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Alternative Strategies to Hedge Longevity Risk in Defined Contribution Pension Plans for Loss-Averse Individuals

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Abstract

This study aims to determine the optimal investment strategy in defined contribution pension plans by extending beyond the traditional expected utility maximisation framework, and explicitly modelling investor behaviour under loss aversion to capture more realistic decision-making dynamics. While determining the optimal investment strategy for loss-averse individuals, the longevity risk arising from the decrease in mortality probabilities observed all over the world in recent years should also be considered. Accordingly, the optimal investment strategy for loss-averse individuals is derived by incorporating longevity risk into the model and employing stochastic dynamic programming as the optimisation technique. The results indicate that a loss-averse individual should follow a more aggressive investment strategy in the accumulation period to hedge longevity risk during the distribution period. Given that loss-averse individuals are generally reluctant to engage in riskier investment strategies, this study explores alternative methods to hedge longevity risk. From the results obtained, it is concluded that determining the appropriate contribution rate and the appropriate minimum fund guarantee for the loss-averse individuals reduces the risk in the optimal investment strategy. These findings underscore the importance of tailoring investment strategies in defined contribution pension plans to align with individual risk preferences and the financial challenges posed by increasing life expectancy.

Keywords Defined contribution pension plan · Loss aversion · Optimal investment strategy · Longevity risk

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Alternative Strategies to Hedge Longevity Risk in Defined Contribution Pension Plans for Loss-Averse Individuals

Pension plans are contracts constituted to ensure that their participants receive regular pension payments when they retire. The pension plans regulate the rights and obligations of the participants, how the contributions will be evaluated, and on what basis the pension payments will be made in detail. The purpose of the pension plans is to ensure that people continue their welfare as at least in their active working life when they retire and to prevent them from being dependent on others in their last years by providing a reasonable standard of living (Blake,1999).

Pension plans are divided into defined benefit (DB) pension plans and defined contribution (DC) pension plans.

DB pension plans are plans in which retirement income is calculated using a formula, although there is no certainty in advance. In these plans, which are generally common in the public sector, the participant usually contributes at a predetermined rate, while the contribution of the plan sponsor depends on the return on investment. Therefore, the investment risk is assumed by the plan sponsor (Aitken, 1996).

Recently, the transition from DB pension plans to DC pension plans has become quite common, and DC pension plans play an important role in the social security system. When the literature is examined, it is realised that studies on DC pension plans have increased considerably in recent years.

DC pension plans are plans in which the amount of contribution to be made by the participant and plan sponsor each year is predetermined. The participants' savings at retirement are a function of the contributions made during the savings period, the period of time to contribute, and the return on investment (Aitken, 1996). DC pension plans have two periods: the accumulation period, in which the contributions are made, and the distribution period, in which the fund generated as a result of the contributions is taken as retirement income. In these plans, unlike the DB pension plans, the participant has to bear the investment risk, and it is very important to determine the optimal investment strategy.

When the studies on the optimal investment strategy in DC pension plans are examined in the literature, it is seen that these studies date back to very old years. The first studies on this subject belong to Merton (1969, 1975) and Samuelson (1975), and there are still studies on determining the optimal investment strategy in DC pension plans. Boulier et al. (2001), Vigna and Haberman (2001), Blake et al. (2001), Cairns et al. (2006), Battocchio et al. (2007), Yang and Huang (2009), Emms (2012), Blake et al. (2013), and Blake et al. (2014) are the main papers in which the optimal investment strategy is studied in DC pension plans.

Generally, the classical approach of maximising expected utility function is used while determining the optimal investment strategy in DC pension plans. However, Rabin and Thailer (2001) stated that the expected benefit criterion is not suitable for most risk behaviours. Maximisation of expected utility does not reflect the real world well, especially when the individual is loss-averse, and most investors are actually loss-averse individuals. For this reason, it is crucial to determine the optimal investment strategy for loss-averse individuals.

Loss aversion is defined by the loss or gain in the asset relative to a predefined reference point or income, rather than the change in the absolute value of the total asset. The concept of loss aversion was first defined by Kahneman and Tversky (1979) in the prospect theory, which is the cornerstone of behavioural finance. Prospect theory is a theory that claims that the motivation that determines the behaviour of the individual is the expectations because of this behaviour. According to this theory, losses affect investors more emotionally than gains. In other words, losses are more important to investors than gains.

Some studies in which the optimal investment strategy has been identified for loss-averse individuals in DC pension plans are Berkelaar et al. (2004), Gomes (2005), and Blake et al. (2013). In these studies, instead of maximising the expected utility, prospect theory which allows minimising the difference between the targeted fund and the actual fund size is utilized. Minimising the difference between the targeted fund and the actual fund size is not a new idea in the literature. Vigna and Haberman (2001,2002) determined the optimal investment strategy to minimise the discounted sum of cost functions, which is defined as the square of the difference between the terminal targeted fund size and the actual fund size. However, in this study, positive deviations from the target are penalised as well as negative deviations. Since a positive deviation from the target fund size is desirable, it is necessary to determine the optimal strategy so that only negative deviations are penalised. Therefore, the optimal investment strategy should be determined under the framework of loss aversion.

In many studies in which the optimal investment strategy has been determined in DC pension plans, longevity risk has not been taken into account. However, the individual may wish to receive the fund amount at the end of the savings period as a whole-life annuity, as in many DC pension plans. In this case, the individual should determine the optimal investment strategy by considering the longevity risk. Yang and Huang (2009), Donnely (2014), Yao et al. (2014), Yao et al. (2016), Huang and Milevsky (2016), Harlow and Brown (2016), and De Kort and Vellekoop (2017) are some studies that have addressed the longevity risk.

Considering the longevity risk, an individual should follow a more aggressive investment strategy in the accumulation period to hedge longevity risk during the distribution period. Since the loss-averse individuals will not prefer a riskier investment strategy, alternative ways have been explored to the longevity risk hedging. In this context, the motivation of the study is to explore alternative ways to the aggressive investment strategy that is not desirable to hedge longevity risk. Since determining the appropriate contribution rate and the appropriate minimum guarantee reduces the risk in the investment strategy, it is hypothesised that these alternative ways could be determining the optimal investment strategy with the appropriate contribution rate and the appropriate minimum fund guarantee amount.

Determining the optimal investment strategy is extremely important for the participant in the DC pension plans. Another important decision is the determination of the contribution rate that is; how much of his salary will be invested in the DC pension plan. The more the participant contributes to the pension plan, the greater the benefit from the plan. On the other hand, transferring a higher percentage of earnings to the pension plan will decrease the participant's standard of living. For this reason, it is critical to determine the appropriate contribution rate for the DC pension plans. Delong et al. (2008), Hainaut and Deelstra (2011), Owadally et al. (2013), Guan and Liang (2014), and Blake et al. (2014) are some of the studies that determine the optimal investment strategy as well as the optimal contribution in DC pension plans. In these studies, the optimal values were determined to maximise the expected utility of the individual.

While determining the appropriate contribution amount, it is necessary to set a constraint regarding the actual fund size. In DC pension plans, the minimum guarantee can be requested by the participant, and if the participant is a loss-averse individual, the concept of the minimum guarantee becomes more important. The concept of guarantee in DC pension plans was first introduced by Deelstra et al. (2003). Guarantee means that the accumulated pension fund at the end of the period is at least equal to a predetermined amount. This guarantee given to the participant provides a kind of derivative product protection and is essential in terms of risk hedging. Deelstra et al. (2003, 2004) contributed to obtaining the optimal investment strategy in the case of minimum guarantees in continuous time in certain pension plans. Di Giacinto et al. (2011), Di

Giacinto et al. (2014) and Basimanebotlhe and Xue (2015) are other studies on the minimum fund guarantee in pension funds. In these studies, the optimal investment strategy is determined by taking the minimum guarantee amount into account.

