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On Contra πgs-Continuity

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Abstract
In this work, a novel form of contra continuity entitled as contra πgs-continuity is examined, which has connections to πgs-closed sets. Furthermore, correlations between contra πgs-continuity and several previously established forms of contra continuous functions are further explored, as well as basic features of contra πgs-continuous functions are disclosed.

Keywords: *πgs-closed sets, Contra πgs-continuity, Contra continuity*

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1. Introduction

After defining semi-open sets [1] in 1963, Levine introduced the concept of g-closed sets [2] in 1970. This interesting new set type has led to the emergence of different types of generalized closed sets. Dontchev and Noiri defined πg-closed sets [3] in 2000. In 2006, Aslım et al. introduced the πgs-closed set [4] definition, which has an important place in this study, to the literature.

The idea of LC-continuous functions was first introduced and analyzed by Ganster and Reilly [5] in 1989. Dontchev [6] produced contra-continuity, as a more robust variant of LC-continuity in 1996. As a very interesting subject, contra continuous functions have continued to attract the attention of many researchers over the years. After Ekici gave the definition of contra πg-continuous functions [7] in 2008, contra πgs-continuous [8] functions were also defined in Caldas et al.'s studies in 2010, which essentially introduced and examined contra πgp-continuous functions [8].

The requirement that every open set in the codomain possesses a preimage that is πgs-closed in the domain identifies contra πgs-continuous functions [8]. A milder version of contra-continuity [6] and contra gs-continuity [9] is contra πgs-continuity. Crucial characteristics of contra πgs-continuous functions are also examined.

2. Preliminaries

Unless otherwise specified, topological spaces in this work always refer to on which no separation axioms are required; Ψ will stand for the topological space $(\Psi, \top)$ and Φ will stand for the topological space $(\Phi, \bot)$; $\aleph$ will

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stand for any subset of the space Ψ. The interior of $\aleph$ is indicated as $int(\aleph)$ and the closure of $\aleph$ in indicated as $cl(\aleph)$. Whenever $\aleph = int(cl(\aleph))$ (correspondingly, $\aleph = cl(int(\aleph))$), afterwards $\aleph$ is a regular closed set (correspondingly, regular open set) [10]. Whenever $\aleph \subset cl(int(\aleph))$, afterwards $\aleph$ is considered as a semi-open set [1]. Whenever $\aleph$ could be expressed as union of regular open sets, afterwards it is accepted as a δ-open set [11]. Complementary of semi-open set (correspondingly δ-open set) is introduced as semi-closed (correspondingly δ-closed). The intersection of whole semi-closed sets involving $\aleph$ is known as semi-closure [12] of $\aleph$ which is expressed by $scl(\aleph)$. Dually the semi-interior [12] of $\aleph$ is characterized as union of whole semi-open sets involved in $\aleph$ and indicated by $sint(\aleph)$.

$\nu \in \Psi$ is termed δ-cluster point [11] of $\aleph$, when $int(cl(F)) \cap \aleph \neq \emptyset$ for every $F \in O(\nu, \Psi)$, where $O(\nu, \Psi)$ stands for all open subsets of Ψ containing the point ν. Whole δ-cluster points of $\aleph$ composes δ-closure [11] of $\aleph$ that is shown with $cl_\delta(\aleph)$.

When $\aleph \subset cl(int(cl_\delta(\aleph)))$, then $\aleph$ is named as an e^*-open set [13]. We speak of an e^*-closed [13] set as complementary of an e^*-open. The e^*-closure [13] of $\aleph$ is the intersection of whole e^*-closed sets involving subset $\aleph$ and it is symbolized by $e^*\text{-}cl(\aleph)$.

Whenever $e^*\text{-}cl(F) \cap \aleph \neq \emptyset$ for each e^*-open set F involving point ν, afterwards ν is identified as e^*-θ-cluster point [14] of $\aleph$. The e^*-θ-closure [14] of $\aleph$ is the set of whole e^*-θ-cluster points of $\aleph$, and is expressed by $e^*\text{-}cl_\theta(\aleph)$. For $\aleph = e^*\text{-}cl_\theta(\aleph)$, then $\aleph$ is e^*-θ-closed [15]. $e^*\text{-}\theta C(\Psi)$ is the notion for the collection of whole e^*-θ-closed subsets of space Ψ.

When for every ν in $\aleph$, if there exists an e^*-open set F comprising ν such that $F \setminus \aleph$ is countable, then $\aleph$ is termed we^*-open [16]. A we^*-closed [16] set is the complementary of an we^*-open.

When $\aleph \subset cl(int(\aleph)) \cup int(cl(\aleph))$, subsequently $\aleph$ is named as b-open [17] (or sp-open [18] or γ-open [19]). A b-closed [17] (or γ-closed [20, 21]) set is the complementary of a b-open (or γ-open). The b-closure [17] (or γ-closure [20]) of $\aleph$ is expressed as $bcl(\aleph)$ (or $\gamma cl(\aleph)$) and it is the intersection of whole b-closed (or γ-closed) sets comprising $\aleph$. The set $\aleph$ is said to be pre-closed [22] if $cl(int(\aleph)) \subset \aleph$. The intersection of all pre-closed sets containing $\aleph$ is called pre-closure [20] of $\aleph$ and denoted by $pcl(\aleph)$.

A subset $\aleph$ of a space Ψ is characterized as a $\hat{g}$-closed [23] set, if $cl(\aleph) \subset F$, whenever F is a semi-open set satisfying the condition $\aleph \subset F$. $\hat{g}$-open sets [23] are the complement of $\hat{g}$-closed sets. When $bcl(\aleph) \subset F$ whenever $\aleph \subset F$ and F is a $\hat{g}$-open set in Ψ, $\aleph$ is a $b\hat{g}$-closed [24] set. A $b\hat{g}$-open [25] is the complementary of a $b\hat{g}$-closed set. When $scl(\aleph) \subset F$ whenever $\aleph \subset F$ and F is a $b\hat{g}$-open set in Ψ, $\aleph$ is called as a $sb\hat{g}$-closed [26] set.

π-open [27] corresponds to the finite union of regular open sets. π-closed represents the complementary of a π-open. When $\aleph \subset F$ and F is open (correspondingly, π-open), afterwards $\aleph$ is regarded as a generalized closed (briefly, g-closed) [2] (correspondingly, πg-closed [17]) if $cl(\aleph) \subset F$. g-open [24] (correspondingly, πg-open [7]) is the complementary of g-closed (correspondingly, πg-closed). While $\aleph \subset F$ and F is open (correspondingly, π-open), afterwards $\aleph$ is regarded to be generalized semi-closed (briefly, gs-closed) [28] (correspondingly, πgs-closed [4]) if $scl(\aleph) \subset F$. gs-open [24] (correspondingly, πgs-open) constitutes the complementary of a gs-closed (correspondingly, πgs-closed) set. If $pcl(\aleph) \subset F$ for all F which are π-open sets containing $\aleph$, then $\aleph$ is called as πgp-closed [29]. The set $\aleph$ is called as $\pi g\gamma$-closed [20], if $\gamma cl(\aleph) \subset F$ for all π-open sets F containing $\aleph$.

The entire πgs-closed (correspondingly, πgs-open, πgp-closed, $\pi g\gamma$-closed, gs-closed, gs-open, closed, semi-closed, semi-open, γ-open, π-open, πg-open, regular open, regular closed, g-closed, πg-closed, we^*-closed, e^*-closed, $e^*\theta$-closed, $b\hat{g}$-closed, $sb\hat{g}$-closed) subsets of Ψ are expressed by $\pi GSC(\Psi)$ (correspondingly, $\pi GSO(\Psi)$, $\pi GPC(\Psi)$, $\pi G\gamma C(\Psi)$, $GSC(\Psi)$, $GSO(\Psi)$, $C(\Psi)$, $SC(\Psi)$, $SO(\Psi)$, $\gamma O(\Psi)$, $\pi O(\Psi)$, $\pi GO(\Psi)$, $RO(\Psi)$, $RC(\Psi)$, $GC(\Psi)$, $\pi GC(\Psi)$, $we^*C(\Psi)$, $e^*C(\Psi)$, $e^*\theta C(\Psi)$, $b\hat{g}C(\Psi)$, $sb\hat{g}C(\Psi)$).

$\pi GSC(\nu, \Psi)$ (correspondingly, $\pi GSO(\nu, \Psi)$, $RO(\nu, \Psi)$, $C(\nu, \Psi)$, $SO(\nu, \Psi)$, $O(\nu, \Psi)$) means the collection of whole πgs-closed (correspondingly, πgs-open, regular open, closed, semi open, open) sets of Ψ comprising point $\nu \in \Psi$.

πgs-closure of the set $\aleph$ is denoted by $cl_{\pi gs}(\aleph)$, which is the intersection of whole πgs-closed sets involving $\aleph$. On the other hand, πgs-interior of a set $\aleph$ is expressed by $int_{\pi gs}(\aleph)$, which corresponds to the union of whole πgs-open sets included in $\aleph$.

Definition 2.1. A topological space Ψ is said to be:
(ι_i) strongly S-closed [6] while a finite subcover matching could found for each closed cover of Ψ,
(ι_{ii}) strongly countably S-closed [7] when a finite subcover matching found for each countable cover of Ψ consisting of closed sets,
(ι_{iii}) strongly S-Lindelöf [7] when a countable subcover matching could found for each closed cover of Ψ,
(ι_{iv}) ultra normal [30] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets,
(ι_v) ultra Hausdorff [30] if for each couple of distinct points, ν_1 and ν_2 in Ψ there exist clopen sets $\aleph_1$ and $\aleph_2$ comprising ν_1 and ν_2 correspondingly, providing $N_1 \cap N_2 = \emptyset$ equality.

