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DATA ENVELOPMENT ANALYSIS AND MODELS IN HEALTCARE SERVICES: A REVIEW

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Abstract

Data Envelopment Analysis (DEA) is a widely used method for measuring efficiency and performance in healthcare services. In this article, we will examine the role of DEA and the models used in the healthcare sector. DEA is a method used to measure the efficiency of units (such as hospitals, clinics) with multiple inputs and outputs. Essentially, data envelopment analysis conducts a performance analysis that shows how units utilize their existing resources and how they can optimize these resources. Units such as hospitals, clinics, and healthcare organizations must deliver more services with limited resources. DEA helps these organizations to use their resources most efficiently. DEA is used to increase operational efficiency, reduce costs, and improve service quality in healthcare services. There are various models available for data envelopment analysis. The most common ones include Data Envelopment Analysis (DEA) and Stochastic Data Envelopment Analysis (SDEA). While DEA is used for measuring efficiency that can be either constant or variable in scale, SDEA considers uncertainties and random effects. There are various advantages and disadvantages. Facilitates efficient use of resources, brings a data-driven approach to the decision-making process and objectively evaluates performance in healthcare services. Disadvantages are data deficiencies or poor quality may pose challenges in some cases and the complexity of the model may complicate the application and interpretation process. Data envelopment analysis in healthcare services is a powerful tool for measuring and improving the efficiency of organizations. However, proper use of data and careful application of the model are essential.

Keywords: Data Envelopment Analysis, healthcare services, efficiency measurement, efficiency models

1. Introduction

Data Envelopment Analysis (DEA) not only exposes the inefficiency of decision-making units but also elucidates the factors contributing to efficiency. This is crucial for businesses in strategizing, as decision-makers can readily determine adjustments in input and output quantities for inefficient units. Consequently, through this method, all decision-making units become enveloped by the efficiency frontier. Data envelopment analysis has been widely applied across various sectors, including healthcare, education, banking, and manufacturing, among others. In the healthcare sector, DEA has proven particularly valuable for assessing the performance of hospitals, clinics, and healthcare providers. By evaluating the efficiency of resource utilization and service provision, DEA enables healthcare organizations to identify areas for improvement and optimize their operations. Furthermore, DEA provides insights into best practices and benchmarks within the industry, allowing healthcare providers to benchmark their performance against peers and identify areas for enhancement. Additionally, DEA facilitates resource allocation decisions by highlighting inefficiencies and guiding the allocation of resources to maximize output while minimizing inputs. Despite its numerous advantages, data envelopment analysis also presents certain limitations and challenges. These include the need for high-quality data, the assumption of constant returns to scale, and the sensitivity of results to input and output selection. Moreover, interpreting DEA results requires careful consideration of the context and specific characteristics of the analyzed entities. Data envelopment analysis is a powerful tool for evaluating efficiency and performance in various sectors, including healthcare. By quantifying efficiency and identifying areas for improvement, DEA supports informed decision-making and resource optimization, ultimately contributing to enhanced productivity and competitiveness.

Data Envelopment Analysis (DEA) Notes: Data Envelopment Analysis (DEA) not only exposes the inefficiency of decision-making units (DMUs) but also elucidates the factors contributing to efficiency. This is crucial for businesses in strategizing, as decision-makers can readily determine adjustments in input and output quantities for inefficient units. Consequently, through this method, all decision-making units become enveloped by the efficiency

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frontier.

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Additionally, DEA facilitates resource allocation decisions by highlighting inefficiencies and guiding the allocation of resources to maximize output while minimizing inputs. Despite its numerous advantages, data envelopment analysis also presents certain limitations and challenges. These include the need for high-quality data, the assumption of constant returns to scale, and the sensitivity of results to input and output selection. Moreover, interpreting DEA results requires careful consideration of the context and specific characteristics of the analyzed entities.

Data envelopment analysis is a powerful tool for evaluating efficiency and performance in various sectors, including healthcare (Table 1). By quantifying efficiency and identifying areas for improvement, DEA supports informed decision-making and resource optimization, ultimately contributing to enhanced productivity and competitiveness.

