I. INTRODUCTION

In the fields of theoretical computer science, automata, coding theory, and formal languages, as well as in the solutions of graph theory and optimization theory, semigroups serve as the fundamental algebraic structure. Ideals are crucial to the advanced study of algebraic structures and their applications. Further study of algebraic structures requires the generalization of ideals in algebraic structures. Numerous mathematicians demonstrated significant findings and characterized algebraic structures by introducing various extensions of the idea of ideals in algebraic structures. The idea of almost left, right, and two-sided ideals of semigroups were first presented by Grosek and Satko [1] in 1980. When there are no proper left, right, or two-sided ideals in a semigroup, they investigated how these ideals are characterized. As an extension of bi-ideals, Boganovic [2] developed the idea of almost bi-ideals in semigroups later in 1981. In 2018, Wattanatripop et al. [3] proposed the notion of almost quasi-ideals by utilizing the notions of quasi-ideals of semigroups and almost ideals. Using the ideas of almost ideals and interior ideals of semigroups, Kaopusek et al. [4], in 2020, proposed the notions of almost interior ideals and weakly almost interior ideals of semigroups and examined their features. Iampan et al. [5] in 2021; Chinram and Nakhasen [6], in 2022; Gaketem [7] in 2022; introduced the notion of almost subsemigroups of semigroups; almost bi-quasi interior ideals of semigroups; almost bi-interior ideal of semigroups, respectively. Additionally, different types of almost ideals’ fuzzification were studied by many researchers in [3, 5-9].

Molodtsov [10], in 1999, proposed the idea of the soft set as a function from the parameter set $E$ to the power set of $U$ to model uncertainty. Since then, soft set has attracted the attention of researchers in many fields. In [11-26],
soft set operations, the basic concept of the theory, are studied. Çağman and Enginoğlu [27] modified the definition of soft set and soft set operations. Moreover, several soft algebraic structures were inspired by the notion of soft intersection groups, introduced by Çağman et al. [28]. The usage of soft sets in semigroups came up with the notion of soft intersection substructures of semigroups. Sezer et al. [29, 30] introduced and studied soft intersection subsemigroups, left (right/two-sided ideals), (generalized) bi-ideals, interior ideals, and quasi-ideals of semigroups. Sezgin and Orbay [31] characterized semisimple semigroups, duo semigroups, right (left) zero semigroups, right (left) simple semigroups, semilattice of left (right) simple semigroups, semilattice of left (right) groups and semilattice of groups in terms of soft intersection substructures of semigroups. Soft sets were studied as a wide range of algebraic structures in [32-43]. Recently, Rao [44-47] introduced some new types of ideals of semigroups such as bi-interior ideals, bi-quasi ideals, quasi-interior ideals, weak-interior ideals, and bi-quasi-interior ideals, respectively. Baupradist [48] defined essential ideals of semigroups.

In this study, we introduced the notion of soft intersection almost left (resp. right) ideals, which is a generalization of the nonnull soft intersection left (resp. right) ideals of a semigroup. Furthermore, we show that every idempotent soft intersection almost (left/right) ideal is a soft intersection almost subsemigroup. We obtain that the collection of soft intersection almost left (resp. right) ideals of a semigroup constructs a semigroup under the binary operation of union for soft sets, but not under the binary operation of intersection for soft sets. Furthermore, we demonstrated the connection between almost left (resp. right) ideals and soft intersection almost left (resp. right) ideals of a semigroup corresponding with minimality, primeness, semiprimeness, and strongly primeness.

II. PRELIMINARY

In this section, we review several fundamental notions related to semigroups and soft sets.

**Definition 2.1.** Let $U$ be the universal set, $E$ be the parameter set, $P(U)$ be the power set of $U$ and $K \subseteq E$. A soft set $f_K$ over $U$ is a set-valued function such that $f_K : E \rightarrow P(U)$ such that for all $x \notin K$, $f_K(x) = \emptyset$. A soft set over $U$ can be represented by the set of ordered pairs $f_K = \{(x, f_K(x)) : x \in E, f_K(x) \in P(U)\}$ [10, 27]. Throughout this paper, the set of all the soft sets over $U$ is designated by $S_E(U)$.

**Definition 2.2.** Let $f_A \in S_E(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then $f_A$ is called a null soft set and denoted by $\emptyset_E$. If $f_A(x) = U$ for all $x \in E$, then $f_A$ is called an absolute soft set and denoted by $U_E$ [27].

**Definition 2.3.** Let $f_A, f_B \in S_E(U)$. If for all $x \in E$, $f_A(x) \subseteq f_B(x)$, then $f_A$ is a soft subset of $f_B$ and denoted by $f_A \subseteq f_B$. If $f_A(x) = f_B(x)$ for all $x \in E$, then $f_A$ is called soft equal to $f_B$ and denoted by $f_A = f_B$ [27].

**Definition 2.4.** Let $f_A, f_B \in S_E(U)$. The union of $f_A$ and $f_B$ is the soft set $f_A \cup f_B$, where $(f_A \cup f_B)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$. The intersection of $f_A$ and $f_B$ is the soft set $f_A \cap f_B$, where $(f_A \cap f_B)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$ [27].

**Definition 2.5.** For a soft set $f_A$, the support of $f_A$ is defined by
\[ supp(f_A) = \{ x \in A : f_A(x) \neq \emptyset \} \quad [15] \]

It is obvious that a soft set with an empty support is a null soft set, otherwise the soft set is nonnull.

**Note 2.6.** If \( f_A \subseteq f_B \), then \( supp(f_A) \subseteq supp(f_B) \) \[49\].

A semigroup \( S \) is a nonempty set with an associative binary operation and throughout this paper, \( S \) stands for a semigroup and all the soft sets are the elements of \( S_S(U) \) unless otherwise specified.

