 Journal of Polytechnic, 2016; 19 (4) : 421-426

Exact Solution to Elastic Behavior of Periodic Heat Generating Solid Cylinder

Tunç APATAY

Faculty of Engineering, Department of Mechanical Engineering, Gazi University, Ankara, Turkey

(Received : 22.01.2016 ; Accepted :19.03.2016)

ABSTRACT

The objective of this study is to develop analytical solution of thermally induced stress and deformation in a periodic heat generating solid cylinder. The cylinder is initially at zero temperature, but for times greater than zero, heat is generated and consumed internally at a time dependent rate. The solution of this complicated problem consists of two parts. In the first part, transient heat conduction equation is formulated and solved by the use of Duhamel's theorem. The second part on the other hand consisted of the formulation of governing elastic equation coupled with the temperature gradient. This equation is nothing but the thermoelastic equation of the cylinder. Analytical solution of the thermoelastic equation reveals the distributions of stress, strain and displacement within the cylinder at a given time instant. Two different periodic functions are handled to describe periodic heat generation in the cylinder. Although periodic, the second function eventually decays to zero.

Keywords: Transient heat generation, periodic heating, Duhamel's theorem, thermoelasticity, generalized plane strain

1. INTRODUCTION

The analysis of thermoelastic deformations in cylindrical coordinates such as cylinders and rods are of great importance in engineering especially in nuclear reactors, aerospace industry and chemical applications [1-4]. Because of the temperature gradient changes, thermal stresses arise in the cylinder, and they must be taken into account in the design to save production costs. For this reason many researchers have investigated the subject in the past under different conditions and different assumptions.

But most of the studies on the thermoelastic behaviors of the cylinders in the literature are steady-state. However, for some applications like laser heating, nuclear reactors and chemical applications, the heat conduction problem is transient. Tu and Lee [5] solved the heat conduction problem analytically by using the shifting functions for hollow cylinders with transient boundary condition. Cossali [6] obtained the analytical solution of periodic heat conduction in a homogeneous cylinder. Fazeli et al. [7] solved the two-dimensional heat transfer problem by using the Duhamel's theorem for a hollow cylinder subjected to transient boundary condition.

Orcan [8] obtained the steady-state stress distribution in an elastic-ideally plastic cylindrical rod with uniform internal energy generation. Later, he also calculated the residual stresses and secondary plastic flow in the rod under similar assumptions [9]. Arslan et al. [10] studied the elastic-plastic stress distribution in a rotating solid shaft subjected to a temperature cycle. Eraslan and Orcan [11] investigated the transient thermoelastic-plastic deformation of a heat generating tube. Deshmukh et al. [12] discussed the thermal stresses in a hollow circular cylinder subjected to arbitrary initial temperature and time dependent heat flux which is applied at the outer circular boundary whereas the inner circular boundary is isolated. Most recently Eraslan and Apatay [13] obtained the thermoelastic behavior of a solid cylinder subjected to time-dependent periodic boundary condition by using Duhamel's theorem. Bhongade and Durge discussed the thermal stresses in a hollow cylinder with internal heat generation [14], and Walde et al. [15] discussed the thermoelastic response of a solid cylinder, in which sources are generated according to a linear function of the temperature, with boundary conditions of the radiation type, by applying integral transform techniques.

In this paper, the thermoelastic response of a long solid cylinder subjected to periodic heat generation is studied by analytical means, applying the uncoupled theory of thermoelasticity. The heat transfer equation is solved by the use of Duhamel's theorem. Two different heat generation functions are handled. The thermoelastic stress and displacement distributions are obtained using the temperature distribution. The results relevant to engineering practice are presented graphically.

2. FORMULATION OF THE PROBLEM

2.1. Transient Temperature Distribution in the Cylinder

A long solid cylinder with radius b is considered. The cylinder is initially at zero temperature and for times $t > 0$ heat is generated or consumed within the cylinder at a time-dependent rate of $g(t)$ while its surface $r = b$ is kept at zero temperature. The transient temperature distribution in the cylinder is described by the heat conduction equation [1]. In terms of dimensionless variables it reads

$$\frac{\partial \bar{T}}{\partial \tau} = \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + G(\tau); \quad 0 \le \bar{r} < 1, \quad \tau > 0 \qquad (1)$$