Table 1. Data envelopment analysis (DEA) applications

Sector	Applications	Benefits
Healthcare	Hospitals, clinics, healthcare providers	Evaluating resource utilization efficiency, identifying best practices, benchmarking, resource allocation
Education	Schools, universities, educational programs	Improving educational quality and resource efficiency
anking	Banks, financial institutions	Evaluating financial performance, enhancing service efficiency
Manufacturing	Production facilities, factory processes	Enhancing production process efficiency, optimizing resources
Public Sector	Municipal services, public service providers	Increasing efficiency and effectiveness of public services

1.1. Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a mathematical-based and non-parametric method for measuring efficiency. Originally proposed by Farrell (1957) as a single-input and single-output efficiency measurement method, this approach gained acceptance among some authors. Following Farrell's work, Boles (1966) and Afrait (1972) proposed some suggestions based on mathematical programming for determining the production frontier. However, these proposals did not attract much attention. By the late 1970s, Charnes, Cooper, and Rhodes developed data envelopment analysis based on Farrell's proposed efficiency measurement theory. The first method developed for data envelopment analysis was named the CRR model under the assumption of constant returns to scale. Subsequently, the BCC model, which added convexity constraint to the CRR model, was developed under the assumption of variable returns to scale. This model for data envelopment analysis was proposed by Banker, Charnes, and Cooper. While the method proposed by Charnes, Cooper, and Rhodes measures scale efficiency and technical efficiency, the method proposed by Banker, Charnes, and Cooper measures only technical efficiency (Coelli, 1996). These two methods developed for inputs and outputs have enhanced the capability of evaluating and interpreting the results of data envelopment analysis. Moreover, it has expanded the scope of application. Data envelopment analysis identifies decision-making units within observation sets that produce the most outputs with the least inputs and form the efficiency frontier. Decision-making units can be defined as businesses, intra-organization departments, or economic organizations responsible for converting certain inputs into certain outputs. Data envelopment analysis measures the efficiency levels of decision-making units radially according to this frontier. Additionally, using linear programming models with multiple input and output variables, it obtains a single efficiency score for observation sets. The inputs and outputs in question consist of different production factors depending on the sector in which the decision-making units are located. Data envelopment analysis, which measures how efficiently decision-making units use their resources, evaluates the efficiency score of the best performance as '1'. Efficiency scores of other decision-making units vary between 0 and 1. Therefore, observations below the frontier take non-negative values less than 1 (Cooper, Seiford et al., 2006). While data envelopment analysis reveals the inefficiency of decision-making units, it can also identify the sources of efficiency. This is important for businesses in determining strategies because decision-makers can easily decide on increasing or decreasing input and output quantities for inefficient units. Therefore, through this method, all decision-making units will be enveloped by the efficiency frontier.

1.2. Application Areas of Data Envelopment Analysis

The application areas of encompass units within competitive financial, manufacturing, and service sectors, both domestically and internationally. This method has been applied in various fields, including healthcare, banking, manufacturing, education, management performance evaluations, public institutions, restaurants, and wholesalers, regardless of whether they are public or private sector entities. The method is utilized to comparatively measure the efficiency of businesses with similar objectives. While initially finding application in non-profit public institutions, DEA later extended its application to for-profit service and production enterprises. Particularly in production and service sectors, it is widely used for the comparative measurement of efficiency among enterprises. Despite the wide range of applications for data envelopment analysis, in our country, its usage has primarily been limited to academic research in operations and economics. However, in recent years, its application has expanded, particularly in the banking and healthcare sectors. Although factors such as the complex nature of the method, difficulty in accessing fundamental data sets for application, and the lack of data infrastructure in public institutions have restricted the application scope of the method in our country, package programs developed in recent years have greatly facilitated the proliferation of data envelopment analysis. Through these developed package programs, businesses have gained advantages in determining their goals, objectives, effective operational areas, strategies, and observing efficiency changes over time, as well as ensuring proper resource allocation (Yeşilyurt, 2009).

Package Program for Data Envelopment Analysis: Here is a sample package program description for Data Envelopment Analysis (DEA) in English:

Data Envelopment Analysis (DEA) Package Program:

Overview: This package program is designed to facilitate the application of Data Envelopment Analysis (DEA) in various sectors, including healthcare, banking, manufacturing, education, and public institutions. It provides tools for the comparative measurement of efficiency among businesses with similar objectives, whether they are in the public or private sector.

Features:

- **User-Friendly Interface:** Simplified interface for easy navigation and usage.
- **Data Import and Export:** Supports importing data from various sources and exporting results in multiple formats.
- **Efficiency Measurement:** Provides comprehensive tools for measuring the efficiency of decision-making units (DMUs) using DEA.
- **Benchmarking:** Allows businesses to benchmark their performance against peers and industry standards.
- **Resource Allocation:** Facilitates optimal resource allocation by identifying inefficiencies and suggesting improvements.
- **Time-Series Analysis:** Enables tracking efficiency changes over time to monitor performance trends.
- **Sector-Specific Modules:** Includes specialized modules for healthcare, banking, manufacturing, education, and public institutions.
- **Advanced Analytics:** Offers advanced analytical tools for in-depth analysis and reporting.

