

Stability of Adaptive Cruise Control of Automated Vehicle Platoon under Constant Time Headway Policy

Duc Lich Luu^{1*} , Thanh Long Phan¹ , Huu Truyen Pham^{2*} , Hoang Thang¹  and Minh Tien Le¹ 

¹Faculty of Transportation Mechanical Engineering, the University of Danang - University of Science and Technology, 54 Nguyen Luong Bang, 550000, Da Nang, Vietnam.

²Vinh University of Technology Education, 117 Nguyen Viet Xuan Street, Vinh City, 43000, Nghe An Province, Vietnam.

Abstract

Traffic congestion is becoming more prevalent as the number of vehicles on the roads continues to rise. To shorten travel times and enhance driver comfort, a range of Advanced Driver Assistance Systems (ADAS) has been developed to assist drivers in urban areas and on highways. The demand for increasing road capacity has introduced the concept of vehicle platooning, where the use of the Adaptive Cruise Control (ACC) system is a key function within the advanced driver assistance (ADAS) technology, this technology manages the vehicle's longitudinal control in specific driving conditions. Cars equipped with ACC can efficiently maintain a set distance from the vehicle ahead, easing the driver's workload while offering advantages such as improved road capacity, lower fuel consumption, and reduced pollution emissions. However, they can be susceptible to string instability, resulting in the amplification of oscillations caused by speed variations along the platoon's rear. This paper presents a string stability analysis of car platoons equipped with ACC system based on a heuristic method by choosing of the constant time headway policy (CTHP). The constant time headway policy selection for string stability is based on the Nyquist diagram of the transfer function of the spacing errors between two cars. A platoon operated by using distance-based ACC control structures is implemented. These structures employ a linear quadratic regulator (LQR) using a dual integrator. The simulation results were acquired by modeling and simulating the studied platoon within Matlab/Simulink.

Keywords: String Stability; ADAS; Linear Quadratic Integral Regulator; Platooning; Adaptive Cruise Control;

Research Article

History

Received 17.04.2024
Revised 10.06.2024
Accepted 16.07.2024

Contact

* Corresponding author:
Duc Lich Luu
ldlich@dut.udn.vn
Address: University of
Danang - University of Sci-
ence and Technology, Da
Nang, Vietnam.
Tel: (+84.236) 3736945/
Huu Truyen Pham
phtruyen2019@gmail.com
Address: Vinh University of
Technology Education,
Nghe An, Vietnam
Tel: (+84. 0238) 3842753

To cite this paper: Luu, D.L., Phan, T.L., Pham, H.T., Thang, H., Le, M.T. Stability of Adaptive Cruise Control of Automated Vehicle Platoon under Constant Time Headway Policy. International Journal of Automotive Science and Technology. 2024; 8 (3): 397-403. <https://doi.org/10.30939/ijastech.1469674>

1. Introduction

A prominent issue in today's society is the escalating number of vehicles leading to congested roads and heightened environmental pollution. Numerous solutions are being researched and developed worldwide to address these challenges faced by humanity. One solution that can be used is the Intelligent Transport System (ITS) [1]. Automated Vehicle Control System (AVCS) plays a pivotal role in autonomous vehicles and is a fundamental component of Intelligent Vehicle Systems [2], [3]. All cars equipped with ACC system into a platoon are close to each other, can also be seen as an AVCS designed to increase the capacity of roads, even in the presence of uncertainties in the system and the environment over extended periods [4]. This involves in utilizing multi sensors, a micro-controller, and suitable software to process sensor data and generate the necessary system output to follow the car in front in a safe manner [5]. The ACC systems have evolved into advanced features within the automobile industry [6]. ACC systems have been applied since the mid of the

twentieth century in many companies such as Chrysler, Audi, BMW, etc.

Car platooning is a highly effective approach that not only boosts traffic throughput but also lowers fuel consumption significantly [7], [8]. When establishing a car platoon, there are two primary properties to aim for: internal stability and string stability. Internal stability refers to the stability of each individual car, describing its ability to converge to the desired trajectories [9]. String stability is mentioned in [10], [11], [12], consistently emphasizes that disturbances must not amplify as they propagate along the string, i.e during acceleration, deceleration, or any other disturbances affecting the cars ahead, distance errors are not amplified as they propagate along the line. This reduces traffic congestion and increases the benefits of platooning [13], [14], this is essential for car platoons.

It is well-known that the spacing policy, which defines the distance between two consecutive cars, may affect a platoon's ability to attenuate disturbances and achieve string stability. Two significant classes of spacing policies exist: the constant

spacing policy [15], [16] and the constant time headway spacing Policy [6], [17], [18]. In the constant spacing policy, the desired inter-car distance remains constant. In contrast, the constant time headway spacing policy uses a linear function of velocity, with a proportional gain called the time headway (h_i), to dictate the desired inter car spacing. There are also more sophisticated spacing policies in the literature, such as the delay-based spacing policy [19] and nonlinear spacing policies [20].