The boundary conditions and the initial condition that complete the heat transfer model are

*Corresponding Author
e-mail: tapay@gazi.edu.tr
Digital Object Identifier (DOI) : 10.2339/2016.19.4 421-426

$$\bar{T}(0,\tau) = \text{finite},$$
$$\bar{T}(1,\tau) = 0, \tag{2}$$
$$\bar{T}(\bar{r},0) = 0.$$

In the above, $\bar{r} = r/b$ is the dimensionless radial coordinate, $\bar{T} = T/T_0$ the dimensionless temperature, $\tau = \alpha_T t/b^2$ the normalized time, $G(\tau) = g(t)b^2/kT_0$ the dimensionless heat generation rate, T_0 a reference temperature, α_T the thermal diffusivity and k the thermal conductivity. Assuming that $G(\tau)$ has no discontinuities, the solution of the heat conduction equation is obtained by the use of Duhamel's theorem as [1]

$$\bar{T}(\bar{r},\tau) = \int_0^\tau G(\beta)\frac{\partial}{\partial\tau}\Phi(\bar{r},\tau-\beta)d\beta \tag{3}$$

in which $\Phi(\bar{r},\tau)$ represents the solution of the auxiliary problem defined by

$$\frac{\partial\Phi}{\partial\tau} = \frac{1}{\bar{r}}\frac{\partial\Phi}{\partial\bar{r}} + \frac{\partial^2\Phi}{\partial\bar{r}^2} + 1; \quad 0 \le \bar{r} < 1, \quad \tau > 0 \tag{4}$$

$$\Phi(0,\tau) = \text{finite},$$
$$\Phi(1,\tau) = 0, \tag{5}$$
$$\Phi(\bar{r},0) = 0.$$

The nonhomogeneity in the governing equation for $\Phi(\bar{r},\tau)$ is handled by proposing a solution of the form

$$\Phi(\bar{r},\tau) = Y(\bar{r},\tau) + Z(\bar{r}) \tag{6}$$

Substituting this proposed solution into Eq. (4) the differential equation for $\Phi(\bar{r},\tau)$ is split into two differential equations in the forms

$$\frac{d^2Z}{d\bar{r}^2} + \frac{1}{\bar{r}}\frac{dZ}{d\bar{r}} + 1 = 0; \quad Z(0) = \text{finite}, \quad Z(1)=0 \tag{7}$$

and

$$\frac{\partial Y}{\partial\tau} = \frac{1}{\bar{r}}\frac{\partial Y}{\partial\bar{r}} + \frac{\partial^2 Y}{\partial\bar{r}^2}; \qquad 0 < \bar{r} < 1, \quad \tau > 0 \tag{8}$$

Subject to

$$Y(0,\tau) = \text{finite},$$
$$Y(1,\tau) = 0, \tag{9}$$
$$Y(\bar{r},0) = -Z(\bar{r}).$$

The solutions are then obtained as

$$Z(\bar{r}) = \frac{1-\bar{r}^2}{4} \tag{10}$$

and

$$Y(\bar{r},\tau) = -2\sum_{n=1}^{\infty} e^{-\lambda_n^2\tau}\frac{J_0(\lambda_n\bar{r})}{\lambda_n^3 J_1(\lambda_n)} \tag{11}$$

in which J_0, J_1 represent Bessel functions of the first kind of order zero and one, respectively, and λ_n for $n = 1,2,\ldots$ are the positive roots of the eigenvalue equation

$$J_0(\lambda_n) = 0 \tag{12}$$

By superposition, the solution of the auxiliary equation is written as

$$\Phi(\bar{r},\tau) = \frac{1-\bar{r}^2}{4} - 2\sum_{n=1}^{\infty} e^{-\lambda_n^2\tau}\frac{J_0(\lambda_n\bar{r})}{\lambda_n^3 J_1(\lambda_n)} \tag{13}$$

and consequently, the solution to transient temperature distribution in the cylinder takes the final form

$$\bar{T}(\bar{r},\tau) = 2\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\bar{r})}{\lambda_n J_1(\lambda_n)}\int_0^\tau G(\beta)e^{-\lambda_n^2(\tau-\beta)}d\beta \tag{14}$$

2.1.1 Integrals

The integral

$$I(\tau,\lambda_n) = \int_0^\tau G(\beta)e^{-\lambda_n^2(\tau-\beta)}d\beta \tag{15}$$

in Eq. (14) is referred to as generation integral. Given the generation function $G(\beta)$ it is evaluated and used in the temperature distribution afterwards. In this work, two different generation rates are taken into consideration. These are as follows.

