 2013	6728	7473
May 2013	7576	7344
June 2013	7232	7555
July 2013	8212	7327
August 2013	8173	7051
September 2013	7215	7701
October 2013	6345	7100
November 2013	6426	7408
December 2013	6660	7425
January 2014	6991	7473
February 2014	6330	7360
March 2014	7116	7555

These two series tested with independent samples *t* test in order to decide whether the population means are equal or not. The p-value of the test statistic was found 0,12, which means that the population means are equal ( $\alpha=0,05 \leq p=0,12$  so null hypothesis cannot be rejected). This result indicates that Monte Carlo Estimates can be used by the managers of the water company.

Following figure presents the graphical view of the actual values and the estimated values of Monte Carlo Simulation for the months between April 2013 and March 2014 Period.



**Figure 2. Graphical Presentation of the Actual Values and MC Simulation Estimates**



In the figure above, both monthly actual and monthly Monte Carlo Estimate values are seen close to each other and their values spread around 7000. Particularly, actual and Monte Carlo Estimate values are seen very close to each other between October 2013 and March 2014 Period.

### 3.2. Second Stage

Here, the same procedure that was applied in the first stage was operated for the data between January 2011 and March 2014 Period. At first, the data (39 months-between January 2011 and March 2014 Period) is sorted in the ascending order as follows:

**Table 8. Ascending Order of the Monthly Data (January 2011-March 2014 Period)**

No	Data	No	Data	No	Data
1	5313	14	7030	27	7943
2	5958	15	7077	28	7955
3	6330	16	7083	29	8005
4	6345	17	7116	30	8031
5	6426	18	7215	31	8166
6	6582	19	7232	32	8173
7	6660	20	7244	33	8212
8	6728	21	7267	34	8396
9	6733	22	7320	35	8405
10	6745	23	7398	36	8458
11	6827	24	7576	37	8622
12	6905	25	7621	38	8628
13	6991	26	7894	39	9374

The difference between the maximum and minimum value is computed (9374-5313=4061). Then, the numbers of intervals is calculated with the following formula:  $\sqrt{n} = \sqrt{39} = 6,24$ . Next, 4061 is divided by 6 and found 676,83. The number 676,83 shows the margin between upper and lower bound for each one of the 6 intervals. The calculated interval values are found as in the following table:

**Table 9: Interval Values for January 2011-March 2014 Period**

Interval No.	Interval Values
1	5313-5989
2	5990-6666
3	6667-7343
4	7344-8020
5	8021-8697
6	8698-9374

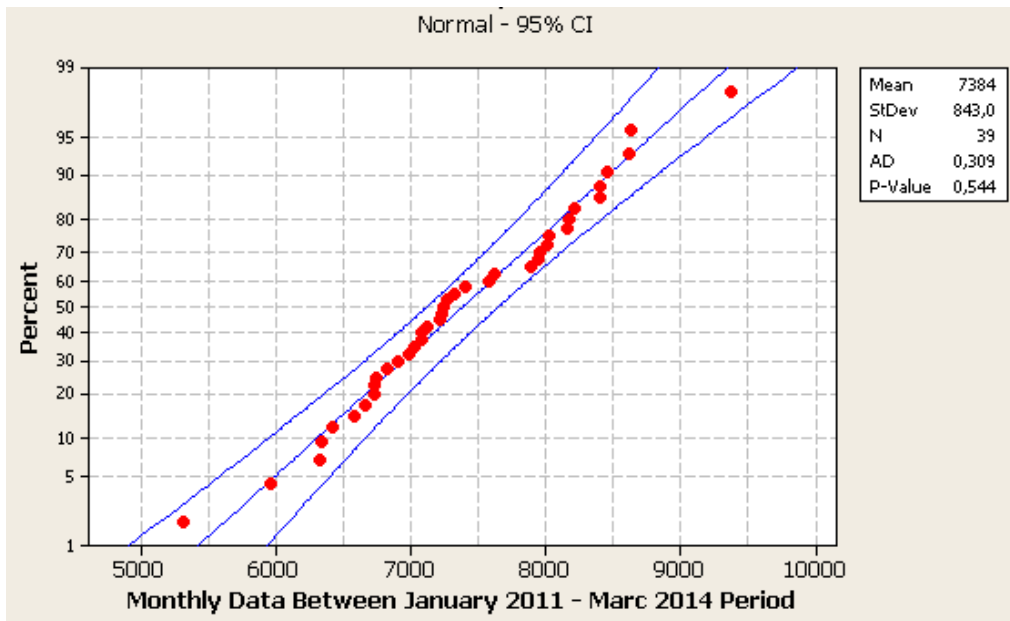


Then, by using the table above frequency distribution is computed and the values of the frequency distribution are converted into the probability values and then cumulative probability distribution is computed as is seen in the Table below.