In summary, our study is different from previous studies in three important aspects:

• The optimal investment strategy in DC pension plans for a loss-averse individual is determined with the longevity risk.

• To hedge longevity risk, an optimal investment strategy is obtained with an appropriate contribution rate for loss-averse individuals in DC pension plans.

• The minimum guarantee constraint is added while the optimal investment strategy and appropriate contribution rate for loss-averse individuals are obtained together.

The rest of study is organised as follows. Section 2 introduces the model to be used under longevity risk and gives the assumptions made in the model. The parameter values used are given and the results are obtained and interpreted in Section 3. Section 4 obtains the optimal investment strategy with the appropriate contribution rate. The minimum fund guarantee constraint is included in the model, and the results with the appropriate contribution rate and minimum fund guarantee are given in Section 5. In the final section of the study, which is Section 6, the conclusions and recommendations are presented.

Model under Longevity Risk

Determining Targeted and Actual Fund Sizes

This section describes the assumptions and the model used in solving the optimisation problem. In this study, we have based the model of Blake et al. (2013). The assumptions of this model are as follows:

- The fund is invested in two assets, one with a high-risk and the other with a risk-free asset.
- Evaluation is done annually. This study is practised in separate time.
- All members enter the system at the age of 20 and retire at the age of 65.
- The targeted fund size at the end of the period is determined by considering the 2/3 replacement ratio.

In this study, since the effect of the decrease in mortality rates will be examined, studies in the literature that take the longevity risk in DC pension plans into account are investigated. One of these studies is Yang and Huang (2009), in which the optimal investment strategy and contribution rate in the accumulation period are determined in a way to longevity risk hedging in the distribution period. They minimised the second moment of the difference between the actual value of the pension fund and its target value. They concluded that either a more aggressive investment strategy or a higher contribution is appropriate for hedging longevity risk. In our study, in order to examine the effect of the longevity risk on the optimal investment strategy for a loss-averse individual, longevity risk is included in the model, as in Yang and Huang (2009). The results were obtained using different mortality models, namely base and projected mortality rates.

In this study, it is assumed that the base mortality probabilities follow the PMA92 table. Given a retirement age of 65 years and a risk-free return rate of r=0.02, the value of a whole-life annuity is calculated as follows:

$$\ddot{a_{65}} = \Sigma_{s=0}^{120-65} {}_s p_{65} \exp(-rs) = 15.8382 \tag{1}$$

Considering the impact of declining mortality rates in many countries, a reduction factor was used to model the expected future mortality probabilities. To calculate the projected mortality probabilities, the first step is to select the mortality table from which the base mortality probabilities will be derived. A reduction factor is then applied to these probabilities.

Let q_x represent the mortality probability based on the base table, and let $RF_{x,t}$, denote the projected reduction factor based on a 20-year time span for an individual aged *x* at time *t*. Accordingly, the probability that an individual aged *x* at time *t* will die within one year is calculated as follows:

$$q_{x,t} = q_x{}^* RF_{x,t} \tag{2}$$

The reduction factor, determined on the basis of the data observed between 1991 and 1994, as reported in the Continuous Mortality Investigation Reports (CMIR, 1999) published in the United Kingdom, is as follows:

$$RF_{x,t} = \alpha(x) + [1-\alpha(x)]^*[1-f(x)]^{t/20} \tag{3}$$

where

$$\begin{aligned} \alpha(x) &= c; x \leq 60 \\ &= 1 + (1-c)\frac{x-110}{50}; 60 \leq x < 110 \\ &= 1; x \geq 110 \end{aligned} \tag{4}$$

and

$$\begin{aligned} f(x) &= h; x \leq 60 \\ &= \frac{(110-x)h + (x-60)k}{50}; 60 \leq x < 110 \\ &= k; x \geq 110 \end{aligned} \tag{5}$$

In this study, the parameter values *c*=0.13, *h*=0.55, *t*=20 and *k*=0.29 were used, as in the Continuous Mortality Investigation Reports published in the United Kingdom. The projected mortality probabilities, obtained using the baseline mortality rates from the PMA92 table along with these parameter values, are presented in Table 1.

Table 1

Base and Projected Mortality Rates

Age	Base Mortality Rates	Projected Mortality Rates
20	0.000188	0.000098042
30	0.000184	0.000095956
40	0.000245	0.000127768
50	0.000729	0.000380174
60	0.003277	0.001708956
70	0.016213	0.010593444
80	0.059223	0.045435175
90	0.156976	0.135452707
100	0.303666	0.285595444
110	0.444014	0.444014
120	1	1

The value of the whole-life annuity, calculated for a retirement age of 65 using the projected mortality probabilities provided in Table 1 and a risk-free return rate of *r*=0.02, is as follows:

$$\ddot{a}_{65} = \Sigma_{s=0}^{120-65} {}_s p_{65} \exp(-rs) = 17.2194 \tag{6}$$

After the annuity values are determined, the targeted and actual fund sizes will be determined. The targeted fund size at the end of the accumulation period is obtained as follows:

$$f(65) = \frac{2}{3} {*} Y_{65} \tag{7}$$

where the annuity value is 15.838 for the base mortality rates and 17.2124 for the projected mortality rates.

The targeted fund sizes for the interim period are obtained recursively using the value of *f(65)*

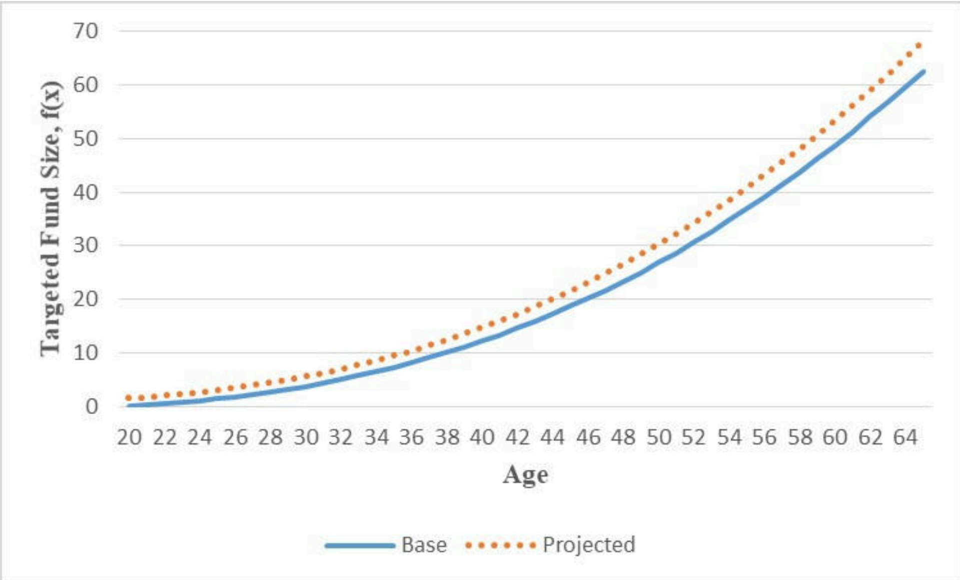
$$f(s) = \frac{f(s+1)}{exp(d)} - \pi {*} Y_s;\, s = 64, 63, \ldots 20 \tag{8}$$

where *d* represents the discount rate used when determining the interim fund targets.