Rate 1. It is sinusoidal described by

$$G(\beta) = A\sin\tau \tag{16}$$

in which A is the load parameter. The corresponding generation integral is evaluated as

$$I = I_1 = \frac{A\left[e^{-\lambda_n^2\tau} + \lambda_n^2\sin\tau - \cos\tau\right]}{1+\lambda_n^4} \tag{17}$$

Rate 2. The periodic and decaying generation rate and the corresponding generation integral are

$$G(\beta) = A\tau e^{-\tau/2}\cos\tau \tag{18}$$

and

$$I = I_2$$
$$= -\frac{4Ae^{-\lambda_n^2\tau}(3+4\lambda_n^2-4\lambda_n^4)[+\lambda_n^2\sin\tau-\cos\tau]}{(5-4\lambda_n^2+4\lambda_n^4)^2}$$
$$+\frac{4Ae^{-\tau/2}[4+5\tau+4\lambda_n^4-4\lambda_n^2(2+\tau)]\sin\tau}{(5-4\lambda_n^2+4\lambda_n^4)^2}$$
$$+\frac{2Ae^{-\tau/2}[6-5\tau+8\lambda_n^6-4\lambda_n^2(2+3\tau)+2\lambda_n^2(4+7\tau)]\cos\tau}{(5-4\lambda_n^2+4\lambda_n^4)^2} \tag{19}$$

where A represents a load parameter as before.

2.2 Thermoelastic Solution

Small deformations and a state of generalized plane strain are assumed. In order to determine transient stresses and deformations in the solid cylinder, dimensionless forms of the basic equations are utilized [2,3]. These are the following.

The equation of equilibrium

$$\frac{d\bar{\sigma}_r}{d\bar{r}} + \frac{\bar{\sigma}_r - \bar{\sigma}_\theta}{\bar{r}} = 0 \tag{20}$$

the strain-displacement relations

$$\bar{\epsilon}_\theta = \frac{\bar{u}}{\bar{r}}, \; \bar{\epsilon}_r = \frac{d\bar{u}}{d\bar{r}} \tag{21}$$

and the equations of the generalized Hooke's law

$$\bar{\epsilon}_r = \bar{\sigma}_r - \nu(\bar{\sigma}_\theta + \bar{\sigma}_z) + \bar{\alpha}\bar{T}, \tag{22}$$
$$\bar{\epsilon}_\theta = \bar{\sigma}_\theta - \nu(\bar{\sigma}_r + \bar{\sigma}_z) + \bar{\alpha}\bar{T}, \tag{23}$$
$$\bar{\epsilon}_z = \bar{\sigma}_z - \nu(\bar{\sigma}_r + \bar{\sigma}_\theta) + \bar{\alpha}\bar{T}, \tag{24}$$

In these equations $\bar{\sigma}_j = \sigma_j/\sigma_Y$ represents a dimensionless stress component, $\bar{\epsilon}_j = \epsilon_j E/\sigma_Y$ a normalized strain component, $\bar{u} = Eu/\sigma_Y b$ the dimensionless radial displacement, $\bar{\alpha} = E\alpha T_0/\sigma_Y$ the dimensionless coefficient of thermal expansion, ν the Poisson's ratio, E the modulus of elasticity and σ_Y the uniaxial yield stress of the material. In a state of generalized plain strain $\bar{\epsilon}_z = \epsilon_0$ is constant and Eq. (24) can be solved for the axial stress to give

$$\bar{\sigma}_z = \epsilon_0 + \nu(\bar{\sigma}_r + \bar{\sigma}_\theta) - \bar{\alpha}\bar{T}, \tag{25}$$