**Table 10: Probability Distribution of Dispenser Size Water Bottles Demand**

Interval No.	Interval Values	Frequency	Probability of Occurrence	Cumulative Probability
1	5313-5989	2	0,05	0,05
2	5990-6666	5	0,13	0,18
3	6667-7343	15	0,38	0,56
4	7344-8020	7	0,18	0,74
5	8021-8697	9	0,23	0,97
6	8698-9374	1	0,03	1,00

Then, the distribution of the monthly data that was between January 2011 and March 2013 period is tested.



**Figure 3. Probability Plot and Anderson Darling Test Values**

In the probability plot, all the points are observed between the lower and upper bounds which means that the data follows a normal probability distribution. Additionally, as a formal test, the result of Anderson Darling test shows that null hypothesis cannot be rejected which means that the data follows a normal probability distribution ( $p\text{-value}=0,544 \geq \alpha=0,05$ ). For this reason, fifty random numbers which follow normal probability distribution were generated for twelve months (April 2014 to March 2015) by using Minitab. The generated random numbers are seen in the table below:



**Table 11: Generated Random Numbers for Twelve Months (50 Trials)**

Random Numbers Months	RN1	RN2	.....	RN50
April 2014	0,883	0,491	.....	0,946
May 2014	0,980	0,610	.....	0,980
June 2014	0,142	0,940	.....	0,506
July 2014	0,046	0,291	.....	0,948
August 2014	0,624	0,703	.....	0,021
September 2014	0,596	0,715	.....	0,345
October 2014	0,637	0,436	.....	0,270
November 2014	0,961	0,706	.....	0,910
December 2014	0,473	0,560	.....	0,245
January 2015	0,416	0,531	.....	0,110
February 2015	0,998	0,228	.....	0,806
March 2015	0,694	0,452	.....	0,977

Then cumulative probability distribution was used for creating the probability intervals (the upper and lower intervals) probabilities as seen in the table below:

**Table 12: Probability Intervals**

Interval No.	Probability Intervals
1	0-0,0499
2	0,05-0,0179
3	0,018-0,559
4	0,56-0,739
5	0,74-0,969
6	0,97-1,0

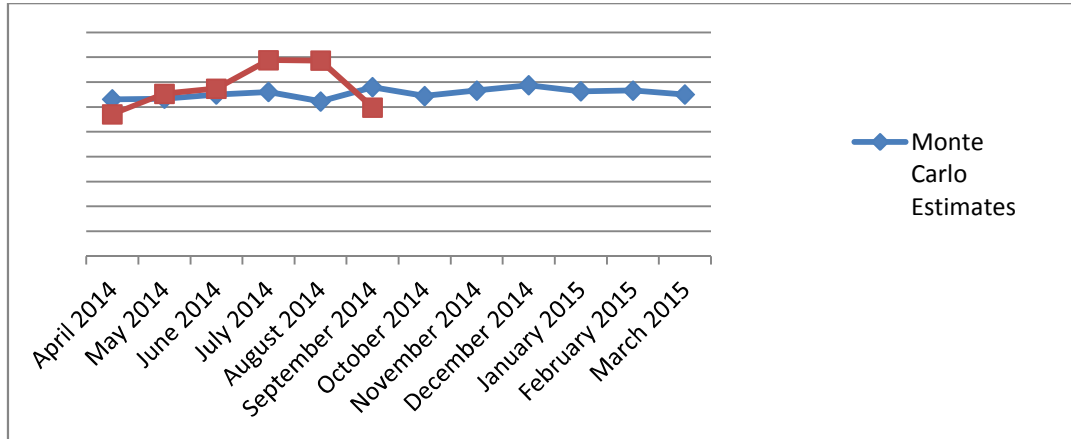
Averages of upper and lower bound values (interval values) are computed for each one of the interval that the random number was classified. The following table presents the average values for each classification.