Definition 2.2. When $\aleph$ in Ψ is strongly S-closed as a subspace, then $\aleph$ is named strongly S-closed [6].

Definition 2.3. $\aleph$ in Ψ is called:
(ι_i) α-open [31] whenever $\aleph \subset int(cl(int(\aleph)))$,
(ι_{ii}) preopen [22] or nearly open [5] whenever $\aleph \subset int(cl(\aleph))$,
(ι_{iii}) β-open [32] or semi-preopen [33] whenever $\aleph \subset cl(int(cl(\aleph)))$.

Complement of an α-open (correspondingly, preopen, β-open) set is introduced as α-closed (correspondingly, preclosed, β-closed) set [7]. $\alpha O(\Psi)$ (correspondingly, $PO(\Psi), \beta O(\Psi)$) stands for the collection of whole α-open (correspondingly, preopen, β-open) subsets of Ψ.

Lemma 2.1. *Whenever* $\aleph \subset \Psi$,
(ι_i) $cl_{\pi gs}(\Psi \backslash \aleph) = \Psi \backslash int_{\pi gs}(\aleph)$;
(ι_{ii}) $\nu \in cl_{\pi gs}(\aleph) \Leftrightarrow \forall \digamma \in \pi GSO(\nu, \Psi), \aleph \cap \digamma \neq \emptyset$.

Proof. Before starting the proof, let's remind the definitions of πgs-interior and πgs-closure of a set in a topological space. Let $(\Psi, \top)$ be a topological space, $\aleph \subset \Psi$. Then, πgs-closure of $\aleph$ is $cl_{\pi gs}(\aleph) = \bigcap\{\Theta : \aleph \subset \Theta, \Theta \in \pi GSC(\Psi)\}$ and πgs-interior of $\aleph$ is $int_{\pi gs}(\aleph) = \bigcup\{\partial : \partial \subset \aleph, \partial \in \pi GSO(\Psi)\}$. Now we can start the proof.
(ι_i): We will complete the proof by showing that the sets claimed to be equal include each other.
Let $(\Psi, \top)$ be a topological space and $\aleph \subset \Psi$.
$(\Rightarrow)$: Let $\nu \in cl_{\pi gs}(\Psi \backslash \aleph)$. Assume that $\nu \notin \Psi \backslash int_{\pi gs}(\aleph)$. Since $\nu \in int_{\pi gs}(\aleph) = \bigcup\{\partial : \partial \subset \aleph, \partial \in \pi GSO(\Psi)\}$, it can be said that there exists a set $\digamma \in \pi GSO(\nu, \Psi)$ such that $\digamma \subset \aleph$. So $\Theta = \Psi \backslash \digamma \in \pi GSC(\Psi)$, $\nu \notin \Theta$ and $\Psi \backslash \aleph \subset \Theta$. This brings us to the contradiction $\nu \notin cl_{\pi gs}(\Psi \backslash \aleph)$ contrary to our assumption. Hence as a result $cl_{\pi gs}(\Psi \backslash \aleph) \subset \Psi \backslash int_{\pi gs}(\aleph)$.
$(\Leftarrow)$: Let $\nu \in \Psi \backslash int_{\pi gs}(\aleph)$. So it can be clearly seen that $\nu \notin int_{\pi gs}(\aleph) = \bigcup\{\partial : \partial \subset \aleph, \partial \in \pi GSO(\Psi)\}$. Then for all of the sets $\partial \in \pi GSO(\Psi)$ such that $\partial \subset \aleph$ we have $\nu \notin \partial$. This means that for all sets $\Psi \backslash \partial \in \pi GSC(\Psi)$ such that $\Psi \backslash \aleph \subset \Psi \backslash \partial$ we have $\nu \in \Psi \backslash \partial$. So $\nu \in cl_{\pi gs}(\Psi \backslash \aleph)$. Hence as a result $\Psi \backslash int_{\pi gs}(\aleph) \subset cl_{\pi gs}(\Psi \backslash \aleph)$.
Now we will give the proof of (ι_{ii}).
(ι_{ii}):
$(\Rightarrow)$: Let $\nu \in cl_{\pi gs}(\aleph)$. Assume that there exists a set $\partial \in \pi GSO(\nu, \Psi)$ such that $\partial \cap \aleph = \emptyset$. Under this assumption, for the set $\Theta = \Psi \backslash \partial$ it can be said that $\nu \notin \Theta$ and $\aleph \subset \Theta$. These results brings us to the contradiction $\nu \notin cl_{\pi gs}(\aleph)$ contrary to our assumption.
$(\Leftarrow)$: Let $\nu \in \Psi$ and let for all sets $\partial \in \pi GSO(\nu, \Psi)$ we have $\partial \cap \aleph \neq \emptyset$. Assume that $\nu \notin cl_{\pi gs}(\aleph)$. Then using (ι_i) we have $\nu \in \Psi \backslash cl_{\pi gs}(\aleph) = \Psi \backslash (\Psi \backslash int_{\pi gs}(\Psi \backslash \aleph)) = int_{\pi gs}(\Psi \backslash \aleph)$. So there exists a set $\digamma \in \pi GSO(\nu, \Psi)$ such that $\digamma \subset \Psi \backslash \aleph$, which means that $\digamma \cap \aleph = \emptyset$ which is a contradiction. So $\nu \in cl_{\pi gs}(\aleph)$.
Thus the proof is completed. □

While $\aleph$ is πgs-closed, then $cl_{\pi gs}(\aleph) = \aleph$. Typically, the opposite of this implication doesn't hold true, as demonstrated in the subsequent example:

Example 2.1. Consider the subset $\aleph = \{\nu_1, \nu_2\}$ of the set $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4, \nu_5\}$ and the topological space $(\Psi, \top)$, where $\top = \{\emptyset, \{\nu_1\}, \{\nu_2\}, \{\nu_1, \nu_2\}, \{\nu_1, \nu_2, \nu_3\}, \Psi\}$. Then the set $\aleph$ is an acceptable sample that fits the given situation just above, since $\aleph = cl_{\pi gs}(\aleph)$, while $\aleph \notin \pi GSC(\Psi)$.

$ker(\mho)$ [34] means $\bigcap\{\digamma \in \top : \mho \subset \digamma\}$ which is known as the kernel of $\mho$.

Lemma 2.2. *[35] The subsequent characteristics apply to subsets $\digamma$ and $\mho$ of Ψ:*
(ι_i) $\nu \in ker(\digamma) \Leftrightarrow (\forall \Theta \in C(\nu, \Psi))(\digamma \cap \Theta \neq \emptyset)$;
(ι_{ii}) $\digamma \subset ker(\digamma)$;
(ι_{iii}) $\digamma \in \Psi \Rightarrow \digamma = ker(\digamma)$;
(ι_{iv}) $\digamma \subset \mho \Rightarrow ker(\digamma) \subset ker(\mho)$.

3. Contra πgs-continuous functions

In this section, first the characterization of contra πgs-continuous functions is presented. Afterwards, the relationships between some types of contra continuous functions and contra πgs-continuous functions were examined. In addition, some new definitions in relation with πgs-open sets are given in order to examine various properties of contra πgs-continuous functions, and these properties are presented through theorems and results.

Definition 3.1. $\Delta : (\Psi, \top) \to (\Phi, \bot)$ is referred as contra πgs-continuous [8], whenever $\Delta^{-1}(\mho) \in \pi GSC(\Psi)$ for each $\mho \in \bot$.

Theorem 3.1. *Under the assumption $\pi GSO(\Psi)$ is closed under arbitrary unions (or likewise $\pi GSC(\Psi)$ is closed under arbitrary intersections), subsequent statements are coequal for $\Delta:(\Psi,\top)\to(\Phi,\bot)$.*
(ι_i) *Δ is contra πgs-continuous;*
(ι_{ii}) $\mho\in C(\Phi)\Rightarrow\Delta^{-1}(\mho)\in\pi GSO(\Psi)$;
(ι_{iii}) $(\forall\nu\in\Psi)(\forall\Theta\in C(\Delta(\nu),\Phi))(\exists F\in\pi GSO(\nu,\Psi))(\Delta(F)\subset\Theta)$;
(ι_{iv}) $\aleph\subset\Psi\Rightarrow\Delta(cl_{\pi gs}(\aleph))\subset ker(\Delta(\aleph))$;
(ι_v) $\Omega\subset\Phi\Rightarrow cl_{\pi gs}(\Delta^{-1}(\Omega))\subset\Delta^{-1}(ker(\Omega))$.