Combination of this equation with strain-displacement relations, Eq. (21) and the equations of generalized Hooke's law, Eqs. (22)-(23) allow one to formulate the stress displacement relations as

$$\bar{\sigma}_r = \frac{1}{(1+\nu)(1-2\nu)}\left[\nu\epsilon_0 + \frac{\nu\bar{u}}{\bar{r}} + (1-\nu)\bar{u}'\right] - \frac{\bar{\alpha}\bar{T}}{1-2\nu}, \tag{26}$$

$$\bar{\sigma}_\theta = \frac{1}{(1+\nu)(1-2\nu)}\left[\nu\epsilon_0 + \frac{(1-\nu)\bar{u}}{\bar{r}} + \nu\bar{u}'\right] - \frac{\bar{\alpha}\bar{T}}{1-2\nu}, \tag{27}$$

A prime above denotes differentiation with respect to the dimensionless radial coordinate r. Substituting these stresses in the equation of equilibrium, Eq. (20), leads to the thermoelastic equation in terms of radial displacement

$$\bar{r}^2\frac{d^2\bar{u}}{d\bar{r}^2} + \bar{r}\frac{d\bar{u}}{d\bar{r}} - \bar{u} = \frac{(1+\nu)}{(1-\nu)}\bar{\alpha}\bar{r}^2\frac{\partial\bar{T}}{\partial\bar{r}} \tag{28}$$

This is a second order nonhomogeneous Cauchy-Euler type differential equation which assumes the exact solution

$$\bar{u}(\bar{r}) = C_1\bar{r} + \frac{C_2}{\bar{r}} + \frac{(1+\nu)}{(1-\nu)}\left(\frac{\bar{\alpha}}{\bar{r}}\right)\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta \tag{29}$$

It is to be noted that

$$\lim_{\bar{r}\to 0}\frac{1}{\bar{r}}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta = \lim_{\bar{r}\to 0}\bar{r}\bar{T}(\bar{r},\tau) = 0 \tag{30}$$

Since the radial displacement $\bar{u}$ must be finite at the center of the solid cylinder ($r = 0$) C_2 must be zero. Hence, we list the equations for the displacement and the stresses

$$\bar{u} = C_1\bar{r} + \frac{(1+\nu)}{(1-\nu)}\left(\frac{\bar{\alpha}}{\bar{r}}\right)\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta, \tag{31}$$

$$\bar{\sigma}_r = \frac{\nu\epsilon_0 + C_1}{(1+\nu)(1-2\nu)} - \frac{\bar{\alpha}}{1-\nu}\frac{1}{\bar{r}^2}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta, \tag{32}$$

$$\bar{\sigma}_\theta = \frac{\nu\epsilon_0 + C_1}{(1+\nu)(1-2\nu)} + \frac{\bar{\alpha}}{1-\nu}\frac{1}{\bar{r}^2}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta - \frac{\bar{\alpha}\bar{T}(\bar{r},\tau)}{1-\nu}, \tag{33}$$

$$\bar{\sigma}_z = \frac{(1-\nu)\epsilon_0 + 2\nu C_1}{(1+\nu)(1-2\nu)} - \frac{\bar{\alpha}\bar{T}(\bar{r},\tau)}{1-\nu}. \tag{34}$$

These equations contain two unknowns; namely C_1 and ϵ_0 to be determined. Since the surface of the solid cylinder is free of stress we have the condition

$$\bar{\sigma}_r(1) = 0 \tag{35}$$

Another condition can be formulated by making use of the fact that the total axial force F_z must vanish as the ends of the solid cylinder are free. This leads to

$$F_z = \int \bar{\sigma}_z dA = 2\pi\int_0^1 \bar{r}\bar{\sigma}_z d\bar{r} = 0 \tag{36}$$

Application of these conditions reveal that

$$C_1 = \frac{\bar{\alpha}(1-3\nu)}{1-\nu}\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} \tag{37}$$

$$\epsilon_0 = 2\bar{\alpha}\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} \tag{38}$$

Finally, the complete thermoelastic solution of the solid cylinder takes the form

$$\bar{\sigma}_r = \frac{\bar{\alpha}}{1-\nu}\left(\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} - \frac{1}{\bar{r}^2}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta\right), \tag{39}$$

$$\bar{\sigma}_\theta = \frac{\bar{\alpha}}{1-\nu}\left(\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} + \frac{1}{\bar{r}^2}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta - \bar{T}(\bar{r},\tau)\right), \tag{40}$$

$$\bar{\sigma}_z = \frac{\bar{\alpha}}{1-\nu}\left(2\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} - \bar{T}(\bar{r},\tau)\right), \tag{41}$$

$$\bar{u} = \frac{\bar{\alpha}}{1-\nu}\left[\bar{r}(1-3\nu)\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} + \frac{1+\nu}{\bar{r}}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta\right]. \tag{42}$$