String stability with constant spacing policy has been guaranteed only when information from the lead vehicle is communicated to all other vehicles in the platoon. However, this communication requirement becomes burdensome, especially as the length of the platoon increases [21].

Moreover, string instability will inevitably occur when each vehicle only has information from a finite number of vehicles ahead [22]. So, the constant time headway spacing policy is considered in this paper, the inter-car spacing increases with velocity, which can lead to a reduction in traffic capacity. For that reason a small value for the time headway is necessary to maximize road capacity. However, excessively small time headways could be dangerous for the platoon, since it could lead to internal and/or string instability and, hence, a collision. Therefore, determining the minimum acceptable time headway (h_i) that ensures string stability while optimizing road capacity is crucial in the context of platooning. To the best of the authors' knowledge, the impacts of the time headway on the string stability of a platoon of ACC-equipped cars have been largely neglected in the aforementioned works.

The purpose of this paper is to systematically examine the effects of the time headway. To this end, a platoon of ACC-equipped cars is proposed considering the influencing factor above. This is demonstrated through illustrative examples in the numerical evaluations. The main contributions of this paper are listed below:

- (i) Development of LQR using a dual integrator for homogeneous ACC car platoon. Tracking performance is achieved by incorporating a dual integrator into the control law, ensuring the rejection of parabolic disturbances generated by the front car's position.
- (ii) String stability and the minimum time headway (h_i) by using a method through Nyquist diagrams of spacing errors transfer functions to ensure string stability.
- (iii) Finally, considering a homogeneous platoon with a leader and n followers, the string stability is tested by simulation.

The rest of this brief is organized as follows: In Section 2, we first consider the mathematical modeling. Then the ACC algorithm implementation is presented in Section 3. The string stability analysis of the platoon will be described in Section 4. In Section 5, simulation check is implemented and in Section 6, conclusions will close this study.

2. Mathematical Modeling

The desired traction force can easily be obtained by using a lookup table method and appropriately commanding the throttle

angle or fuel injection rate. It is considered that the friction force of the tires does not pose a limiting factor. Thus, It is assumed that the traction force may be directly manipulated. The balance of forces acting on the car's longitudinal axis is given [23], [24]:

$$m\dot{v} = F_x - F_R \quad (1)$$

where, m indicates the car mass, v is the forward velocity, F_x is the actuation force produced by the engine and drivetrain, and resistance forces are defined as:

$$F_R = \frac{1}{2} \rho_v C_v A_v (v + v_w)^2 + mg \sin \alpha + C_r mg \cos \alpha \quad (2)$$

with g the gravity constant, α the road slope, ρ_v the air density, A_v the frontal car area, C_v the air drag coefficient, v_w the wind speed, C_r the modified resistance coefficient to take account of cornering resistances.

The Eq.1 may be linearized by taking into account a nominal operating condition, as $\frac{dv}{dt} = 0$. At equilibrium, Eq. (1) is solved as described in [23], [24] and the solution is repeated here by Eq. (3):

$$F_{x0} = \frac{1}{2} \rho_v A_v C_v (v_0 + v_w)^2 + C_r mg \cos \alpha_0 + mg \sin \alpha_0 \quad (3)$$

where F_{x0} can be determined by assuming appropriate values for v_0 , α_0 , m , C_r , A_v , C_v , v_w , ρ_v . By linearizing Eq.1 in relation to the specified operation state utilising a Taylor series, the relation in Eq. (4) is derived:

$$\tau_v \dot{\bar{v}} + \bar{v} = K_v (\bar{F}_x + \Gamma) \quad (4)$$

where the variables subject to increment or disturbance, as well as the model parameters, are specified as follows (see in [23], [24]):

$$\begin{aligned} v &= \bar{v} + v_0; F_x = \bar{F}_x + F_{x0}; \alpha = \bar{\alpha} + \alpha_0 \\ \tau_v &= m / (\rho_v A_v C_v (v_0 + v_w)^2) \\ K_v &= 1 / \rho_v A_v C_v (v_0 + v_w)^2 \\ \Gamma &= mg(C_r \sin \alpha_0 - \cos \alpha_0) \alpha \end{aligned} \quad (5)$$

The transfer function of the car model may be expressed by taking the Laplace transforms of Eq. (4) as follows:

$$G(s) = \frac{v}{F_x} = \frac{K_v}{\tau_v s + 1} \quad (6)$$

Eq. 4 captures the relation between the forward velocity and the tractive force. Γ can be ignored for a flat road (no inclination).

The car model depicted in Figure 2 is built on the basis of the linearized equations and may be represented by a transfer function.