The graph of the targeted fund sizes calculated from Eq. 2.7 and Eq. 2.8 for the base and projected mortality rates is given in Figure 1.

Figure 1

Targeted fund size with base and projected mortality rates



It can be seen from Figure 1 that improvement in mortality rates increases the targeted fund size. After determining the targeted fund sizes, the actual fund sizes are obtained at any time in the accumulation period. Let

θ_{x-1}: the ratio of the fund evaluated in the high-risk investment instrument at the age of *x-1*,

Y_x: income at age *x*,

π: the constant contribution rate,

r: the annual return of the risk-free asset,

R_x: the annual return of the high-risk asset during the age of *x* to *x+1*,

$R_x = r + \left(\mu - \frac{1}{2}\sigma^2\right) + \sigma Z_x$: the annual return of the high-risk asset during the age of *x* to *x+1*, where

μ: the annual risk premium of the high-risk asset,

σ: the annual standard deviation of the high-risk asset,

$\{Z_x\}$: independent and identically random variables with the standard normal distribution

Then F_x, which is the actual fund value at the age of *x*, is calculated recursively starting from the value of $F_{20}=0$ as follows:

$$F_x = (F_{x-1} + \pi Y_{x-1}) \exp\left[r + \theta_{x-1}\left(\left(\mu - \frac{1}{2}\sigma^2\right) + \sigma Z_x\right)\right]; x = 21, 22, \ldots, 65 \tag{9}$$

Income

In this study, the model of Cairns et al. (2006) is used without stochastic shocks. l_x represents the rate of growth in earnings and is defined as:

$$l_x = r_1 + \frac{S_x - S_{x-1}}{S_{x-1}} \tag{10}$$

where r_1 is the average annual growth rate of the national average earnings. S_x is the career salary profile at age x and is defined as

$$S_x = 1 + h_1\left[-1 + \frac{x-20}{45}\right] + h_2[-1 + \frac{4(x-20)}{45} - \left\{\frac{\sqrt{3}(x-20)}{45}\right\}^2] \tag{11}$$

where h_1=-0.1865 and h_2=0.7537. These parameter estimations have been reported by Cairns et al. (2006) from the 2005 Annual Survey of Hours and Earning data by the UK male salary, using the least squares method.

Thus, starting from Y_{20}=1, the expected income of the member aged x is obtained recursively as

$$Y_x = Y_{x-1} \exp(l_x) \ ; \ x=21,22,\dots,65 \tag{12}$$

Determining the Objective Function

Using the prospect theory, the utility function for each age from employment to retirement is defined as

$$U_x(F_x) = \frac{(F_x - f(x))^{v_1}}{v_1}; F_x \geq f(x)$$

$$= -\lambda \frac{(f(x) - F_x)^{v_2}}{v_2}; F_x < f(x) \tag{13}$$

where F_x is the actual fund size, $f(x)$ is the targeted fund size, v_1 is the curvature parameter for the gain, v_2 is the curvature parameter for the losses, and λ is the loss aversion ratio.

Since reaching the final fund target is more important than reaching the interim fund targets for the participant, a lower coefficient w<1 is applied to the interim fund targets. Hence, the sum of the discounted utility function from age x to retirement is:

$$V_x = [\Sigma_{s=0}^{65-x-1} \beta^s w U_{x+s}(F_{x+s})] + \beta^{65-x} U_{65}(F_{65}) = wU_x(F_x) + \beta V_{x+1} \tag{14}$$

where β is the participant discount factor.

Optimisation

The individual aims to maximise the expected value of the sum of the discounted value of the utility function up to retirement. The optimal equity rate θ_x for all ages x=20,21,…,64 can be obtained from the solution of the following problem:

$$\max_{\theta_x} E_x(V_x) \overset{\max}{\underset{\theta_x}{=}} E_x[\left\{\sum_{s=0}^{65-x-1} \beta^s w U_{x+s}(F_{x+s})\right\} + \beta^{65-x} U_{65}(F_{65})] \tag{15}$$

subject to

- $Y_x = Y_{x-1} \exp(l_x)$
- $F_x = (F_{x-1} + \pi Y_{x-1}) \exp\left[r + \theta_{x-1}\left(\left(\mu - \frac{1}{2}\sigma^2\right) + \sigma Z_x\right)\right] \geq 0; x = 21, 22, \dots, 65$
- $0 \leq \theta_x \leq 1$

Obtaining Optimal Investment Strategy Using Base and Projected Mortality Rates

For the solution of the optimisation problem given in Eq. 2.15, the θ_x values maximising $E_x(V_x)$ should be obtained for each age from 20 years old, which is the beginning of the accumulation period, to 64 years, which is the end of the accumulation period. In the solution to this optimisation problem, the stochastic dynamic programming method is used because it determines the optimal investment strategy separately for each age. The structure of the problem is recursively decomposed into interrelated sub-problems as in Blake et al. (2013).

While obtaining the values of the utility function, the parameter values in Blake et al. (2013) based on an experiment conducted on a group of 25 graduate students, Tversky and Kahneman (1992) are used and they are given in Table 2. In this study, neither parameter estimation nor sensitivity analysis is conducted; instead, previously established parameters are utilized to ensure consistency in the results and to make comparisons with other studies.

Table 2

Parameter values

Loss Aversion Parameter		**Income Parameter**	
Loss aversion ratio λ	4.5	r_1	0.02
Curvature parameter for gain v_1	0.44	σ_1	0.05
Curvature parameter for loss v_2	0.88	h_1	−0.1865
		h_2	0.7537
Asset Returns		**Other Parameters**	
Risk-free rate r	0.02	Contribution rate π	0.15
Equity risk premium μ	0.04	Weights for the interim targets w	0.5
Volatility of equity returns σ	0.18	c	0.13
Discount factor β	0.96	h	0.55
		k	0.29

In order to solve the problem with the stochastic dynamic programming method, the expected value expression $E_{64}(V_{64})$ should be made distinct. For this purpose, the actual fund sizes at the age of 64 F_{64}, are divided into 201 parts between the possible value range [0,200] and the results are obtained for each value. The expected value is determined by the Gauss-Hermite quadrature method. After determining the expected value, it is seen that the expression $E_{64}(V_{64})$ is a function that only depends on F_{64} an θ_{64}. For each value in the [0,200] range, θ_{64} values that maximise the objective function are obtained. By using these θ_{64} values obtained, θ_{63} values that maximise $E_{63}(V_{63})$ value are found. This process continues recursively until the age of 20 to obtain θ_{20} values that maximise the $E_{20}(V_{20})$ value. For more detailed information about solving the optimisation problem, readers can refer to Blake et al. (2013).

