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Fractional Order Mathematical Modeling of COVID-19 Dynamics with Mutant and Quarantined Strategy

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Abstract – Mathematical models provide a common language for communicating ideas, theories, and findings across disciplines. They allow researchers to represent complex concepts in a concise and precise manner, facilitating collaboration and interdisciplinary research. Additionally, visual representations of models help in conveying insights and understanding complex relationships. Mathematical modeling finds applications in various areas across science, engineering, economics, and other fields. Recently disease models have helped us understand how infectious diseases spread within populations. By studying the interactions between susceptible, infected, and recovered individuals, we can identify key factors influencing transmission, such as contact patterns, population density, and intervention strategies. The incorporation of fractional order modeling in studying disease models such as COVID-19 dynamics holds significant importance, offering a more accurate and efficient portrayal of system behavior compared to conventional integer-order derivatives. So in this study, we adopt a fractional operator-based approach to model COVID-19 dynamics. The existence and uniqueness of solutions are crucial properties of mathematical models that ensure their reliability, stability, and relevance for real-world applications. These properties underpin the validity of predictions, the interpretability of results, and the effectiveness of models in informing decision-making processes. Our investigation focuses on positivity of solutions, the existence and uniqueness of solutions within the model equation system, thereby contributing to a deeper understanding of the pandemic's dynamics. Finally, we present a numerical scheme for our model.

1. Introduction

There are various approaches to modeling the spread of infectious diseases, including compartmental models like the SIR (Susceptible-Infectious-Recovered) model, SEIR (Susceptible-Exposed-Infectious-Recovered) model, and their variations (Alkahtani and Koca, 2021), (Anderson and May, 1991), (Kermack and McKendrick, 1927). One of the simplest and most widely used models is the SIR model. It divides the population into three compartments: susceptible, infectious, and recovered. Strengths include simplicity, ease of interpretation, and applicability to large populations. However, it assumes homogeneous mixing, constant parameters and does not consider demographic or spatial heterogeneity. Extending the SIR model by adding an exposed compartment to account for the latent period between infection and becoming infectious, SEIR model better captures the incubation period of the disease.

In contemporary mathematical modeling, there has been a noticeable shift from employing classical derivatives to embracing fractional derivatives. This transition is reflected in recent research, where mathematicians have increasingly incorporated fractional differential operators into their models. These operators, encompassing exponential, Mittag-Leffler kernels, and power-law distributions, offer alternative frameworks for describing diverse phenomena (Atangana and Baleanu, 2016), (Caputo and Fabrizio, 2016), (Podlubny, 1999). Fractional order models are particularly useful for describing systems with long-range interactions or non-local effects. By considering fractional derivatives, these models can more accurately represent the underlying physics or biology (Koca and Ozalp, 2013), (Koca, 2018).

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Modeling the spread of COVID-19 has been a critical area of research since the pandemic began. Since the beginning of the COVID-19 pandemic, there has been a significant surge in research and publications related to various aspects of the disease (Dokuyucu and Celik, 2021). Researchers from diverse fields including epidemiology, virology, public health, medicine, mathematics, computer science, and social sciences have contributed to the growing body of knowledge on COVID-19.

In a recent publication (Yu et al., 2024), researchers introduce a novel nonlinear dynamics model called SEIMQR (Susceptible-Exposed-Infected-Mutant-Quarantined-Recovered), designed specifically to delve into the intricacies of COVID-19 transmission dynamics and to forecast its future trends with greater precision. In this paper, different from publication (Yu et al., 2024), we consider their model as fractional order to describe system behavior with greater accuracy and efficiency compared to traditional integer-order derivatives.

2. Preliminaries

Our focus in this section is to provide clear and concise definitions of non-integer fractional derivatives and integrals (Kilbas et al., 2006).

Definition 1 Riemann-Liouville definition of fractional order derivative:

$$ {}_{a}^{RL}D_t^{\varepsilon} f(t) = \frac{1}{\Gamma(n-\varepsilon)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\varepsilon-1} f(\tau) d\tau, \tag{1}$$

where

$$ n-1 < \varepsilon \leqslant n, n \in \mathbb{N} \tag{2}$$

and $\varepsilon \in R$ is a fractional order of the differ-integral of the function $f(t)$.