**Table 13: Average Values for Each Classification**

	RN1	RN2	.....	RN50	Row Average
April 2014	8359	7005	.....	8359	7154
May 2014	9036	7682	.....	9036	7168
June 2014	6328	8359	.....	7005	7249
July 2014	5651	7005	.....	8359	7303
August 2014	7682	7682	.....	5651	7113
September 2014	7682	7682	.....	7005	7398
October 2014	7682	7005	.....	7005	7222
November 2014	8359	7682	.....	8359	7330
December 2014	7005	7005	.....	7005	7438
January 2015	7005	7005	.....	6328	7316
February 2015	9036	7005	.....	8359	7330
March 2015	7682	7005	.....	9036	7249



At the end, the averages of rows for each month are computed. These computed values are treated as the estimates of Monte Carlo Simulation for the months between April 2014 and March 2015 period. Following figure presents the estimated values of Monte Carlo Simulation (between April 2014 and March 2015 Period).



**Figure 4. Graphical Presentation of the MC Simulation Estimates**

In the figure above, available actual monthly values and monthly Monte Carlo estimates are seen close to each other.

### 4.3. Third Stage

The data between January 2011 and March 2013 were divided into three seasons. The first season is composed of January, February and March. The second season is composed of April, May, June, July, and August. The third season is composed of September, October, November and December. Same procedures, which were performed in the monthly data, were also performed for the seasonal data. The probability values and then cumulative probability distribution computed and obtained as seen in the following table for each season, respectively:

**Table 14: Probability Distribution of Dispenser Size Water Bottles Demand for Season 1**

Interval No.	Interval Values	Frequency	Probability of Occurrence	Cumulative Probability
1	5313-6361	2	0,22	0,22
2	6362--7410	4	0,44	0,67
3	7411-8458	3	0,33	1,00

**Table 15: Probability Distribution of Dispenser Size Water Bottles Demand for Season 2**

Interval No.	Interval Values	Frequency	Probability of Occurrence	Cumulative Probability
1	5313-6361	2	0,22	0,22
2	6362--7410	4	0,44	0,67
3	7411-8458	3	0,33	1,00



**Table 16: Probability Distribution of Dispenser Size Water Bottles Demand for Season 3**

Interval No.	Interval Values	Frequency	Probability of Occurrence	Cumulative Probability
1	6581-7306	4	0,50	0,50
2	7307-8031	4	0,50	1,00

Then cumulative probability distributions used for the creation of the probability intervals (the upper and lower intervals) as seen in the table below:

**Table 17: Probability Intervals For Season 1, Season 2 and Season 3**

Interval No.	Probability Intervals For Season 1	Probability Intervals For Season 2	Probability Intervals For Season 3
1	0-0,2199	0-0,4399	0-0,4999
2	0,22-0,6699	0,44-0,8099	0,50-1,00
3	0,67-1,00	0,81-1,00	-

Averages of upper and lower bound values (interval values) are computed for each one of the interval that the random number was classified. The following table presents the average values for each classification.

**Table 18: Average Values for Each Classification**

	RN1	RN2	.....	RN50	Row Average
Season 1	7934	6886	.....	5837	6948
Season 2	8202	7421	.....	7421	7905
Season 3	6944	7669	.....	6944	7263

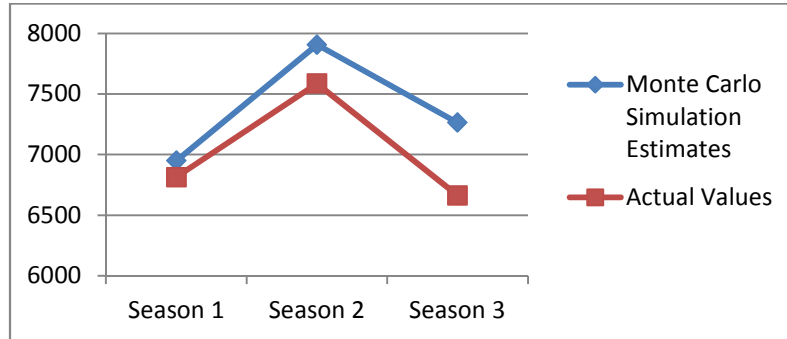
At the end, the averages of rows for each season are computed. These computed values are treated as the estimates of Monte Carlo Simulation for the Season 1, Season 2 and Season 3. These values can be compared with the actual values as shown in the table below:

**Table 19: Comparisons of Actual Values and Monte Carlo Simulation Estimates**

	Monte Carlo Simulation Estimates	Actual Values
Season 1	6948	6812
Season 2	7905	7584
Season 3	7263	6662



Monte Carlo estimates and actual values for Season 1, Season 2 and Season 3 for 2013-2014 are shown in the Figure 5 below.