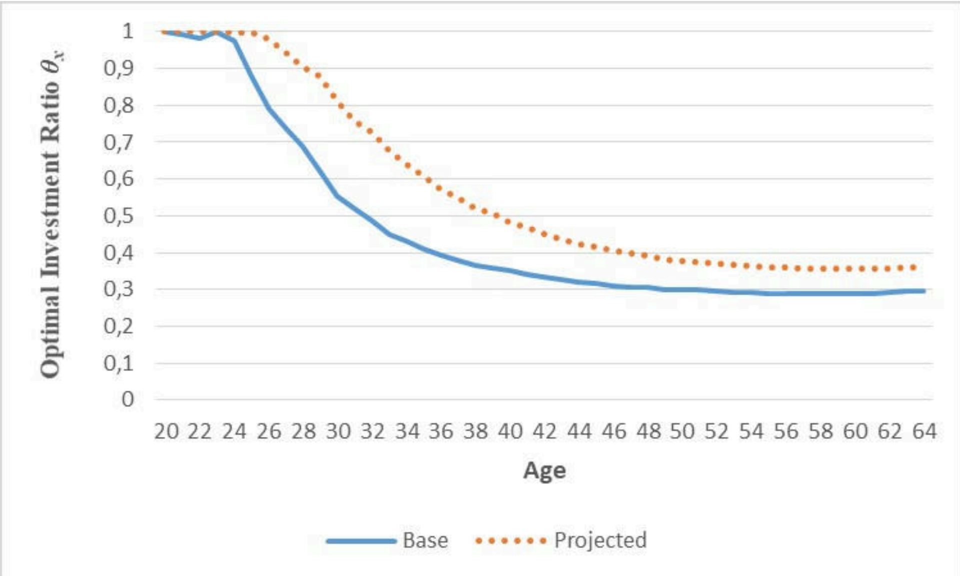
θx values, which are the optimal investment ratios for all ages, are obtained for 201 values corresponding to each possible actual fund value in the range of [0,200]. In order to obtain a single optimal investment rate for each age, the actual fund sizes at all ages must be known. While determining the actual fund size, the return of the risky asset, that is, the θ_x values that are tried to be optimised are needed. As priori information about the θ_x values, the actual fund sizes are determined by using the θ_x values obtained in Kırkağaç and Gençtürk (2016).

Since the optimal investment ratios are obtained only at 201 points in the range of [0,200], the optimal ratios for non-integer values of the fund size are found by the interpolation method.

Later, 100000 simulations are generated for the actual fund size as possible scenarios. An optimal investment ratio is found for each fund size obtained in these scenarios, and the average of these investment ratios is designated as the final optimal investment ratio. The optimal investment ratio for the loss-averse individuals determined by using the base and projected mortality rates from the age of 20 to the age of 64 are shown in Figure 2.

Figure 2

Optimal investment strategy with base and projected mortality rates



It is observed from Figure 2 that for the base and projected mortality rates, the ratio of the fund evaluated in the high-risk investment asset takes the value of 1 at the beginning of the maturity and gradually decreases towards the end of the maturity. The strategy obtained in both models is actually similar to the traditional lifestyle strategy that has been accepted and widely used in many retirement plans. According to this strategy, at the beginning of the accumulation period, the ratio of the fund evaluated in the high-risk investment instrument takes the value of 1, that is, the entire fund is valued in the high-risk investment instrument. As the maturity progresses, the ratio of the fund evaluated in the high-risk investment instrument decreases while the ratio evaluated in the low-risk investment instrument increases. At the end of the accumulation period, a large part of the fund is valued in low-risk investment instruments.

It can also be seen from Figure 2 that the strategy obtained when using projected mortality rates involving longevity risk is similar to the strategy obtained when using base mortality rates. However, as expected, when projected mortality rates are used, it is seen that both the rate of the fund evaluated in high-risk investment asset and the number of years in which the fund evaluated in high-risk investment asset are higher. In other words, the optimal investment strategy becomes more aggressive and riskier when projected mortality rates are used. Accordingly, a more aggressive investment strategy should be followed in the accumulation period in order to hedge longevity risk in the distribution period.

Due to the fact that an aggressive investment strategy is not desired by the loss-averse individuals, alternative ways to the aggressive investment strategy, which are determining the optimal investment strategy with the appropriate contribution rate and determining optimal investment strategy with minimum fund guarantee constraint, will be sought in the following parts of the study.

Determining the Optimal Investment Strategy with the Appropriate Contribution Rate

It is extremely important for the participant to determine the optimal investment strategy in the DC pension plans. In addition, another important decision for the participant is to find out the amount of his salary to the pension plan, that is, the contribution rate. The more the participant contributes to the pension plan, the greater the benefit from the plan. On the other hand, transferring a higher percentage of earnings to the pension plan will lead to a decrease in the participant's standard of living. For this reason, it is critical to determine the appropriate contribution rate for the DC pension plans.

The appropriate contribution rate for the loss-averse individuals is determined by considering the contribution rate that yields the strategy that maximises the discounted sum of the prospect theory utility function with the least risk. In order to determine the appropriate contribution rate, the optimal investment strategy for each rate is obtained by increasing the contribution rate from 0 to 1 by 1%. Optimal investment strategies obtained with different contribution rates from the age of 20 to the age of 64 for base mortality rates are summarised in Table 3 and Figure 3.

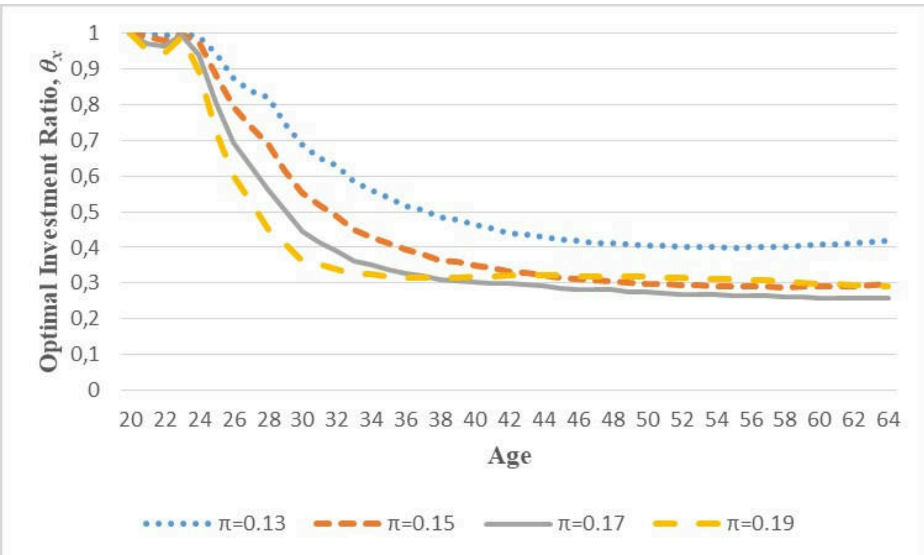
Table 3

Optimal investment ratios with different contribution rates

Age	π=0.13	π=0.14	π=0.15	π=0.16	π=0.17	π=0.18	π=0.19	π=0.20
20	1	1	1	1	**1**	1	1	1
25	0.9414	0.9810	0.8810	0.8450	**0.8059**	0.7645	0.7214	0.6776
30	0.6884	0.7842	0.5525	0.4958	**0.4449**	0.3996	0.3643	0.3387
35	0.5417	0.5646	0.4118	0.3686	**0.3372**	0.3239	0.3198	0.3284
40	0.4641	0.4563	0.3497	0.3182	**0.3014**	0.3031	0.3167	0.3417
45	0.4233	0.3938	0.3164	0.2928	**0.2853**	0.2963	0.3204	0.3518
50	0.4046	0.3610	0.2977	0.2780	**0.2746**	0.2897	0.3170	0.3546
55	0.3995	0.3463	0.2901	0.2673	**0.2662**	0.2804	0.3101	0.3496
60	0.4070	0.3423	0.2899	0.2659	**0.2592**	0.2715	0.2988	0.3362
64	0.4186	0.3478	0.2959	0.2652	**0.2560**	0.2645	0.2893	0.3255

Figure 3

Optimal investment strategies with different contribution rates



It can be concluded from Table 3 and Figure 3 that the increase in the contribution rate makes the optimal investment strategy more conservative up to a point and more aggressive after the point. The point where this change occurs is the point where the contribution rate is taken as π=0.17.