Definition 2 Caputo's definition of fractional order derivative:

$$ {}_{a}^{C}D_t^{\varepsilon} f(t) = \frac{1}{\Gamma(n-\varepsilon)} \int_a^t (t-\tau)^{n-\varepsilon-1} f^n(\tau) d\tau. \tag{3}$$

Here, $n-1 < \varepsilon \leqslant n$, $n \in \mathbb{N}$, $\varepsilon \in \mathbb{R}$ is a fractional order of the derivative of the function $f(t)$.

Definition 3 According to Riemann-Liouville's perspective, the Riemann-Liouville fractional integral of order $\varepsilon > 0$ for a function $f: (0, \infty) \to R$ is defined as the antiderivative of f with respect to a fractional exponent :

$$ I_t^{\varepsilon} f(t) = \frac{1}{\Gamma(\varepsilon)} \int_0^t (t-\tau)^{\varepsilon-1} f(\tau) d\tau. \tag{4}$$

3. Model Derivation

Herein, we undertake the examination of a Covid-19 model, incorporating a standard incidence specified as follows:

$$\begin{aligned}
\frac{ds(t)}{dt} &= k_1 - k_2 s(t)(i(t)+e(t)) - k_3 s(t) m(t) - k_4 s(t), \qquad (5)\\
\frac{de(t)}{dt} &= k_2 s(t)(i(t)+e(t)) + k_3 s(t) m(t) - k_5 e(t) - k_6 e(t) - k_4 e(t),\\
\frac{di(t)}{dt} &= \frac{1}{2} k_5 e(t) - k_7 i(t) - k_6 i(t) - k_8 i(t) - k_1 i(t),\\
\frac{dm(t)}{dt} &= \frac{1}{2} k_5 e(t) + k_7 i(t) - k_8 m(t) - k_6 m(t) - k_4 m(t),\\
\frac{dq(t)}{dt} &= k_6 (e(t) + i(t) + m(t)) - k_1 q(t),
\end{aligned}$$

where s, e, i, m, q and $n = s + e + i + m + q$ is the number of total population individuals.

$$ s(t_0) = s_0, e(t_0) = e_0, i(t_0) = i_0, m(t_0) = m_0 \text{ and } q(t_0) = q_0. $$

The entire population within the SEIMQ (Susceptible-Exposed-Infected-Mutant-Quarantined) model can be classified into six distinct groups, each with its respective characteristics outlined as susceptible (s) denotes individuals who have not contracted the virus but are at risk of infection when in contact with carriers, exposed (e) refers to individuals who have been infected with the virus but have yet to display symptoms, infected (i) represents those who have contracted the virus and are exhibiting symptoms, mutant (m) is attributed to individuals infected with a variant strain of the virus and quarantined (q) is designated for individuals isolated to prevent viral transmission to the broader community. The parameters within the system (5) are characterized by positive constants.

The field of calculus, encompassing fractional derivatives and integrals, has garnered growing attention from researchers. Fractional operators have been recognized for their superior ability to depict system behavior compared to integer-order derivatives. Given the significant advantage in memory properties, we propose enhancing the aforementioned system by substituting the integer-order time derivative with the Caputo fractional derivative as presented below:

$$\begin{aligned}
{}_0^C D_t^\alpha s(t) &= k_1 - k_2 s(t)(i(t)+e(t)) - k_3 s(t)m(t) - k_4 s(t), \\
{}_0^C D_t^\alpha e(t) &= k_2 s(t)(i(t)+e(t)) + k_3 s(t)m(t) - k_5 e(t) - k_6 e(t) - k_4 e(t), \\
{}_0^C D_t^\alpha i(t) &= \frac{1}{2}k_5 e(t) - k_7 i(t) - k_6 i(t) - k_8 i(t) - k_1 i(t), \\
{}_0^C D_t^\alpha m(t) &= \frac{1}{2}k_5 e(t) + k_7 i(t) - k_8 m(t) - k_6 m(t) - k_4 m(t), \\
{}_0^C D_t^\alpha q(t) &= k_6(e(t)+i(t)+m(t)) - k_1 q(t),
\end{aligned} \tag{6}$$

with the initial conditions

$$s(t_0) = s_0, e(t_0) = e_0, i(t_0) = i_0, m(t_0) = m_0 \text{ and } q(t_0) = q_0. \tag{7}$$

