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***Research Article / Araştırma Makalesi***

# On Neutrosophic Continuity

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## Abstract

In this paper, we defined the neutrosophic continuous function, neutrosophic open function, neutrosophic closed function and neutrosophic homeomorphism on neutrosophic topological spaces. Then, we give some characteristics of these functions; neutrosophic closed function is a neutrosophic continuous function, neutrosophic open function is a neutrosophic continuous function.

***Keywords***: Neutrosophic set, neutrosophic topological space, neutrosophic continuous function, neutrosophic open function, neutrosophic homeomorphism.

# Neutrosophic Süreklilik Üzerine

## Öz

Bu çalışmada, neutrosophic topolojik uzaylarda neutrosophic sürekli fonksiyon, neutrosophic açık fonksiyon, neutrosophic kapalı fonksiyon ve neutrosophic homeomorfizm tanımlandı. Daha sonra, bu fonksiyonların bazı karakteristik özellikleri hakkında bilgi verildi.

***Anahtar Kelimeler***: Neutrosophic küme, neutrosophic topolojik uzay, neutrosophic sürekli fonksiyon, neutrosophic açık fonksiyon, neutrosophic homeomorfizm.

# 1. Introduction

The concept of neutrosophic sets was first introduced by Smarandache (Smarandache, 2005), as a generalization of intuitionistic fuzzy sets (Atanassov, 1986) where we have the degree of membership, the degree of indeterminacy and the degree of non-membership of each element in X. After the introduction of the neutrosophic sets, neutrosophic set operations have been investigated. Topology of neutrosophic sets have been studied intensively by researchers, such as Samarandache (Samarandache, 2002), Lupianez (Lupianez, 2008), (Lupianez, 2009(1)), (Lupianez, 2009(2)) and (Lupianez, 2010).

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In this study, we define the neutrosophic continuous function and its features neutrosophic closed function, neutrosophic open function. Finally, we describe the neutrosophic homeomorphism with an its expository example.

# 2. Preliminaries

In this section, we give some information about the neutrosophic sets and the neutrosophic topological spaces (Karataş ve Kuru, 2016). Let $A$ neutrosophic set $A$ on the universe of discourse $X$ be defined as

$$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$$

where $\mu_A, \sigma_A, \gamma_A : X \to ]^{-}0,1+[$ and $^{-}0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^{+}$. The neutrosophic have the values on non-standard (or real standard) on interval $]^{-}0,1^{+}[$. Thus, we take into account the neutrosophic set which takes the values on subsets of $[0,1]$ real interval. Set of all neutrosophic sets over $X$ is denoted by $\mathrm{N}(X)$, (Smarandache, 2005).

**Definition 2.1** *Let* $\tau \subseteq \mathrm{N}(X)$, *then* $\tau$ *is called a neutrosophic topology on* $X$ *if*

**1**. $\tilde{X}$ and $\tilde{\emptyset}$ belong to $\tau$,
**2**. The union of any number neutrosophic sets in $\tau$ belongs to $\tau$,
**3**. The intersection of any two neutrosophic sets in $\tau$ belongs to $\tau$.

The pair $(X, \tau)$ is called a neutrosophic topological space over $X$. Moreover, the members of $\tau$ are said to be neutrosophic open sets in $X$. If $A^c \in \tau$, then $A \in \mathrm{N}(X)$ is said to be neutrosophic closed set in $X$. Set of all neutrosophic closed sets over $X$ is denoted by $\kappa(\tau)$, (Karataş ve Kuru, 2016).

**Definition 2.2** *Let* $(X, \tau)$ *be a neutrosophic topological space over* $X$ *and* $A \in \mathrm{N}(X)$.

**1.** The neutrosophic interior of $A$, denoted by $int(A)$ is the union of all neutrosophic open subsets of $A$. So $int(A)$ is the biggest neutrosophic open set over $X$ containing $A$,
**2.** The neutrosophic closure of $A$, denoted by $cl(A)$ is the intersection of all neutrosophic closed super subsets of $A$. So $cl(A)$ is the smallest neutrosophic closed set over $X$ which containing $A$,
**3.** The neutrosophic boundary of a neutrosophic set $A$ over $X$ is denoted by $fr(A)$ and is defined as $fr(A) = cl(A) \cap (int(A))^c$. It must be noted that $fr(A) = fr(A^c)$ (Karataş ve Kuru, 2016).

**Theorem 2.1** *Let* $(X, \tau)$ *be a neutrosophic topological space over* $X$ *and* $A, B \in \mathrm{N}(X)$. *Then ,we have* (Karataş ve Kuru, 2016).