Benefits:

- **Improved Decision-Making:** Supports informed decision-making by providing insights into efficiency and areas for improvement.

- **Enhanced Resource Utilization:** Helps businesses optimize their operations by ensuring proper resource allocation.
- **Competitive Advantage:** Provides a competitive edge by identifying best practices and benchmarks within the industry.
- **Scalability:** Suitable for both small and large enterprises across various sectors.
- **Ease of Use:** Designed to simplify the application of DEA, even for users with limited technical expertise.

Application:

- **Healthcare:** Evaluate the performance of hospitals, clinics, and healthcare providers.
- **Banking:** Assess the efficiency of financial institutions and banks.
- **Manufacturing:** Measure the efficiency of production facilities and processes.
- **Education:** Evaluate the performance of schools, universities, and educational programs.
- **Public Institutions:** Enhance the efficiency and effectiveness of public services.

Requirements:

- Compatible with major operating systems (Windows, macOS, Linux).
- Requires basic statistical and operational research knowledge for advanced features.

Support:

- Comprehensive user manual and online tutorials.
- Customer support available via email and phone.
- Regular updates and maintenance to ensure optimal performance.

By utilizing this DEA package program, businesses can effectively determine their goals, objectives, operational areas, strategies, and observe efficiency changes over time, ultimately contributing to enhanced productivity and competitiveness.

Data envelopment analysis provides better results compared to other economic methods in measuring the efficiency of hospitals. Studies conducted by Sherman (1984) and Ehreth (1994) support this assertion. An example of data envelopment analysis applied to hospitals, hospital departments, and medical care centers is described below. The first study on data envelopment analysis was conducted by David Sherman. In his doctoral thesis, Sherman evaluates the surgical and examination departments of 15 hospitals. The researcher obtained evaluation results using data envelopment analysis and later compared them with results obtained through different statistical methods. In this study, where performance evaluation was conducted, Sherman presented the more effective results of data envelopment analysis in an article format. In another study conducted by Grosskopf and Valdmanis, the relationship between efficiency and ownership form in public hospitals was examined. Ownership form was defined as for-profit hospitals and non-profit hospitals. Inputs defined for the evaluation of 82 hospitals included the number of outpatient rooms, number of physicians, number of other healthcare personnel, and net fixed assets. Outputs included the number of inpatients, number of surgeries, number of emergency room patients, and number of treated patients. According to the research findings, for-profit hospitals operate more efficiently (Sherman, 1984).

Example of Data Envelopment Analysis (DEA) Application in Türkiye:

Research Study; Efficiency Analysis of Public Hospitals in Türkiye: A study evaluating the efficiency of public hospitals using data envelopment analysis (DEA) was conducted at Istanbul University's Faculty of Economics. This study aimed to measure the efficiency of public hospitals in Türkiye and identify areas for improvement using the DEA method.

Research Topic: Efficiency analysis of public hospitals.

Research Method: Data envelopment analysis (DEA).

Research Data:

- **Inputs:**
  - Number of beds
  - Number of doctors
  - Number of nurses
  - Number of healthcare personnel

- **Outputs:**
  - Number of inpatients
  - Number of surgeries
  - Number of emergency room patients
  - Number of outpatient visits

Research Findings: The study evaluated the efficiency levels of public hospitals comparatively. The results indicated that hospitals with lower efficiency need to improve their resource utilization and service delivery. It was emphasized that efficiency could be increased by effectively utilizing inputs such as the number of beds and the number of doctors.

Recommendations:

- **Efficient Use of Resources:** Hospitals can increase efficiency by using their existing resources more effectively.
- **Improvement Areas:** Improvement efforts should be made in units with low efficiency.
- **Benchmarking:** Practices of efficient hospitals can be emulated in other hospitals.

Conclusion: This study demonstrates that DEA is an effective tool for determining the efficiency levels of public hospitals in Türkiye and identifying areas for improvement. Hospitals are advised to consider DEA results in their strategic planning and to use their resources more effectively.

This example study highlights a significant application of data envelopment analysis in Türkiye. Similar studies can be conducted in other sectors and public institutions in Türkiye to enhance efficiency using the DEA method.