3.1. The positivity and boundedness of solutions

The aim of this section is to illustrate the positivity of the solutions of the system concerning with $\forall t \geqslant 0$, we define the norm

$$\|\Phi\|_\infty = \sup_{t\in[0,T]} |\Phi(t)|. \tag{8}$$

We begin by defining the system and then proceed to address the first equation:

$$\begin{aligned}
\frac{ds(t)}{dt} &= k_1 - k_2 s(t)(i(t)+e(t)) - k_3 s(t)m(t) - k_4 s(t), \forall t \geqslant 0, \\
&\geq -k_2(i(t)+e(t))s(t) - k_3 m(t)s(t) - k_4 s(t), \forall t \geqslant 0, \\
&\geq \left(-k_2(\sup_{t\in[0,T]}|i(t)| + \sup_{t\in[0,T]}|e(t)|) - k_3 \sup_{t\in[0,T]}|m(t)| - k_4\right)s(t), \forall t \geqslant 0, \\
&\geq -(k_2(\|i\|_\infty + \|e\|_\infty) + k_3\|m\|_\infty + k_4)s(t), \forall t \geqslant 0.
\end{aligned} \tag{9}$$

Then this provides that

$$s(t) \geq s_0 e^{-(k_2(\|i\|_\infty + \|e\|_\infty) + k_3\|m\|_\infty + k_4)t}, \forall t \geqslant 0. \tag{10}$$

Secondly for the function $e(t)$, we obtain

$$\begin{aligned}
\frac{de(t)}{dt} &= k_2 s(t)(i(t)+e(t)) + k_3 s(t)m(t) - k_5 e(t) - k_6 e(t) - k_4 e(t), \forall t \geqslant 0, \\
&\geq -(k_5 + k_6 + k_4)e(t), \forall t \geqslant 0.
\end{aligned}$$

So this dictates that

$$e(t) \geq e_0 e^{-(k_5+k_6+k_4)t}, \forall t \geqslant 0. \tag{11}$$

Here we assume that $s(t)$ and $m(t)$ are nonnegative solutions. For equation $i(t)$, we obtain

$$\frac{di(t)}{dt} = \frac{1}{2}k_5 e(t) - k_7 i(t) - k_6 i(t) - k_8 i(t) - k_1 i(t), \forall t \geqslant 0, \tag{12}$$

$$\geq -(k_7 + k_6 + k_8 + k_1) i(t), \forall t \geqslant 0.$$

This dictates that

$$i(t) \geq i_0 e^{-(k_7+k_6+k_8+k_1)t}, \forall t \geqslant 0. \tag{13}$$

Now let us check for the fourth equation is given by

$$\frac{dm(t)}{dt} = \frac{1}{2}k_5 e(t) + k_7 i(t) - k_8 m(t) - k_6 m(t) - k_4 m(t), \forall t \geqslant 0 \tag{14}$$

$$\geq -(k_8 + k_6 + k_4) m(t), \forall t \geqslant 0.$$

So we have

$$m(t) \geq m_0 e^{-(k_8+k_6+k_4)t}, \forall t \geqslant 0. \tag{15}$$

For final equation of model we get

$$\frac{dq(t)}{dt} = k_6(e(t) + i(t) + m(t)) - k_1 q(t), \forall t \geqslant 0 \tag{16}$$

$$\geq -k_1 q(t), \forall t \geqslant 0.$$

So we have

$$q(t) \geq q_0 e^{-k_1 t}, \forall t \geqslant 0. \tag{17}$$

4. Existence and Uniqueness

Existence and uniqueness conditions are fundamental for establishing the mathematical validity, predictability, and stability of solutions to ordinary differential equations, thereby enabling their application in various scientific and engineering domains. The importance of existence and uniqueness for ordinary differential equations lies in their fundamental role in ensuring the well-posedness of mathematical models and the predictability of solutions. In this section, we provide an in-depth examination of the existence and uniqueness of the equation system. To accomplish this objective, we verify the following theorem (Atangana, 2021).