3. ACC Algorithm Implementation

In this paper, the car platoon under consideration consists of one lead car designed by a cruise control system (CC), and multiple follower cars installed with ACC systems, as shown in Figure 1. The ACC systems enable the followers to stay a desired distance to the preceding car based on their own speed. The reference of the ACC control system in close loop is determined by the desired spacing, $D_{i,des}$, which is defined employing the CTHP.

$$D_{i,des} = d_0 + h_i v_i \quad (7)$$

where d_0 , h_i , v_i are denoted the spacing to be kept at zero speed, the time headway, the actual speed of the car i .

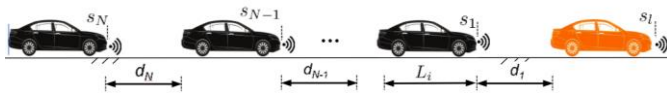


Fig. 1. Conceptual illustration of a N cars platoon

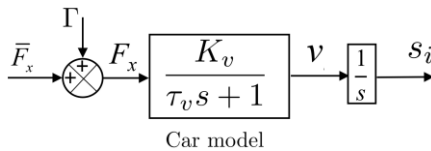


Fig. 2. Block diagram of linearized car longitudinal dynamics

Taking the measured distance into account D_i , which is obtained from the devices, and the desired distance $D_{i,des}$, the spacing error for the controller based on the desired spacing and the actual spacing can be expressed as follows [15]:

$$\delta_i = D_{i,des} - D_i = D_{i,des} - (s_{i-1} - s_i - L_i) \quad (8)$$

with s_i , s_{i-1} the longitudinal position of the car, i , $i-1$ respectively, L_i is the car length.

The extended state space equations can be derived by employing the model defined by Eq. (6), then adding the actual distance and the two integrators in the error channel:

$$\begin{cases} \dot{x}_1 = \dot{d}_i = v_{i-1} - v_i \\ \dot{x}_2 = \dot{v}_i = -(1/\tau_v)v_i + (K_v/\tau_v)F_{xi} \\ \dot{x}_3 = \delta_i \\ \dot{x}_4 = x_3 \\ y_i = d_i \end{cases} \Rightarrow \begin{cases} \dot{x}_i = A_i x_i + B_i F_{xi} \\ y_i = C_i^T x_i \end{cases} \quad (9)$$

in which, x_1 is actual spacing between two cars, x_2 is the velocity of car, x_3 is the integral of the error and x_4 is the double integral of the error, which $x_i \in \mathbb{R}^4$ are the system state. $A_i \in \mathbb{R}^{4 \times 4}$, $B_i \in \mathbb{R}^{4 \times 1}$, $C_i \in \mathbb{R}^{1 \times 4}$ are system matrices. Two integrators are added to the error channel of the control system to compensate for the disturbance caused by the location s_{i-1} of the

front car, which is not directly measured. This helps in maintaining the desired distance for two consecutive cars in the spacing-based ACC system.

where: F_{xi} is the control command. The inclusion of the two integrators in the car dynamics, as shown in Eq. (10), leads to the development of the ACC car. For this system, the feedback LQR control law with a dual integrator was found:

$$F_{xi} = -K_i x_i = k_1 v_i + k_2 D_i + k_3 \int \delta_i(\tau) d\tau + k_4 \int \left(\int \delta_i(\tau) d\tau \right) d\tau \quad (10)$$

where the coefficients k_1 to k_4 are the elements of the gain matrix K_i obtained after the optimization of the quadratic cost function (11).

$$J(x_i, F_{xi}) = \int_0^\infty (x_i^T Q_i x_i + F_{xi}^T R_i F_{xi}) d\tau \quad (11)$$

where $Q_i = Q_i^T \geq 0$ and $R_i = R_i^T \geq 0$ are performance weights. To use the LQR, two extra weight matrices have to be defined. They are defined in Eq. (12) and Eq. (13) where q_1 , q_2 , q_3 and r_i are user-selected constants.

$$Q_i = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{bmatrix} \quad (12)$$

$$R_i = [r_i] \quad (13)$$

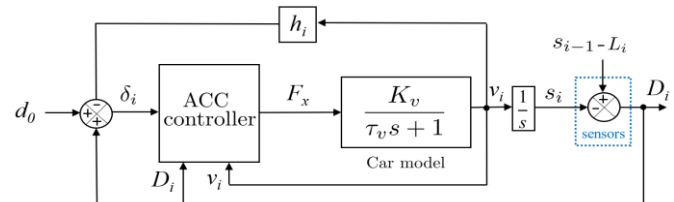


Fig. 3. Spacing-based ACC system for car configuration

4. String Stability Analysis of platoon

To determine the stability of an ACC car platoon seen on Figure 3, it is necessary to develop a method that can assess whether the string is stable or unstable. The proposed approach for string stability analysis, as presented in [10] and [15], relies on the transfer function of the spacing error for two cars.

$$H_i(s) = \frac{\Delta_i(s)}{\Delta_{i-1}(s)} \quad (14)$$

Let $\Delta_i(s)$ be the Laplace transform of $\delta_i(t)$.

