Politeknik Dergisi | Journal of Polytechnic
Cilt: 5 Sayı: 2 s. 129-136, 2002 | Vol: 5 No: 2 pp. 129-136, 2002

MINIMUM SAMPLING RATE FOR CALCULATION OF MEAN RMS AND CRMS VALUES OF A PERIODIC WAVEFORM HAVING ODD-ORDER HARMONICS

Aydın ÇETİN
Gazi University, Institute of Science and Technology
06570 Maltepe, ANKARA

ABSTRACT

This paper discusses calculation of mean, rms and complex rms values of h-order harmonics of a periodic waveform having odd-order harmonics. Nyquist rate suggests that 2h+1 samples should be taken in a period to calculate mean, rms and complex rms values of the waveform so that the original waveform can be reconstructed with Fourier series. In this paper, it is proved that only h+2 samples are required to perform same calculations. The recursive running discrete Fourier series approach (RRDFS) is used for computation. Results show that exact information about the waveform can be obtained below the Nyquist rate.

Keywords: Harmonics, waveform sampling, Nyquist

TEK DERECELİ HARMONİKLERE SAHİP BİR PERİYODİK DALGANIN ORTALAMA RMS VE CRMS DEĞERLERİNİN HESAPLANMASI İÇİN MİNİMUM ÖRNEKLEME ORANI TAYİNİ

ÖZET

Bu makalede h. dereceden tek dereceli harmonikler içeren bir dalga şeklinde ortalama, etkin ve kompleks etkin değerlerin hesabı incelenmektedir. Nyquist örnekleme kriterine göre, dalga şeklini elde etmek ve ortalama, etkin ve kompleks etkin değerleri hesaplamak için bir periyot içerisinde 2h+1 örnek alınması gerekmektedir. Bu makalede aynı hesaplamalar için bir periyot içerisinde h+2 örneğin yeterli olduğu gösterilmektedir. Hesaplamalar için ardışıl ayrık Fourier serileri yaklaşımı kullanılmaktadır. Elde edilen sonuçlar Nyquist örnekleme kriterinin altında bir değerle dalga şekli hakkında tüm bilgilerin elde edilebileceğini göstermektedir.

Anahtar kelimeler: Harmonik, dalga şekli örnekleme, Nyquist

1. INTRODUCTION

Harmonics produced by nonlinear loads (e.g. devices with electrical arc, ballasts of fluorescent lamps, transmission system transformers and electrical machines) are well known. The use of power electronic semiconductors increased rapidly with the development of semiconductor technology. The most power electronic equipment, such as switch-mode dc power supplies, uninterruptible power supplies, and ac and dc motor drives, controlled and uncontrolled ac-to-dc, dc-to-ac converters can add inherent power line disturbances by distorting the utility waveform due to harmonic currents injected into the utility grid, and by producing electromagnetic interface causing deterioration of supply quality and power factor (1). Due to internal impedance of supply, the voltage waveform at the point of common coupling to the other loads will become distorted, which may cause harmful effects in other equipment such as, transformers, rotating machines, switch gear, capacitor banks, fuses, protective relays, computers, and computer-like measuring and control equipment (2-4). Overall distortion to the waveform is expressed as the total harmonic distortion (THD) that is limited by the IEEE standard 519 (5) to less than 5.0 percent for voltage and, 3.0 percent for current.

The new standards legislated by governments place increasingly stringent requirements on electrical systems. New generations of equipment and methods must have higher performance parameters such as better efficiency and reduced electromagnetic interference. To meet these standards and requirements, harmonics must be compensated or decreased to a certain level described by standards. Harmonics first are to be identified and then to be analyzed for compensation and suppression. Fourier series analysis is common for identification of harmonics. However, analysis of such harmonics produced by electronic equipment which use fast semiconductor switches may require high sampling rate, therefore, high computational efforts. It may be achieved by the use of fast computing devices such as computers, microprocessors and digital signal processors.