Theorem 1 With the presence of positive constants p_i and $\overline{p}_i$ satisfying the following:

(i) $\forall i \in \{1,2,3,4,5\}$,

$$|\Phi_i(t, x_i) - \Phi_i(t, x_i',)|^2 \leq p_i |x_i - x_i'|^2. \tag{18}$$

(ii) $\forall (x, t) \in R \times [0, T]$,

$$|\Phi_i(t, x_i)|^2 \leq \overline{p}_i (1 + |x_i|^2). \tag{19}$$

Then the system of equations has a unique system of solutions. Let us revisit our model with taking right side of model as follows:

$$\frac{ds(t)}{dt} = \Phi_1(t, s), \tag{20}$$

$$\frac{de(t)}{dt} = \Phi_2(t, e),$$

$$\frac{di(t)}{dt} = \Phi_3(t, i),$$

$$\frac{dm(t)}{dt} = \Phi_4(t, m),$$

$$\frac{dq(t)}{dt} = \Phi_5(t, q).$$

Here we consider

$$\Phi_1(t, s) = k_1 - k_2 s(t)(i(t) + e(t)) - k_3 s(t) m(t) - k_4 s(t), \tag{21}$$

$$\Phi_2(t,e) = k_2 s(t)(i(t)+e(t)) + k_3 s(t)m(t) - k_5 e(t) - k_6 e(t) - k_4 e(t),$$

$$\Phi_3(t,i) = \frac{1}{2}k_5 e(t) - k_7 i(t) - k_6 i(t) - k_8 i(t) - k_1 i(t),$$

$$\Phi_4(t,m) = \frac{1}{2}k_5 e(t) + k_7 i(t) - k_8 m(t) - k_6 m(t) - k_4 m(t),$$

$$\Phi_5(t,q) = k_6(e(t)+i(t)+m(t)) - k_1 q(t).$$

We commence by examining the function $\Phi_1(t,s)$. Subsequently, we will illustrate that

$$|\Phi_1(t,s) - \Phi_1(t,s_1)|^2 \le p_1|s-s_1|^2. \tag{22}$$

Afterwards, we express

$$|\Phi_1(t,s) - \Phi_1(t,s_1)|^2 = \left|\begin{matrix}-k_2(i(t)+e(t))(s(t)-s_1(t)) \\ -k_3 m(t)(s(t)-s_1(t)) - k_4(s(t)-s_1(t))\end{matrix}\right|^2, \tag{23}$$

$$= |(-k_2(i(t)+e(t)) - k_3 m(t) - k_4)(s(t)-s_1(t))|^2,$$

$$\le \{3k_2^2(|i(t)|^2 + |e(t)|^2) + 3k_3^2|m(t)|^2 + 3k_4^2\}|s(t)-s_1(t)|^2,$$

$$\le \begin{Bmatrix}3k_2^2(\sup_{t\in[0,T]}|i(t)|^2 + \sup_{t\in[0,T]}|e(t)|^2) \\ +3k_3^2\sup_{t\in[0,T]}|m(t)|^2 + 3k_4^2\end{Bmatrix}|s(t)-s_1(t)|^2$$

$$\le \{3k_2^2(\|i\|_\infty^2 + \|e\|_\infty^2) + 3k_3^2\|m\|_\infty^2 + 3k_4^2\}|s(t)-s_1(t)|^2,$$

$$\le p_1|s(t)-s_1(t)|^2,$$

where

$$p_1 = \{3k_2^2(\|i\|_\infty^2 + \|e\|_\infty^2) + 3k_3^2\|m\|_\infty^2 + 3k_4^2\}. \tag{24}$$