According to the definition, the string stability condition in the platoon may be expressed in the frequency domain as follows:

$$\|H_i(j\omega)\|_{\infty} \leq 1 \quad (15)$$

here, $\|H_i(j\omega)\|_{\infty} = \max |H_i(j\omega)|$.

The condition from Eq.15 ensures that $\|\delta_{i-1}\|_2 \geq \|\delta_i\|_2$ which implies that the distance errors reduce successively as they propagate from one car to another within the platoon, eventually converging at the last car. Simultaneously, this observation serves as evidence that the errors do not amplify in the upstream direction [10].

To conduct the string stability analysis, the heuristic approach proposed by [10] is employed. This method involves in selecting various values for the CTHP h_i and examining whether the condition specified in Eq.15 is satisfied or unsatisfied. Beginning from 0, the CTHP h_i is systematically increased in ascending order, and the corresponding Nyquist diagrams $H_i(j\omega)$ are made. For the diagram created based on Eq.15, it is necessary to consider its complex domain form: $\|H_i(j\omega)\|_{\infty} \leq 1$ where $\omega \geq 0$. When the Nyquist diagrams are contained within the unit radius circle centered at the origin of the coordinate axes, Eq.15 is satisfied, indicating that the string is stable for a specific h_i value. Of interest is the minimum value of h_i at which the Nyquist diagram lies entirely within the unit radius circle.

Given the CTHP, the spacing error may be determined utilizing Eq.8.

$$\delta_i = d_0 + h_i v_i - D_i \quad (16)$$

The time derivative of the Eq.16 is:

$$\begin{aligned} \dot{\delta}_i &= h_i \dot{v}_i - v_{i-1} + v_i \Rightarrow v_i = \dot{\delta}_i + v_{i-1} - h_i \dot{v}_i \\ \ddot{\delta}_i &= h_i \ddot{v}_i - \dot{v}_{i-1} + \dot{v}_i \Rightarrow \dot{v}_i = \ddot{\delta}_i + \dot{v}_{i-1} - h_i \ddot{v}_i \\ \ddot{\delta}_i &= h_i \ddot{v}_i - \ddot{v}_{i-1} + \ddot{v}_i \Rightarrow \ddot{v}_i = \ddot{\delta}_i + \ddot{v}_{i-1} - h_i \ddot{v}_i \end{aligned} \quad (17)$$

Bring the equation Eq.17 into the Eq.9 leads to:

$$\tau_v \ddot{\delta}_i + \dot{\delta}_i = K_v (h_i \dot{T}_{xi} - T_{x(i-1)} + T_{xi}) \quad (18)$$

After a calculation, it results:

$$\begin{aligned} \tau_v \ddot{\delta}_i + (1 - K_v k_{3d} \ddot{\delta}_i + K_v (k_2 - h_i k_3) \ddot{\delta}_i - K_v (k_3 + k_4 h_i) \dot{\delta}_i \\ - K_v k_4 \dot{\delta}_i = K_v k_2 \ddot{\delta}_{i-1} - K_v k_3 \dot{\delta}_{i-1} - K_v k_4 \dot{\delta}_{i-1} \end{aligned} \quad (19)$$

Finally, by applying the Laplace transform to Eq.19 with zero initial conditions, we obtain the transfer function describing the tracking errors:

$$H_i(s) = \frac{\Delta_i(s)}{\Delta_{i-1}(s)} = \frac{K_v k_2 s^2 - K_v k_3 s - K_v k_4}{\tau_v s^4 + (1 - K_v k_3) s^3 + K_v (k_2 - h_i k_3) s^2 - K_v (k_3 + k_4 h_i) s - K_v k_4} \quad (20)$$

5. Simulation Check

String stability analysis was conducted through simulation using Matlab/Simulink. The platoon setup consisted of a lead car

controlled by a CC functions, and 10 followers controlled by ACC functions.

All cars within the platoon are identical in terms of their construction and dynamics. The parameters for each car in a homogeneous platoon are shown in Table 1.