Proof. Let $\Delta:(\Psi,\top)\to(\Phi,\bot)$ be a function, where $(\Psi,\top)$ and $(\Phi,\bot)$ are two topological spaces and let $\pi GSO(\Psi)$ be closed under arbitrary unions (or likewise $\pi GSC(\Psi)$ be closed under arbitrary intersections).
$(\iota_i)\Rightarrow(\iota_{ii})$: Let $\Theta\in C(\Phi)$. Then $\Phi\backslash\Theta$ is open in Φ. Since Δ is contra πgs-continuous, $\Psi\backslash\Delta^{-1}(\Theta)=\Delta^{-1}(\Phi\backslash\Theta)$ is πgs-closed in Ψ. Therefore, $\Delta^{-1}(\Theta)$ is πgs-open in Ψ.
$(\iota_{ii})\Rightarrow(\iota_i)$: Obvious.
$(\iota_i)\Rightarrow(\iota_{iii})$: Let $\nu\in\Psi$ and $\Theta\in C(\Delta(\nu),\Phi)$. Then by (ι_i), we have $\Delta^{-1}(\Theta)\in\pi GSO(\Psi)$. Choosing $F=\Delta^{-1}(\Theta)$ we obtain that $F\in\pi GSO(\nu,\Psi)$ and $\Delta(F)\subset\Theta$.
$(\iota_{iii})\Rightarrow(\iota_{ii})$: Let $\Theta\in C(\Phi)$ and $\nu\in\Delta^{-1}(\Theta)$. Since $\Delta(\nu)\in\Theta$, by (ι_{iii}) there exist a πgs-open set $F_\nu\in\pi GSO(\nu,\Psi)$ such that $\Delta(F_\nu)\subset\Theta$. So we have $\nu\in F_\nu\subset\Delta^{-1}(\Theta)$ and hence $\Delta^{-1}(\Theta)=\bigcup\{F_\nu:\nu\in\Delta^{-1}(\Theta)\}$ is πgs-open in Ψ since $\pi GSO(\Psi)$ is closed under arbitrary unions.
$(\iota_{ii})\Rightarrow(\iota_{iv})$: Let $\aleph$ be any subset of Ψ. Suppose that there exist an element μ of $\Delta(cl_{\pi gs}(\aleph))$ such that $\mu\notin ker(\Delta(\aleph))$. Then there exists an open set F of Φ such that $\Delta(\aleph)\subset F$ and $\mu\notin F$. Hence, there exists $\Theta=\Phi\backslash F\in C(\mu,\Phi)$ such that $\Delta(\aleph)\cap\Theta=\emptyset$ and $cl_{\pi gs}(\aleph)\cap\Delta^{-1}(\Theta)=\emptyset$. From here we obtain that $\Delta(cl_{\pi gs}(\aleph))\cap\Theta=\emptyset$ and $\mu\notin\Delta(cl_{\pi gs}(\aleph))$ which is a contradiction.
$(\iota_{iv})\Rightarrow(\iota_v)$: Let Ω be any subset of Φ. Then $\Delta^{-1}(\Omega)\subset\Psi$. By (ι_{iv}), $\Delta(cl_{\pi gs}(\Delta^{-1}(\Omega)))\subset ker(\Delta(\Delta^{-1}(\Omega)))\subset ker(\Omega)$. Hence, $cl_{\pi gs}(\Delta^{-1}(\Omega))\subset\Delta^{-1}(ker(\Omega))$.
$(\iota_v)\Rightarrow(\iota_i)$: Let F be any open subset of Φ. Then by (ι_v) and by Lemma 2.2, $cl_{\pi gs}(\Delta^{-1}(F))\subset\Delta^{-1}(ker(F))=\Delta^{-1}(F)$. So we have $cl_{\pi gs}(\Delta^{-1}(F))=\Delta^{-1}(F)$. Since $\pi GSO(\Psi)$ is closed under arbitrary unions, $\pi GSC(\Psi)$ is closed under arbitrary intersections and hence $\Delta^{-1}(F)=cl_{\pi gs}(\Delta^{-1}(F))$ is πgs-closed. □

Remark 3.1. Statements (ι_i) and (ι_{ii}) in Theorem 3.1 are identical even if $\pi GSO(\Psi)$ is not closed under arbitrary unions (or likewise, $\pi GSC(\Psi)$ is not closed under arbitrary intersections).

Definition 3.2. $\Delta:(\Psi,\top)\to(\Phi,\bot)$ is categorized as:
(ι_1) perfectly continuous [36] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in\top\cap C(\Psi))$,
(ι_2) RC-continuous [9] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in RC(\Psi))$,
(ι_3) strongly continuous [37] $:\Leftrightarrow(F\subset\Phi\Rightarrow\Delta^{-1}(F)\in\top\cap C(\Psi))$ (identically $(\aleph\subset\Psi\Rightarrow\Delta(cl(\aleph))\subset\Delta(\aleph))$),
(ι_4) contra-continuous [6] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in C(\Psi))$,
(ι_5) contra-super-continuous [38]$:\Leftrightarrow(\forall\nu\in\Psi)(\forall\Theta\in C(\Delta(\nu),\Phi)(\exists F\in RO(\nu,\Psi))(\Delta(F)\subset\Theta)$,
(ι_6) contra-semicontinuous [9] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in SC(\Psi))$,
(ι_7) contra g-continuous [39] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in GC(\Psi))$,
(ι_8) contra gs-continuous [9] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in GSC(\Psi))$,
(ι_9) contra πg-continuous [7] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in\pi GC(\Psi))$,
(ι_{10}) contra we^*-continuous [16] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in we^*C(\Psi))$,
(ι_{11}) contra $e^*\theta$-continuous [40] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in e^*\theta C(\Psi))$,
(ι_{12}) contra e^*-continuous [41] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in e^*C(\Psi))$,
(ι_{13}) almost contra e^*-continuous [42] $:\Leftrightarrow(F\in RO(\Phi)\Rightarrow\Delta^{-1}(F)\in e^*C(\Psi))$,
(ι_{14}) almost contra $e^*\theta$-continuous [42] $:\Leftrightarrow(F\in RO(\Phi)\Rightarrow\Delta^{-1}(F)\in e^*\theta C(\Psi))$,
(ι_{15}) contra $b\hat{g}$-continuous [25] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in b\hat{g}C(\Psi))$,
(ι_{16}) contra sb$\hat{g}$-continuous [43] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in sb\hat{g}C(\Psi))$.
(ι_{17}) contra πgp-continuous function [8] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in\pi GPC(\Psi))$,
(ι_{18}) contra $\pi g\gamma$-continuous function [20] $:\Leftrightarrow(F\in\bot\Rightarrow\Delta^{-1}(F)\in\pi G\gamma C(\Psi))$.

Remark 3.2.

$$\begin{array}{rcccccccccc} & \iota_6 & \longleftarrow & \iota_4 & \longleftarrow & \iota_5 & \longleftarrow & \iota_2 & \longleftarrow & \iota_1 & \longleftarrow \ \iota_3 \\ & \downarrow & & \downarrow & & & & & & & \\ & \iota_8 & \longleftarrow & \iota_7 & & & & & & & \\ & \downarrow & & \downarrow & & & & & & & \\ \text{contra } \pi\text{gs-continuous} \leftarrow & & & \iota_9 & & & & & & & \\ & \downarrow & & \downarrow & & & & & & & \\ & \iota_{18} & \leftarrow & \iota_{17} & & & & & & & \end{array}$$

Remark 3.3. As can be seen from the samples below, reversibility of the consequences in the above diagram need not to be true.

Example 3.1. $\top = \{\emptyset, \{\nu_2\}, \{\nu_1, \nu_4\}, \{\nu_2, \nu_3\}, \{\nu_1, \nu_2, \nu_4\}, \{\nu_1, \nu_2, \nu_3, \nu_4\}, \Psi\}$ is the topology on $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4, \nu_5\}$. Since mappings under $\Delta : \Psi \to \Psi$ are listed as $\Delta(\nu_1) = \nu_1$, $\Delta(\nu_2) = \nu_2$, $\Delta(\nu_3) = \nu_3$, $\Delta(\nu_4) = \nu_5$, $\Delta(\nu_5) = \nu_4$ the contra πgs-continuity of Δ is evident. However, it is neither contra πg-continuous nor contra gs-continuous since $\Delta^{-1}(\{\nu_2\}) = \{\nu_2\} \notin \pi GC(\Psi)$ and $\Delta^{-1}(\{\nu_2\}) = \{\nu_2\} \notin \pi GSC(\Psi)$.

Example 3.2. Let $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}$, $\top = \{\emptyset, \{\nu_1\}, \{\nu_2\}, \{\nu_1, \nu_2\}, \Psi\}$. Match-ups for $\Delta : \Psi \to \Psi$ are

$$\Delta(\nu_1) = \Delta(\nu_2) = \Delta(\nu_3) = \nu_1, \Delta(\nu_4) = \nu_3.$$

Δ is contra πgs-continuous, but it is not contra $e^*\theta$-continuous since $\Delta^{-1}(\{\nu_1\}) = \Delta^{-1}(\{\nu_1, \nu_2\}) = \{\nu_1, \nu_2, \nu_3\}$ is not $e^*\theta$-closed w.r.t. $\top$.

Example 3.3. Given $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}$, $\top = \{\emptyset, \{\nu_1\}, \{\nu_2\}, \{\nu_1, \nu_2\}, \Psi\}$. Match-ups for $\Delta : \Psi \to \Psi$ are

$$\Delta(\nu_1) = \nu_3, \Delta(\nu_2) = \nu_1, \Delta(\nu_3) = \Delta(\nu_4) = \nu_4.$$

Although Δ is contra πgs-continuous, it is not almost contra e^*-continuous,since $\{\nu_1, \nu_3\}$ is regular open and $\Delta^{-1}(\{\nu_1, \nu_3\}) = \{\nu_1, \nu_2\}$ is not an e^*-closed. By checking the connections between these class of functions in [42] we can easily state that Δ cannot be almost contra $e^*\theta$-continuous, contra $e^*\theta$-continuous and contra e^*-continuous.