Proceeding further with the function $\Phi_2(t,e)$, we obtain

$$|\Phi_2(t,e) - \Phi_2(t,e_1)|^2 = |k_2 s(t)(e(t)-e_1(t)) - (k_5+k_6+k_4)(e(t)-e_1(t))|^2 \tag{25}$$

$$\le \{2k_2^2|s(t)|^2 + 2(k_5+k_6+k_4)^2\}|(e(t)-e_1(t))|^2$$

$$\le \left\{2k_2^2\sup_{t\in[0,T]}|s(t)|^2 + 2(k_5+k_6+k_4)^2\right\}|(e(t)-e_1(t))|^2$$

$$\le \{2k_2^2\|s\|_\infty^2 + 2(k_5+k_6+k_4)^2\}|(e(t)-e_1(t))|^2$$

$$\le p_2|(e(t)-e_1(t))|^2$$

where

$$p_2 = \{2k_2^2\|s\|_\infty^2 + 2(k_5+k_6+k_4)^2\}. \tag{26}$$

Similary we get,

$$|\Phi_3(t,i) - \Phi_3(t,i_1)|^2 = |(-k_7-k_6-k_8-k_1)(i(t)-i_1(t))|^2, \tag{27}$$

$$\le \{(k_7+k_6+k_8+k_1)^2\}|(i(t)-i_1(t))|^2,$$

$$\le p_3|(i(t)-i_1(t))|^2$$

where

$$p_3 = \{k_7+k_6+k_8+k_1\}^2. \tag{28}$$

Similary we get,

$$|\Phi_4(t,m) - \Phi_4(t,m_1)|^2 = |(-k_8-k_6-k_4)(m(t)-m_1(t))|^2, \tag{29}$$

$$= \{(k_8+k_6+k_4)^2\}|(m(t)-m_1(t))|^2,$$

$$\leq p_4|(m(t)-m_1(t))|^2$$

where

$$p_4=\{k_8+k_6+k_4\}^2. \tag{30}$$

Finally we get,

$$|\Phi_5(t,q)-\Phi_5(t,q_1)|^2=|-k_1(q(t)-q_1(t))|^2, \tag{31}$$

$$=\{k_1^2\}|(q(t)-q_1(t))|^2,$$

$$\leq p_5|(q(t)-q_1(t))|^2$$

where

$$p_5=\{k_1^2\}. \tag{32}$$

Now that we have checked the first condition for all functions, we move on to verifying the second condition for our model.

$$|\Phi_1(t,s)|^2=|k_1-k_2s(t)(i(t)+e(t))-k_3s(t)m(t)-k_4s(t)|^2, \tag{33}$$

$$=|k_1-(k_2(i(t)+e(t))+k_3m(t)+k_4)s(t)|^2,$$

$$\leq 2k_1^2+2(k_2^2(|i(t)|^2+|e(t)|^2)+k_3^2|m(t)|^2+k_4^2)|s(t)|^2,$$

$$\leq 2k_1^2+2\begin{pmatrix}k_2^2\left(\sup_{t\in[0,T]}|i(t)|^2+\sup_{t\in[0,T]}|e(t)|^2\right)\\ +k_3^2\sup_{t\in[0,T]}|m(t)|^2+k_4^2\end{pmatrix}|s(t)|^2,$$

$$\leq 2k_1^2+2(k_2^2(\|i\|_\infty^2+\|e\|_\infty^2)+k_3^2\|m\|_\infty^2+k_4^2)|s(t)|^2$$

$$\leq 2k_1^2\left(1+\frac{2\left(k_2^2(\|i\|_\infty^2+\|e\|_\infty^2)+k_3^2\|m\|_\infty^2+k_4^2\right)}{2k_1^2}|s(t)|^2\right)$$

$$\leq \overline{p}_1(1+|s(t)|^2)$$

under the condition

$$\frac{k_2^2(\|i\|_\infty^2+\|e\|_\infty^2)+k_3^2\|m\|_\infty^2+k_4^2}{k_1^2}<1. \tag{34}$$

$$|\Phi_2(t,e)|^2=\begin{vmatrix}k_2s(t)i(t)+k_3s(t)m(t)\\ +(k_2s(t)-(k_5+k_6+k_4))e(t)\end{vmatrix}^2, \tag{35}$$

$$\leq 3k_2^2|s(t)|^2|i(t)|^2+3k_3^2|s(t)|^2|m(t)|^2$$

$$+3(k_2^2|s(t)|^2+(k_5+k_6+k_4)^2)e(t),$$

$$\leq 3k_2^2\sup_{t\in[0,T]}|s(t)|^2\sup_{t\in[0,T]}|i(t)|^2+3k_3^2\sup_{t\in[0,T]}|s(t)|^2\sup_{t\in[0,T]}|m(t)|^2$$