**Definition 2.7.** A nonempty subset \( A \) of \( S \) is called,

(1) a subsemigroup of \( S \) if \( AA \subseteq A \),

(2) a right ideal of \( S \) if \( AS \subseteq A \); and a left ideal of \( S \) if \( SA \subseteq A \); and an ideal of \( S \) when is both a left ideal of \( S \) and a right ideal of \( S \),

(3) an almost subsemigroup of \( S \) if \( AA \cap A \neq \emptyset \),

(4) an almost left ideal of \( S \) if \( sA \cap A \neq \emptyset \) for all \( s \in S \); and an almost right ideal of \( S \) if \( As \cap A \neq \emptyset \) for all \( s \in S \); and an almost ideal of \( S \) when is both an almost left ideal of \( S \) and an almost right ideal of \( S \).

**Definition 2.8.** An almost left (resp. right) ideal \( A \) of \( S \) is called minimal almost left (resp. right) ideal of \( S \) if for any almost left (resp. right) ideal \( B \) of \( S \) if whenever \( B \subseteq A \), then \( A = B \).

**Definition 2.9.** Let \( P \) be an almost left (resp. right) ideal of \( S \). Then \( P \) is called,

(1) a prime almost left (resp. right) ideal of \( S \) if for any almost left (resp. right) ideals \( A \) and \( B \) of \( S \) such that \( AB \subseteq P \) implies that \( A \subseteq P \) or \( B \subseteq P \),

(2) a semiprime almost left (resp. right) ideal of \( S \) if for any almost left (resp. right) ideal \( A \) of \( S \) such that \( AA \subseteq P \) implies that \( A \subseteq P \),

(3) a strongly prime almost left (resp. right) ideal of \( S \) if for any almost left (resp. right) ideals \( A \) and \( B \) of \( S \) such that \( AB \cap BA \subseteq P \) implies that \( A \subseteq P \) or \( B \subseteq P \).

**Definition 2.10.** Let \( f_S \) and \( g_S \) be soft sets over the common universe \( U \). Then, soft intersection product \( f_S \circ g_S \) is defined by \[29\]

\[
(f_S \circ g_S)(x) = \begin{cases} 
\bigcup_{x=yz} (f_S(y) \cap g_S(z)), & \text{if } \exists y, z \in S \text{ such that } x = yz \\
\emptyset, & \text{otherwise}
\end{cases}
\]

**Theorem 2.11.** Let \( f_S, g_S, h_S \in S_S(U) \). Then,

i) \( (f_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S) \).

ii) \( f_S \circ g_S \neq g_S \circ f_S \), generally.

iii) \( f_S \circ (g_S \cup h_S) = (f_S \circ g_S) \cup (f_S \circ h_S) \) and \( (f_S \circ g_S) \circ h_S = (f_S \circ h_S) \cup (g_S \circ h_S) \).

iv) \( f_S \circ (g_S \cap h_S) = (f_S \circ g_S) \cap (f_S \circ h_S) \) and \( (f_S \cap g_S) \circ h_S = (f_S \circ h_S) \cap (g_S \circ h_S) \).

v) If \( f_S \subseteq g_S \), then \( f_S \circ h_S \subseteq g_S \circ h_S \) and \( h_S \circ f_S \subseteq h_S \circ g_S \).
vi) If \( t_S, k_S \in S_S(U) \) such that \( t_S \subseteq f_S \) and \( k_S \subseteq g_S \), then \( t_S \circ k_S \subseteq f_S \circ g_S \) [29].

**Lemma 2.12.** Let \( f_S \) and \( g_S \) be soft sets over \( U \). Then, \( f_S \circ g_S = \emptyset_S \iff f_S = \emptyset_S \) or \( g_S = \emptyset_S \).

**Definition 2.13.** Let \( A \) be a subset of \( S \). We denote by \( S_A \) the soft characteristic function of \( A \) and define as

\[
S_A(x) = \begin{cases} 
U, & \text{if } x \in A \\
\emptyset, & \text{if } x \in S \setminus A 
\end{cases}
\]

The soft characteristic function of \( A \) is a soft set over \( U \), that is, \( S_A: S \rightarrow P(U) \) [29].

**Corollary 2.14.** \( \text{supp}(S_A) = A \) [49].

**Theorem 2.15.** Let \( X \) and \( Y \) be nonempty subsets of \( S \). Then, the following properties hold [29]:

i) If \( X \subseteq Y \) if and only if \( S_X \subseteq S_Y \)

ii) \( S_X \cap S_Y = S_{X \cap Y} \) and \( S_X \cup S_Y = S_{X \cup Y} \)

iii) \( S_X \circ S_Y = S_{X \circ Y} \)

**Proof:** In [29], (i) is given as if \( X \subseteq Y \), then if \( S_X \subseteq S_Y \). In [49], it was also shown that if \( S_X \subseteq S_Y \), then \( X \subseteq Y \). Let \( S_X \subseteq S_Y \) and \( x \in X \). Then, \( S_X(x) = U \) and this implies that \( S_Y(x) = U \) since \( S_X \subseteq S_Y \). Hence, \( x \in Y \) and so \( X \subseteq Y \). Now let \( x \notin Y \). Then, \( S_Y(x) = \emptyset \), and this implies that \( S_X(x) = \emptyset \) since \( S_X \subseteq S_Y \). Hence, \( x \notin X \) and so \( Y' \subseteq X' \), implying that \( X \subseteq Y \).

**Definition 2.16.** Let \( x \) be an element in \( S \). We denote by \( S_x \) the soft characteristic function of \( x \) and define as

\[
S_x(y) = \begin{cases} 
U, & \text{if } y = x \\
\emptyset, & \text{if } y \neq x 
\end{cases}
\]

The soft characteristic function of \( x \) is a soft set over \( U \), that is, \( S_x: S \rightarrow P(U) \) [49].

**Corollary 2.17.** Let \( x \in S \), \( f_S \) and \( S_x \) be soft sets over \( U \). Then,

\[
f_S \circ S_x = \emptyset_S \iff f_S = \emptyset_S \iff S_x = \emptyset_S.
\]

**Proof:** By Lemma 2.12, \( f_S \circ S_x = \emptyset_S \iff f_S = \emptyset_S \) or \( S_x = \emptyset_S \). By Definition 2.16, \( S_x \neq \emptyset_S \); hence the rest of the proof is obvious.