**Figure 5. Graphical Presentation of the Actual Values and MC Simulation Estimates**

In the figure above, actual seasonal values and seasonal Monte Carlo estimates are seen close to each other.

#### 4.4. Fourth Stage

The data between January 2011 and March 2014 were divided into three seasons. First season is composed of January, February and March. Second season is composed of April, May, June, July, and August. Third season is composed of September, October, November and December. The probability values and then cumulative probability distribution computed and obtained as seen in the following table for each season respectively:

**Table 20: Probability Distribution of Dispenser Size Water Bottles Demand for Season 1**

Interval No.	Interval Values	Frequency	Probability of Occurrence	Cumulative Probability
1	5313-6361	3	0,25	0,25
2	6362--7410	6	0,5	0,75
3	7411-8458	3	0,25	1

For the rest of seasons (Season 2 and Season 3) same procedure is implemented and concerning values are obtained as seen in the Table 16 and 17.

**Table 21: Probability Distribution of Dispenser Size Water Bottles Demand for Season 2**

Interval No.	Interval Values	Frequency	Probability of Occurrence	Cumulative Probability
1	6727-7609	6	0,40	0,40
2	7610-8492	6	0,40	0,80
3	8493-9374	3	0,20	1,00



**Table 22: Probability Distribution of Dispenser Size Water Bottles Demand for Season 3**

Interval No.	Interval Values	Frequency	Probability of Occurrence	Cumulative Probability
1	6344-6906	5	0,42	0,42
2	6907-7469	4	0,33	0,75
3	7470-8031	3	0,25	1,00

Then, the cumulative probability distribution that was computed in the table above is used for creating the probability intervals (the upper and lower intervals) probabilities as seen in the table below:

**Table 23: Probability Intervals For Season 1, Season 2 and Season 3**

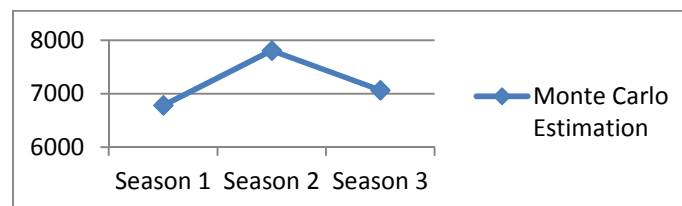
Interval No.	Probability Intervals For Season 1	Probability Intervals For Season 2	Probability Intervals For Season 3
1	0-0,2499	0-0,3999	0-0,4199
2	0,25-0,7499	0,40-0,7999	0,42-0,7499
3	0,75-1,00	0,80-1,00	0,75-1,00

Averages of upper and lower bound values (interval values) are computed for each one of the interval that the random number was classified. The following table presents the average values for each classification.

**Table 24: Average Values for Each Classification**

	RN1	RN2	.....	RN50	Row Average
Season 1	6886	6886	.....	6886	6781
Season 2	8051	8051	.....	8933	7804
Season 3	7750	6626	.....	7750	7064

Monte Carlo estimates and actual values for Season 1, Season 2 and Season 3 for 2014-2015 are shown in the Figure 6 below.



**Figure 6. Graphical Presentation of the MC Simulation Estimates**





#### 4. CONCLUSION

This study focuses on the presentation of how Monte Carlo Simulation Method is used for forecasting the demand practically and for forecasting the future demands that would help managerial decisions. In the first and second sections theoretical explanations were presented. Then, for reaching the goal of this study, several Monte Carlo Simulations were performed in the application part. The summary of the research results is:

- The monthly data between April 2013 and March 2014 were forecasted and they compared with the actual data. Monthly actual and monthly Monte Carlo Estimated values were found close to each other by observing the visual graph and estimated values. Then actual values and forecasted values were tested with independent samples t test in order to decide whether the population means are equal or not. Test result suggests that population means are equal and Monte Carlo Estimates can be used by the managers of the water company.
- The monthly data between April 2014 and March 2015 were forecasted. Then, these forecasted values and available actual data (between April 2014-September 2014 Period) were drawn in a time series plot. Similarly, it was observed that they approximated each other.
- Seasonal data between 2013 and 2014 were forecasted. Actual seasonal values and seasonal Monte Carlo estimates were found close to each other.
- Finally, seasonal data between 2014 and 2015 were forecasted.