$$+3\left(k_2^2\sup_{t\in[0,T]}|s(t)|^2+(k_5+k_6+k_4)^2\right)e(t)$$

$$\leq 3k_2^2\|s\|_\infty^2\|i\|_\infty^2+3k_3^2\|s\|_\infty^2\|m\|_\infty^2$$

$$+3(k_2^2\|s\|_\infty^2+(k_5+k_6+k_4)^2)e(t)$$

$$\leq (3k_2^2\|s\|_\infty^2\|i\|_\infty^2+3k_3^2\|s\|_\infty^2\|m\|_\infty^2)\left(1+\frac{3\left(k_2^2\|s\|_\infty^2+(k_5+k_6+k_4)^2\right)}{3k_2^2\|s\|_\infty^2\|i\|_\infty^2+3k_3^2\|s\|_\infty^2\|m\|_\infty^2}e(t)\right),$$

$$\leq \overline{p}_2(1+|e(t)|^2)$$

under the condition

$$\frac{k_2^2\|s\|_\infty^2+(k_5+k_6+k_4)^2}{k_2^2\|s\|_\infty^2\|i\|_\infty^2+k_3^2\|s\|_\infty^2\|m\|_\infty^2}<1. \tag{36}$$

$$|\Phi_3(t,i)|^2=\left|\frac{1}{2}k_5e(t)-(k_7+k_6+k_8+k_1)i(t)\right|^2, \tag{37}$$

$$\leq \frac{1}{2}k_5^2|e(t)|^2+2(k_7+k_6+k_8+k_1)^2|i(t)|^2,$$

$$\leq \frac{1}{2}k_5^2\sup_{t\in[0,T]}|e(t)|^2+2(k_7+k_6+k_8+k_1)^2|i(t)|^2,$$

$$\leq \frac{1}{2}k_5^2\|e\|_\infty^2+2(k_7+k_6+k_8+k_1)^2|i(t)|^2,$$

$$\leq \left(\frac{1}{2}k_5^2\|e\|_\infty^2\right)\left(1+\frac{2(k_7+k_6+k_8+k_1)^2}{\frac{1}{2}k_5^2\|e\|_\infty^2}|i(t)|^2\right),$$

$$\leq \overline{p}_3(1+|i(t)|^2)$$

under the condition

$$\frac{4(k_7+k_6+k_8+k_1)^2}{k_5^2\|e\|_\infty^2}<1. \tag{38}$$

$$|\Phi_4(t,m)|^2=\left|\frac{1}{2}k_5e(t)+k_7i(t)-(k_8+k_6+k_4)m(t)\right|^2, \tag{39}$$

$$\leq \frac{3}{4}k_5^2|e(t)|^2+3k_7^2|i(t)|^2+3(k_7+k_8+k_6+k_4)^2|m(t)|^2,$$

$$\leq \frac{3}{4}k_5^2\sup_{t\in[0,T]}|e(t)|^2+3k_7^2\sup_{t\in[0,T]}|i(t)|^2$$

$$+3(k_7+k_8+k_6+k_4)^2|m(t)|^2,$$

$$\leq \frac{3}{4}k_5^2\|e\|_\infty^2+3k_7^2\|i\|_\infty^2+3(k_7+k_8+k_6+k_4)^2|m(t)|^2,$$

$$\leq \left(\frac{3}{4}k_5^2\|e\|_\infty^2+3k_7^2\|i\|_\infty^2\right)\left(1+\frac{3(k_7+k_8+k_6+k_4)^2}{\frac{3}{4}k_5^2\|e\|_\infty^2+3k_7^2\|i\|_\infty^2}|m(t)|^2\right)$$

$$\leq \overline{p}_4(1+|m(t)|^2)$$

under the condition

$$\frac{4(k_7+k_8+k_6+k_4)^2}{k_5^2\|e\|_\infty^2+3k_7^2\|i\|_\infty^2}<1. \tag{40}$$