The use of fast computing devices does not decrease the calculation effort to identify harmonics and thus, may not be effective in real time applications. To overcome this problem, computing devices should be faster than they are or, same computing devices should do same job with less computational effort, if possible. The sampling rate is to be reduced to decrease computational effort. The reduction in the sampling rate may result in loss of information. In balanced three phase power systems, the characteristic harmonics are all odd-order. Even harmonics are not characteristics of a balanced power system. Positive and negative sequence odd-order harmonics circulate between the phases, while the zero sequences circulate between phase and neutral or ground. Thus, sampling rate can be decreased below the Nyquist rate without loss of any information about harmonics of the waveform.

In this paper, it is shown that sampling rate can be reduced to almost half of the Nyquist rate for periodic waveforms having odd-order harmonics, and then the original waveform can be reconstructed without loss of any information on the waveform. The mean, rms and complex rms values of h-order harmonics can be calculated without error, if discretization related errors are neglected.

2. MINIMUM NUMBER OF SAMPLES

A *band-limited signal* is a signal, *x(t)*, which has no spectral components beyond a frequency f Hz; that is, $F(s) = 0$ for $|s| > 2.\pi.f$. The *sampling theorem* states that a real signal, *f(t)*, which is band-limited to *f* Hz can be reconstructed without error from samples taken uniformly at a rate $R > 2f$ samples per second. This minimum sampling frequency, $f_s = 2f$ Hz, is called the *Nyquist rate* or the *Nyquist frequency* (6). The corresponding sampling interval, $T = 1/2f$ (where $t = nT$), is called the *Nyquist interval*. A signal band limited to *f* Hz which is sampled at less than the Nyquist frequency of 2*f*, *i.e.*, which was sampled at an interval $T > 1/2f$, is said to be *undersampled*.

A number of practical difficulties are encountered in reconstructing a signal from its samples. The sampling theorem assumes that a signal is band limited. In practice, however, signals are time-limited rather than band-limited. As a result, determining an adequate sampling frequency that does not lose desired information can be difficult. To create a digital representation of a waveform, the sampling process takes “snapshots” of the instantaneous value of the wave at regular intervals: the sample rate. Afterward, no information is available about what happened between those samples. If the set of sampled data is to be an accurate representation of the actual waveform, the samples must be spaced closely enough to capture all the significant details. When a signal is undersampled, its spectrum has overlapping tails; that is *F(s)* no longer has complete information about the spectrum and it is no longer possible to recover x*(t)* from the sampled signal. In this case, the tailing spectrum does not go to zero, but is folded back onto the apparent spectrum. This inversion of the tail is called *spectral folding* or a*liasing*

Suppose the true input signal has frequency *f*, and the sampling frequency is f_s. The following rules allow the alias frequency (f_a) to be calculated.

$$f < \frac{1}{2} f_s \tag{1}$$

As the signal frequency lies below the limit set by the sampling theorem, no aliasing occurs; the sampled signal has the correct frequency.

$$\frac{1}{2} f_s < f < f_s \tag{2}$$

The signal undergoes aliasing, with an alias frequency

$$f_a = f_s - f \tag{3}$$

where $f < f_s$.

Aliasing again occurs. The aliasing or folding property of a band-limited signal can be used to determine frequency components of the sampled signal with undersampling if signal contains only odd-order harmonics.

Let the signal has the highest harmonic frequency (f_h). Then, the folding frequency (f_c) near the fundamental frequency (f_1) is defined as

$$f_c = \frac{f_h}{2} + f_1 . \tag{4}$$

The folding index, h_x for the harmonic pairs become

$$h_x = \frac{f_h}{f_1} \pm \frac{1}{2} n \qquad (5)$$

where n is an odd number (n = 1, 3, 5, ... , n_h) and, n_h for the harmonic pairs defined as

$$n_h = \frac{f_h}{f_1} - 2. \qquad (6)$$

Therefore, minimum number of samples that should be taken in a period for reconstruction of the signal is

$$N = \frac{f_h}{f_1} + 2. \qquad (7)$$

Since highest harmonic component is $h = \frac{f_h}{f_1}$, equation (7) becomes

$$N = h + 2 \qquad (8)$$

Let the signal be sampled with N samples,

then, at any sampling instant k, for k=0,1,2,...N-1

$$\mathbf{W}_{(h,k+1)} = e^{-j\, hk2\pi/N} \qquad (9)$$

is always folded near the fundamental component as seen in Fig. 1.