Example 3.4. $\top = \{\emptyset, \{\nu_1\}, \{\nu_2\}, \{\nu_2, \nu_1\}, \{\nu_3, \nu_1\}, \{\nu_1, \nu_3, \nu_2\}, \{\nu_1, \nu_2, \nu_4\}, \Psi\}$ is a topology on $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}$. Match-ups of $\Delta : \Psi \to \Psi$ are

$$\Delta(\nu_1) = \nu_1, \Delta(\nu_2) = \nu_2, \Delta(\nu_3) = \Delta(\nu_4) = \nu_4.$$

Since $\Delta^{-1}(\{\nu_1, \nu_2\}) = \Delta^{-1}(\{\nu_1, \nu_2, \nu_3\}) = \{\nu_1, \nu_2\} \notin \pi GSC(\Psi)$, Δ is not contra πgs-continuous. However, it is contra $e^*\theta$-continuous. So it is contra e^*-continuous, almost contra $e^*\theta$-continuous and almost contra e^*-continuous.

As seen from the examples above contra πgs-continuity does not require almost contra $e^*\theta$-continuity, almost contra e^*-continuity, contra $e^*\theta$-continuity and contra e^*-continuity. It is also clear that almost contra $e^*\theta$-continuity, almost contra e^*-continuity, contra $e^*\theta$-continuity and contra e^*-continuity does not require contra πgs-continuity. As another result we can state that contra we^*-continuity does not require contra πgs-continuity.

Research Question Does contra πgs-continuity require contra we^*-continuity?

Example 3.5. $\top = \{\emptyset, \{\nu_1\}, \{\nu_2\}, \{\nu_1, \nu_2\}, \{\nu_3, \nu_1\}, \{\nu_1, \nu_3, \nu_2\}, \{\nu_2, \nu_1, \nu_4\}, \Psi\}$ is a topology on $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}$. Match-ups of $\Delta : \Psi \to \Psi$ are

$$\Delta(\nu_1) = \nu_3, \Delta(\nu_2) = \nu_2, \Delta(\nu_3) = \nu_1, \Delta(\nu_4) = \nu_2$$

Δ is contra πgs-continuous, but it is not contra $b\hat{g}$-continuous since $\Delta^{-1}(\{\nu_1, \nu_3\}) = \{\nu_1, \nu_3\}$ is not $b\hat{g}$-closed. So it cannot be contra $sb\hat{g}$-continuous.

Example 3.6. $\top = \{\emptyset, \{\nu_1, \nu_5\}, \{\nu_2, \nu_4\}, \{\nu_1, \nu_2, \nu_4, \nu_5\}, \Psi\}$ is a topology on $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4, \nu_5\}$. Match-ups of $\Delta : \Psi \to \Psi$ are

$$\Delta(\nu_1) = \nu_1, \Delta(\nu_2) = \nu_2, \Delta(\nu_3) = \Delta(\nu_4) = \nu_3, \Delta(\nu_5) = \nu_5$$

Δ is contra $b\hat{g}$-continuous. However, since $\Delta^{-1}(\{\nu_1, \nu_2, \nu_4, \nu_5\}) = \{\nu_1, \nu_2, \nu_5\} \notin \pi GSC(\Psi)$, it is not contra πgs-continuous.

As seen from the examples above there is no relation between contra $b\hat{g}$-continuity and contra πgs-continuity. As another result we see that a contra πgs-continuity does not require contra $sb\hat{g}$-continuity.

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Example 3.7. [8] Let $\top = \{\emptyset, \{\nu_1\}, \{\nu_2\}, \{\nu_1, \nu_2\}, \{\nu_3, \nu_2\}, \{\nu_3, \nu_2, \nu_1\}, \Psi\}$ and $\perp = \{\emptyset, \{\nu_1\}, \Psi\}$ be two topologies on $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}$. The identity function $\Delta : (\Psi, \top) \to (\Psi, \perp)$ is contra πgs-continuous, but it is not contra πgp-continuous.

Example 3.8. [8] Let $\top = \{\emptyset, \{\nu_2\}, \{\nu_3, \nu_2\}, \{\nu_1, \nu_4\}, \{\nu_1, \nu_2, \nu_4\}, \{\nu_1, \nu_2, \nu_4, \nu_3\}, \Psi\}$ and $\perp = \{\emptyset, \{\nu_4\}, \Psi\}$ be two topologies on $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4, \nu_5\}$. The identity function $\Delta : (\Psi, \top) \to (\Psi, \perp)$ is contra πgp-continuous and contra $\pi g\gamma$-continuous, but it is not contra πgs-continuous.

As seen from Example 3.7 and Example 3.8 there is no connection between contra πgp-continuity and contra πgs-continuity. Example 3.8 also shows that contra $\pi g\gamma$-continuity does not require contra πgs-continuity.

Theorem 3.2. *[4] Let $\aleph \subset \Psi$, afterwards $\aleph \in RO(\Psi)$ if and only if $\aleph \in \pi O(\Psi) \cap \pi GSC(\Psi)$.*

Definition 3.3. $\Delta : \Psi \to \Phi$ is called as:
(ι_1) π-continuous [3] $:\Leftrightarrow (F \in \perp \Rightarrow \Delta^{-1}(F) \in \pi O(\Psi))$,
(ι_2) πg-continuous [3] $:\Leftrightarrow (F \in \perp \Rightarrow \Delta^{-1}(F) \in \pi GO(\Psi))$,
(ι_3) πgs-continuous [4] $:\Leftrightarrow (F \in C(\Phi) \Rightarrow \Delta^{-1}(F) \in \pi GSC(\Psi))$,
(ι_4) completely continuous [44] $:\Leftrightarrow (F \in \perp \Rightarrow \Delta^{-1}(F) \in RO(\Psi))$.

Theorem 3.3. *Whenever $\Delta : \Psi \to \Phi$, afterwards the statement below is satisfied:*
Δ is contra πgs-continuous and π-continuous if and only if Δ is completely continuous.

Proof. Obvious from Theorem 3.2. □

Theorem 3.4. *Under the circumstance $\pi GSO(\Psi)$ is closed under arbitrary unions, it can be stated that whenever $\Delta : \Psi \to \Phi$ is contra πgs-continuous and Φ is regular, afterwards Δ is πgs-continuous.*

Definition 3.4. Whenever $\pi GSC(\Psi) \subset SC(\Psi)$ afterwards Ψ is accepted as πgs-$T_{\frac{1}{2}}$ [4].

Theorem 3.5. *Whenever Ψ is considered as πgs-$T_{\frac{1}{2}}$ space afterwards, contra πgs-continuity, contra-semicontinuity and contra gs-continuity of $\Delta : \Psi \to \Phi$ are identical.*

Proof. Assume that Ψ as a πgs-$T_{\frac{1}{2}}$ space. Since $SC(\Psi) \subset \pi GSC(\Psi)$, we have $SC(\Psi) = \pi GSC(\Psi)$. Using the relation $SC(\Psi) \subset GSC(\Psi)$, we obtain $\pi GSC(\Psi) \subset GSC(\Psi)$. Since $GSC(\Psi) \subset \pi GSC(\Psi)$, we have $GSC(\Psi) = \pi GSC(\Psi)$. Therefore $\pi GSC(\Psi) = SC(\Psi) = GSC(\Psi)$. □

Theorem 3.6. *For each $i \in I$, p_i stands for projection of $\prod \Phi_i$ onto Φ_i. If $\Delta : \Psi \to \prod \Phi_i$ is contra πgs-continuous, then $p_i \circ \Delta : \Psi \to \Phi_i$ is contra πgs-continuous for each $i \in I$.*

Proof. Since p_i is continuous and Δ is contra πgs-continuous, we can state that $p_i^{-1}(U_i)$ is open in $\prod \Upsilon_i$ for any $U_i \in \perp_i$ and $(p_i \circ \Delta)^{-1}(U_i) = \Delta^{-1}(p_i^{-1}(U_i)) \in \pi GSC(\Psi)$. Hereby, $p_i \circ \Delta$ is contra πgs-continuous. □

Definition 3.5. A topological space Ψ is said to be locally πgs-indiscrete if $\pi GSO(\Psi) \subset C(\Psi)$.

Theorem 3.7. *The fact that Ψ is locally πgs-indiscrete for contra πgs-continuous $\Delta : \Psi \to \Phi$ requires that Δ is continuous.*

Proof. Allow $F \in \perp$. Since Δ is contra πgs-continuous, $\Delta^{-1}(F) \in \pi GSC(\Psi)$. Since Ψ is locally πgs-indiscrete, $\Delta^{-1}(F) \in \top$. □

Theorem 3.8. *Whenever Ψ is a πgs-$T_{\frac{1}{2}}$ for any $\Delta : \Psi \to \Phi$, afterwards following are equivalent :*
(ι_1) *Δ is completely continuous;*
(ι_2) *Δ is π-continuous and contra πgs-continuous;*
(ι_3) *Δ is π-continuous and contra gs-continuous;*
(ι_4) *Δ is π-continuous and contra-semicontinuous.*

Proof. Equivalence of (ι_2), (ι_3) and (ι_4) is obvious from Theorem 3.5 and the equivalence of (ι_1) and (ι_2) can be easily seen from Theorem 3.2. □

Definition 3.6. The topological space $(\Psi, \top)$ is called:
(ι_1) submaximal [45] $:\Leftrightarrow (\forall \aleph \subset \Psi)(cl(\aleph) = \Psi \Rightarrow \aleph \in \top)$,
(ι_2) extremally disconnected [45] $:\Leftrightarrow (\forall \aleph \subset \Psi)(\aleph \in \top \Rightarrow cl(\aleph) \in \top)$.