$$|\Phi_5(t,q)|^2=|k_6(e(t)+i(t)+m(t))-k_1q(t)|^2, \tag{41}$$

$$\leq 2k_6^2(|e(t)|^2+|i(t)|^2+|m(t)|^2)+2k_1^2|q(t)|^2,$$

$$\leq 2k_6^2\left(\sup_{t\in[0,T]}|e(t)|^2+\sup_{t\in[0,T]}|i(t)|^2+\sup_{t\in[0,T]}|m(t)|^2\right)+2k_1^2|q(t)|^2,$$

$$\leq 2k_6^2(\|e\|_\infty^2+\|i\|_\infty^2+\|m\|_\infty^2)+2k_1^2|q(t)|^2$$

$$\leq 2k_6^2(\|e\|_\infty^2+\|i\|_\infty^2+\|m\|_\infty^2)\left(1+\frac{2k_1^2}{2k_6^2(\|e\|_\infty^2+\|i\|_\infty^2+\|m\|_\infty^2)}|q(t)|^2\right)$$

$$\leq \overline{p}_5(1+|q(t)|^2)$$

under the condition

$$\frac{k_1^2}{k_6^2(\|e\|_\infty^2+\|i\|_\infty^2+\|m\|_\infty^2)}<1. \tag{42}$$

Provided that the condition for linear growth is satisfied, such that

$$\max\left\{\begin{array}{l}\frac{k_2^2(\|i\|_\infty^2+\|e\|_\infty^2)+k_3^2\|m\|_\infty^2+k_4^2}{k_1^2}\\ \frac{k_2^2\|s\|_\infty^2+(k_5+k_6+k_4)^2}{k_2^2\|s\|_\infty^2\|i\|_\infty^2+3k_3^2\|s\|_\infty^2\|m\|_\infty^2}\\ \frac{4(k_7+k_6+k_8+k_1)^2}{k_5^2\|e\|_\infty^2}\\ \frac{4(k_7+k_8+k_6+k_4)^2}{k_5^2\|e\|_\infty^2+3k_7^2\|i\|_\infty^2}\\ \frac{k_1^2}{k_6^2(\|e\|_\infty^2+\|i\|_\infty^2+\|m\|_\infty^2)}\end{array}\right\}<1, \quad (43)$$

there exists only one solution set for the system of equations.

5. Numerical Scheme For Model With Riemann-Liouville Derivative

In the forthcoming section, we offer an analysis of the model under consideration within the realm of fractional calculus employing the Riemann-Liouville derivative. When implementing the numerical scheme, we employ the Atangana-Toufik numerical rules (Toufik and Atangana, 2017). Let us now express the model utilizing the Riemann-Liouville derivative as follows:

$$\begin{aligned} {}_0^{RL}D_t^\alpha s(t) &= \Phi_1(t,s), \quad (44)\\ {}_0^{RL}D_t^\alpha e(t) &= \Phi_2(t,e),\\ {}_0^{RL}D_t^\alpha i(t) &= \Phi_3(t,i),\\ {}_0^{RL}D_t^\alpha m(t) &= \Phi_4(t,m),\\ {}_0^{RL}D_t^\alpha q(t) &= \Phi_5(t,q),\end{aligned}$$

$s(t_0)=s_0$, $e(t_0)=e_0$, $i(t_0)=i_0$, $m(t_0)=m_0$, and $q(t_0)=q_0$.

Here

$$\begin{aligned}\Phi_1(t,s) &= k_1 - k_2 s(t)(i(t)+e(t)) - k_3 s(t)m(t) - k_4 s(t), \quad (45)\\ \Phi_2(t,e) &= k_2 s(t)(i(t)+e(t)) + k_3 s(t)m(t) - k_5 e(t) - k_6 e(t) - k_4 e(t),\\ \Phi_3(t,i) &= \frac{1}{2}k_5 e(t) - k_7 i(t) - k_6 i(t) - k_8 i(t) - k_1 i(t),\\ \Phi_4(t,m) &= \frac{1}{2}k_5 e(t) + k_7 i(t) - k_8 m(t) - k_6 m(t) - k_4 m(t),\\ \Phi_5(t,q) &= k_6(e(t)+i(t)+m(t)) - k_1 q(t).\end{aligned}$$