The integrals in the solution are evaluated as

$$\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} = 2\sum_{n=1}^{\infty}\frac{1}{\lambda_n^2}I(\tau,\lambda_n) \tag{43}$$

$$\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta = 2\sum_{n=1}^{\infty}\frac{\bar{r}J_1(\lambda_n\bar{r})}{\lambda_n^2 J_1(\lambda_n)}I(\tau,\lambda_n) \tag{44}$$

in which $I(\tau,\lambda_n)$ refers to the generation integral in Eq. (15). For the first generation rate it is to be replaced by I_1 in Eq. (17) and for the second by I_2 in Eq. (19).

3. NUMERICAL RESULTS AND DISCUSSION

First, verification of the derived solution is performed against a finite element numerical solution [16]. The distributions of the von Mises stress in the cylinder at various time instants are calculated and compared with the results of the numerical solution for this purpose. The von Mises stresses $\bar{\sigma}_{vM}$ are obtained from [17]

$$\bar{\sigma}_{vM} = \sqrt{\frac{1}{2}[(\bar{\sigma}_r - \bar{\sigma}_\theta)^2 + (\bar{\sigma}_r - \bar{\sigma}_z)^2 + (\bar{\sigma}_\theta - \bar{\sigma}_z)^2]} \tag{45}$$

It is well known that the deformation is elastic as long as $\bar{\sigma}_{vM} \le 1$. Since $\bar{\sigma}_{vM}$ is a function of all three stress components a small error in one of them may result in apparent discrepancies in the results. The parameter values $\nu = 0.3, A = 3.0, \bar{\alpha} = 1.75$ are used in these calculations. The results of the verification calculations are presented in Figure 1a and Figure 1b on which solid lines belong to the results of this work and dots to the results of numerical solution [16]. Figure 1a is based on Rate 1 as described by Eq. (16) while Figure 1b on Rate 2 given by Eq. (18).

Corresponding to the same values of the parameters ν, A and $\bar{\alpha}$, variation of the temperature $\bar{T}$ and its gradient $d\bar{T}/d\bar{r}$ with time at the radial position $\bar{r} = 0.5$ are calculated and plotted in Figure 2a and Figure 2b. The behavior corresponding to Rate 1 is depicted in Figure 2a. One period of generation is covered Figure 2a. In Figure 2b the decaying nature of the generation rate with time is visualized.

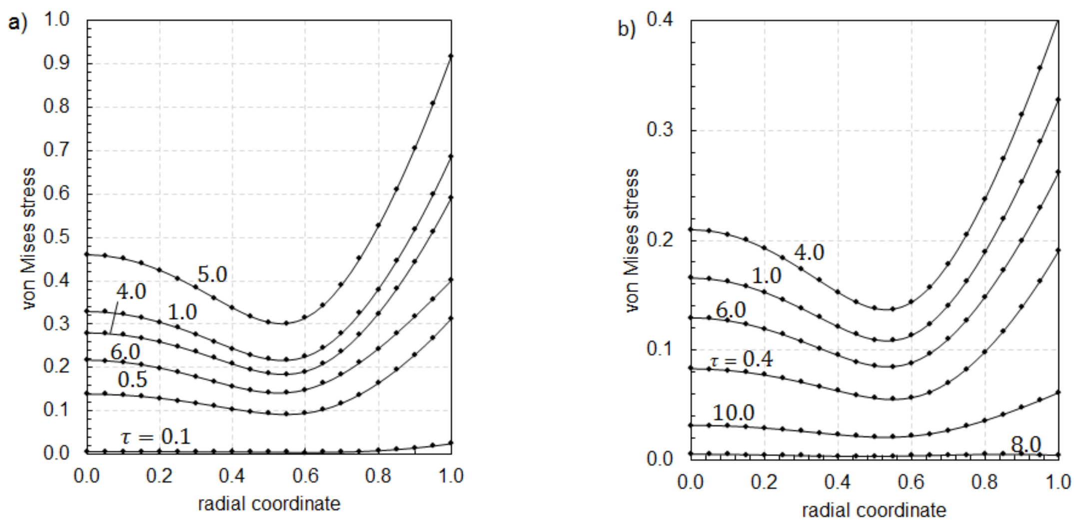

Figure 1. Distributions of the von Mises stress in the cylinder at various time instants based on a) Rate, 1 b) Rate 2, for the parameter set $\nu = 0.3, A = 3.0$ and $\bar{\alpha} = 1.75$.