Table 1. The vehicle parameter values [23], Q_i and R_i matrices

Symbol	Value	Unit
m	1000	[kg]
v_0	25	[m/s]
v_{ω}	2	[m/s]
α	5	[°]
g	9.81	[m/s]
ρ_v	1.202	[kg/m ³]
A_v	1.5	[m ²]
C_v	0.5	[-]
C_r	0.015	[-]
d_0	2	[m]
L_i	5	[m]
q_i	0.1	[-]
R_i	0.001	[-]

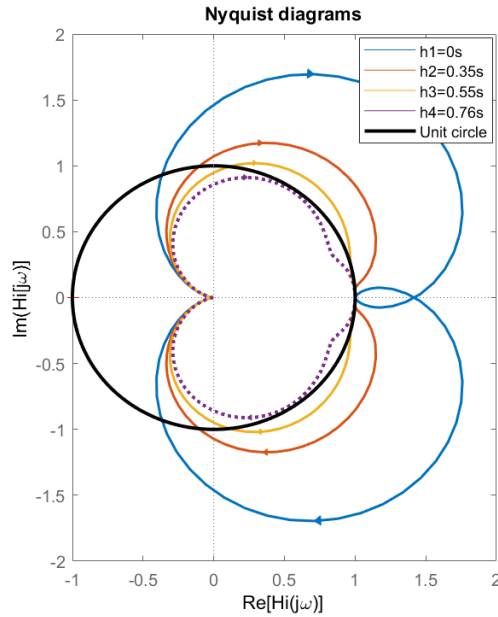
The matrix K_i is derived by minimizing the cost function in Eq.11, leading to:

$$H_i(s) := \frac{371.40s^2 + 294.10s + 102.00}{[75.60s^4 + 237.50s^3 + (294.16 + 371.40h_i)s^2 + (294.10 + 120.00h_i)s + 102.00]} \quad (21)$$

String stability analysis of the spacing errors for two consecutive cars, as described by Eq.21, based on the string stability conditions from inequality Eq.15. This conditions are examined by plotting the Nyquist diagrams, as shown in Figure 4, by the transfer function $H_i(s)$, the h_i is systematically increased in ascending order to a value that satisfies the stability condition.

We can see that the time headway is increased with different values $h_i = [0s; 0.35s; 0.55s]$, none of the Nyquist diagrams fulfills the stability condition as the representation of $H_i(s)$ in the complex domain is not contained within the unit circle, suggesting that the approach of maintaining these values does not lead to a stable string. Increasing the time headway to $h_i = 0.75s$, the calculated value satisfying the string stability condition from Eq.15 is observed in Figure 4, with the Nyquist diagram situated inside the unit circle.

A comparative analysis of the string stability of a platoon using ACC structures was performed through simulations with various speed profiles. The simulation results showcase the effectiveness and advantages of the proposed method. The speed reference of leader car employed in this simulation study is the driving cycle shown in Figure 5.

Fig. 4. String stability plots for different values of headway time h_i

With h_i of $0.35s$, the signals obtained through simulation are illustrated in the next figures as follows:

Figure 6(a) depicts both the platoon traveling distance and the car locations, Figure 6(b) contains the car velocities with 10 followers, Figure 6(c) illustrates the inter-car spacing between successive pairs of cars, Figure 6(d) depicts the spacing errors as

the difference between the actual spacing and the desired inter-car spacing.

In the case of h_i of $0.75s$ with LQR controllers for 10 followers, the signals resulted after simulation is performed as follows:

In Figure 7(a), the path of the cars' movement, indicated by their positions, along with the total traveling distance, can be observed. The car velocities are shown in Figure 7(b), where it can be observed that they follow the velocity profile set by the leader's velocity reference, Figure 7(c) displays the spacings between successive pairs of cars during their movement, the spacing errors as being the difference between the real spacing and the spacing reference is depicted in Figure 7(c).