Thus,

$$\mathbf{W}_{(h+1)} = \mathbf{W}^*_{(N-h-1)} \qquad (10)$$

where, * donates complex conjugate and h is the order of harmonics.

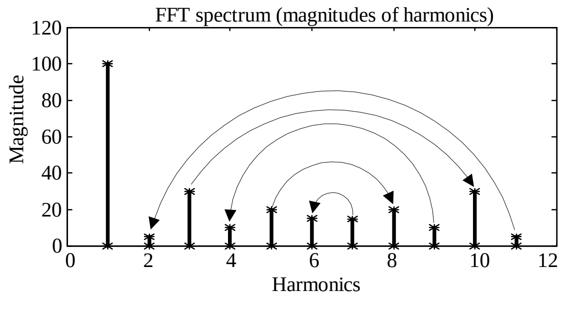

Fig. 1. Folding of the harmonics near the fundamental harmonics

Summation of index of folded numbers always gives the number of samples. For example, if a signal contains odd-order harmonics up to 11th harmonic, then the number of samples is N = h+2 = 13. Since N is below the Nyquist rate which is 2h+1=23, folding near the fundamental occurs. At points x, (11 and 2, 9 and 4, 7 and 6, 5 and 8, 3 and 10) quantities of **W** are complex conjugate of each other. Thus, even-order harmonics occurred due to sampling below the Nyquist rate are to be ignored.

3. CALCULATION OF FUNCTIONALS

Common functionals for describing properties of electric circuits with periodic waveforms of voltages and currents having period T, e.g. mean, Root Mean Square, (rms) and Complex Root Mean Square (CRMS) can be calculated if both the waveform is sampled with N number of samples and Nyquist criteria is satisfied. Let periodic signal *x(t)* with period T is sampled periodically with the sampling period T_S, in the period T with N number of samples. Then, functionals the mean, the rms and the CRMS of the h-order harmonic can be defined as,

$$\bar{x} := \frac{1}{N} . \sum_{n=0}^{N-1} x_n \qquad (11)$$

$$\|x\| := \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} x_n^2} . \qquad (12)$$

$$X_h := \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n e^{-j \frac{2\pi}{N} nh} . \qquad (13)$$

where; $\bar{x}$ is the mean value, $\|x\|$ is rms value and X_h is the CRMS value of the *h*-order harmonic component of the periodic waveform *x(t).*

If sampling is dense enough to meet Nyquist criteria, then $x(t)$ can be reconstructed with the Fourier series. If the number of samples taken is an even number, then reconstruction is given by,

$$x(t) = \bar{x} + \sqrt{2}\,\mathrm{Re} \sum_{h=1}^{\frac{N}{2}-1} \mathbf{X}_h e^{jh\omega_1 t} . \qquad (14)$$

else,

$$x(t) = \bar{x} + \sqrt{2}\,\mathrm{Re} \sum_{h=1}^{\frac{N-1}{2}} \mathbf{X}_h e^{jh\omega_1 t} . \qquad (15)$$

The Recursive Running Discrete Fourier Series (RRDFS) approach, also referred to as running approach, can be used to calculate mean, rms and complex rms values of the waveform. With the running approach, functionals of the waveform can be calculated at each sampling interval. Consequently, any change in the waveform can be detected when next sample is taken. The functionals the mean, the rms and the CRMS of the h-order harmonic can be defined at discrete time k with N number of samples in the interval {*k*...*k*-N+1} as follows;

$$x = x_{k-1} + \frac{1}{N}(x_k - x_{k-N}). \qquad (16)$$

$$||x||_k = \sqrt{||x||_{k-1} + \frac{1}{N}(x_k^2 - x_{k-N}^2)}. \qquad (17)$$

The CRMS value of the h-order harmonic:

$$\mathbf{X}_{hk} = \mathbf{X}_{h,k-1} + (x_k - x_{k-N})\mathbf{W}^{kh}, \qquad (18)$$

where

$$\mathbf{W} := \frac{\sqrt{2}}{N} e^{-j\frac{2\pi}{N}}. \qquad (19)$$

then $x(t)$ can be reconstructed in the interval {*k*...*k*-N+1} with the Fourier series,

$$x(t) = \tilde{x}_k + \mathrm{Re}\sum_{h=1}^{\frac{N-1}{2}} \tilde{\mathbf{X}}_{hk} e^{jh\omega_1 t}. \qquad (20)$$