1. $int(A) \subseteq A$,
2. $A \subseteq cl(A)$,
3. $int(A^c) = (cl(A))^c$,

4. $\mathrm{c}l(A^c) = (\mathrm{i}nt(A))^c$,
5. $\mathrm{c}l(A) = \mathrm{i}nt(A) \int \ \mathrm{f}r(A)$.

**Definition 2.3** *Let* $X$ *and* $Y$ *be two non empty set,* $f : X \to Y$ *be a function,* $A \in \mathrm{N}\,(X)$ *and* $B \in \mathrm{N}\,(Y)$. *Then, we have* (Salama ve ark., 2014).

**1.** Image of $A$ under $f$ is defined by

$$f(A) = \{\langle y, f(\mu_A)(y), (1 - f(1-\sigma_A))(y), (1 - f(1-\nu_A))(y)\rangle : y \in Y\}$$

where

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \ \mu_A(x), & f^{-1}(y) \neq \varnothing, \\ 0, & f^{-1}(y) = \varnothing, \end{cases}$$

$$(1 - f(1-\sigma_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \ \sigma_A(x), & f^{-1}(y) \neq \varnothing, \\ 1, & f^{-1}(y) = \varnothing, \end{cases}$$

$$(1 - f(1-\nu_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \ \nu_A(x), & f^{-1}(y) \neq \varnothing, \\ 1, & f^{-1}(y) = \varnothing. \end{cases}$$

**2.** Pre-image $B$ under $f$ is defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\nu_B)(x)\rangle : x \in X\}.$$

**Theorem 2.2** *Let* $f : X \to Y$ *be a function,* $A_1, A_2 \in \mathrm{N}\,(X)$ *and* $B_1, B_2 \in \mathrm{N}\,(Y)$. *Then followings are provided:* (Salama ve ark., 2014).

1. If $A_1 \,|\, A_2$, then $f(A_1) \,|\, f(A_2)$,
2. $A \,|\, f^{-1}(f(A))$ (If $f$ is an injective function, then equality holds.),
3. If $B_1 \,|\, B_2$, then $f^{-1}(B_1) \,|\, f^{-1}(B_2)$,
4. $f(f^{-1}(B)) \,|\, B$ (If $f$ is surjective function, then equality holds.) (If $f$ is a injective function, then equality holds).

# 3. Neutrosophic Continuity

**Example 3.1** *Let* $X=\{x_1,x_2,x_3\}$, $Y=\{y_1,y_2,y_3\}$, $A\in \mathrm{N}(X)$ *and* $B\in \mathrm{N}(Y)$ *such that*

$$A=\{\langle x_1,0.1,0.2,0.3\rangle,\langle x_2,0.7,0.6,0.5\rangle,\langle x_3,0.3,0.4,0.7\rangle\}$$
$$B=\{\langle y_1,0.2,0.5,0.7\rangle,\langle y_2,0.3,0.8,0.6\rangle,\langle y_3,0.1,0.7,0.9\rangle\}.$$

Moreover, let $f:X\to Y$ be a function such that $f(x_1)=y_2$, $f(x_2)=y_1$ and $f(x_3)=y_1$. Then, we have

$$f(A)=\{\langle y_1,0.7,0.4,0.5\rangle,\langle y_2,0.1,0.2,0.3\rangle,\langle y_3,0.0,1.0,1.0\rangle\}$$
$$f^{-1}(B)=\{\langle x_1,0.3,0.8,0.6\rangle,\langle x_2,0.2,0.5,0.7\rangle,\langle x_3,0.2,0.5,0.7\rangle\}.$$

**Definition 3.1** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces and* $f:X\to Y$ *be a function. If* $f^{-1}(G)\in\tau$ *for all* $G\in\sigma$*, then* $f$ *is called neutrosophic continuous function (Salama ve ark., 2014).*

**Example 3.2** *Let* $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$, $A\in \mathrm{N}(X)$, $B\in \mathrm{N}(Y)$ *and* $f:X\to Y$ *be a function such that*

$$A=\{\langle x_1,0.4,0.2,0.2\rangle,\langle x_2,0.5,0.4,0.6\rangle\},\ B=\{\langle y_1,0.2,0.4,0.8\rangle,\langle y_2,0.5,0.7,0.1\rangle\}$$

and

$$f(x_1)=y_1,\ f(x_2)=y_2.$$

Then, $\tau=\{\tilde{X},\tilde{\emptyset},A\}$ and $\sigma=\{\tilde{Y},\tilde{\emptyset},B\}$ are two neutrosophic topologies over $X$ and over $Y$, respectively. Hence, $f:(X,\tau)\to(Y,\sigma)$ is a neutrosophic continuous function.