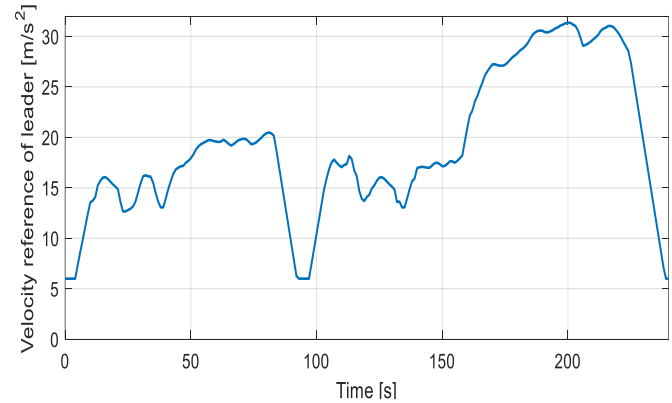


Fig. 5. The velocity profiles

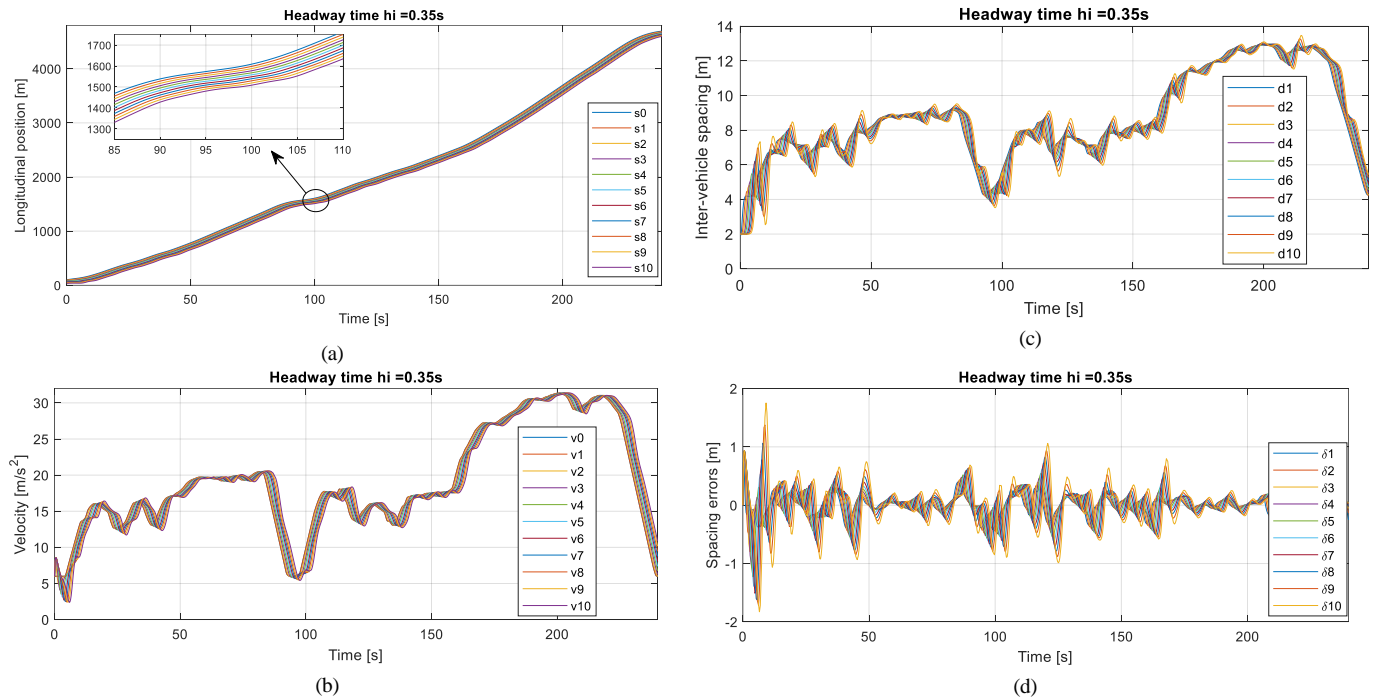


Fig. 6. Simulation result with headway time $h_i = 0.35s$: (a) Longitudinal position of the cars; (b) The velocity; (c) Inter-car distance; (d): Spacing errors