4. SIMULATION RESULTS

Several simulations have been performed to demonstrate and to verify the effectiveness of the proposed method. First, data have been obtained from practical equipment (An air conditioner, a PC supply, a transformer magnetizing current, a six-pulse and twelve-pulse current source converter (CSC) currents). Then, harmonic analysis has been carried out by the use of Nyquist sampling rate. During the test, harmonics up to 50th are taken into account. Next, waveform has been reconstructed. Same current waveforms have been sampled with h+2 samples. Original waveform again has been reconstructed from its samples and harmonic spectrum has been obtained.

During the simulation fist two periods are considered. The waveform has been sampled with 2550 Hz sampling frequency instead of 4950 Hz Nyquist sampling frequency. Higher order harmonics are not considered since in power applications higher harmonics have small magnitudes.

Example 1:

A square voltage waveform having components up to 50th harmonic is considered. Fig. 2 shows the original, reconstructed and the error between original and reconstructed waveform. Fig. 3 shows the harmonic spectrum of the waveform with their angles. The original waveform has harmonics up to 2450Hz.

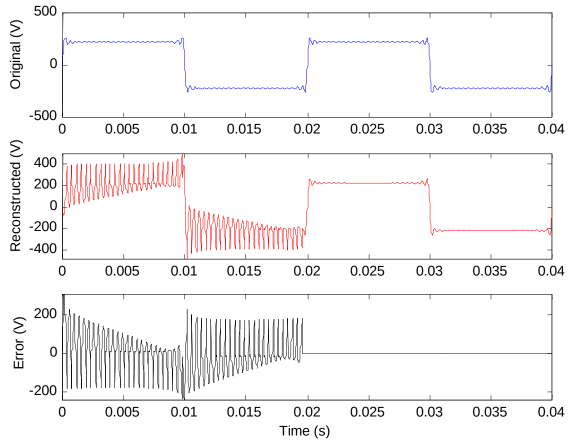

Fig. 2. Original, reconstructed and the error between original and reconstructed waveforms of a square voltage

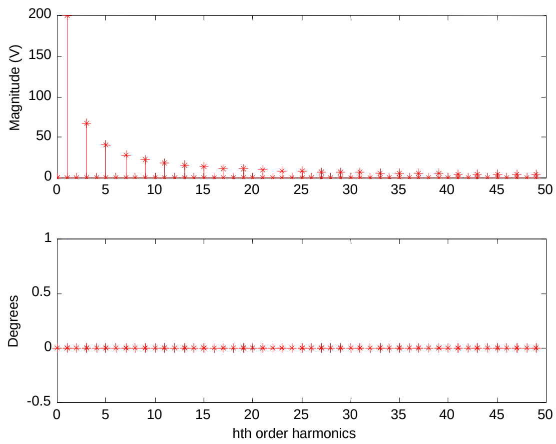

Fig. 3. Harmonic spectrum of the square voltage waveform with their angles

Example 2:

Utility side one phase current waveform of an 180kVA six-pulse current source converter

(CSC) is considered. The effects of the transformer magnetizing current are neglected. Fig. 4 shows the original, reconstructed and the error between original and reconstructed waveform. Fig. 5 shows the harmonic spectrum up to 50th harmonic of the current waveform with their angles. The original waveform is band limited with 2450Hz., and all harmonic components have 180 degrees phase angle.