**Theorem 3.1** *Let* $(X,\tau)$, $(Y,\sigma)$ *and* $(Z,\rho)$ *be three neutrosophic topological spaces. If* $f:X\to Y$ *and* $g:Y\to Z$ *are two neoutrosophic continuous functions, then* $gof:X\to Z$ *is a neutrosophic continuous function.*

*Proof.* Let $G\in\rho$ is a neutrosophic continuous so $g^{-1}(G)\in\sigma$ and $g$ is a neutrosophic continuous so $f^{-1}\left(g^{-1}(G)\right)\in\tau$ and $f^{-1}\left(g^{-1}(G)\right)=\left(gof\right)^{-1}(G)$ so $\left(gof\right)^{-1}(G)\in\tau$. Then, $\left(gof\right)$ is a neutrosophic continuous function.

**Theorem 3.2** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces.* $f:X\to Y$ *is a neutrosophic continuous if and only if* $f^{-1}(F)\in\kappa(\tau)$ *for all* $F\in\kappa(\sigma)$.

*Proof.* $(\Rightarrow)$: Let $A\in\kappa(\tau)$. Then we have $f^{-1}(A)\in\kappa(\tau)$. Because $f$ is neutrosophic continuous, $f^{-1}(A)$ set is neutrosophic closed set. Thence, $f^{-1}$ function is neutrosophic closed.

$(\Leftarrow)$: Let $A \in \kappa(\tau)$. Then we have $f^{-1}(A) \in \kappa(\tau)$. $f^{-1}(A)$ set is neutrosophic closed set because of $f^{-1}$ is neutrosophic closed. Thence, $f^{-1}$ function is neutrosophic continuous.

**Theorem 3.3** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces and* $f: X \to Y$ *be a function. Then,* $f$ *is a neutrosophic continuous function if and only if* $f(\mathrm{cl}(A)) \subseteq \mathrm{cl}(f(A))$ *for all* $A \in \mathrm{N}(X)$.

*Proof.* $(\Rightarrow)$: Let $A \in \mathrm{N}(X)$ and $f$ be a neutrosophic continuous function.
From Theorem 2.1, we know that

$$A \subseteq \mathrm{cl}(A) \Rightarrow f(A) \subseteq \mathrm{cl}(f(A))$$

Then, apploying 2 Theorem 2.2 , we have

$$A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\mathrm{cl}(f(A))) \text{ and } \mathrm{cl}(A) \subseteq \mathrm{cl}(f^{-1}(\mathrm{cl}(f(A)))).$$

*Because* $f$ *is a neutrosophic continuous function and* $\mathrm{cl}(f(A))$ is a neutrosophic closed set, we know $\mathrm{cl}(f^{-1}(\mathrm{cl}(f(A))) = f^{-1}(\mathrm{cl}(f(A)))$.
Hence, $f(\mathrm{cl}(A) \subseteq \mathrm{cl}(f(A))$.

$(\Leftarrow)$: Let $f(\mathrm{cl}(A)) \subseteq \mathrm{cl}(f(A))$ for all $A \in \mathrm{N}(X)$. $F \in \kappa(\sigma)$ is given. Then, $\mathrm{cl}(f(f^{-1}(F))) \subseteq \mathrm{cl}(F) = F$ and so $f^{-1}(F) \in \kappa(\tau)$. From Theorem 3.2, $f$ is a neutrosophic continuous function.

**Theorem 3.4** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces,* $f: X \to Y$ *be a function. Then,* $f$ *is a neutrosophic continuous function if and only if* $\mathrm{cl}(f^{-1}(B)) \subseteq f^{-1}(\mathrm{cl}(B))$ *for all* $B \in \mathrm{N}(Y)$.