We concluded that by decreasing the uncertainty of future, company managers had the advantage of making healthy future decisions about controlling their inventory, purchasing new equipment, hiring a new worker and other sources.

As stated before, there are numerous forecasting methods in the literature. Although the objective of this study is not that, those various methods can also be used for the water forecasts in future studies. Then the results can be compared with each other.

#### REFERENCES

- Almutaz, I, Ali, E., Kahlid, Y, and Ajbar, A. H., (2013), “A long-term forecast of water demand for a desalinated dependent city: case of Riyadh City in Saudi Arabia”, *Desalination and Water Treatment*, Vol. 51, pp. 5934-5941.
- Aygören, H. and İlem, M., (2006), “Türkiye’de Özelleştirme Sonrası Araç Muayene İstasyonları Sermaye Bütçelemesinin Monte Carlo Simülasyonu Yöntemi ile Analizi”, *Kara Ulaştırması Genel Müdürlüğü Araç Muayene İstasyonları Projesi Tanıtım Dokümanı*, Ankara, s. 75-88.



- Banks J., Carson J.S., Nelson B.L. and Nicol D.M., (2005), Discrete Event System Simulation, Pearson Prentice Hall, Upper Saddle River.
- Barnett, T. P. (1995), “Monte Carlo Climate Forecasting”, Journal of Climate, Vol. 8, pp. 1005-1022.
- Bruno, M. and Clarke, (2003), “The hidden value of air transportation infrastructure”, 7th International Conference on Technology Policy and Innovation in Monterrey, Mexico, June 10th-13th.
- Carson, J. S., (2004), “Introduction to Modeling and Simulation”, Proceedings of the 2004 Winter Simulation Conference, pp. 9-16.
- Chatfield C. (1996), The Analysis of Time Series An Introduction, Chapman & Hall, London.
- Cubasch, U., Santer, B. D., Hellbach, A., Hegerl, G., Hck, H., Reimer, E. M., Mikolajewicz, U., Stössel, A. and Voss, R., (1994), “Monte Carlo climate forecasts with a global coupled ocean-atmosphere model”, Climate Dynamics, Vol. 10, pp. 1-19.
- Demirdöğen, O., (1998), “Talep Tahmininde Monte Carlo Simülasyon Tekniğinin Kullanılması”, Atatürk Üniversitesi İİBF Dergisi, Cilt 12, Sayı 1-2, s. 229-240.
- Desrochers, G., Blanchard, M. and Sud, S., (1986), “A Monte Carlo Simulation Method for the Economic Assessment of the contribution of Wind Energy to Power Systems”, IEEE Transactions on Energy Conversion, Vol. EC-1, No. 4, pp. 50-56.
- Dobrican, O., (2013), “Forecasting Demand for Automotive Aftermarket Inventories”, Informatica Economica, Vol. 17, No. 2, pp. 119-129.
- Hague, M., Rahman, A., Hagare, D. and Kibria, G., (2014), “Probabilistic Water Demand Forecasting Using rejected Climatic Data for Blue Mountains Water Supply System in Australia”, Water Resour Manage, Vol. 28, pp. 1959-1971.
- Hanke J. E. and Wichern D. W. (2005), Business Forecasting, Prentice Hall, New Jersey.
- Heizer J. and Render B. (2000), Operation Management, Prentice Hall, New Jersey.
- Hujala, M. and Hilmola, O., (2009), “Forecasting long-term paper demand in emerging markets”, Foresight, Vol. 11, No. 6, pp.56-73.
- Kalar, M., (2013), “ Monte Carlo or bust?”, Public Finance, Vol. 43, pp.43-44.
- Kiely, d., (2004), “The State of Pharmaceutical Industry Supply Planning and Demand Forecasting”, The Journal of Business Forecasting, Vol. 23, No. 3, pp. 20-22.
- Kitamura, R., Chen, C., Pendyala, R. M. and Narayanan, R., (2000), “Micro-simulation of daily activity-travel patterns for travel demandforecasting”, Transportation, Vol. 27, pp. 25-51.
- Křižanová, A., Majerčák, P., Masárová, G. and Bc, D., (2013), “Monte Carlo Cost Simulation in the Supply Chain in E-Business”, Nase More, Vol. 60, No. 5-6, pp. 99-104.