**Definition 2.18.** A soft set \( f_S \) over \( U \) is called a soft intersection subsemigroup of \( S \) over \( U \) if \( f_S(xy) \supseteq f_S(x) \cap f_S(y) \) for all \( x, y \in S \); and is called a soft intersection left (resp. right) ideal of \( S \) over \( U \) if \( f_S(xy) \supseteq f_S(x) \) (\( f_S(xy) \supseteq f_S(y) \)) for all \( x, y \in S \). A soft set \( f_S \) over \( U \) is called a soft intersection ideal of \( S \) if it is both a soft intersection left ideal of \( S \) over \( U \) and a soft intersection right ideal of \( S \) over \( U \) [29].

It is easy to see that if \( f_S(x) = U \) for all \( x \in S \), then \( f_S \) is a soft intersection (left/right) ideal. We denote such a kind of soft intersection (left/right) ideal by \( \overline{S} \). It is obvious that \( \overline{S} = S_S \), that is, \( \overline{S}(x) = U \) for all \( x \in S \) [29].
Theorem 2.19. Let $f_S$ be a soft set over $U$. Then, $f_S$ is a soft intersection subsemigroup of $S$ over $U$ if and only if $f_S \circ f_S \subseteq f_S$; and $f_S$ is a soft intersection left (resp. right) ideal of $S$ over $U$ if and only if $S \circ f_S \subseteq f_S$ ($f_S \circ S \subseteq f_S$) [29].

Definition 2.20. Let $f_S$ be a soft set over $U$. Then, $f_S$ is a soft intersection almost subsemigroup of $S$ over $U$ if

$$(f_S \circ f_S) \cap f_S \neq \emptyset_S$$

Definition 2.21. Let $f_S$ be a soft set over $U$. Then, $f_S$ is a soft intersection almost left ideal of $S$ over $U$ if for all $x \in S$,

$$(S_x \circ f_S) \cap f_S \neq \emptyset_S$$

Definition 2.22. Let $f_S$ be a soft set over $U$. Then, $f_S$ is a soft intersection almost right ideal of $S$ over $U$ if for all $x \in S$,

$$(f_S \circ S_x) \cap f_S \neq \emptyset_S$$

Definition 2.23. Let $f_S$ be a soft set over $U$. $f_S$ is called a soft intersection almost ideal of $S$ if for all $x \in S$,

$$(S_x \circ f_S) \cap f_S \neq \emptyset_S \text{ and } (f_S \circ S_x) \cap f_S \neq \emptyset_S$$

Inspired by the divisibility of determinants, we refer to [50] for the considerations of graph applications and network analysis.

III. RESULTS ON SOFT INTERSECTION ALMOST IDEALS OF SEMIGROUPS

Definition 3.1. Let $f_S$ be a soft set over $U$. $f_S$ is called a soft intersection almost left ideal of $S$ if for all $x \in S$,

$$(S_x \circ f_S) \cap f_S \neq \emptyset_S$$

Definition 3.2. Let $f_S$ be a soft set over $U$. $f_S$ is called a soft intersection almost right ideal of $S$ if for all $x \in S$,

$$(f_S \circ S_x) \cap f_S \neq \emptyset_S$$

Definition 3.3. Let $f_S$ be a soft set over $U$. $f_S$ is called a soft intersection almost ideal of $S$ if for all $x \in S$,

$$(S_x \circ f_S) \cap f_S \neq \emptyset_S \text{ and } (f_S \circ S_x) \cap f_S \neq \emptyset_S$$

Hereafter, for brevity, soft intersection almost left ideal and soft intersection almost right ideal of $S$ are denoted by SI-almost L-ideal and SI-almost R-ideal, respectively. The similar arguments and abbreviations are valid for almost left and right ideals and soft intersection left and right ideals of $S$.

Example 3.4. Let $S = \{\ell, g, n\}$ be the semigroup with the following Cayley Table.

<table>
<thead>
<tr>
<th></th>
<th>$\ell$</th>
<th>$g$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell$</td>
<td>$\ell$</td>
<td>$g$</td>
<td>$n$</td>
</tr>
<tr>
<td>$g$</td>
<td>$g$</td>
<td>$n$</td>
<td>$\ell$</td>
</tr>
<tr>
<td>$n$</td>
<td>$n$</td>
<td>$\ell$</td>
<td>$g$</td>
</tr>
</tbody>
</table>

Let $f_S$ and $h_S$ be soft sets over $U = \{(a, 0) \mid a \in \mathbb{Z}_4\}$ as follows:

$$f_S = \{(\ell, \{[\begin{array}{c} 1 \\ 0 \end{array}]_0, [\begin{array}{c} 2 \\ 0 \end{array}]_2\}), (g, \{[\begin{array}{c} 2 \\ 0 \end{array}]_0, [\begin{array}{c} 0 \\ 3 \end{array}]_1\}), (n, \{[\begin{array}{c} 1 \\ 0 \end{array}]_0, [\begin{array}{c} 3 \\ 0 \end{array}]_3\})\}$$

$$h_S = \{(\ell, \{[\begin{array}{c} 1 \\ 0 \end{array}]_0, [\begin{array}{c} 3 \\ 0 \end{array}]_3\}), (g, \{[\begin{array}{c} 0 \\ 0 \end{array}]_0, [\begin{array}{c} 1 \\ 0 \end{array}]_1\}), (n, \{[\begin{array}{c} 0 \\ 0 \end{array}]_0, [\begin{array}{c} 2 \\ 0 \end{array}]_2\})\}$$
and let \( g_S \) be soft set over \( U = D_S = \{< x, y >: x^3 = y^2 = e, \ xy = yx^2 \} = \{ e, x, x^2, y, yx, yx^2 \} \) as follows:

\[
g_S = \{(f, \epsilon) . (g, \{x, x^2\}). (n, \{y, yx, yx^2\})\}
\]

Here, \( f_S \) and \( h_S \) are both SI-almost ideals. Let’s first show that \( f_S \) is an SI-almost L-ideal, that is, for all \( x \in S \)

\((S_x \circ f_S) \cap f_S \neq \emptyset \):