The targeted fund size at the end of the period calculated from Eq. 2.7 is not dependent on the contribution rate. The targeted fund sizes for the interim periods calculated from Eq. 2.8 are taken as π=0.15 so that they do not change with regard to the contribution rate. In this way, it is ensured that the end-of-period and targeted fund sizes are the same for all contribution rates. However, the actual fund sizes calculated from Eq. 2.9 vary depending on the contribution rate. Considering that the participant is a loss-averse individual, it can be said that the investment strategy that takes the least risk with the same targeted fund size is suitable for the participant because the participant will not want to take more risk while maximising the expected benefit with the same target fund size.

When Table 3 and Figure 3 are examined in this direction, it is seen that the investment strategy in which the individual takes the least risk occurs when the contribution rate is π=0.17.

From Table 3 and Figure 3, it is seen that the investment strategy obtained is less risky when the contribution rate up to a certain age is taken as π=0.19. On the other hand, when the contribution rate is taken as π=0.17, it is seen that the number of years with less risk is higher. Therefore, it is concluded that the optimal investment strategy for the participant is the one corresponding to the situation where the appropriate contribution rate is π=0.17.

It has been stated in the previous sections that this aggressive investment strategy is an undesirable situation for the loss-averse individuals. Loss-averse individuals want to implement an investment strategy that maximises the expected value of the discounted sum of the prospect theory utility functions by taking as little risk as possible. Depending on the result that determining the appropriate contribution rate reduces the risk in the optimal investment strategy, the appropriate contribution amount is taken as π=0.17. Later, the results obtained with this contribution rate both π=0.17 and π=0.15 are shown in Table 4 and Figure 4.

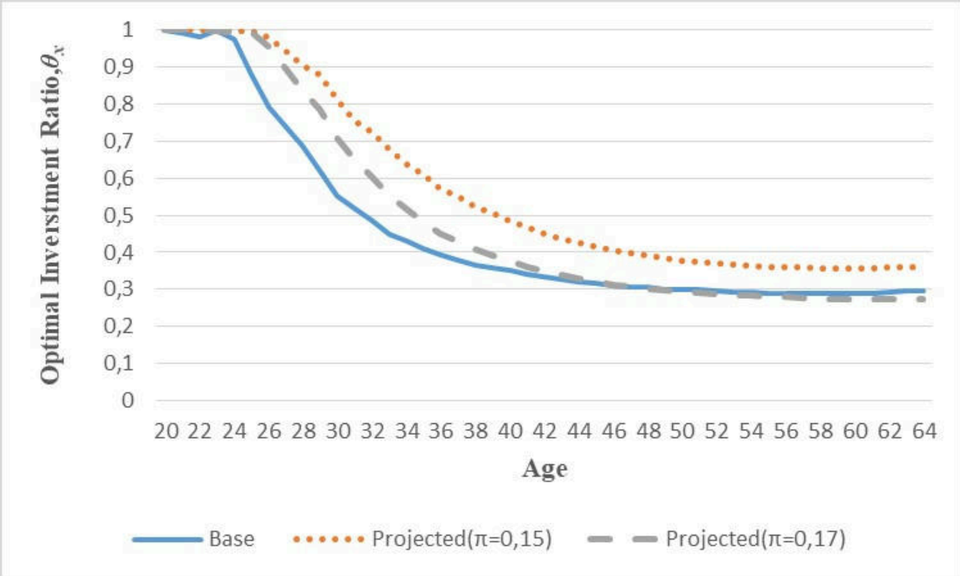
Table 4

Optimal investment ratios with base and projected mortality rates when appropriate contribution rate are used

Age	Base (π=0.15)	Projected (π=0.15)	Projected (π=0.17)
20	1	1	1
25	0.8810	0.9980	0.9935
30	0.5525	0.8106	0.7082
35	0.4118	0.6079	0.4818
40	0.3497	0.4837	0.3757
45	0.3164	0.4159	0.3217
50	0.2977	0.3772	0.2944
55	0.2901	0.3614	0.2791
60	0.2899	0.3557	0.2738
64	0.2959	0.3616	0.2739

Figure 4

Optimal investment strategy with base and projected mortality rates using the appropriate contribution rate



From Table 4 and Figure 4, it is seen that the optimal investment strategy obtained with the use of projected mortality rates including longevity risk is more conservative, that is, less risky, when an appropriate contribution rate π=0.17 is used. The optimal investment ratios obtained with the appropriate contribution rate are close to the optimal investment ratios obtained when the base mortality rates are used. However, towards the end of the accumulation period, it is seen that the ratio of the fund evaluated in the high-risk investment asset is lower, that is, less risk is taken. According to the results, either a more aggressive investment strategy should be followed or the appropriate contribution rate should be determined in order to hedge longevity risk in the distribution period. Considering that a loss-averse individual would not prefer an aggressive investment strategy, it is a better alternative to determine the appropriate contribution rate instead of following a more aggressive investment strategy in the accumulation period. In this way, the participant will not have to choose a more aggressive investment strategy during the accumulation period and will be able to hedge longevity risk in the distribution period by determining the appropriate contribution rate.

Optimal Investment Strategy with a Minimum Fund Guarantee

In the previous sections, as it was assumed that the participant did not want a minimum fund guarantee, the minimum fund guarantee was not taken into account. However, in DC pension plans, the minimum guarantee can be requested by the participant, and if the participant is a loss-averse individual, the concept of the minimum guarantee becomes more important. The results are obtained by including the minimum fund guarantee in the model.

When the studies on the minimum fund guarantee are examined, it is seen that the simplest form of guarantee is a fixed amount. If the pension is to be received as an annuity, the guarantee amount is determined according to the value of this annuity at the end of the accumulation period. In this case, when the participant retires, he will be entitled to a predetermined minimum amount, thus providing some degree of protection against the decline in investment performance.