Definition 3.7. $\Delta : \Psi \to \Phi$ is called contra α-continuous [46] (correspondingly contra precontinuous [46], contra β-continuous [47], contra γ-continuous [48]) if the preimage of every open subsets of Φ is α-closed (correspondingly preclosed, β-closed, γ-closed) in Ψ.

Lemma 3.1. *For any $(\Psi, \top)$, if $\pi GSC(\Psi)$ is closed under finite unions then, πgs-$\top = \{U \subset \Psi : cl_{\pi gs}(\Psi \backslash U) = \Psi \backslash U\}$.*

Theorem 3.9. *Whenever Ψ is extremally disconnected, submaximal and πgs-$T_{\frac{1}{2}}$ for any $\Delta : \Psi \to \Phi$, afterwards the following are equivalent:*
(ι_1) *Δ is contra πgs-continuous;*
(ι_2) *Δ is contra gs-continuous;*
(ι_3) *Δ is contra-semicontinuous;*
(ι_4) *Δ is contra-continuous;*
(ι_5) *Δ is contra precontinuous;*
(ι_6) *Δ is contra β-continuous;*
(ι_7) *Δ is contra α-continuous;*
(ι_8) *Δ is contra γ-continuous.*

Proof. In an extremally disconnected submaximal space $(\Psi, \top)$,

$$\top = \alpha O(\Psi) = SO(\Psi) = PO(\Psi) = \gamma O(\Psi) = \beta O(\Psi).$$

From this fact we can say that $(\iota_3), (\iota_4), (\iota_5), (\iota_6), (\iota_7), (\iota_8)$ are equivalent. The equivalence of $(\iota_1), (\iota_2), (\iota_3)$ is obvious from Theorem 3.5. □

Theorem 3.10. *Whenever Ψ is said to be extremally disconnected, afterwards any $\Delta : \Psi \to \Phi$ is contra πgs-continuous and πgs-continuous.*

Definition 3.8. $\Delta : \Psi \to \Phi$ is said to be πgs-irresolute [4] if $\Delta^{-1}(F) \in \pi GSO(\Psi)$ for each $F \in \pi GSO(\Phi)$.

Theorem 3.11. *For $\Delta : \Psi \to \Phi$ and $\rho : \Phi \to \zeta$ following properties hold:*
(ι_1) *If Δ is πgs-irresolute and ρ is contra πgs-continuous, then $\rho \circ \Delta$ is contra πgs-continuous;*
(ι_2) *If Δ is contra πgs-continuous and ρ is continuous, then $\rho \circ \Delta$ is contra πgs-continuous;*
(ι_3) *If Δ is contra πgs-continuous and ρ is RC-continuous, then $\rho \circ \Delta$ is πgs-continuous;*
(ι_4) *If Δ is πgs-continuous and ρ is contra continuous, then $\rho \circ \Delta$ is contra πgs-continuous;*
(ι_5) *If Δ is πgs-irresolute and ρ is RC-continuous (correspondingly contra π-continuous, contra-continuous, contra g-continuous, contra πg-continuous, contra-semicontinuous, contra gs-continuous), then $\rho \circ \Delta$ is contra πgs-continuous.*

Definition 3.9. $\Delta : \Psi \to \Phi$ is characterized as πgs-open if $\Delta(\aleph)$ is πgs-open in Φ for each πgs-open subset $\aleph$ of Ψ.

Theorem 3.12. *$\Delta : \Psi \to \Phi$ and $\rho : \Phi \to \zeta$ be two functions and suppose that $\pi GSC(\Phi)$ is closed under arbitrary intersections. Whenever Δ is surjective πgs-open function and $\rho \circ \Delta$ is contra πgs-continuous, afterwards ρ is contra πgs-continuous.*

Proof. Suppose $\mu \in \Phi$ and $\Theta \in C(\rho(\mu), \zeta)$. Since Δ is surjective, existence of $\nu \in \Psi$ satisfying $\Delta(\nu) = \mu$ is clear. Naturally, $\Theta \in C(\rho \circ \Delta(\nu), \zeta)$. Since $\rho \circ \Delta$ is contra πgs-continuous, $\eth \in \pi GSO(\nu, \Psi)$ naturally appears satisfying $\rho \circ \Delta(\eth) \subset \Theta$ relation. Since Δ is πgs-open, $\Delta(\eth)$ is an element of $\pi GSO(\mu, \Phi)$. Hence, for each $\mu \in \Phi$ and for each $\Theta \in C(\rho(\mu), \zeta)$, existence of $\Delta(\eth) = F \in \pi GSO(\mu, \Phi)$ is natural satisfying $\rho(F) \subset \Theta$. By Theorem 3.1 ρ is contra πgs-continuous. □

Corollary 3.1. *Whenever $\pi GSC(\Phi)$ is closed under arbitrary intersections and $\Delta : \Psi \to \Phi$ is surjective πgs-irresolute and πgs-open, afterwards for any $\rho : \Phi \to \zeta$, $\rho \circ \Delta$ is contra πgs-continuous if and only if ρ is contra πgs-continuous.*

Proof. Obvious from Theorems 3.11 and 3.12. □

Definition 3.10. $\Delta : \Psi \to \Phi$ is characterized as weakly contra πgs-continuous whenever $\nu \in \Psi$ and $\Theta \in C(\Delta(\nu), \Phi)$, afterwards a set $F \in \pi GSO(\nu, \Psi)$ exists satisfying $int(\Delta(F)) \subset \Theta$.

Definition 3.11. A function $\Delta : \Psi \to \Phi$ is called as $(\pi gs\text{-}s)$-$open$ whenever $\Delta(F) \in SO(\Phi)$ for all $F \in \pi GSO(\Psi)$.

Theorem 3.13. *Whenever $\Delta : \Psi \to \Phi$ is a weakly contra πgs-continuous and $(\pi gs\text{-}s)$-$open$ and $\pi GSO(\Psi)$ is closed under arbitrary unions, afterwards Δ is contra πgs-continuous.*

Proof. Whenever $\nu \in \Psi$ and $\Theta \in C(\Delta(\nu), \Phi)$, with the weakly contra πgs-continuity of Δ, as a result the set $F \in \pi GSO(\nu, \Psi)$ appears satisfying $int(\Delta(F)) \subset \Theta$. Since Δ is $(\pi gs\text{-}s)$-$open$, $\Delta(F)$ is semi-open in Φ. Hence, $\Delta(F) \subset cl(int(\Delta(F))) \subset cl(\Theta) = \Theta$. □

Definition 3.12. $fr_{\pi gs}(\aleph)$ stands for πgs-frontier of $\aleph \in \Psi$ and characterized as $cl_{\pi gs}(\aleph) \cap cl_{\pi gs}(\Psi \backslash \aleph)$.

Theorem 3.14. *Let $\Delta : \Psi \to \Phi$ be a function. Whenever $\pi GSC(\Psi)$ is closed under arbitrary intersections then, the set of whole points $\nu \in \Psi$ at which Δ is not contra πgs-continuous is equal to $\bigcup\{fr_{\pi gs}(\Delta^{-1}(\Theta)) : \Theta \in C(\Delta(\nu), \Phi)\}$.*

Proof. Let ν be any element of Ψ at which Δ is not contra πgs-continuous. Then, there exists a closed subset Θ of Φ comprising $\Delta(\nu)$ such that $\Delta(F)$ is not contained in Θ for every $F \in \pi GSO(\nu, \Psi)$. So $F \cap (\Psi \backslash \Delta^{-1}(\Theta)) \neq \emptyset$. Then, we have $\nu \in cl_{\pi gs}(\Psi \backslash \Delta^{-1}(\Theta))$. Since $\nu \in \Delta^{-1}(\Theta) \subset cl_{\pi gs}(\Delta^{-1}(\Theta))$, $\nu \in fr_{\pi gs}(\Delta^{-1}(\Theta))$.
For the converse, assume that Δ is contra πgs-continuous at $\nu \in \Psi$ and $\Theta \in C(\Delta(\nu), \Phi)$. Naturally a set $F \in \pi GSO(\nu, \Psi)$ appears satisfying $F \subset \Delta^{-1}(\Theta)$. Therefore, $\nu \in int_{\pi gs}(\Delta^{-1}(\Theta))$. Hence, $\nu \notin fr_{\pi gs}(\Delta^{-1}(\Theta))$. □

Corollary 3.2. *For any $\Delta : \Psi \to \Phi$, whenever $\pi GSC(\Psi)$ is closed under arbitrary intersections, afterwards Δ is not contra πgs-continuous at ν if and only if $\Theta \in C(\Delta(\nu), \Phi)$ appears satisfying $\nu \in fr_{\pi gs}(\Delta^{-1}(\Theta))$.*

4. Preservation theorems

In this section, new separation axioms, connected spaces, compact spaces, covers and graphs related to πgs-open sets are defined and various results are presented by examining the properties of these new concepts.