As seen in these figures, perfect agreement is obtained between the results of this work and finite element solution verifying the present analytical model for both rates. Note also that the largest values of $\bar{\sigma}_{vM}$ occur at the surface of the cylinder, hence failure with respect to plastic deformation occurs at the surface of the cylinder as the load parameter A is further increased.

On the other hand, variations of the stress components, the von Mises stress and the radial displacement with time at the same radial location ($\bar{r} = 0.5$) are plotted in Figure 3a and Figure 3b. It is obvious in these figures that the deformation is purely elastic as $\bar{\sigma}_{vM} \leq 1$ throughout and compressive as well as tensile stresses may be found

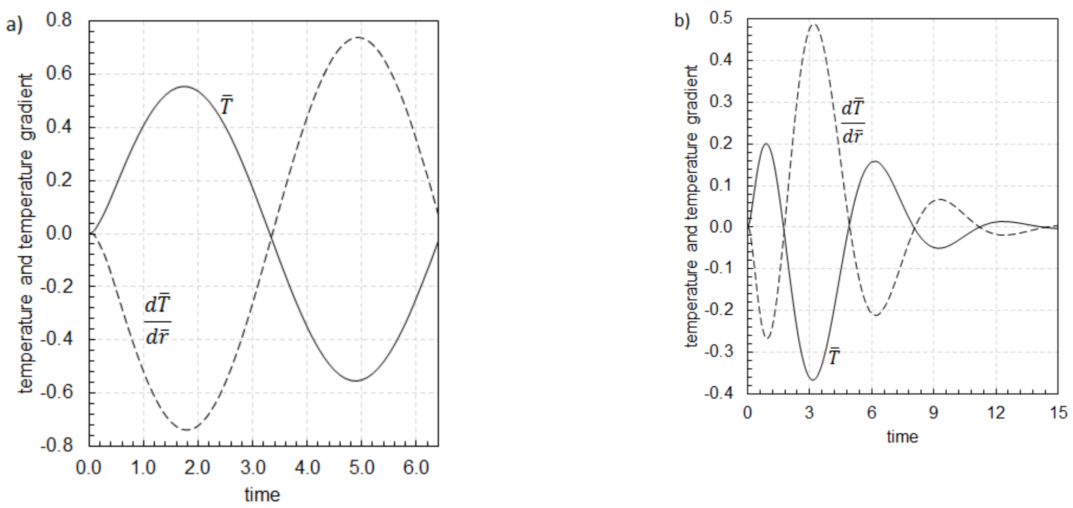

Figure 2. Variations of temperature and temperature gradient with time at $\bar{r} = 0.5$ based on a) Rate 1, b) Rate 2, for the parameter set $\nu = 0.3, A = 3.0$ and $\bar{\alpha} = 1.75$.

in the cylinder according as the temperature gradient is negative or positive.