*Proof.* $(\Rightarrow)$: Let $B \in \mathrm{N}(Y)$ and $f$ be a neutrosophic continuous fonction. From Theorem 2.2 and Theorem 2.1, we have $f^{-1}(B) \subseteq f^{-1}(\mathrm{cl}(B))$. Then, $\mathrm{cl}(f^{-1}(B)) \subseteq \mathrm{cl}(f^{-1}(\mathrm{cl}(B)))$. Because we know $\mathrm{cl}(B) \in \kappa(\sigma)$ by Theorem 3.2, $f^{-1}(\mathrm{cl}(B)) \in \kappa(\tau)$. Thus,

$$\mathrm{cl}(f^{-1}(B)) \subseteq \mathrm{cl}(f^{-1}(\mathrm{cl}(B))) = f^{-1}(\mathrm{cl}(B)).$$

$(\Leftarrow)$: Let $\mathrm{cl}(f^{-1}(B)) \subseteq f^{-1}(\mathrm{cl}(B)$ for all $B \in \mathrm{N}(Y)$. $F \in \kappa(\sigma)$ is given. Then,

$$\mathrm{cl}(f^{-1}(F)) \subseteq f^{-1}(\mathrm{cl}(F)) = f^{-1}(F).$$

From Theorem 3.2, $f$ is a neutrosophic continuous function.

**Theorem 3.5** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces,* $f:(X,\tau) \to (Y,\sigma)$ *be a bijective function.* $f$ *is neutrosophic continuous if and only if*

$int(f(A)) \subseteq f(int(A))$ *for all* $A \in \mathcal{N}(X)$.

*Proof.* $(\Rightarrow)$: Let, $A \in \mathcal{N}(X)$ and $f$ be a bijective and neutrosophic continuous function. $f(A) = B$ is given. From Theorem 2.2 and Theorem 2.1 we know that $f^{-1}(int(B)) \subseteq f^{-1}(B)$. Since $f$ is an injective function we know $f^{-1}(B) = A$, so that $f^{-1}(int(B)) \subseteq A$. Therefore, $int(f^{-1}(int(B))) \subseteq int(A)$. Here, $f^{-1}(int(B)) \in \tau$ and $f^{-1}(int(B)) \subseteq int(A)$ then, $f(f^{-1}(int(B)) \subseteq f(int(A))$. Since $f$ is a surjective function we know that $f(f^{-1}(int(B))) = int(B)$. Hence, $int(f(A)) \subseteq f(int(A))$.

$(\Leftarrow)$: Let $int(f(A)) \subseteq f(int(A))$ for all $A \in \mathcal{N}(X)$. Because $V \in \sigma$, $f$ is surjective we know that $V = int(V) = int(f(f^{-1}(V))) \subseteq (f(int(f^{-1}(V)))$. $f^{-1}(V) \subseteq int(f^{-1}(V))$ then $f^{-1}(V) \in \tau$ by injective $f$. Hence, $f$ is a neutrosophic continuous function.

**Theorem 3.6** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces and* $f: X \to Y$ *be a function. Then,* $f$ *is a neutrosophic continuous function if and only if* $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ *for all* $B \in \mathcal{N}(Y)$.

*Proof.* $(\Rightarrow)$: Let $B \in \mathcal{N}(Y)$ and $f$ be a neutrosophic continuous function.
$int(B) \subseteq B \Rightarrow f^{-1}(int(B)) \subseteq f^{-1}(B) \Rightarrow int(f^{-1}(int(B)) \subseteq int(f^{-1}(B))$
Since $int(B) \in \sigma$ and $f^{-1}(int(B)) \in \tau$. So that, $int(f^{-1}(int(B))) = f^{-1}(int(B)) \subseteq int(f^{-1}(B))$.

$(\Leftarrow)$: Let $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for all $B \in \mathcal{N}(Y)$ and $G \in \tau$. Then, $f^{-1}(G) \subseteq int(f^{-1}(G))$ and $f^{-1}(G) = int(f^{-1}(G))$ so that $f^{-1}(G) \in \tau$. Hence $f$ is a neutrosophic continuous function.

**Theorem 3.7** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces and* $f: X \to Y$ *be a bijective function. Then* $f$ *is a neutrosophic continuous function if and only if* $f(fr(A)) \subseteq fr(f(A))$ *for all* $A \in \mathcal{N}(X)$.