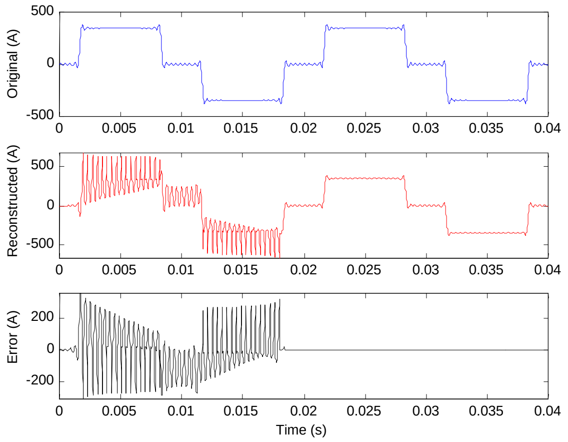

Fig. 4. Original, reconstructed and the error between original and reconstructed waveforms of a six-pulse CSC current

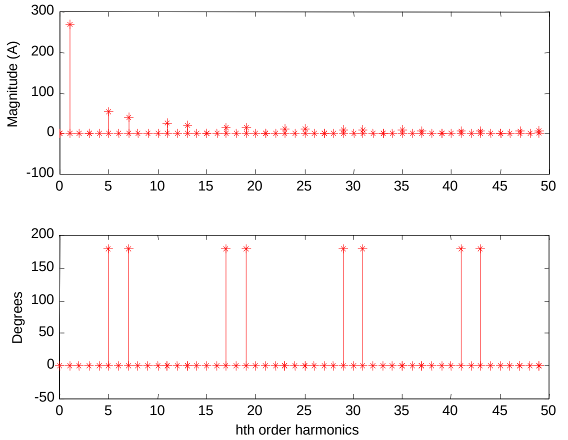

Fig. 5. Harmonic spectrum and their angles for a six-pulse CSC current

At the beginning, the sampling error (Fig. 4) is maximum and goes to zero when h+2 samples are acquired. Total harmonic distortion (THD) of the waveform is 30 percent. All harmonics beyond the fundamental (Fig.5) have 180 degrees phase angle.

Example 3:

The same six-pulse current source converter in Example 2 with 30 degrees phase shift is considered. Phase shift is used to remove phase angles of harmonics so that parallel operation of the converter can be achieved. Fig. 6 shows the original, reconstructed and the error waveforms. Fig. 7 shows the harmonic spectrum up to 50th harmonic of the waveform with their angles.

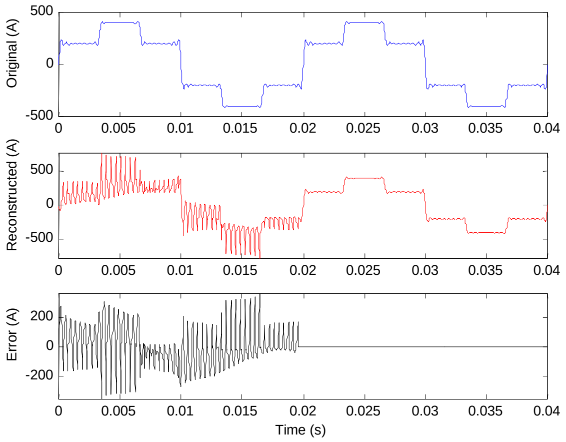

Fig. 6. Original, reconstructed and the error waveforms of a six-pulse CSC current with 30 degrees phase shift

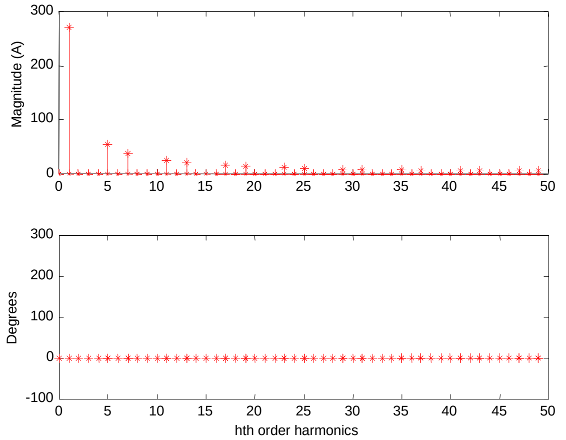

Fig. 7. Harmonic spectrum and their angles for a six-pulse CSC current with 30 degrees phase shift

As seen in Fig. 6 and 7, again zero error was obtained after initiating period. And, harmonic spectrum with their phase angles is obtained correctly.