Definition 4.1. Ψ is said to be πgs-T_1 whenever ν and μ in Ψ are distinct points, sets $F \in \pi GSO(\nu, \Psi)$ and $\mho \in \pi GSO(\mu, \Psi)$ naturally appears satisfying $\mu \notin F$ and $\nu \notin \mho$.

Definition 4.2. Ψ is said to be πgs-T_2 whenever ν and μ in Ψ are distinct points, sets $F \in \pi GSO(\nu, \Psi)$ and $\mho \in \pi GSO(\mu, \Psi)$ naturally appears satisfying $F \cap \mho = \emptyset$.

Theorem 4.1. *Under the assumption $\mho$ is an Uryshon space, whenever ν and μ are distinct points in Ψ a function $\Delta : \Psi \to \Phi$ naturally appears that is contra πgs-continuous at ν and μ and for which $\Delta(\nu) \neq \Delta(\mu)$, afterwards Ψ is πgs-T_2.*

Proof. Assume that ν and μ as distinct points in Ψ. Also, let $\Delta : \Psi \to \Phi$ be contra πgs-continuous at ν and μ such that $\Delta(\nu) \neq \Delta(\mu)$. Letting $\nu' = \Delta(\nu)$ and $\mu' = \Delta(\mu)$ with the knowlegde of Φ is Urysohn, existence of $\partial \in O(\nu', \Phi)$ and $F \in O(\mu', \Phi)$ guaranteed such that $cl(\partial) \cap cl(F) = \emptyset$. Since Δ is contra πgs-continuous at ν and μ, there exist πgs-open subsets $\aleph$ and Ω of Ψ comprising ν and μ, correspondingly, such that $\Delta(\aleph) \subset cl(\partial)$ and $\Delta(\Omega) \subset cl(F)$. Hereby, $\Delta(\aleph \cap \Omega) \subset \Delta(\aleph) \cap \Delta(\Omega) \subset cl(\partial) \cap cl(F) = \emptyset$ which implies that $\aleph \cap \Omega = \emptyset$. Hence, Ψ is πgs-T_2. □

Corollary 4.1. *Whenever $\Delta : \Psi \to \Phi$ is contra πgs-continuous injection and Φ is an Urysohn space, afterwards Ψ is πgs-T_2.*

Definition 4.3. The topological space Ψ is called as,
(ι_1) πgs-connected space $:\Leftrightarrow$ Ψ is not the union of two disjoint non-empty πgs-open sets,
(ι_2) gs-connected space [15] $:\Leftrightarrow$ Ψ is not the union of two disjoint non-empty gs-open sets.

Remark 4.1. Although πgs-connected spaces are gs-connected, the contrary implication is not valid in general.

Example 4.1. Let $\Psi = \{\nu, \mu\}$ and $\top = \{\emptyset, \{\nu\}, \Psi\}$. Ψ is gs-connected, but it is not πgs-connected since $\{\nu\}$ and $\{\mu\}$ are non-empty disjoint πgs-open subsets of Ψ.

Theorem 4.2. *For a topological space Ψ the following are equivalent:*
(ι_1) *Ψ is πgs-connected;*
(ι_2) *The only subsets of Ψ which are both πgs-open and πgs-closed are $\emptyset$ and Ψ;*
(ι_3) *Each πgs-continuous function of Ψ into a discrete space Φ with at least two points is a constant function.*

Proof. Firstly let Ψ be a topological space.
$(\iota_1) \Rightarrow (\iota_2)$ Suppose that $\aleph$ is a proper non-empty subset of Ψ which is both πgs-open and πgs-closed. Then, $\Psi \backslash \aleph$ is a proper non-empty subset of Ψ which is both πgs-open and πgs-closed, $\aleph \cap (\Psi \backslash \aleph) = \emptyset$ and $\aleph \cup (\Psi \backslash \aleph) = \Psi$. But this result contradicts with the πgs-connectedness of Ψ. Hence, the only subsets of Ψ which are both πgs-open and πgs-closed $\emptyset$ and Ψ.
$(\iota_2) \Rightarrow (\iota_1)$ Suppose that Ψ is not πgs-connected. Then as a result two non-empty disjoint πgs-open subsets $\aleph$ and Ω of Ψ appears such that $\aleph \cup \Omega = \Psi$. Since $\aleph = \Psi \backslash \Omega$ and $\Omega = \Psi \backslash \aleph$, $\aleph$ and Ω are proper non-empty subsets of Ψ which are both πgs-open and πgs-closed, but this is a contradiction. Hereby, Ψ is πgs-connected.
$(\iota_2) \Rightarrow (\iota_3)$ Let Φ be any discrete space with at least two elements and $\Delta : \Psi \to \Phi$ be any contra πgs-continuous function. Since Φ is discrete, $\{\mu\}$ is clopen in Φ for each $\mu \in \Phi$. Therefore, $\{\mu\}$ is both πgs-open and πgs-closed in Φ for each $\mu \in \Phi$. We also have $\Psi = \Delta^{-1}(\Phi) = \Delta^{-1}(\bigcup\{\{\mu\} : \mu \in \Phi\}) = \bigcup\{\Delta^{-1}(\{\mu\}) : \mu \in \Phi\}$. By (ι_2), $\Delta^{-1}(\{\mu\}) = \emptyset$ or $\Delta^{-1}(\{\mu\}) = \Psi$ for each $\mu \in \Phi$. If $\Delta^{-1}(\{\mu\}) = \emptyset$ for some $\mu \in \Phi$ then, Δ would not be a function anymore. If there exist at least two distinct elements a and b in Φ such that $\Delta^{-1}(\{a\}) = \Psi = \Delta^{-1}(\{b\})$, then Δ would not be a function anymore. Therefore, there exists only one element μ of Φ such that $\Delta^{-1}(\{\mu\}) = \Psi$, which means that $\Delta(\Psi) = \{\mu\}$. Hence, Δ is a constant function.
$(\iota_3) \Rightarrow (\iota_2)$ Let P be a non-empty set such that $P \in \pi GSO(\Psi) \cap \pi GSC(\Psi)$, Φ be any discrete space with at least two elements and contra πgs-continuous $\Delta : \Psi \to \Phi$ defined as $\Delta(P) = \{\varsigma\}$ and $\Delta(\Psi \backslash P) = \{\eta\}$, for distinct elements ς and η of Φ. Since Δ is constant by (ι_3), $\Psi \backslash P = \emptyset$. Therefore, $P = \Psi$. □

Theorem 4.3. *Let $\Delta : \Psi \to \Phi$ be a surjective contra πgs-continuous function. While Ψ is πgs-connected, Φ cannot be a discrete space.*

Proof. Assume Φ as a discrete space. Let $\aleph$ be any proper non-empty subset of Φ. Since $\aleph$ is clopen in Φ and $\Delta : \Psi \to \Phi$ is contra πgs-continuous surjection, $\Delta^{-1}(\aleph) \in \pi GSO(\Psi) \cap \pi GSC(\Psi)$ is a proper non-empty subset of Ψ. But this result contradicts with the πgs-connectedness of Ψ. Hence, Φ is not a discrete space. □

Theorem 4.4. *While whole contra πgs-continuous functions with a domain Ψ into any T_0 space Φ is constant, Ψ has to be πgs-connected.*

Proof. Assume that Ψ is not πgs-connected. So, at least one proper non-empty subset $\aleph \in \pi GSO(\Psi) \cap \pi GSC(\Psi)$ appears. Let $\Phi = \{\varsigma, \eta\}$ and $\perp = \{\emptyset, \{\varsigma\}, \{\eta\}, \Phi\}$. Let $\Delta : \Psi \to \Phi$ be a function such that $\Delta(\aleph) = \{\varsigma\}$ and $\Delta(\Psi \backslash \aleph) = \{\eta\}$. Then, Φ is a T_0 space and Δ is a contra πgs-continuous function which is not constant. But this is a contradiction. Hereby, Ψ has to be πgs-connected. □

Theorem 4.5. *Whenever $\Delta : \Psi \to \Phi$ is surjective contra πgs-continuous function and Ψ is πgs-connected, afterwards Φ has to be connected.*

Proof. Suppose that Φ as a disconnected space. So two non-empty disjoint open sets $\aleph$ and Ω of Φ appear, so that $\aleph \cup \Omega = \Phi$. So $\Delta^{-1}(\aleph) \neq \emptyset$, $\Delta^{-1}(\Omega) \neq \emptyset$, $\Delta^{-1}(\aleph) \cap \Delta^{-1}(\Omega) = \emptyset$, $\Delta^{-1}(\aleph) \cup \Delta^{-1}(\Omega) = \Psi$ since Δ is surjective. Since Δ is contra πgs-continuous, $\Delta^{-1}(\aleph)$ and $\Delta^{-1}(\Omega)$ are both πgs-open and πgs-closed in Ψ. Therefore, we reach the result that Ψ is not πgs-connected which is a contradiction. Hereby, Φ is connected. □

Theorem 4.6. *The projection functions $p_\Psi : \Psi \times \Phi \to \Psi$ and $p_\Phi : \Psi \times \Phi \to \Phi$ are πgs-irresolute.*