Let’s start with \( S_f \):

\[
[S_f \circ f_S] \cap f_S(f) = (S_f \circ f_S)(f) \cap (S_f(f) \cap f_S(f)) \cap f_S(f) = f_S(f) = \left\{ [0, 1], [0, 2] \right\}
\]

\[
[S_f \circ f_S] \cap f_S(g) = (S_f \circ f_S)(g) \cap f_S(g) = (S_f(g) \cap f_S(g)) \cap (S_f(n) \cap f_S(n)) \cap f_S(g) = f_S(g) = \left\{ [2, 0], [3, 0] \right\}
\]

\[
[S_f \circ f_S] \cap f_S(n) = (S_f \circ f_S)(n) \cap f_S(n) = (S_f(n) \cap f_S(n)) \cap f_S(n) = f_S(n) = \left\{ [1, 0], [3, 0] \right\}
\]

Thus,

\[
(S_f \circ f_S) \cap f_S = \left\{ (f, \epsilon), (g, \{x, x^2\}). (n, \{y, yx, yx^2\}) \right\} \neq \emptyset.
\]

Let’s continue with \( S_g \):

\[
[S_g \circ f_S] \cap f_S(f) = (S_g \circ f_S)(f) \cap (S_g(f) \cap f_S(f)) \cap f_S(f) = f_S(f) = \left\{ [1, 0] \right\}
\]

\[
[S_g \circ f_S] \cap f_S(g) = (S_g \circ f_S)(g) \cap f_S(g) = (S_g(g) \cap f_S(g)) \cap (S_g(n) \cap f_S(n)) \cap f_S(g) = f_S(g) = \left\{ [2, 0] \right\}
\]

\[
[S_g \circ f_S] \cap f_S(n) = (S_g \circ f_S)(n) \cap f_S(n) = (S_g(n) \cap f_S(n)) \cap f_S(n) = f_S(n) = \left\{ [3, 0] \right\}
\]

Thus,

\[
(S_g \circ f_S) \cap f_S = \left\{ (f, \epsilon), (g, \{x, x^2\}). (n, \{y, yx, yx^2\}) \right\} \neq \emptyset.
\]

Let’s continue with \( S_n \):

\[
[S_n \circ f_S] \cap f_S(f) = (S_n \circ f_S)(f) \cap (S_n(f) \cap f_S(f)) \cap f_S(f) = f_S(f) = \left\{ [2, 0] \right\}
\]

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Thus, let's continue with \( R \)-ideal, that is, for all 
\[
\left( S_n \circ f_\ell \right) \cap f_\ell(g) = \left( S_n \circ f_\ell \right) \cap f_\ell(g) = \left( S_n \cap f_\ell \right) \cup \left( S_n \cap f_\ell \right) \cup \left( \left( S_n \cap f_\ell \right) \cap f_\ell(n) \cup f_\ell(n) \right) \cap f_\ell(g) = f_\ell(n) \cap f_\ell(g) = \left[ \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \right]
\]
\[
\left( S_n \circ f_\ell \right) \cap f_\ell(n) = \left( S_n \circ f_\ell \right) \cap f_\ell(n) = \left( S_n \cap f_\ell \right) \cup \left( S_n \cap f_\ell \right) \cup \left( \left( S_n \cap f_\ell \right) \cap f_\ell(n) \cup f_\ell(n) \right) \cap f_\ell(n) = f_\ell(n) \cap f_\ell(n) = \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]
\]
Consequently,
\[
\left( S_n \circ f_\ell \right) \cap f_\ell = \left( \left( \ell, \left[ \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \right] \right), \left( \ell, \left[ \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \right] \right), \left( \ell, \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \right) \right) \neq \emptyset
\]
Therefore, for all \( x \in S, \left( S_x \circ f_\ell \right) \cap f_\ell \neq \emptyset \), so \( f_\ell \) is an SI-almost L-ideal. Now let's show that \( f_\ell \) is an SI-almost R-ideal, that is, for all \( x \in S, \left( f_\ell \circ S_x \right) \cap f_\ell \neq \emptyset \).

Let's start with \( S_\ell \):
\[
\left( f_\ell \circ S_\ell \right) \cap f_\ell = \left( f_\ell \circ S_\ell \right) \cap f_\ell = \left( f_\ell \cap S_\ell \right) \cup \left( f_\ell \cap S_\ell \right) \cup \left( \left( f_\ell \cap S_\ell \right) \cup f_\ell \cap f_\ell(n) \right) \cap f_\ell = f_\ell(n) \cap f_\ell(n) = \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]
\]
\[
\left( f_\ell \circ S_\ell \right) \cap f_\ell(n) = \left( f_\ell \circ S_\ell \right) \cap f_\ell(n) = \left( f_\ell \cap S_\ell \right) \cup \left( f_\ell \cap S_\ell \right) \cup \left( \left( f_\ell \cap S_\ell \right) \cup f_\ell \cap f_\ell(n) \right) \cap f_\ell(n) = f_\ell(n) \cap f_\ell(n) = \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]
\]
Thus,
\[
\left( f_\ell \circ S_\ell \right) \cap f_\ell = \left( \left( \ell, \left[ \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \right] \right), \left( \ell, \left[ \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \right] \right), \left( \ell, \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \right) \right) \neq \emptyset
\]
Let's continue with \( S_\ell \):
\[
\left( f_\ell \circ S_\ell \right) \cap f_\ell = \left( f_\ell \circ S_\ell \right) \cap f_\ell = \left( f_\ell \cap S_\ell \right) \cup \left( f_\ell \cap S_\ell \right) \cup \left( \left( f_\ell \cap S_\ell \right) \cup f_\ell \cap f_\ell(n) \right) \cap f_\ell = f_\ell(n) \cap f_\ell(n) = \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]
\]
\[
\left( f_\ell \circ S_\ell \right) \cap f_\ell(n) = \left( f_\ell \circ S_\ell \right) \cap f_\ell(n) = \left( f_\ell \cap S_\ell \right) \cup \left( f_\ell \cap S_\ell \right) \cup \left( \left( f_\ell \cap S_\ell \right) \cup f_\ell \cap f_\ell(n) \right) \cap f_\ell(n) = f_\ell(n) \cap f_\ell(n) = \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]
\]
Hence,
Let's continue with $S_n$:

$$(f_s \circ S_n) \cap f_s = \left\{ (\ell, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}), (g, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}), (n, \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}) \right\} \neq \emptyset_s$$

Similarly,

$$\left( f_s \circ S_{n} \right) \cap f_s = \left\{ (\ell, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}), (g, \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}), (n, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}) \right\} \neq \emptyset$$

Consequently,

$$\left( f_s \circ S_{n} \right) \cap f_s = \left\{ (\ell, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}), (g, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}), (n, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}) \right\} \neq \emptyset$$

Therefore, for all $x \in S$, $(f_s \circ S_n) \cap f_s \neq \emptyset$, so $f_s$ is an SI-almost R-ideal, thus $f_s$ is an SI-almost ideal.