Example 4:

Current waveform of a twelve-pulse current source converter is considered. Twelve-pulse current source converter is obtained with parallel connection of two six-pulse CSCs with 30 degrees phase shift of Example 3. Fig. 8 shows the original, reconstructed and the error between original and reconstructed waveform. Fig. 9 shows the harmonic spectrum up to 50^{th} harmonic of the waveform with their angles.

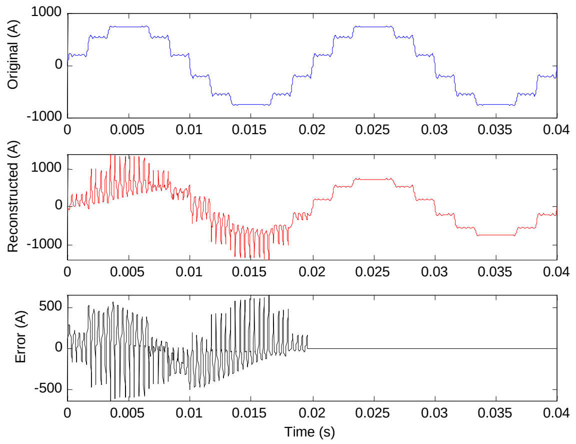

Fig. 8. Original, reconstructed and the error waveforms of a twelve-pulse CSC current

As seen in Fig. 8, again zero error was obtained when h+2 samples are acquired. Only $12n \pm 1$ harmonics are occurred in the waveform, where n is any integer. The THD is decreased to 14.18 percent from 30 percent.

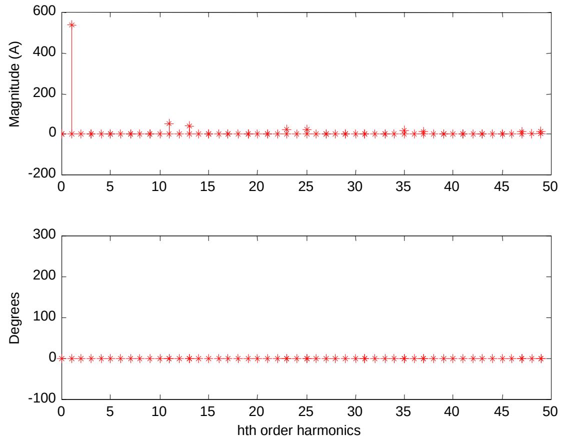

Fig. 9. Harmonic spectrum and their angles for a twelve-pulse CSC current

Example 5:

Magnetizing current waveform of a power system distribution transformer is considered. Fig. 10 shows the original, reconstructed and the error between original and reconstructed waveform. Fig. 11 shows the harmonic spectrum up to 50^{th} harmonic of the waveform with their angles.

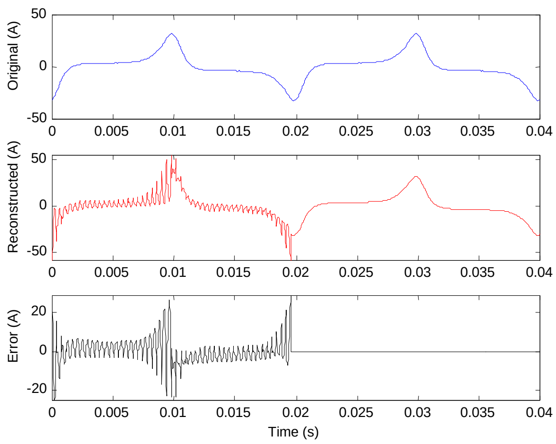

Fig. 10. Original, reconstructed and the error waveforms of a transformer magnetizing current