Proof. Let $p_\Psi : \Psi \times \Phi \to \Psi$ be the projection function from $\Psi \times \Phi$ onto Ψ and $\aleph$ be any πgs-closed subset of Ψ. Then, ${p_\Psi}^{-1}(\aleph) = \aleph \times \Phi$. Let $\digamma$ be any π-open subset of $\Psi \times \Phi$ involving $\aleph \times \Phi$. Then, there exists a π-open subset $\mho$ of Ψ involving $\aleph$ such that $\digamma = \mho \times \Phi$. Since $\aleph$ is πgs-closed in Ψ, $scl(\aleph) \subset \mho$. Therefore, $scl(\aleph) \times \Phi \subset \mho \times \Phi = \digamma$. Since $scl(\aleph \times \Phi) \subset scl(\aleph) \times \Phi$, we have $scl(\aleph \times \Phi) \subset \digamma$. So $\aleph \times \Phi = p_\Psi^{-1}(\aleph)$ is πgs-closed in $\Psi \times \Phi$. Hence, projection function $p_\Psi : \Psi \times \Phi \to \Psi$ is πgs-irresolute. The proof for the other projection function $p_\Phi : \Psi \times \Phi \to \Phi$ is similar. □

Theorem 4.7. *Whenever $\Delta : \Psi \to \Phi$ is a πgs-irresolute surjection and Ψ is πgs-connected, afterwards Φ has to be πgs-connected.*

Proof. Assume that Φ is not πgs-connected. Naturally, two non-empty disjoint πgs-open subsets $\digamma$ and Ω of Φ appears so that $\digamma \cup \Omega = \Phi$. Then $\Delta^{-1}(\digamma)$ and $\Delta^{-1}(\Omega)$ are non-empty πgs-open subsets of Ψ, since Δ is surjective and πgs-irresolute. Besides, $\emptyset = \Delta^{-1}(\digamma \cap \Omega) = \Delta^{-1}(\digamma) \cap \Delta^{-1}(\Omega)$ and $\Psi = \Delta^{-1}(\digamma) \cup \Delta^{-1}(\Omega)$. Therefore, we reach the result that Ψ is not πgs-connected which is a contradiction. Hereby, Φ is πgs-connected. □

Theorem 4.8. *Whenever the product space of two non-empty spaces is πgs-connected, each factor space has to be πgs-connected.*

Proof. Accept Ψ and Φ as non-empty topological spaces and the product space $\Psi \times \Phi$ as πgs-connected. Since the projection functions are πgs-irresolute and surjective, by Theorem 4.7, Ψ and Φ are πgs-connected. □

Definition 4.4. A topological space Ψ is called as:
(ι_1) πgs-compact if every πgs-open cover of Ψ has a finite subcover,
(ι_2) countably πgs-compact if every countable cover of Ψ by πgs-open sets has a finite subcover,
(ι_3) πgs-Lindelöf if every πgs-open cover of Ψ has a countable subcover.

Definition 4.5. $\aleph \in \Psi$ is characterized to be πgs-compact relative to Ψ whenever every πgs-open cover of $\aleph$ by πgs-open sets of Ψ has a finite subcover.

Theorem 4.9. *Whenever $\Delta : \Psi \to \Phi$ is contra πgs-continuous and $\aleph \subset \Psi$ is πgs-compact relative to Ψ, afterwards $\Delta(\aleph)$ has to be strongly S-closed.*

Proof. Let $\{\Theta_i : i \in I\}$ be a closed cover of $\Delta(\aleph)$ by closed subsets of the subspace $\Delta(\aleph)$. Then for each $i \in I$, there exits a closed set $\aleph_i$ in Φ such that $\Delta(\aleph) = \bigcup\{\Theta_i : i \in I\} = \bigcup\{\aleph_i \cap \Delta(\aleph) : i \in I\} = (\bigcup\{\aleph_i : i \in I\}) \cap \Delta(\aleph)$ and $\Theta_i = \aleph_i \cap \Delta(\aleph)$. Since for each $\nu \in \aleph$, we have $\Delta(\nu) \in \Delta(\aleph)$ and since Δ is contra πgs-continuous, for each $\nu \in \aleph$ there exists $i(\nu) \in I$ and there exists $F_\nu \in \pi GSO(\nu, \Psi)$ such that $\Delta(\nu) \in \aleph_{i(\nu)}$ and $\Delta(F_\nu) \subset \aleph_{i(\nu)}$. Then, $\{F_\nu : \nu \in \aleph\}$ is a cover of $\aleph$ by πgs-open sets of Ψ. Since $\aleph$ is πgs-compact relative to Ψ, there exists a finite subset $\aleph_0$ of $\aleph$ such that $\aleph \subset \bigcup\{F_\nu : \nu \in \aleph_0\}$. Then, we obtain $\Delta(\aleph) \subset \bigcup\{\aleph_{i(\nu)} : \nu \in \aleph_0\}$. Therefore, $\Delta(\aleph) = \Delta(\aleph) \cap (\bigcup\{\aleph_{i(\nu)} : \nu \in \aleph_0\}) = \bigcup\{\Delta(\aleph) \cap \aleph_{i(\nu)} : \nu \in \aleph_0\} = \bigcup\{\Theta_{i(\nu)} : \nu \in \aleph_0\}$ and this means that $\{\Theta_{i(\nu)} : \nu \in \aleph_0\}$ is a finite subcover of $\{\Theta_i : i \in I\}$. Hence, $\Delta(\aleph)$ is strongly S-closed. □

Corollary 4.2. *Whenever $\Delta : \Psi \to \Phi$ is a contra πgs-continuous surjection and Ψ is πgs-compact, afterwards Φ has to be strongly S-closed.*

Theorem 4.10. *Whenever the product space of two non-empty spaces is πgs-compact, afterwards each factor space has to be πgs-compact.*

Proof. Let $\Psi \times \Phi$ be the product space of the non-empty topological spaces Ψ and Φ and $\Psi \times \Phi$ be πgs-compact. Let $\{\eth_i : i \in I\}$ be any πgs-open cover of Ψ. Then, $\Psi \times \Phi = p_\Psi^{-1}(\Psi) = p_\Psi^{-1}(\bigcup\{\eth_i : i \in I\}) = \bigcup\{p_\Psi^{-1}(\eth_i) : i \in I\}$. Since p_Ψ is πgs-irresolute, $p_\Psi^{-1}(\eth_i) = \eth_i \times \Phi$ is πgs-open in $\Psi \times \Phi$ for each $i \in I$. Therefore, $\{\eth_i \times \Phi : i \in I\}$ is a πgs-open cover of $\Psi \times \Phi$. Since $\Psi \times \Phi$ is πgs-compact, there exists a finite subset I_0 of I such that $\bigcup\{\eth_i \times \Phi : i \in I_0\} = \Psi \times \Phi$. Then, $\Psi = p_\Psi(\Psi \times \Phi) = p_\Psi(\bigcup\{\eth_i \times \Phi : i \in I_0\}) = p_\Psi((\bigcup\{\eth_i : i \in I_0\}) \times \Phi) = \bigcup\{\eth_i : i \in I_0\}$. Hence, Ψ is πgs-compact. The proof for the space Φ is similar. □

Theorem 4.11. *Contra πgs-continuous images of πgs-Lindelöf (correspondingly countably πgs-compact) spaces are strongly S-Lindelöf (correspondingly strongly countably S-closed).*

Proof. Let Ψ be a πgs-Lindelöf space and $\Delta : \Psi \to \Phi$ be a surjective contra πgs-continuous function. Let $\{\Theta_i : i \in I\}$ be a closed cover of Φ. Since Δ is contra πgs-continuous, $\{\Delta^{-1}(\Theta_i) : i \in I\}$ is a πgs-open cover of Ψ. Since Ψ is πgs-Lindelöf, there exists a countable subset I_0 of I such that $\bigcup\{\Delta^{-1}(\Theta_i) : i \in I_0\} = \Psi$. Since Δ is surjective, $\Phi = \Delta(\Psi) = \Delta(\bigcup\{\Delta^{-1}(\Theta_i) : i \in I_0\}) = \bigcup\{\Delta(\Delta^{-1}(\Theta_i)) : i \in I_0\} = \bigcup\{\Theta_i : i \in I_0\}$ and then $\Phi = \bigcup\{\Theta_i : i \in I_0\}$. Hence, Φ is strongly S-Lindelöf. The proof for the contra πgs-continuous images of countably πgs-compact spaces is similar. □

Definition 4.6. The graph $G(\Delta)$ of $\Delta : \Psi \to \Phi$ is said to be a contra πgs-graph if for each (ν, μ) in $(\Psi \times \Phi)\backslash G(\Delta)$, there exist a set $\aleph$ in $\pi GSO(\nu, \Psi)$ and a set Ω in $C(\mu, \Phi)$ such that $(\aleph \times \Omega) \cap G(\Delta) = \emptyset$.