Similarly, $h_s$ is an SI-almost ideal. In fact;

$$(S_{F} \circ h_s) \cap h_s = \left\{ (\ell, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}), (g, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}), (n, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}) \right\} \neq \emptyset_s$$

Hence, $h_s$ is an SI-almost L-ideal. And,

$$(h_s \circ S_{L}) \cap h_s = \left\{ (\ell, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}), (g, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}), (n, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}) \right\} \neq \emptyset_s$$

Thus, $h_s$ is an SI-almost R-ideal, thus $h_s$ is an SI-almost ideal.

One can also show that $g_s$ is neither an SI-almost L-ideal nor an SI-almost R-ideal. In fact;

$$\left( S_{g} \circ g_s \right) \cap g_s = \left\{ (\ell, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}), (g, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}), (n, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}) \right\} \neq \emptyset_s$$

$$\left( S_{g} \circ g_s \right) \cap g_s = \left\{ (\ell, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}), (g, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}), (n, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}) \right\} \neq \emptyset_s$$

Thus, $g_s$ is an SI-almost R-ideal, thus $h_s$ is an SI-almost ideal.
\[(S_\rho \circ g_\text{S}) \cap g_\text{S})(\rho) = (S_\rho \circ g_\text{S})(\rho) \cap g_\text{S}(\rho) = \left[ (S_\rho(\ell) \cap g_\text{S}(\rho)) \cup (S_\rho(\rho) \cap g_\text{S}(\ell)) \cup (S_\rho(n) \cap g_\text{S}(n)) \right] \cap g_\text{S}(\rho) = g_\text{S}(\ell) \cap g_\text{S}(\rho) = \emptyset \]

\[(S_\rho \circ g_\text{S}) \cap g_\text{S}(n) = (S_\rho \circ g_\text{S})(n) \cap g_\text{S}(n) = \left[ (S_\rho(\ell) \cap g_\text{S}(n)) \cup (S_\rho(\rho) \cap g_\text{S}(\ell)) \cup (S_\rho(n) \cap g_\text{S}(\rho)) \right] \cap g_\text{S}(n) = g_\text{S}(\rho) \cap g_\text{S}(n) = \emptyset \]

Hence, for \( g \in S; \) \((S_\rho \circ g_\text{S}) \cap g_\text{S} = \{(\ell, \emptyset), (\rho, \emptyset), (n, \emptyset)\} = \emptyset_S\), thus \( g_\text{S} \) is not an SI-almost L-ideal. Similarly, for \( n \in S; \) \((g_\text{n} \circ S_\rho) \cap g_\text{n} = \{(\ell, \emptyset), (\rho, \emptyset), (n, \emptyset)\} = \emptyset_S\), thus, \( g_\text{n} \) is not an SI-almost R-ideal. It is obvious that \( g_\text{S} \) is not an SI-ideal.

From now on, the proofs are given for only SI-almost L-ideal, since the proofs for SI-almost R-ideal can be shown similarly.

**Proposition 3.5.** If \( f_\text{S} \) is an SI-L (resp. R)-ideal such that \( f_\text{S} \neq \emptyset_S \), then \( f_\text{S} \) is an SI-almost L (resp. R)-ideal.

**Proof:** Let \( f_\text{S} \neq \emptyset_S \) be an SI-L-ideal, thus \( S_\rho \circ f_\text{S} \subseteq f_\text{S} \). Since \( f_\text{S} \neq \emptyset_S \), by Corollary 2.17 it follows that \( S_\rho \circ f_\text{S} \neq \emptyset_S \). We need to show that for all \( x \in S \),

\[(S_\rho \circ f_\text{S}) \cap f_\text{S} \neq \emptyset_S.\]

Since \( S_\rho \circ f_\text{S} \subseteq S_\rho \circ f_\text{S} \subseteq f_\text{S} \), it follows that \( S_\rho \circ f_\text{S} \subseteq f_\text{S} \). Thus,

\[(S_\rho \circ f_\text{S}) \cap f_\text{S} = S_\rho \circ f_\text{S} \neq \emptyset_S \]

implying that \( f_\text{S} \) is an SI-almost L-ideal.

Here it is obvious that \( \emptyset_S \) is an SI-L-ideal as \( S_\rho \circ \emptyset_S \subseteq \emptyset_S \); but it is not SI-almost L-ideal since \( S_\rho \circ \emptyset_S \cap \emptyset_S = \emptyset_S \cap \emptyset_S = \emptyset_S \).

Here note that if \( f_\text{S} \) is an SI-almost L (resp. R)-ideal, then \( f_\text{S} \) needs not to be an SI-L (resp. R)-ideal as shown in the following example:

**Example 3.6.** In Example 3.4, it is shown that \( f_\text{S} \) and \( h_\text{S} \) are SI-almost L (resp. R)-ideals; however \( f_\text{S} \) and \( h_\text{S} \) are not SI-L (resp. R)-ideals. In fact,

\[(S_\rho \circ f_\text{S})(\rho) = \left[ (S_\rho(\ell) \cap f_\text{S}(\rho)) \cup (S_\rho(\rho) \cap f_\text{S}(\ell)) \cup (S_\rho(n) \cap f_\text{S}(n)) \right] \cap f_\text{S}(\rho) = f_\text{S}(\ell) \cup f_\text{S}(n) \cup f_\text{S}(\rho) \subseteq f_\text{S}(\rho) \]

thus, \( f_\text{S} \) is not an SI-L-ideal. Similarly,

\[(f_\text{S} \circ S_\rho)(\ell) = \left[ (f_\text{S}(\ell) \cap S_\rho(\ell)) \cup (f_\text{S}(\rho) \cap S_\rho(\ell)) \cup (f_\text{S}(n) \cap S_\rho(n)) \right] \cap S_\rho(\ell) = f_\text{S}(\ell) \cup f_\text{S}(n) \cup f_\text{S}(\rho) \subseteq f_\text{S}(\ell) \]

thus, \( f_\text{S} \) is not an SI-R-ideal. It is obvious that \( f_\text{S} \) is not an SI-ideal.
Similarly,

\[(S \circ h_S)(\ell) = \left[ (S(\ell) \cap h_S(\ell)) \cup (S(\ell) \cap h_S(n)) \right] \]

\[= h_S(\ell) \cup h_S(n) \cup h_S(\ell) \subseteq h_S(\ell) \]

thus, \(h_S\) is not an SI-L-ideal. Similarly,

\[(h_S \circ S)(\ell) = \left[ (h_S(\ell) \cap S(\ell)) \cup (h_S(\ell) \cap S(n)) \right] \]

\[= h_S(\ell) \cup h_S(n) \cup h_S(\ell) \subseteq h_S(\ell) \]

thus, \(h_S\) is not an SI-R-ideal. It is clear that \(h_S\) is not an SI-ideal.

**Proposition 3.7.** Let \(f_S\) be an idempotent soft set. If \(f_S\) is an SI-almost (L/R)-ideal, then \(f_S\) is an SI-almost subsemigroup.

**Proof:** Assume that \(f_S\) is an idempotent SI-almost L-ideal, then \(f_S \circ f_S = f_S\) and \((S_S \circ f_S) \upharpoonright f_S \neq \emptyset_S\) for all \(x \in S\). We need to show that \(f_S\) is an SI-almost subsemigroup, that is \((f_S \circ f_S) \upharpoonright f_S \neq \emptyset_S\).

\[(S \circ f_S) \upharpoonright f_S = [(S \circ f_S) \upharpoonright f_S] \upharpoonright f_S \]

\[= [(S_S \circ f_S) \upharpoonright (f_S \circ f_S)] \upharpoonright f_S \]

\[\subseteq (f_S \circ f_S) \upharpoonright f_S\]

Since \((S \circ f_S) \upharpoonright f_S \neq \emptyset_S\), it is obvious that \((f_S \circ f_S) \upharpoonright f_S \neq \emptyset_S\). Thus, \(f_S\) is an SI-almost subsemigroup.

**Theorem 3.8.** Let \(f_S \subseteq h_S\) such that \(f_S\) is an SI-almost L (resp. R)-ideal, then \(h_S\) is an SI-almost L (resp. R)-ideal.

**Proof:** Assume that \(f_S\) is an SI-almost L-ideal. Hence, for all \(x \in S\), \((S \circ f_S) \upharpoonright f_S \neq \emptyset_S\). We need to show that \((S \circ h_S) \upharpoonright h_S \neq \emptyset_S\). In fact,

\[(S \circ h_S) \upharpoonright h_S \subseteq (S \circ h_S) \upharpoonright h_S\]

Since \((S \circ f_S) \upharpoonright f_S \neq \emptyset_S\), it is obvious that \((S_S \circ h_S) \upharpoonright h_S \neq \emptyset_S\). This completes the proof.

**Theorem 3.9.** Let \(f_S\) and \(h_S\) be SI-almost L (resp. R)-ideals. Then, \(f_S \cup h_S\) is an SI-almost L (resp. R)-ideal.

**Proof:** Since \(f_S\) is an SI-almost L-ideal by assumption and \(f_S \subseteq f_S \cup h_S\), \(f_S \cup h_S\) is an SI-almost L-ideal by Theorem 3.8.

**Corollary 3.10.** The finite union of SI-almost L (resp. R)-ideals is an SI-almost L (resp. R)-ideal.

**Corollary 3.11.** Let \(f_S\) or \(h_S\) be SI-almost L (resp. R)-ideal. Then, \(f_S \cup h_S\) is an SI-almost L (resp. R) ideal.

Here note that if \(f_S\) and \(h_S\) are SI-almost L (resp. R)-ideals, then \(f_S \cup h_S\) needs not to be an SI-almost L (resp. R)-ideal.

**Example 3.12.** Consider the SI-almost L (resp. R)-ideals \(f_S\) and \(h_S\) in Example 3.4. Since,
\[ f_S \mathcal{R} h_S = \{(\ell, \emptyset), (g, \emptyset), (n, \emptyset)\} = \emptyset_S \]

\( f_S \mathcal{R} h_S \) is not an SI-almost L (resp. R)-ideal.

Now, we give the relationship between almost L (resp. R)-ideal and SI-almost L (resp. R)-ideal of \( S \). But first of all, we give the following lemma to use it in Theorem 3.14.

**Lemma 3.13.** Let \( x \in S \) and \( Y \) be nonempty subset of \( S \). Then, \( S^\circ_s Y = S_{xy} \).

**Proof:** Let \( s \in S \) such that \( s \in xY \). Then, \( s = xn \) for some \( n \in Y \) and \( x \in S \). Thus,

\[
\begin{align*}
(S^\circ_s Y)(s) &= \bigcup_{s=xn} \{S(x) \cap S_Y(q)\} \\
&\supseteq S(x) \cap S_Y(n) \\
&= U \cap U \\
&= U
\end{align*}
\]

so \( (S^\circ_s Y)(s) = U \) . Since \( s = xn \in xY \), it follows that \( S_{xy}(s) = U \). Hence, \( S^\circ_s Y = S_{xy} \).

In another case, let \( s \in S \) such that \( s \notin xY \). Then, if \( s = mn \) for some \( m, n \in S \), we have that \( m \neq x \) or \( n \notin Y \).