- Korn, G. A., (2007), *Advanced Dynamic-System Simulation*, John Wiley & Sons, Inc., New Jersey, USA.
- Law, A. M., (2005), “How to build valid and credible simulation models”, *Proceedings of the 2005 Winter Simulation Conference*, pp. 24-32.
- Law, A. M. (2007), *Simulation Modeling and Analysis*, Mcgraw Hill, New York.
- Makridakis, S. and Wheelwright, S. C., (1989), *Forecasting Methods for Management*, John Wiley & Sons, New York, USA.
- McQueen, D. H. O., Hyland, P. R. and Watson, S. J., (2004), “Monte Carlo Simulation of Residential Electricity Demand for Forecasting Maximum Demand on Distribution Networks”, *IEE Transactions On Power Systems*, Vol. 19, No. 3, pp. 1685-1689.
- Mun J. (2006), *Modelling Risk: Applying Monte Carlo Simulation, Real Options Analysis, Forecasting and Optimization Techniques*, John Wiley & Sons Inc., Hoboken, New Jersey.
- Park, D. W. and Tomek, W. G., (2014), “An Appraisal of Composite Forecasting Methods”, *North Central Journal of Agricultural Economics*, Vol. 10, No. 1, pp. 1-11.
- Patır, S. and Yıldız, M. S., (2003), “Talep Tahmininde Monte Carlo Simulasyonunun Uygulanması”, *EKEV Akademi Dergisi*, Yıl 7, Sayı 17, s. 327-336.
- Pegden C. D., and Shannon R.E. (1990), *Sadowski R.P., Introduction to Simulation Using Siman*, Mcgraw-Hill, New York.
- Raychaudhuri, S., (2008), “Introduction to Monte Carlo Simulation”, *Proceedings of the 2008 Winter Simulation Conference*.
- Robert, C. P. and Casella, G., (2004), *Monte Carlo Statistical Methods*, Springer, New York, USA.
- Sargent, R. G., (2004), “Validation and Verification of Simulation Models”, *Proceedings of the 2004 Winter Simulation Conference*, pp. 17-28.
- Sezen H. K., and Günal M. (2009), *Yöneylem Araştırmasında Benzetim*, Ekin Yayınevi, Bursa.
- Shahbandarzadeh, H., Salimifard, K. and Moghdani, R., (2013), “Application of Monte Carlo Simulation in the Assesment of European Call Options”, *Iranian Journal of Management Studies*, Vol. 6, No. 1, pp. 9-27.
- Tadeu, H. F. B. and Silva, J. T. M., (2013), “The Determinants of the Long Term Private Investment in Brazil: An Empirical Analysis Using Cross-Section and a Monte Carlo Simulation”, Vol. 18, pp. 11-17.
- Travers, K. J. and Gray, K. G., (1981), “The Monte Carlo Method: A Fresh Approach to Teaching Probabilistic Concepts”, *The Mathematic Teacher*, Vol. 74, No. 5, pp. 327-334.



- Udina, F. and Delicado, P., (2005), “Estimating Parliamentary composition through electoral polls”, J. R. Statist. Soc. A., 168, Part 2, pp. 387-399.
- Valle, M. and Norvell, T., (2013), “Using Monte Carlo Simulation to Teach Students about Forecast Uncertainty”, Business Education Innovation Journal, Vol. 5, No. 2, pp. 35-41.
- Yu, H., Feng, L., Peng, Z., Feng, Z., Shay, D. K. and Yang, W., (2009), “Estimates of the impact of a future influenza pandemic in China”, Influenza and Other Respiratory Viruses, Vol. 3, pp. 223-231.
- Zakhary, A., Atiya, A. F., El-Shishiny, H. and Gayar, N. E., (2009), “Forecasting hotel arrivals and occupancy using Monte Carlo simulation”, Journal of Revenue & Pricing Management, Vol. 10, No. 4, pp. 344-366.