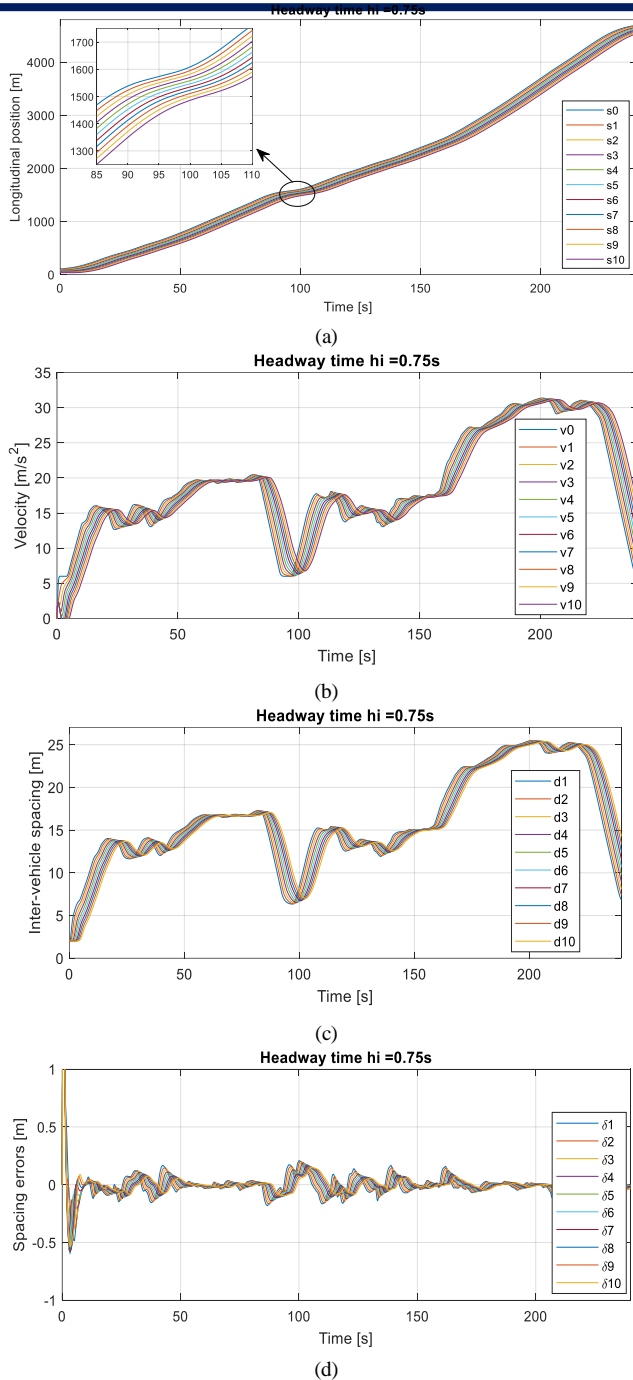


Fig. 7. Simulation result with headway time $h_i = 0.75s$:
 (a) Longitudinal position of the cars; (b) The velocity;
 (c) Inter-car distance; (d): Spacing errors

For h_i of $0.35s$, the driving behaviour of the leader car consists of continuous acceleration and deceleration. In Figure 6(c), it is apparent that the car platoon cannot stay an inter-car distance to the car in front. This instability is notable due to the amplification of spacing errors, as shown in Figure 6(c).

In both cases, it can be seen that the velocity of followers follow the velocity profile introduced as the leader's velocity reference.

The inter-car distances obtained with h_i of $0.75s$ for the platoon structures are shown in Figure 7(c). It is clearly shown that the car platoon under the ACC strategy successfully achieves the desired spacing and maintains the desired velocity of the leading car. The stability of the platoon configuration with h_i of $0.75s$ is reinforced by the fact that the spacing errors shown in Figure 7(d) decrease from the leader car to the last car in the platoon.

We can find that the spacing errors propagating the platoon upstream are not amplified, which is also in good agreement with the results of the previous stability analysis.

As the value of h_i increases, the Nyquist diagram becomes more enclosed within the circle, indicating better stability. However, a higher time headway leads to a larger distance desired for the platoon, resulting in increased distance between cars in a platoon. To achieve stable car platoons with the shortest possible length, it's important to determine the smallest value of that ensures string stability.

Therefore, Figure 7 generally reflects that h_i of $0.75s$ can conduct safe and stable longitudinal control under CTHP Strategy which is also in good agreement with the results of the previous stability analysis.

6. Conclusions

In this paper, the design of a Linear Quadratic Integral Regulator problem is studied for a homogenous platoon of ACC cars with the controller's two integrators ensure tracking performance by rejecting the parabolic disturbances caused by the front car's position, which the system can guarantee the desired tracking performance and string stability requirement. To ensure string stability, a method based on the transfer functions of the spacing errors was proposed. This method guarantees string stability by selecting CTHP using the Nyquist diagram. This method ensures that the Nyquist diagram contains the circle of unit radius centered at the origin of the coordinate axis. The minimum time headway is determined to ensure string stability while optimizing road capacity. A comparative analysis of the string stability of a platoon using ACC structures was performed through simulations with various speed profiles. The simulation results show the effectiveness and advantages of the proposed method.

As a further study, it is worth studying how ACC affects the traffic flow in a simulation environment.

Conflict of Interest Statement

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this work.

CRediT Author Statement

Duc Lich Luu: Validation, writing – review and editing;

Thanh Long Phan: Formal analysis;

Huu Truyen Pham: Writing – original draft preparation;

Hoang Thang: Methodology;

Minh Tien Le: Conceptualization. All authors have read and agreed to the published version of the manuscript.