Theorem 4.12. *The following are equivalent for the graph $G(\Delta)$ of any $\Delta : \Psi \to \Phi$.*
(ι_1) *$G(\Delta)$ is contra πgs-graph;*
(ι_2) *For all $(\nu, \mu) \in (\Psi \times \Phi)\backslash G(\Delta)$, there exist a πgs-open set $\aleph \subset \Psi$ comprising ν and a closed set $\Omega \subset \Phi$ comprising μ such that $\Delta(\aleph) \cap \Omega = \emptyset$.*

Theorem 4.13. *Whenever $\Delta : \Psi \to \Phi$ is contra πgs-continuous and Φ is an Uryshon space, afterwards $G(\Delta)$ has to be a contra πgs-graph.*

Proof. For all $(\nu, \mu) \in (\Psi \times \Phi)\backslash G(\Delta)$, it is clear that $\Delta(\nu) \neq \mu$. Since Φ is Uryshon space, there exist open sets $\eth_\nu$ and $\eth_\mu$ in Φ comprising $\Delta(\nu)$ and μ, correspondingly, such that $cl(\eth_\nu) \cap cl(\eth_\mu) = \emptyset$. Since Δ is contra πgs-continuous, a $\aleph \in \pi GSO(\nu, \Psi)$ appears so that $\Delta(\aleph) \subset cl(\eth_\nu)$. Then, $\Delta(\aleph) \cap cl(\eth_\mu) = \emptyset$. Hereby, $G(\Delta)$ is contra πgs-graph. □

Theorem 4.14. *Let $\Delta : \Psi \to \Phi$ be a function and $\rho : \Psi \to \Psi \times \Phi$ be the graph function of Δ defined as $\rho(\nu) = (\nu, \Delta(\nu))$ for every $\nu \in \Psi$. If ρ is contra πgs-continuous, then Δ is contra πgs-continuous.*

Proof. For all open set $\digamma \subset \Phi$, it is clear that $\Psi \times \digamma$ is open in $\Psi \times \Phi$. Since ρ is a contra πgs-continuous function, $\Delta^{-1}(\digamma) = \rho^{-1}(\Psi \times \digamma)$ is πgs-closed in Ψ. Hence, Δ is contra πgs-continuous. □

Theorem 4.15. *Let $\Delta : \Psi \to \Phi$ and $\rho : \Psi \to \Phi$ be two contra πgs-continuous functions. If Φ is an Uryshon space and $\pi GSO(\Psi)$ is closed under finite intersections then, the set $E = \{\nu \in \Psi : \Delta(\nu) = \rho(\nu)\}$ is πgs-closed in Ψ.*

Proof. If we show that "$\nu \notin E \Rightarrow \nu \notin cl_{\pi gs}(E)$", then the theorem will be proved. Let $\nu \in \Psi \backslash E$. Then, $\Delta(\nu) \neq \rho(\nu)$. Since Φ is Uryshon, there exist open subsets $\digamma$ and $\mho$ of Φ comprising $\Delta(\nu)$ and $\rho(\nu)$, correspondingly, such that $cl(\digamma) \cap cl(\mho) = \emptyset$. Since Δ and ρ are contra πgs-continuous, $\Delta^{-1}(cl(\digamma))$ and $\rho^{-1}(cl(\mho))$ are πgs-open in Ψ. Let $\Delta^{-1}(cl(\digamma)) = \eth_1$ and $\rho^{-1}(cl(\mho)) = \eth_2$. Then, $\nu \in \eth_1 \cap \eth_2$. Let $\aleph = \eth_1 \cap \eth_2$. Since $\pi GSO(\Psi)$ is closed under finite intersections, $\aleph$ is a πgs-open set in Ψ comprising ν. So, $\Delta(\aleph) \cap \rho(\aleph) = \emptyset$. Hence, $\aleph \cap E = \emptyset$. By Lemma 2.1, $\nu \notin cl_{\pi gs}(E)$. □

Definition 4.7. For a subset $\aleph$ of space Ψ, if $cl_{\pi gs}(\aleph) = \Psi$ then $\aleph$ is said to be πgs-dense in Ψ.

Theorem 4.16. *Let $\Delta : \Psi \to \Phi$ and $\rho : \Psi \to \Phi$ be two functions. If*
(ι_1) Φ is an Uryshon space and $\pi GSO(\Psi)$ is closed under finite intersections,
(ι_2) Δ and ρ are contra πgs-continuous,
(ι_3) $\Delta = \rho$ on a πgs-dense subset $\aleph$ of Ψ,
then $\Delta = \rho$ on Ψ.

Proof. By Theorem 4.15, the set $E = \{\nu \in \Psi : \Delta(\nu) = \rho(\nu)\}$ is πgs-closed in Ψ. Since $\Delta = \rho$ on a πgs-dense subset $\aleph$, we have $\aleph \subset E$. Then, $\Psi = cl_{\pi gs}(\aleph) \subset cl_{\pi gs}(E) = E$. Hence, $E = \Psi$. □

Definition 4.8. Ψ is characterized to be weakly Hausdorff [49] if each element of Ψ is an intersection of regular closed sets.

Theorem 4.17. *Let $\Delta : \Psi \to \Phi$ be an injective contra πgs-continuous function. If Φ is weakly Hausdorff then, Ψ is πgs-T_1.*

Proof. Let ν and μ be any two elements in Ψ such that $\nu \neq \mu$. Since Δ is injective, $\Delta(\nu) \neq \Delta(\mu)$. Since Φ is weakly Hausdorff, regular closed subsets Θ_1 and Θ_2 of Φ comprising $\Delta(\nu)$ and $\Delta(\mu)$, correspondingly, appears such that $\Delta(\nu) \notin \Theta_2$ and $\Delta(\mu) \notin \Theta_1$. Since regular closed sets are closed and Δ is contra πgs-continuous, $\Delta^{-1}(\Theta_1)$ and $\Delta^{-1}(\Theta_2)$ are πgs-open subsets of Ψ comprising ν and μ, correspondingly, such that $\mu \notin \Delta^{-1}(\Theta_1)$ and $\nu \notin \Delta^{-1}(\Theta_2)$. Hence, Ψ is πgs-T_1. □

Theorem 4.18. *If $\Delta : \Psi \to \Phi$ is an injective function whose graph $G(\Delta)$ is contra πgs-graph then, Ψ is πgs-T_1.*

Proof. Let ν and μ be any two elements in Ψ such that $\nu \neq \mu$. Since Δ is injective, $(\nu, \Delta(\mu)) \in (\Psi \times \Phi)\backslash G(\Delta)$. Since $G(\Delta)$ is contra πgs-graph, there exists a πgs-open subset $\eth$ of Ψ and a closed subset Θ of Φ comprising ν and $\Delta(\mu)$, correspondingly, such that $\Delta(\eth) \cap \Theta = \emptyset$. Then $\Delta^{-1}(\Theta) \cap \eth = \emptyset$ and $\mu \notin \eth$. Similarly, since $(\Delta(\nu), \mu) \in (\Psi \times \Phi)\backslash G(\Delta)$, there exists a πgs-open subset Ω of Ψ comprising μ such that $\nu \notin \Omega$. Hence, Ψ is πgs-T_1. □

Theorem 4.19. *Let $\Delta : \Psi \to \Phi$ be an injective contra πgs-continuous function. Whenever Φ is an ultra Hausdorff space, Ψ has to be πgs-T_2.*

Proof. Let ν and μ be any two elements in Ψ such that $\nu \neq \mu$. Since Δ is injective, $\Delta(\nu) \neq \Delta(\mu)$. Since Φ is an ultra Hausdorff space, there exist disjoint clopen subsets $\eth_1$ and $\eth_2$ of Φ comprising $\Delta(\nu)$ and $\Delta(\mu)$, correspondingly. Then, $\Delta^{-1}(\eth_1)$ and $\Delta^{-1}(\eth_2)$ are disjoint subsets of Ψ comprising ν and μ, correspondingly, which are both πgs-open and πgs-closed in Ψ since Δ is contra πgs-continuous. Hence, Ψ is πgs-T_2. □

Definition 4.9. A space Ψ is said to be πgs-normal if each pair of non-empty disjoint closed sets can be seperated by disjoint πgs-open sets.

Theorem 4.20. *Let $\Delta : \Psi \to \Phi$ be an injective closed contra πgs-continuous function. If Φ is ultra normal, then Ψ is πgs-normal.*

Proof. Let Θ_1 and Θ_2 be any two non-empty disjoint closed subsets of Ψ. Since Δ is injective and closed, $\Delta(\Theta_1)$ and $\Delta(\Theta_2)$ are non-empty disjoint closed subsets of Φ. Since Φ is ultra normal, there exist disjoint clopen subsets $\eth_1$ and $\eth_2$ of Φ such that $\Delta(\Theta_1) \subset \eth_1$ and $\Delta(\Theta_2) \subset \eth_2$. Since Δ is contra πgs-continuous, $\Delta^{-1}(\eth_1)$ and $\Delta^{-1}(\eth_2)$ are disjoint πgs-open subsets of Ψ such that $\Theta_1 \subset \Delta^{-1}(\eth_1)$ and $\Theta_2 \subset \Delta^{-1}(\eth_2)$. Hence, Ψ is πgs-normal. □

5. Conclusion

It is understood from the studies of many researchers on contra continuity, which is one of the types of continuity that has been frequently studied recently as in the past, still arouses curiosity today. Researchers have not only examined various properties of the different types of contra continuous functions they have identified, but also examined the relationships between different contra continuities. In this study, we not only share the concept of contra πgs-continuity [8] related with πgs-open sets defined by Çaksu [4], but also investigated various properties of contra πgs-continuous functions and examined the relationships between different contra continuities. Remark 3.2 clearly shows that the concept of contra πgs-continuity is weaker than the concepts of contra πg-continuity [7], contra gs-continuity [9], contra g-continuity [39], contra semicontinuity [9], contra super continuity [38], contra continuity [6], strong contra continuity [37], perfect continuity [35] and RC continuity [9]. We also obtained important results by examining various properties related to separation axioms, connectedness, compactness, cover and graph concepts. We believe that our study will shed light on the studies researchers interested in contra continuous functions.

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