Consider the following equations:

\[
(S^\circ_s Y)(s) = \bigcup_{s=mn} \{S_x(m) \cap S_Y(n)\};
\]

**Case 1:** Let \( m \neq x \) and \( n \in Y \). Then,

\[
\bigcup_{s=mn} \{S_x(m) \cap S_Y(n)\} = \bigcup_{s=mn} \{\emptyset \cap U\} = \emptyset
\]

**Case 2:** Let \( m = x \) and \( n \notin Y \). Then,

\[
\bigcup_{s=mn} \{S_x(m) \cap S_Y(n)\} = \bigcup_{s=xn} \{U \cap \emptyset\} = \emptyset
\]

**Case 3:** Let \( m \neq x \) and \( n \notin Y \), then,

\[
\bigcup_{s=mn} \{S_x(m) \cap S_Y(n)\} = \bigcup_{s=mn} \{\emptyset \cap \emptyset\} = \emptyset.
\]

In all cases, \( (S^\circ_s Y)(s) = \emptyset \) when \( s \notin xY \). Since \( s \notin xY \), it follows that \( S_{xy}(s) = \emptyset \). Hence \( S^\circ_s Y = S_{xy} \).

When \( X \) is a nonempty subset of \( S \) and \( Y \in S \), then it is obvious that \( S^\circ_X Y = S_{xy} \).

**Theorem 3.14.** Let \( A \) be a subset of \( S \). Then, \( A \) is an almost L (resp. R)-ideal if and only if \( S_A \), the soft characteristic function of \( A \), is an SI-almost L (resp. R)-ideal, where \( \emptyset \neq A \subseteq S \).
Proof: Assume that $\emptyset \neq A$ is an almost $L$-ideal. Then, $xA \cap A \neq \emptyset$ for all $x \in S$, and so there exist $k \in S$ such that $k \in xA \cap A$. Since,
\[
((S_x \circ S_A) \cap S_A)(k) = (S_{xA} \cap S_A)(k) = (S_{x\cap A})(k) = U \neq \emptyset
\]
it follows that $(S_x \circ S_A) \cap S_A \neq \emptyset$. Thus, $S_A$ is an SI-almost $L$-ideal.

Conversely assume that $S_A$ is an SI-almost $L$-ideal. Hence, we have $(S_x \circ S_A) \cap S_A \neq \emptyset$ for all $x \in S$. To show that $A$ is an almost $L$-ideal, we should prove that $A \neq \emptyset$ and $xA \cap A \neq \emptyset$ for all $x \in S$. $A \neq \emptyset$ is obvious from assumption. Now,
\[
\emptyset \neq (S_x \circ S_A) \cap S_A \Rightarrow \exists k \in S; (S_x \circ S_A)(k) \neq \emptyset
\]
\[
\Rightarrow \exists k \in S; (S_{xA})(k) \neq \emptyset
\]
\[
\Rightarrow \exists k \in S; (S_{x\cap A})(k) = U
\]
\[
\Rightarrow k \in xA \cap A
\]

Hence, $xA \cap A \neq \emptyset$. Consequently, $A$ is an almost $L$-ideal.

Lemma 3.15. Let $f_S$ be a soft set over $U$. Then, $f_S \subseteq S_{supp(f_S)}$ [49].

Theorem 3.16. If $f_S$ is an SI-almost $L$ (resp. $R$)-ideal, then $supp(f_S)$ is an almost $L$ (resp. $R$)-ideal.

Proof: Assume that $f_S$ is an SI-almost $L$-ideal. Thus, $(S_x \circ f_S) \cap f_S \neq \emptyset$ for all $x \in S$. To show that $supp(f_S)$ is an almost $L$-ideal, by Theorem 3.14, it is enough to show that $S_{supp(f_S)}$ is an SI-almost $L$-ideal. By Lemma 3.15,
\[
(S_x \circ f_S) \cap f_S \subseteq (S_x \circ S_{supp(f_S)}) \cap S_{supp(f_S)}
\]
and $(S_x \circ f_S) \cap f_S \neq \emptyset$, it implies that $(S_x \circ S_{supp(f_S)}) \cap S_{supp(f_S)} \neq \emptyset$. Consequently, $S_{supp(f_S)}$ is an SI-almost $L$-ideal and by Theorem 3.14, $supp(f_S)$ is an almost $L$-ideal.

Here note that the converse of Theorem 3.16 is not true in general as shown in the following example.

Example 3.17. We know that $g_S$ is not an SI-almost $L$-ideal in Example 3.4. and it is obvious that $supp(g_S) = \{\ell, g, n\} = S$. Since,
\[
[[\ell]supp(g_S)] \cap supp(g_S) = \{\ell\} \cap \{\ell, g, n\} = \{\ell\} \neq \emptyset
\]
\[
[[g]supp(g_S)] \cap supp(g_S) = \{g\} \cap \{\ell, g, n\} = \{g\} \neq \emptyset
\]
\[
[[n]supp(g_S)] \cap supp(g_S) = \{n\} \cap \{\ell, g, n\} = \{n\} \neq \emptyset.
\]
It is seen that $[[x]supp(g_S)] \cap supp(g_S) \neq \emptyset$ for all $x \in S$. That is to say, $supp(g_S)$ is an almost $L$-ideal; although $g_S$ is not an SI-almost $L$-ideal.
Definition 3.18. Let \( f_S \) and \( h_S \) be SI-almost L (resp. R)-ideals such that \( h_S \subseteq f_S \). If \( \text{supp}(h_S) = \text{supp}(f_S) \), then \( f_S \) is called a minimal SI-almost L (resp. R)-ideal.

Theorem 3.19. \( A \) is a minimal almost L (resp. R)-ideal if and only if \( S_A \), the soft characteristic function of \( A \), is a minimal SI-almost L (resp. R)-ideal, where \( \emptyset \neq A \subseteq S \).

Proof: Assume that \( A \) is a minimal almost L-ideal. Thus, \( A \) is an almost L-ideal, and so \( S_A \) is an SI-almost L-ideal by Theorem 3.14. Let \( f_S \) be an SI-almost L-ideal such that \( f_S \subseteq S_A \). By Theorem 3.15, \( \text{supp}(f_S) \) is an almost L-ideal, and by Note 2.6 and Corollary 2.14,

\[
\text{supp}(f_S) \subseteq \text{supp}(S_A) = A.
\]

Since \( A \) is a minimal almost L-ideal, \( \text{supp}(f_S) = \text{supp}(S_A) = A \). Thus, \( S_A \) is a minimal SI-almost L-ideal by Definition 3.18.