When T is the retirement age, $G(T)$ indicates the minimum fund guarantee. The actual fund F_T at the end of the period must be greater than or equal to the minimum fund guarantee:

$$F_T \geq G(T) \tag{16}$$

The general structure of the optimisation problem under the minimum guarantee is as follows:

$$\max_{\theta_x} E[U(F_T - G(T))] \tag{17}$$

In this case, the utility function given by Eq. 2.13 for the age 65 is as follows:

$$U_{65}(F_{65}) = \frac{(F_{65} - f(65))^{v_1}}{v_1} - G(65); F_{65} \geq f(65) \tag{18}$$

$$= -\lambda \frac{(f(65) - F_{65})^{v_2}}{v_2} - G(65); F_{65} < f(65) \tag{19}$$

Then, the sum of the discounted utility function from age x to retirement is again defined as in Equation 2.14:

$$V_x = [\sum_{s=0}^{65-x-1} \beta^s w U_{x+s}(F_{x+s})] + \beta^{65-x} U_{65}(F_{65}) = wU_x(F_x) + \beta V_{x+1} \tag{20}$$

Considering that there is also a minimum guarantee, the constraint $F_{65} \geq G(65)$ is added to the constraints for the optimisation problem. Under the minimum fund guarantee, the optimisation problem is obtained as follows:

$$\max_{\theta_x} E_x(V_x) \overset{\max}{\underset{\theta_x}{=}} E_x[\left\{ \sum_{s=0}^{65-x-1} \beta^s w U_{x+s}(F_{x+s}) \right\} + \beta^{65-x} U_{65}(F_{65})] \tag{21}$$

with subject to

- $F_x = (F_{x-1} + \pi Y_{x-1}) \exp\left[r + \theta_{x-1}\left(\left(\mu - \frac{1}{2}\sigma^2\right)\right)\right] \geq 0; x = 21, 22, ..., 65$
- $Y_x = Y_{x-1} exp(l_x)$
- $0 \leq \theta_x \leq 1$
- $F_{65} \geq G(65)$ (22)

It is important to determine the minimum guarantee amount. It should be determined by considering the targeted fund size at the end of the period and it should not be higher than the targeted fund size at the end of the period. For this purpose, the minimum fund guarantee amount was increased by one unit, starting from 0 to the end-of-period targeted fund size as calculated in Section 2 of $f(65)$=62.66, and the appropriate minimum fund guarantee amount was determined. The optimal investment strategies obtained under a certain minimum fund guarantee amount with a contribution rate of π=0.15 for base mortality rates are summarised in Table 5 and Figure 5.

Table 5

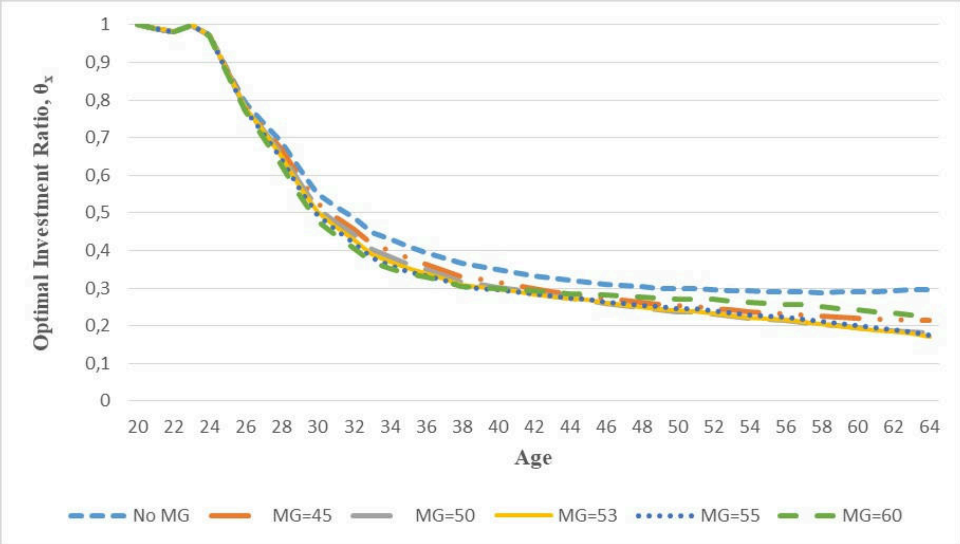
Optimal investment ratios under some minimum fund guarantees

Age	No MG	MG= 35	MG= 40	MG= 45	MG= 50	MG= 53	MG= 55	MG= 60	MG= 62,66
20	1	1	1	1	1	**1**	1	1	1
25	0.8811	0.8810	0.8793	0.8771	0.8740	**0.8726**	0.8710	0.8689	0.8658
30	0.5525	0.5509	0.5427	0.5276	0.5126	**0.5024**	0.4951	0.4790	0.4699
35	0.4119	0.4086	0.3975	0.3803	0.3645	**0.3530**	0.3449	0.3391	0.3366
40	0.3497	0.3452	0.3336	0.3161	0.3021	**0.2986**	0.2954	0.2986	0.3053
45	0.3164	0.3104	0.2970	0.2793	0.2672	**0.2678**	0.2697	0.2841	0.2985
50	0.2978	0.2903	0.2730	0.2524	0.2382	**0.2402**	0.2461	0.2717	0.2935

Age	No MG	MG= 35	MG= 40	MG= 45	MG= 50	MG= 53	MG= 55	MG= 60	MG= 62,66
55	0.2901	0.2814	0.2600	0.2340	0.2166	**0.2184**	0.2253	0.2597	0.2885
60	0.2900	0.2795	0.2536	0.2200	0.1949	**0.1942**	0.2001	0.2425	0.2785
64	0.2960	0.2839	0.2538	0.2136	0.1799	**0.1719**	0.1749	0.2224	0.2708

Figure 5

Optimal investment strategies under some minimum fund guarantees



It is understood from Table 5 and Figure 5 that the optimal investment strategy is similar in cases where there is no minimum fund guarantee amount and there is.

It has been observed that the optimal investment strategy for the values in the range of [1, 35] of the minimum fund guarantee amount is quite similar to the situation where there is no minimum fund guarantee. The optimal investment strategy started to change from the point where the minimum fund guarantee is 35. This change has taken place in a way that makes the investment strategy more conservative. In other words, the optimal investment strategy started to be less risky for the values after 35 of the minimum fund guarantee. The change in the optimal investment strategy to be less risky continued up to the point where the minimum fund guarantee is 53. In this range, the participant who wants a minimum fund guarantee tends to take less risk than the participant who does not want a minimum fund guarantee. In the values after 53 of the minimum fund guarantee amount, the optimal investment strategy started to be more aggressive.

When the minimum fund guarantee is taken as 62.66, which is the targeted fund size at the end of the period, it is seen that the optimal investment strategy is more conservative, that is, less risky than the situation where there is no minimum fund guarantee amount. However, it is seen that this difference is not very high. The results are close to each other.

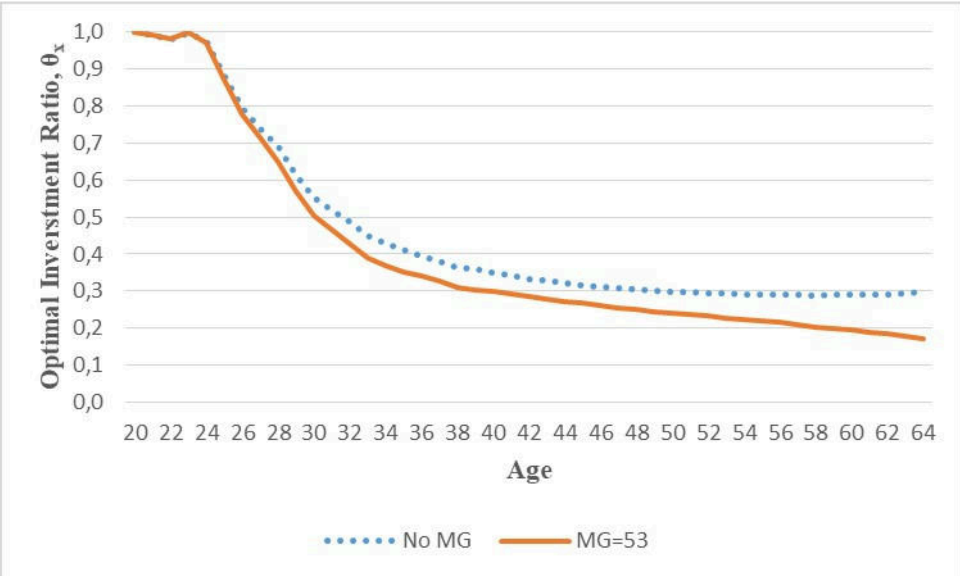
In cases where the minimum fund guarantee amount is higher than the terminal targeted fund size, the optimal investment strategy becomes more aggressive than the one when there is no minimum fund guarantee. As the minimum fund guarantee increases, it has been observed that the rate of the fund used in the high-risk investment asset has increased considerably, especially at the end of the accumulation period, in order to provide this guarantee. This increase also changes the general strategy. In cases where the minimum fund guarantee amount is higher than the targeted fund size, the optimal investment strategy is changed to decrease as opposed to the increase in the ratio of the fund evaluated in the riskless investment asset at the end of the accumulation period. Considering this situation, the minimum fund guarantee amount should not be chosen higher than the targeted fund size. This choice changes the general shape of the optimal investment strategy and leads to much more aggressive investment strategies.