References

- [1] Vegamoor VK, Darbha S, Rajagopal KR. A review of automatic vehicle following systems. *Journal of the Indian Institute of Science*. 2019; 99:567-587. <https://doi.org/10.1007/s41745-019-00143-7>
- [2] Claussmann L, Revilloud M, Gruyer D, Glaser S. A review of motion planning for highway autonomous driving. *IEEE Transactions on Intelligent Transportation Systems*. 2019;21(5):1826-1848. <https://doi.org/10.1109/TITS.2019.2913998>
- [3] Iancu DT, Mihai NA, GHITA SA, Florea AM. Trajectory Prediction Using Video Generation in Autonomous Driving. *Studies in Informatics and Control*. 2022;31(1):37-48. <https://doi.org/10.24846/v31i1y202204>
- [4] Kabasakal B, Üçüncü M. The design and simulation of adaptive cruise control system. *International Journal of Automotive Science And Technology*. 2022;6(3):242-256. <https://doi.org/10.30939/ijastech..1038371>
- [5] Iancu DT, Florea AM. An improved vehicle trajectory prediction model based on video generation. *Studies in Informatics and Control*. 2023;32(1):25-36. <https://doi.org/10.24846/v32i1y202303>
- [6] Adıgüzel F. Doğrusal karesel regülatör ve ileri beslemeli kontrol yöntemi ile otonom araçlar için kooperatif uyarlamalı hız kontrol sistemi. *Politeknik Dergisi*. 2023;27(2):3-12. <https://doi.org/10.2339/politeknik.1170311>
- [7] Ma K, Wang H, Ruan T. Analysis of road capacity and pollutant emissions: Impacts of connected and automated vehicle platoons on traffic flow. *Physica A: Statistical Mechanics and its Applications*. 2021;583:126301. <https://doi.org/10.1016/j.physa.2021.126301>
- [8] Guo G, Yuan WL. Bidirectional platoon control of Arduino cars with actuator saturation and time-varying delay. *Journal of Control Engineering and Applied Informatics*. 2017;19(1):37-48.
- [9] Luu DL, Lupu C. Vehicle string using spacing strategies for cooperative adaptive cruise control system. *UPB Sci. Bull., Series C*. 2021;83(1):91-106.
- [10] Swaroop DV. String stability of interconnected systems: An application to platooning in automated highway systems. *University of California, Berkeley*; 1994.
- [11] Hu SG, Wen HY, Xu L, Fu H. Stability of platoon of adaptive cruise control vehicles with time delay. *Transportation Letters*. 2019;11(9):506-515. <https://doi.org/10.1080/19427867.2017.1407488>
- [12] Xiao L, Gao F. Practical string stability of platoon of adaptive cruise control vehicles. *IEEE Transactions on intelligent transportation systems*. 2011; 12(4):1184-1194. <https://doi.org/10.1109/TITS.2011.2143407>
- [13] Tiganasu A, Lazar C, Caruntu CF, Dosoftei C. Comparative analysis of advanced cooperative adaptive cruise control algorithms for vehicular cyber physical systems. *Journal of Control Engineering and Applied Informatics*. 2021;23(1):82-92.
- [14] Öncü S, Ploeg J, Van de Wouw N, Nijmeijer H. Cooperative adaptive cruise control: Network-aware analysis of string stability. *IEEE Transactions on Intelligent Transportation Systems*. 2014;15(4):1527-1537. <https://doi.org/10.1109/TITS.2014.2302816>
- [15] Luu DL, Lupu C, Alshareefi H. A Comparative Study of Adaptive Cruise Control System based on Different Spacing Strategies. *Journal of Control Engineering and Applied Informatics*. 2022;24(2):3-12.
- [16] Guo XG, Wang JL, Liao F, Teo RS. String stability of heterogeneous leader-following vehicle platoons based on constant spacing policy. In 2016 IEEE Intelligent Vehicles Symposium, 2016, pp. 761-766. <https://doi.org/10.1109/IVS.2016.7535473>
- [17] Yuan H, Liu R, Zhong L, Zhang Y, Lin L, Huang K. Investigation on multi-objective following control algorithm for vehicle adaptive cruise control under cruise state. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*. 2024;238(5):2-16. <https://doi.org/10.1177/09544070241238298>
- [18] Gülden B, Emirler MT. Investigation of Different Communication Topologies for Cooperative Adaptive Cruise Control Systems. *International Journal of Automotive Science And Technology*. 2024;8(1):150-158. <https://doi.org/10.30939/ijastech..1368820>
- [19] Besselink B, Johansson KH. String stability and a delay-based spacing policy for vehicle platoons subject to disturbances. *IEEE Transactions on Automatic Control*. 2017;62(9):4376-4391. <https://doi.org/10.1109/TAC.2017.2682421>
- [20] Hung NV, Luu DL, Dong NS, Lupu C. Reducing time headway for cooperative vehicle following in platoon via information flow topology. *Journal of Control Engineering and Applied Informatics*. 2024;26(2):77-87. <https://doi.org/10.61416/ceai.v26i2.8834>
- [21] Darbha S, Konduri S, Pagilla PR. Vehicle platooning with constant spacing strategies and multiple vehicle look ahead information. *IET Intelligent Transport Systems*. 2020;14(6):589-600. <https://doi.org/10.1049/iet-its.2019.0204>
- [22] Konduri S, Pagilla PR, Darbha S. Vehicle platooning with multiple vehicle look-ahead information. *IFAC-PapersOnLine*. 2017;50(1):5768-5773. <https://doi.org/10.1016/j.ifacol.2017.08.415>
- [23] Ulsoy AG, Peng H, Çakmakci M. *Automotive control systems*. Cambridge University Press; 2012.
- [24] Ibrahim A, Goswami D, Li H, Soroa IM, Basten T. Multi-layer multi-rate model predictive control for vehicle platooning under IEEE 802.11 p. *Transportation Research Part C: Emerging Technologies*. 2021;124:102905. <https://doi.org/10.1016/j.trc.2020.102905>