*Proof.* $(\Rightarrow)$: Let $f$ is a bijective and neutrosophic continuous function and $A \in \mathcal{N}(X)$. From Definition 2.2, we know that $fr(A) = cl(A) \cap (int(A))^c$. Therefore, from Theorem 3.2, $f(int(A)) \subseteq int(f(A))$ and from Theorem 3.5 we find $f(cl(A)) \subseteq cl(f(A))$. Hence,

$$f(fr(A)) = f(cl(A) \cap (int(A))^c)$$

$$\subseteq f(\mathrm{c}l(A)) \cap f((\mathrm{i}nt(A))^c)$$
$$= f(\mathrm{c}l(A)) \cap f((\mathrm{i}nt(A)))^c$$
$$= \mathrm{f}r(f(A))$$

$(\Leftarrow)$: Let $f(\mathrm{f}r(A)) \subseteq \mathrm{f}r(f(A))$ for all $A \in \mathcal{N}(X)$.

$$f(\mathrm{c}l(A)) = f(A) \cup (\mathrm{f}r(A))$$
$$= f(A) \cup f((\mathrm{f}r(A))$$
$$\subseteq f(A) \cup \mathrm{f}r(f(A))$$
$$= \mathrm{c}l(f(A))$$

By Theorem 3.3 we find $f$ is a neutrosophic continuous function.

**Theorem 3.8** *Let $(X,\tau)$ and $(Y,\sigma)$ be two neutrosophic topological spaces and $f : X \to Y$ be a bijective function. Then, $f$ is a neutrosophic continuous function if and only if $\mathrm{f}r(f^{-1}(B)) \subseteq f^{-1}(\mathrm{f}r(B))$ for all $B \in \mathcal{N}(Y)$.*

*Proof.* $(\Rightarrow)$: Let $f$ is a bijective and neutrosophic continuous function and $B \in \mathcal{N}(Y)$. By Theorem 3.4 and Theorem 3.6, we know that $\mathrm{c}l(f^{-1}(B)) \subseteq f^{-1}(\mathrm{c}l(B))$ and

$$f^{-1}(\mathrm{i}nt(B)) \subseteq \mathrm{i}nt(f^{-1}(B)).$$
$$f^{-1}(\mathrm{f}r(B)) = f^{-1}(\mathrm{c}l(B) \cup (\mathrm{i}nt(B))^c)$$
$$= f^{-1}(\mathrm{c}l(B)) \cup f^{-1}((\mathrm{i}nt(B))^c)$$
$$= f^{-1}(\mathrm{c}l(B)) \cap f(^{-1}(\mathrm{i}nt(B)))^c$$

From Theorem 3.3 and Theorem 3.6 we know $\mathrm{c}l(f^{-1}(B)) \subseteq f^{-1}(\mathrm{c}l(B))$ and $(\mathrm{i}nt(f^{-1}(B)))^c \subseteq (f^{-1}(\mathrm{i}nt(B)))^c$; hence, $f^{-1}(B) \subseteq f^{-1}(\mathrm{f}r(B))$.

$(\Leftarrow)$: Let $\mathrm{f}r(f^{-1}(B)) \subseteq f^{-1}(\mathrm{f}r(B))$ for all $B \in \mathcal{N}(Y)$. Then,

$$\mathrm{f}r(f^{-1}(B)) \cup f^{-1}(B) \subseteq f^{-1}(\mathrm{f}r(B)) \cup f^{-1}(B)$$

Hence,

$$\mathrm{c}l(f^{-1}(B)) \subseteq f^{-1}(\mathrm{f}r(B) \cup B)$$
$$= f^{-1}(\mathrm{c}l(B))$$

From Theorem 3.4, $f$ is a neutrosophic contiuous function.

**Definition 3.2** *Let $(X,\tau)$ and $(Y,\sigma)$ be two neutrosophic topological spaces and*

$f:(X,\tau)\to(Y,\sigma)$ *be a function.*

1. If $f(U)\in\sigma$ for all $U\in\tau$, then $f$ is called a neutrosophic open function.
2. If $f(F)\in\kappa(\sigma)$ for all $F\in\kappa(\tau)$, then $f$ is called a neutrosophic closed function.

**Example 3.3** *Let* $X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$, $A\in\mathrm{N}(X)$, $B\in\mathrm{N}(Y)$ *and* $f:X\to Y$ *be a function such that*

$A=\{\langle x_1,0.5,0.4,0.3\rangle,\langle x_2,0.7,0.8,0.2\rangle\}$,

$B=\{\langle y_1,0.1,0.7,0.6\rangle,\langle y_2,0.8,0.9,0.5\rangle\}$,

and

$f(x_1)=y_2$ and $f(x_2)=y_1$.

Then, $\tau=\{\tilde{X},\tilde{\varnothing},A\}$ and $\sigma=\{\tilde{Y},\tilde{\varnothing},B\}$ are two neutrosophic topological spaces. Therefore, $f$ is a neutrosophic open function. But $f$ is not a neutrosophic closed function.