Based on these results, it is concluded that the appropriate minimum fund guarantee amount is 53 because the most appropriate investment strategy for the loss-averse individuals is the one that maximises the expected utility while taking the least risk. The value of 53 is close to the value of 62.66, which is the targeted fund size at the end of the period. Accordingly, it is concluded that while determining the optimal investment strategy with the minimum fund guarantee amount for the loss-averse individuals, the appropriate minimum fund guarantee amount should be smaller than the end-of-period targeted fund size and should be close to this value.

Optimal investment strategies with no minimum fund guarantee and with the appropriate minimum fund guarantee are shown in Figure 6.

Figure 6

Optimal investment strategies with no minimum fund guarantee and with the appropriate minimum fund guarantee



It is realised from Figure 6 that the optimal investment strategy obtained using the appropriate minimum fund guarantee is similar to the strategy obtained without the minimum fund guarantee. Moreover, the optimal investment strategy obtained under the appropriate minimum fund guarantee is more conservative, that is, less risky.

It is concluded that determining the appropriate minimum fund guarantee amount reduces the risk in the optimal investment strategy. Considering that the desired situation for the loss-averse individuals is that the optimal investment strategy is less risky and the appropriate minimum fund guarantee amount makes the optimal investment strategy less risky, the optimal investment strategy has been obtained under the longevity risk and with the minimum fund guarantee amount. The optimal investment strategy obtained with base mortality rates and projected mortality rates in the absence and presence of a minimum fund guarantee are summarised in Table 6 and Figure 7.

Table 6

Optimal investment ratios with base and projected mortality rates with appropriate minimum fund guarantee

Age	Base (No MG)	Projected (No MG)	Projected (MG=53)
20	1	1	1
25	0.8810	0.9980	0.9975

Age	Base (No MG)	Projected (No MG)	Projected (MG=53)
30	0.5525	0.8106	0.7652
35	0.4118	0.6079	0.5156
40	0.3497	0.4837	0.3797
45	0.3164	0.4159	0.3030
50	0.2977	0.3772	0.2512
55	0.2901	0.3614	0.2205
60	0.2899	0.3557	0.1960
64	0.2959	0.3616	0.1796

Figure 7

Optimal investment strategy with base and projected mortality rates with appropriate minimum fund guarantee

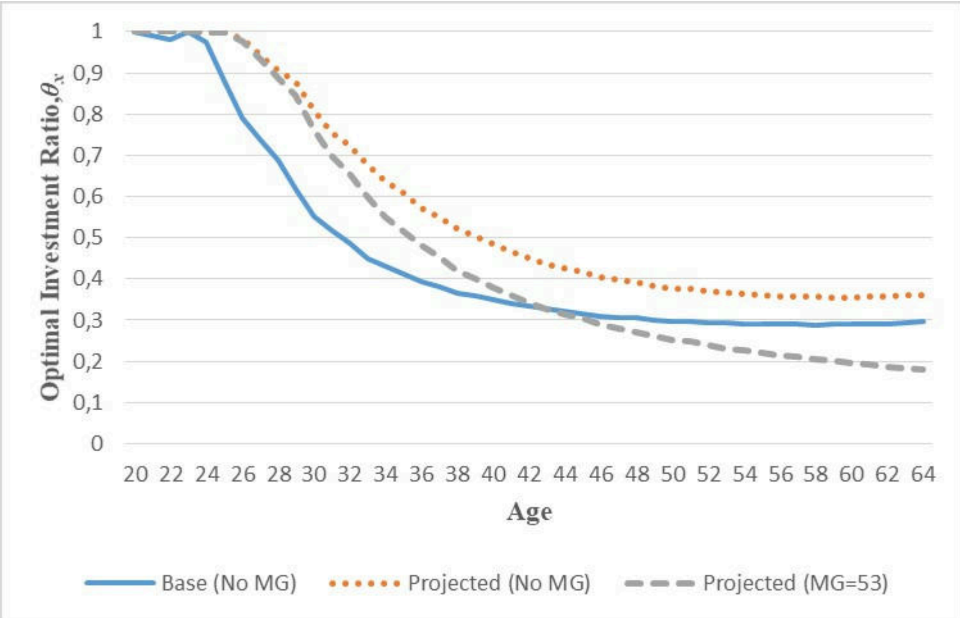


Table 6 and Figure 7 show that the optimal investment strategy obtained by using the appropriate minimum fund guarantee of 53 is more conservative than the optimal investment strategy obtained when using projected mortality rates that include longevity risk. This optimal investment strategy obtained using the minimum fund guarantee is less risky than the optimal investment strategy obtained when base mortality rates are used. Considering these results, it can be said that it is a better alternative to determine the appropriate minimum fund guarantee instead of following a more aggressive investment strategy in the accumulation period in order to hedge longevity risk in the distribution period. If the optimal investment strategy in the accumulation period is achieved with the appropriate minimum fund guarantee amount, the participant will be able to hedge longevity risk in the distribution period by taking even less risk than the base case. Considering the fact that the participant is a loss-averse individual and therefore does not want to follow a more aggressive investment strategy, the appropriate minimum fund guarantee amount can be determined instead of choosing an aggressive investment strategy. In this way, the investment strategy that the participant will follow in the accumulation period becomes more conservative. Therefore, for the loss-averse individuals in DC pension plans, it becomes more important to determine the appropriate minimum fund guarantee amount.