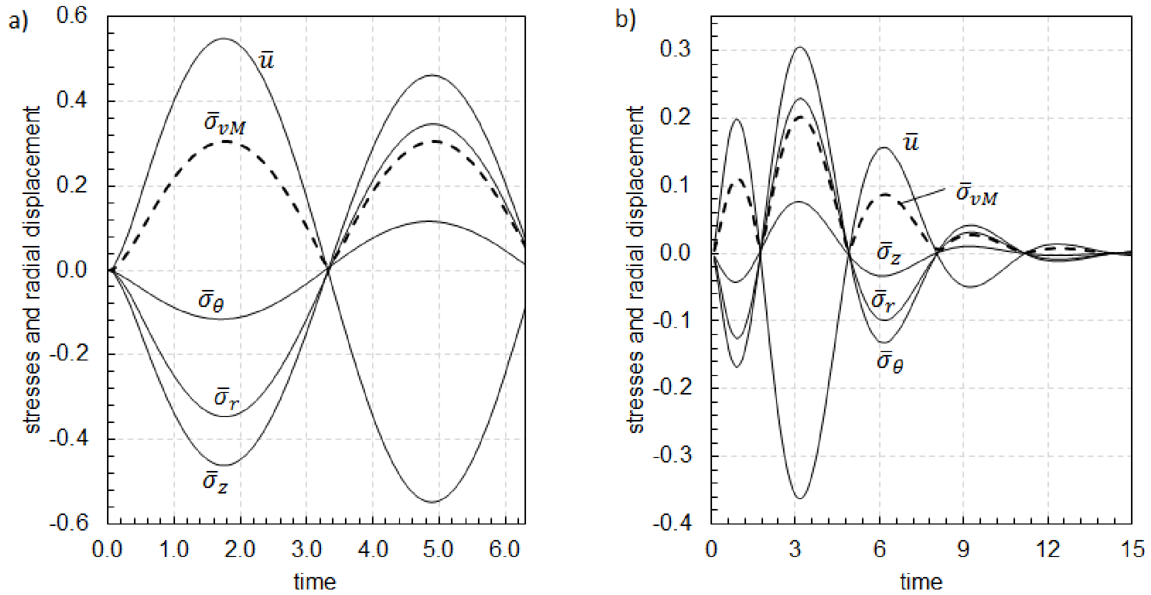

Figure 3. Variations of the stress components, the von Mises stress and the displacement with time at $\bar{r} = 0.5$ based on a) Rate 1, b) Rate 2, for the parameter set $\nu = 0.3$, $A = 3.0$ and $\bar{\alpha} = 1.75$.

The rod may expand or contract in the axial direction as the ends are free. The change in height of the cylinder at any time instant is obtained by multiplying the axial strain with its height. The results of these calculations are depicted in Figure 4a and Figure 4b. The change of height when the heat is generated according to Rate 1 over one period is shown in Figure 4a. As seen in this figure when one period is completed the cylinder returns to its original height. The same is true for Rate 2 as time gets as large as $\tau = 15$ as shown in Figure 4b. It is apparent that the

4. CONCLUSION

In this study, the thermoelastic problem of a solid cylinder subjected to periodic internal heat generation was solved analytically by using Duhamel's theorem. Two different types of periodic function were handled to define the internal heat generation. Although one is periodic, the second function eventually decays to zero. In the first part of the study, the temperature distribution

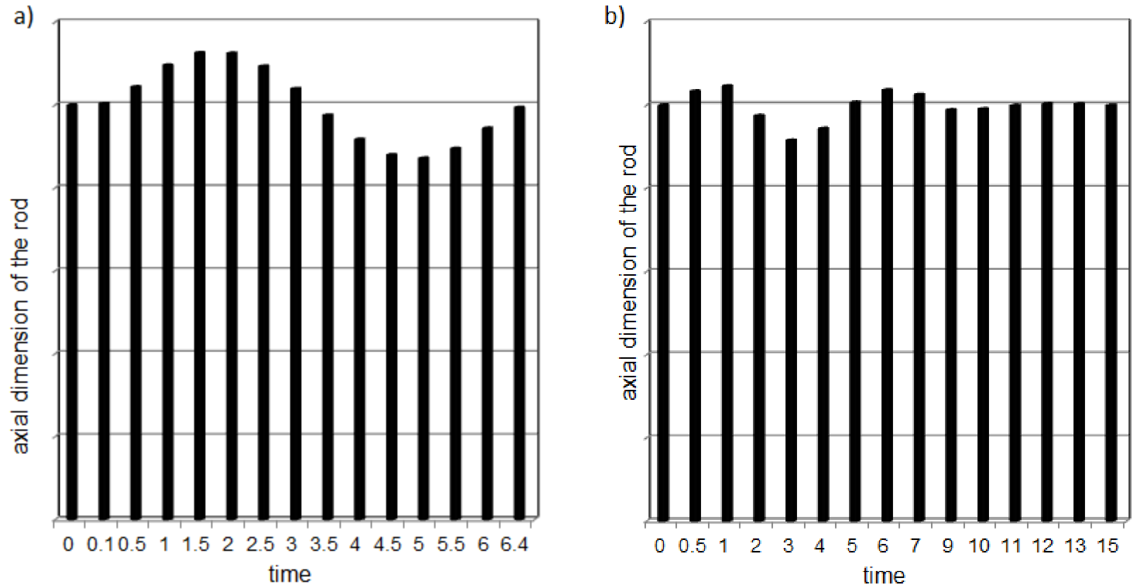

Figure 4. The changes in the height of the cylinder with time based on a) Rate 1, b) Rate 2 for the parameter set $\nu = 0.3, A = 2.0$ and $\bar{\alpha} = 1.75$.

in the cylinder was obtained then this distribution was used for the calculation of the elastic stresses and radial displacement distributions.