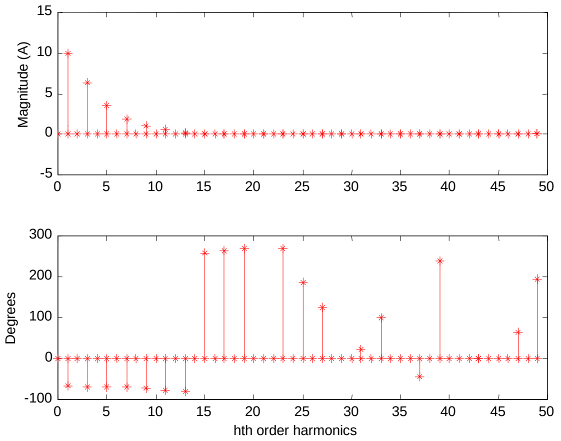

Fig. 11. Harmonic spectrum and their angles for a transformer magnetizing current

The waveform (Fig. 10) was reconstructed without error. Higher harmonics are low in magnitude and all harmonic components have different phase angles (Fig. 11). The THD of the waveform is 76.11 percent.

Example 6:

Current waveform of a PC supply is considered. Fig. 12 shows the original, reconstructed and the error between original and reconstructed waveform. Fig. 13 shows the harmonic spectrum up to 50^{th} harmonic of the waveform with their angles.

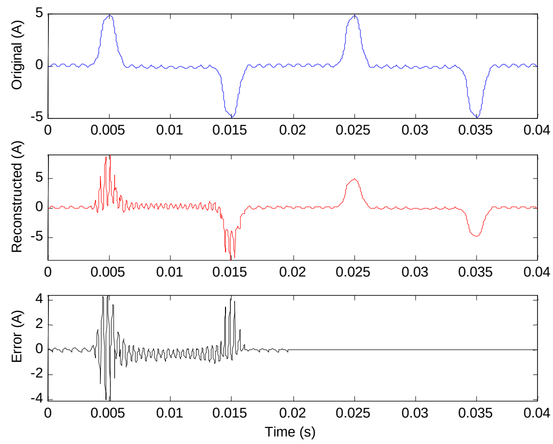

Fig. 12. Original, reconstructed and the error waveforms of a PC current

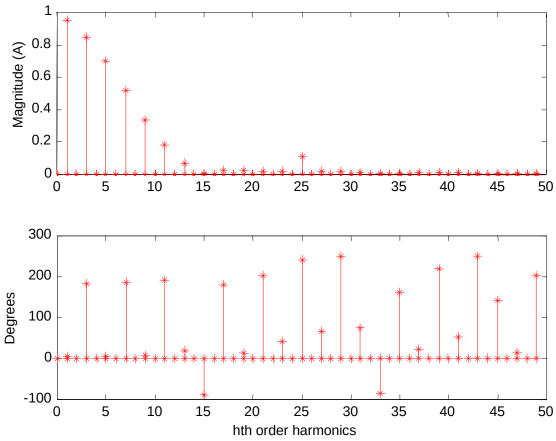

Fig. 13. Harmonic spectrum and their angles for a PC current

As seen in Fig. 12 waveform was reconstructed without error. Higher harmonics are low in magnitude and all harmonic components have different phase angles as seen in Fig. 13, the THD of the waveform is 134.72 percent.

Example 7

Current waveform of an air conditioner is considered. Fig. 14 shows the original, reconstructed and the error waveforms. Fig. 15 shows the harmonic spectrum up to 50^{th} harmonic of the waveform with their angles.

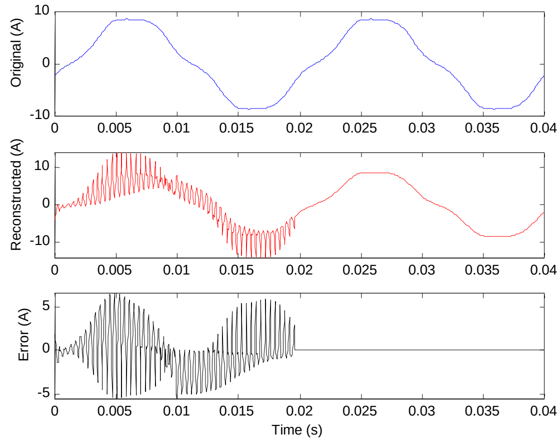

Fig. 14. Original, reconstructed and the error waveforms of an air conditioner current