Conversely, let \( S_A \) be a minimal SI-almost L-ideal. Thus, \( S_A \) is an SI-almost L-ideal, and \( A \) is an almost L-ideal by Theorem 3.14. Let \( B \) be an almost L-ideal such that \( B \subseteq A \). By Theorem 3.14, \( S_B \) is an SI-almost L-ideal, and by Theorem 2.15 (i), \( S_B \subseteq S_A \). Since \( S_A \) is a minimal SI-almost L-ideal,

\[
B = \text{supp}(S_B) = \text{supp}(S_A) = A
\]

by Corollary 2.14. Thus, \( A \) is a minimal almost L-ideal.

Definition 3.20. Let \( f_S \), \( g_S \) and \( h_S \) be any SI-almost L (resp. R)-ideals. If \( h_S \circ g_S \subseteq f_S \) implies that \( h_S \subseteq f_S \) or \( g_S \subseteq f_S \), then \( f_S \) is called an SI-prime almost L (resp. R)-ideal.

Definition 3.21. Let \( f_S \) and \( h_S \) be any SI-almost L (resp. R)-ideals. If \( h_S \circ h_S \subseteq f_S \) implies that \( h_S \subseteq f_S \), then \( f_S \) is called an SI-semiprime almost L (resp. R)-ideal.

Definition 3.22. Let \( f_S \), \( g_S \), and \( h_S \) be any SI-almost L (resp. R)-ideals. If \( (h_S \circ g_S) \circ (g_S \circ h_S) \subseteq f_S \) implies that \( h_S \subseteq f_S \) or \( g_S \subseteq f_S \), then \( f_S \) is called an SI-strongly prime almost L (resp. R)-ideal.

It is obvious that every SI-strongly prime almost L (resp. R)-ideal is an SI-prime almost L (resp. R)-ideal and every SI-prime almost L (resp. R)-ideal is an SI-semiprime almost L (resp. R)-ideal.

Theorem 3.23. If \( S_P \), the soft characteristic function of \( P \), is an SI-prime almost L (resp. R)-ideal, then \( P \) is a prime almost L (resp. R)-ideal, where \( \emptyset \neq P \subseteq S \).

Proof: Assume that \( S_P \) is an SI-prime almost L-ideal. Thus, \( S_P \) is an SI-almost L-ideal and thus, \( P \) is an almost L-ideal by Theorem 3.14. Let \( A \) and \( B \) be almost L-ideals such that \( AB \subseteq P \). Thus, by Theorem 3.14, \( S_A \) and \( S_B \) are SI-almost L-ideals, and by Theorem 2.15 (i) and (iii), \( S_A \circ S_B = S_{AB} \subseteq S_P \). Since \( S_P \) is an SI-prime almost L-ideal and \( S_A \circ S_B \subseteq S_P \), it follows that \( S_A \subseteq S_P \) or \( S_B \subseteq S_P \). Therefore, by Theorem 2.15 (i), \( A \subseteq P \) or \( B \subseteq P \). Consequently, \( P \) is a prime almost L-ideal.

Theorem 3.24. If \( S_P \), the soft characteristic function of \( P \), is an SI-semiprime almost L (resp. R)-ideal, then \( P \) is a semiprime almost L (resp. R)-ideal, where \( \emptyset \neq P \subseteq S \).
Proof: Assume that $S_P$ is an SI-semiprime almost L-ideal. Thus, $S_P$ is an SI-almost L-ideal and thus, $P$ is an almost L-ideal by Theorem 3.14. Let $A$ be an almost L-ideal such that $AA \subseteq P$. Thus, by Theorem 3.14, $S_A$ is an SI-almost L-ideal, and by Theorem 2.15 (i) and (iii), $S_A \circ S_A = S_{AA} \subseteq S_P$. Since $S_P$ is an SI-semiprime almost L-ideal and $S_A \circ S_A \subseteq S_P$, it follows that $S_A \subseteq S_P$. Therefore, by Theorem 2.15 (i), $A \subseteq P$. Consequently, $P$ is a semiprime almost L-ideal.

Theorem 3.25. If $S_P$, the soft characteristic function of $P$, is an SI-strongly prime almost L (resp. R)-ideal, then $P$ is a strongly prime almost L (resp. R)-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that $S_P$ is an SI-strongly prime almost L-ideal. Thus, $S_P$ is an SI-almost L-ideal and thus, $P$ is an almost L-ideal by Theorem 3.14. Let $A$ and $B$ be almost L-ideals such that $AB \cap BA \subseteq P$. Thus, by Theorem 3.14, $S_A$ and $S_B$ are SI-almost L-ideal, and by Theorem 2.15, $(S_A \circ S_B) \cap (S_B \circ S_A) = S_{AB} \cap S_{BA} = S_{AB \cup BA} \subseteq S_P$. Since $S_P$ is an SI-strongly prime almost L-ideal and $(S_A \circ S_B) \cap (S_B \circ S_A) \subseteq S_P$, it follows that $S_A \subseteq S_P$ or $S_B \subseteq S_P$. Thus, by Theorem 2.15 (i), $A \subseteq P$ or $B \subseteq P$. Therefore, $P$ is a strongly prime almost L-ideal.

IV. CONCLUSIONS

In this study, as a generalization of the nonnull soft intersection left (resp. right) ideal of a semigroup, we introduced the idea of the soft intersection almost left (resp. right) ideal of a semigroup. We showed that every idempotent soft intersection almost (left/right) ideal of a semigroup is a soft intersection almost subsemigroup. We obtained that a semigroup can be constructed under the binary operation of union, but not under the binary of operation intersection for soft sets, given the collection of almost (left/right) ideals of a semigroup. Moreover, we showed the relation between soft intersection almost left (resp. right) ideals of a semigroup and almost left (resp. right) ideals of a semigroup in accordance with minimality, primeness, semiprimeness, and strongly primeness. In future studies, many types of soft intersection almost ideals, including quasi-ideal, interior ideal, bi-ideal, bi-interior ideal, bi-quasi-ideal, quasi-interior ideal, and bi-quasi-interior ideal of semigroups can be examined.

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