The elastic results show that the temperature distribution and the radial displacement distribution have similar behavior. On the other hand, the temperature gradient and the stress distributions have similar behavior in the cylinder. Parameter A affects the magnitude of the internal heat generation and $\bar{\alpha}$ is related to the effects of material properties on the calculations. The results show that the equivalent von Mises stress values always occur at the surface of the cylinder when the heat generation magnitude is increased, means that the yielding always starts at the outer surface according to this criteria.

REFERENCES

1. Hahn W.D., Özışık M.N., "Heat Conduction", ***John Wiley & Sons***, New Jersey, (2012)
2. Timoshenko S., Goodier J.N., "Theory of Elasticity", ***McGraw-Hill***, New York, (1970)
3. Ugural A.C., Fenster S.K. "Advanced Mechanics of Materials and Applied Elasticity", ***Prentice-Hall***, London, (2012)
4. Lu X., Tervola P., Viljanen M., "Transient analytical solution to heat conduction in composite circular cylinder", ***Int J Heat Mass Tran***, 49: 341 – 348, (2006)
5. Tu T.W., Lee S.Y., "Analytical solution of heat conduction for hollow cylinders with time-dependent boundary condition and time-dependent heat transfer coefficient", ***J Applied Math,*** vol 2015, Article ID-203404.
6. Cossali G.E., "Periodic heat conduction in a solid homogeneous finite cylinder", ***Int J Therm Sci***, 48: 722-732, (2009)
7. Fazeli, H., Abdous, M.A., Karabi, H., Moallemi, N., Esmaeili M. "Analysis of transient heat conduction in a hollow cylinder using Duhamel theorem", ***Int. J Thermophysics***, 34: 350-365, (2013)
8. Orcan Y., "Thermal stresses in a heat generating elastic-plastic cylinder with free ends", ***Int J Eng Sci***, 32: 883 – 898, (1994)
9. OrcanY., "Residual stresses and secondary plastic flow in a heat generating elastic-plastic cylinder with free ends", ***Int J Eng Sci***, 33: 1689 – 1698, (1995)
10. Arslan E., Mack W., Eraslan A.N., "Effect of a temperature cycle on a rotating elastic-plastic shaft", ***Acta Mech***, 195: 129-140, (2008)
11. Eraslan A.N., Orcan Y., "Computation of transient thermal stresses in elastic-plastic tubes: Effect of coupling and temperature-dependent physical properties", ***J Therm Stresses***, 25: 559-572, (2002)
12. Deshmukh K.C., Warbhe S.D., Kulkarni V.S. "Brief note on heat flow with arbitrary heating rates in a hollow cylinder", ***Therm. Sci.***, 15 (1): 275-280, (2011)
13. Bhongade C.M., Durge M.H., "Transient thermal stresses due to instantaneous internal heat generation in a thin hollow cylinder", ***Int J Math and Stat Invent (IJMSI)***, 2 (3): 83-88, (2014)
14. Eraslan A.N., Apatay T., "Thermoelastic stresses in a rod subjected to periodic boundary condition: An analytical treatment", ***Journal of Multidisciplinary Engineering Science and Technology*** (***JMEST)***, 2 (9): 2438-2444, (2015)
15. Walde R.T., Pathak A.C., Khobragade N.W., "Thermal stresses of a solid cylinder with internal heat source", ***Int J Eng and Innovative Tech (IJEIT)***, 3 (1): 407-410, (2013)
16. Eraslan A.N., Varli E., "Elastic Response of a Heat Generating Rod at a Variable Generation Rate", ***Proceedings of 12th International Conference of Numerical Analysis and Applied Mathematics (ICNAAM),*** vol 1648, Rhodes, Greece, (2014)
17. Mendelson A. "Plasticity: Theory and Application", ***The Macmillan Company***, New York, (1968)