We transform the aforementioned system into its numerical counterpart using Lagrange polynomial interpolation.

$$\begin{aligned} s_{v+1} &= \frac{\alpha(\Delta t)^\alpha}{\Gamma(\alpha+2)}\sum_{k=2}^{v}\Phi_1(t_k,s_k).\begin{bmatrix}(v-k+1)^\alpha(v-k+2+\alpha)\\ -(v-k)^\alpha(v-k+2+2\alpha)\end{bmatrix} \quad (46)\\ &-\frac{(\Delta t)^\alpha}{\Gamma(\alpha+2)}\sum_{k=2}^{v}\Phi_1(t_{k-1},s_{k-1}).\begin{bmatrix}(v-k+1)^{\alpha+1}\\ -(v-k)^\alpha(v-k+1+\alpha)\end{bmatrix},\\ e_{v+1} &= \frac{\alpha(\Delta t)^\alpha}{\Gamma(\alpha+2)}\sum_{k=2}^{v}\Phi_2(t_k,e_k).\begin{bmatrix}(v-k+1)^\alpha(v-k+2+\alpha)\\ -(v-k)^\alpha(v-k+2+2\alpha)\end{bmatrix}\\ &-\frac{(\Delta t)^\alpha}{\Gamma(\alpha+2)}\sum_{k=2}^{v}\Phi_2(t_{k-1},e_{k-1}).\begin{bmatrix}(v-k+1)^{\alpha+1}\\ -(v-k)^\alpha(v-k+1+\alpha)\end{bmatrix},\\ i_{v+1} &= \frac{\alpha(\Delta t)^\alpha}{\Gamma(\alpha+2)}\sum_{k=2}^{v}\Phi_3(t_k,i_k).\begin{bmatrix}(v-k+1)^\alpha(v-k+2+\alpha)\\ -(v-k)^\alpha(v-k+2+2\alpha)\end{bmatrix}\\ &-\frac{(\Delta t)^\alpha}{\Gamma(\alpha+2)}\sum_{k=2}^{v}\Phi_3(t_{k-1},i_{k-1}).\begin{bmatrix}(v-k+1)^{\alpha+1}\\ -(v-k)^\alpha(v-k+1+\alpha)\end{bmatrix}.\end{aligned}$$

$$m_{v+1} = \frac{\alpha(\Delta t)^\alpha}{\Gamma(\alpha+2)} \sum_{k=2}^{v} \Phi_2(t_k, m_k). \begin{bmatrix} (v-k+1)^\alpha (v-k+2+\alpha) \\ -(v-k)^\alpha (v-k+2+2\alpha) \end{bmatrix}$$

$$-\frac{(\Delta t)^\alpha}{\Gamma(\alpha+2)} \sum_{k=2}^{v} \Phi_2(t_{k-1}, m_{k-1}). \begin{bmatrix} (v-k+1)^{\alpha+1} \\ -(v-k)^\alpha (v-k+1+\alpha) \end{bmatrix},$$

$$q_{v+1} = \frac{\alpha(\Delta t)^\alpha}{\Gamma(\alpha+2)} \sum_{k=2}^{v} \Phi_5(t_k, q_k). \begin{bmatrix} (v-k+1)^\alpha (v-k+2+\alpha) \\ -(v-k)^\alpha (v-k+2+2\alpha) \end{bmatrix}$$

$$-\frac{(\Delta t)^\alpha}{\Gamma(\alpha+2)} \sum_{k=2}^{v} \Phi_5(t_{k-1}, q_{k-1}). \begin{bmatrix} (v-k+1)^{\alpha+1} \\ -(v-k)^\alpha (v-k+1+\alpha) \end{bmatrix}.$$

4. Conclusion

In this study, we explore a COVID-19 model alongside its fractional order counterpart. We examine the model under linear growth and Lipschitz rules, deriving conditions for the existence and uniqueness of system solutions. Ultimately, we provide numerical approximations to demonstrate the effectiveness of our method.

Ethics Permissions

This paper does not require ethics committee approval.

Conflict of Interest

Author declares that there is no conflict of interest for this paper.

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