**Theorem 3.9** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces and* $f:X\to Y$ *be a neutrosophic continuous function. Then,* $f$ *is a neutrosophic open function if and only if* $f(\mathrm{i}nt(A))\,\hat{o}\,\mathrm{i}nt(f(A))$ *for all* $A\in N(X)$.

*Proof.* $(\Rightarrow)$: Let $f:(X,\tau)\to(Y,\sigma)$ be a neutrosophic continuous function and $A\in N(X)$

$$\mathrm{i}nt(A)\,\hat{o}\,A$$
$$f(\mathrm{i}nt(A))\,\hat{o}\,f(A)$$
$$\mathrm{i}nt(f(\mathrm{i}nt(A)))\,\hat{o}\,\mathrm{i}nt(f(A))$$
$$f(\mathrm{i}nt(A))\,\hat{o}\,\mathrm{i}nt(f(A)).$$

$(\Leftarrow)$: Let $f(\mathrm{i}nt(A))\,\hat{o}\,\mathrm{i}nt(f(A))$ for all $A\in N(X)$. If $G\in X$ neutrosophic subset is a neutrosophic open function, $f(G)$ is a subset of $f(\mathrm{i}nt(G))$. So, $f(G)$ is a neutrosophic open function.

**Theorem 3.10** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces an* $f:X\to Y$ *be a bijective function. Then,* $f^{-1}$ *is a neutrosophic continuous function if and only if* $f^{-1}$ *is a neutrosophic open function.*

*Proof.* ( $\Rightarrow$ ): Let $U\in\sigma$ and $f$ be a neutrosophic continuous function. Then, we have $f^{-1}(G)=g(G)$. Hence, $f^{-1}$ is a neutrosohic open function.

$(\Leftarrow)$ : Let $f^{-1}$ be a neutrosophic open function. Then, we have $f^{-1}(U)\in\tau$ for all $U\in\sigma$. So, $f^{-1}$ is a neutrosophic continuous function.

**Theorem 3.11** Let $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces an* $f : X \to Y$ *be a bijective function. Then,* $f^{-1}$ *is a neutrosophic continuous function if and only if* $f^{-1}$ *is a neutrosophic closed function.*

*Proof.* ($\Rightarrow$): Let $A \in \kappa(\sigma)$. Then, $f^{-1}(A) \in \kappa(\tau)$ is a neutrosophic continuous function, so $f^{-1}(A)$ is a neutrosophic closed function. From Theorem 3.2, we have $f^{-1}$ is a neutrosophic closed function.
($\Leftarrow$): Let $A \in \kappa(\sigma)$. Then $f^{-1}(A) \in \kappa(\tau)$. $f^{-1}$ is a neutrosophic closed function, so $f^{-1}(A)$ is a neutrosophic closed function. From Theorem 3.2 f *is* a neutrosophic continuous function.

**Definition 3.3** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces and* $f : X \to Y$ *be a function. If following conditions hold, then* $f$ *is called neutrosophic homeomorphism*

1. $f$ is a bijective function,
2. $f$ is a neutrosophic continuous function,
3. $f^{-1}$ is a neutrosophic continuous function.

**Example 3.5** *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$ *and* $A \in \mathrm{N}(X)$ *and* $B \in \mathrm{N}(Y)$ *such that*

$$A = \{\langle x_1, 0.5, 0.4, 0.3\rangle, \langle x_2, 0.7, 0.8, 0.2\rangle\},$$
$$B = \{\langle y_1, 0.1, 0.7, 0.6\rangle, \langle y_2, 0.8, 0.9, 0.5\rangle\}.$$

Then, $\tau = \{\tilde{X}, \tilde{\emptyset}, A\}$ and $\sigma = \{\tilde{Y}, \tilde{\emptyset}, B\}$ are two neurosophic topology over $X$ and $Y$, respectively. Moreover, let $f : X \to Y$ be a function such that $f(x_1) = y_1$ and $f(x_2) = y_2$. It can be seen clearly that $f$ is a neutrosophic homeomorphism.

**Theorem 3.12** *Let* $(X,\tau)$ *and* $(Y,\sigma)$ *be two neutrosophic topological spaces and* $f : X \to Y$ *be a bijective function. Then f is a neutrosophic homeomorphism if and only if f is a neutrosophic continuous and neutrosophic closed function.*

*Proof.* Let *f* is a neutrosophic homeomorphism. From Definition 3.3 we know f is a neutrosophic continuous function. Then, from Theorem 3.10 we have $f^{-1}$ is a neutrosophic closed function. So, $\left(f^{-1}\right)^{-1} = f$ is a neutrosophic closed function.

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