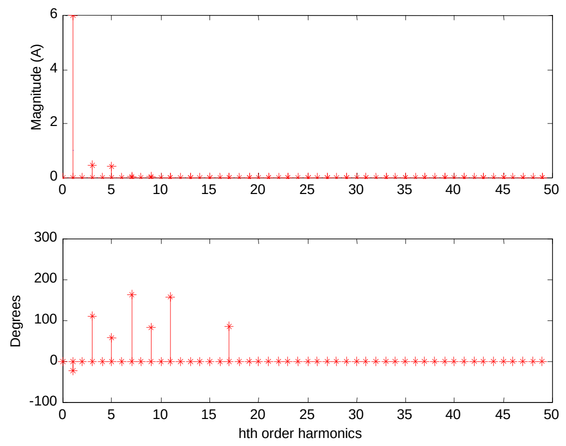

Fig. 15. Harmonic spectrum and their angles for an air conditioner current

As seen in Fig. 14 waveform was reconstructed without error. Higher harmonics are low in magnitude and all harmonic components have different phase angles as seen in Fig. 14, the THD of the waveform is 10.53 percent.

5. DISCUSSION

After sampling is initiated, as seen in Fig. 2-15, the error of the reconstructed waveform is not zero during the first period. In running approach, calculation of the functionals begins immediately after the first sample is acquired. So, the reconstructed waveform contains error until all the required samples are obtained. This situation is the property of the running approach and happens only at first period. Once all the required samples are obtained for the first period, there will be enough data for the continuous reconstruction operation. Thereafter, any changes in the waveform at each sampling instant will be detected without error. If the running approach is not preferred, reconstruction is completed at the end of the first sampling period. However, in this case any changes of the magnitude and phase angle of the waveform cannot be detected at each sampling instant.

If another sampling rate below the Nyquist rate, and different from suggested rate, folding occurs at fractional frequencies. Therefore, although it is possible to find and folding index at fractional frequencies, it is not possible to reconstruct the waveform, and calculate common functionals from its samples. When applying suggested sampling rate it is suggested that first Nyquist rate should be applied to be sure that even-order harmonics does not exist, and if even-order harmonics is not present in the waveform, then sampling rate should be decreased to suggested rate.

6. CONCLUSIONS

In this paper, it is proved that sampling rate can be reduced to almost half of the Nyquist rate for periodic waveforms having odd-order harmonics, and then the original waveform can be reconstructed without loss of any information on the waveform. The mean, rms and CRMS values of h-order harmonics can be calculated without error, if discretization related errors are neglected. It should be noted that if the number of samples is chosen between the suggested rate and Nyquist rate, the waveform could not be reconstructed due to folding at fractional frequencies.

REFERENCES

1. Mohan, N., Undeland, T.M., Robbins, W.P., Power Electronics: Converters, Applications, and Design, John Wiley & Sons, New York, 1989.

2. Dwyer, R., Khan, A.K., McCluskey, R.K., Sung, R., Evaluation of Harmonic Impacts from Compact Fluorescent Lamps On Distribution Systems, IEEE/PES winter Meeting, New York, January/February, pp.1-7, 1995.

3. Reid, W.E and Petrus, K.J., Harmonics and Capacitors in Power System, Pasific Coast Electrical Association Engineering & Operating Conference, Los Angeles, March, pp. 19-20, 1985.

4. Grady, W.M., Chaudry, A.H., Mansoor A., and Samotyj M.J., An Investigation of Harmonics Attenuation and Diversity Among Distributed Single Phase Power Electronic Load, Proceeding of the 1994 Power Engineering Society Transmission and Distribution Conference, Chicago, May, pp. 110-116, 1994.

5. IEEE standard 519 –1992., Recommended Practice for Harmonic Control and Reactive Compensation in Power Systems.

6. Rabiner, L. and Gold, B., Theory and Application of Digital Signal Processing, Englewood Cliffs: Prentice-Hall, New Jersey, 1975.