Thus far, it has been concluded that determining both the appropriate contribution rate and the appropriate minimum fund guarantee is an alternative to a more aggressive investment strategy in the accumulation period in order to hedge longevity risk in the distribution period. At this point, both alternatives are used together. In order to hedge longevity risk, the optimal investment strategy is determined by taking both the appropriate contribution rate π=0.17 and the minimum fund guarantee amount 53 together. The results obtained are summarised in Table 7 and Figure 8

Table 7

Optimal investment ratios with base and projected mortality rates when the appropriate contribution rate and minimum fund guarantee amount are used

Age	Base (π=0.15, No MG)	Projected (π=0.15, No MG)	Projected (π=0.15, MG=53)	Projected (π=0.17, No MG)	Projected (π=0.17, MG=53)
20	1	1	1	1	1
25	0.8810	0.9980	0.9975	0.9935	0.9925
30	0.5525	0.8106	0.7652	0.7082	0.6766
35	0.4118	0.6079	0.5156	0.4818	0.4348
40	0.3497	0.4837	0.3797	0.3757	0.3285
45	0.3164	0.4159	0.3030	0.3217	0.2757
50	0.2977	0.3772	0.2512	0.2944	0.2452
55	0.2901	0.3614	0.2205	0.2791	0.2145
60	0.2899	0.3557	0.1960	0.2738	0.1867
64	0.2959	0.3616	0.1796	0.2739	0.1632

Figure 8

Optimal investment strategy with base and projected mortality rates with appropriate minimum fund guarantee

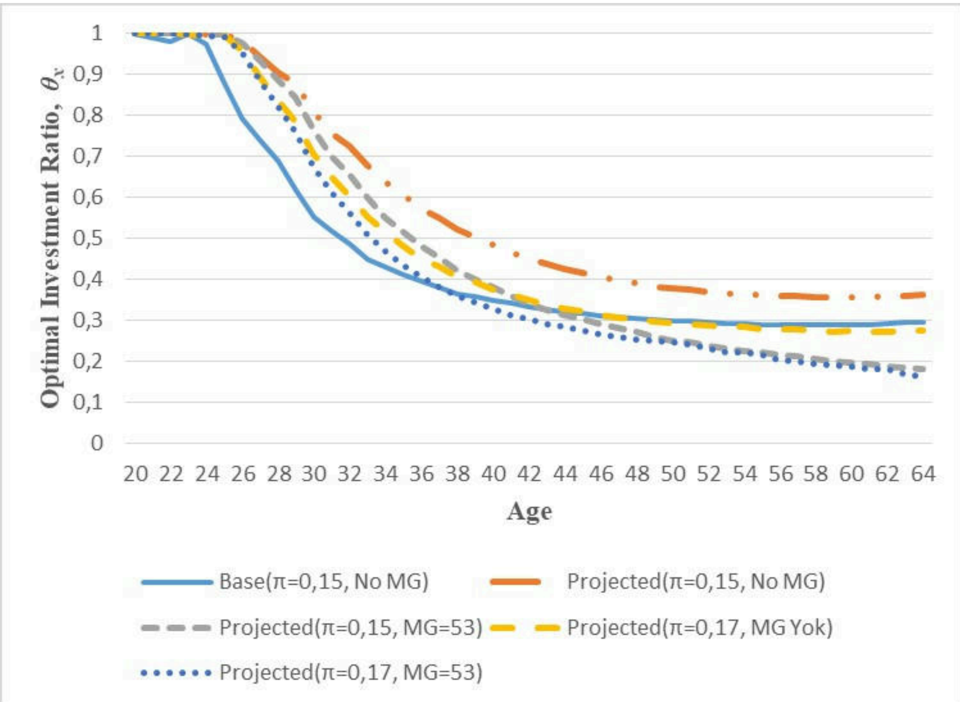


Figure 8 Optimal investment strategies with base and projected mortality rates when appropriate contribution and minimum fund guarantee amount are used

It can be seen from Table 7 and Figure 8 that determining both the appropriate contribution rate and the appropriate minimum fund guarantee amount makes the investment strategy in the accumulation period more conservative, that is, it reduces the risk in the investment strategy. When the optimal investment strategies obtained with the appropriate contribution rate and the appropriate minimum fund guarantee amount are compared, it is seen that determining only the appropriate minimum fund guarantee amount reduces the aggressiveness in the investment strategy more than determining only the appropriate contribution rate. From this point of view, it could be inferred that setting the appropriate minimum fund guarantee amount is a better alternative for the loss-averse individuals.

It is also obtained from Table 7 and Figure 8 that the combination of the appropriate contribution amount and the appropriate minimum fund guarantee amount makes the optimal investment strategy even less risky as expected. Therefore, in order to hedge longevity risk in the distribution period, a loss-averse individual can determine the optimal investment strategy with the appropriate contribution rate and the appropriate minimum fund guarantee amount together, instead of making the investment strategy more risky. In this way, the individuals reduce the investment risk further.

Conclusions and Suggestions

In DC pension plans, determining the optimal investment strategy is crucial because the investment risk in DC plans is on the participant. Another important decision made by the participant in the DC plan is the determination of the appropriate contribution rate. The higher the contribution, the larger the fund. However, contributing too high is also undesirable by the participant. Therefore, it is important to determine the appropriate contribution in DC plans.

In the real world, most investors are loss-averse. Therefore, one important thing is that the pension fund is at least equal to a certain amount. The optimal investment strategy should be obtained with the appropriate minimum fund guarantee amount.

Due to the increasing life expectancy around the world, it is extremely important to determine the optimal investment strategy considering the longevity risk. The optimal investment strategy for the loss-averse individuals becomes more aggressive and riskier when projected mortality rates are used. Accordingly, the loss-averse individuals should follow a more aggressive investment strategy in the accumulation period in order to hedge longevity risk in the distribution period. Since the participant is assumed to be a loss-averse individual, an aggressive investment strategy is undesirable by the participant. For this reason, in this study, alternative ways to an aggressive investment strategy have been sought for longevity risk hedging. These alternative ways could be used to determine the optimal investment strategy with the appropriate contribution rate and minimum fund guarantee.

It is investigated how the longevity risk in the distribution period can be dealt with using the appropriate contribution rate in the accumulation period. It has been concluded that the optimal investment strategy obtained by determining the appropriate contribution rate for the loss-averse individuals is a less risky strategy.

While the optimal investment strategy for the loss-averse individuals is obtained with the minimum fund guarantee, the appropriate minimum fund guarantee should be determined close to the targeted fund size at the end of the period and should be lower than the targeted fund size. It has been seen that the optimal investment strategy obtained with the appropriate minimum fund guarantee amount determined in this way is more conservative and less risky than the optimal investment strategy obtained in the absence of the minimum fund guarantee amount. Therefore, it could be deduced that determining the optimal investment strategy with a minimum guarantee is a better alternative for the loss-averse individuals.

Loss-averse individuals should follow a riskier investment strategy during the distribution period to hedge longevity risk during the distribution period. On the other hand, it has been observed that the loss-averse individuals can determine both the appropriate contribution rate and the appropriate minimum fund guarantee to hedge longevity risk in the distribution period, rather than adopting a more aggressive investment strategy. In this way, the participant will be able to follow a less risky investment strategy. When the optimal investment strategies obtained with the appropriate contribution rate and the appropriate minimum fund guarantee amount are compared, it is observed that determining the appropriate minimum fund guarantee amount reduces the aggressiveness in the investment strategy more than determining the appropriate contribution rate. Accordingly, it is more important to determine the optimal investment strategy with the appropriate minimum fund guarantee amount for the participant. If the loss-averse individuals wish to reduce risk further during the distribution period, they must determine both the appropriate contribution rate and the appropriate minimum fund guarantee amount together

The results obtained in this study are open to improvement in some aspects.

In the model used, the contribution rate was taken as constant. The contribution rate can be determined in variable amounts by considering the difference between the actual fund size and the targeted fund size of the contribution amount to be made by the participant to the fund, instead of a fixed amount.

The results obtained for loss-averse individuals can also be obtained for individuals with different risk attitudes, and the results can be compared with the results obtained for loss-averse individuals. In this way, it can be examined how different risk attitudes change the optimal investment strategy.

In the model used in this study, time is assumed to be distinct. By changing this assumption, the case of continuous time can be handled, and the results obtained can be compared with the results obtained in discrete time.

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