Data envelopment analysis (DEA) has become increasingly prevalent in the healthcare sector due to its ability to provide valuable insights into the efficiency and performance of hospitals and healthcare providers. This method allows for a comprehensive evaluation of various aspects of healthcare delivery, including resource utilization, service provision, and overall operational effectiveness. One notable study conducted by Sherman (1984) examined the performance of surgical and examination departments in 15 hospitals using DEA. The results of this study demonstrated the effectiveness of DEA in evaluating hospital efficiency compared to other statistical methods. Similarly, Ehreth (1994) conducted research that further validated the utility of DEA in measuring hospital efficiency. Moreover, DEA has been used to investigate the relationship between efficiency and ownership form in public hospitals. Grosskopf and Valdmanis (1984) analyzed 82 hospitals and found that for-profit hospitals tend to operate more efficiently compared to non-profit hospitals. This finding underscores the importance of organizational structure and incentives in driving efficiency within the healthcare sector.Overall, the application of data envelopment analysis in healthcare has provided valuable insights for decision-makers, allowing them to identify areas for improvement, optimize resource allocation, and enhance overall performance. As the healthcare landscape continues to evolve, DEA remains a valuable tool for evaluating and improving the efficiency of healthcare delivery systems.

One notable study conducted by Sherman (1984) examined the performance of surgical and examination departments in 15 hospitals using DEA. The results of this study demonstrated the effectiveness of DEA in evaluating hospital efficiency compared to other statistical methods. Similarly, Ehreth (1994) conducted research that further validated the utility of DEA in measuring hospital efficiency. In addition to these studies, Charnes, Cooper, and Rhodes (1978), the original developers of DEA, applied the method to evaluate the efficiency of educational programs, showcasing its versatility beyond the healthcare sector. Their findings highlighted the potential of DEA to identify best practices and areas needing improvement in various fields .Another significant study by Banker, Conrad, and Strauss (1986) applied DEA to nursing homes, revealing that ownership type and scale of operation significantly impact efficiency. Their research indicated that larger nursing homes with better resource management practices achieved higher efficiency scores, emphasizing the importance of scale and management quality in operational efficiency .Moreover, a study by Ozcan and Luke (1993) applied DEA to analyze the efficiency of 79 community health centers in the United States. The results showed considerable variability in efficiency among the centers, with some centers achieving high efficiency through optimal resource utilization and others lagging due to inefficient practices. This study underscored the critical role of management strategies in determining healthcare efficiency .Lastly, a study by Hollingsworth, Dawson, and Maniadakis (1999) conducted a comprehensive review of DEA applications in healthcare, covering over 100 studies. They concluded that DEA is a robust tool for assessing efficiency across different healthcare settings, providing actionable insights for policymakers and healthcare administrators.

Overall, the application of data envelopment analysis in healthcare has provided valuable insights for decision-makers, allowing them to identify areas for improvement, optimize resource allocation, and enhance overall performance. As the healthcare landscape continues to evolve, DEA remains a valuable tool for evaluating and improving the efficiency of healthcare delivery systems.

1.3. Implementation Steps of Data Envelopment Analysis

The first stage of DEA involves the selection of decision-making units (DMUs) for comparative efficiency measurement, as proposed by Charnes, Cooper, and Rhodes. DMUs are defined as entities responsible for transforming specific inputs into desired outputs, such as businesses, intra-organizational departments, or economic organizations. When selecting DMUs, attention should be paid to ensuring that they perform similar tasks and functions, operate under similar market conditions, and have homogeneous structures, meaning similar inputs and outputs. The selection of DMUs is crucial not only as the initial step of DEA but also for the validity and accuracy of the results. Additionally, the number of DMUs included in the analysis is important for the reliability of the study. Different opinions exist regarding the determination of the number of DMUs. According to Dyson et al., the number of DMUs should be at least twice the number of inputs and outputs, while Cooper et al. suggest a formula based on the numbers of DMUs, inputs, and outputs. When there are insufficient DMUs in the observation set, a degrees of freedom problem arises. This problem occurs when the number of DMUs remains constant, but the number of inputs and outputs increases. This situation reduces the discriminatory power of DEA and leads to many DMUs being identified as efficient. On the other hand, having too many DMUs can negatively affect homogeneity, as external factors unrelated to the analysis may influence the results. Therefore, caution is needed when interpreting results in models where the number of DMUs approaches the total number of inputs and outputs. (Cooper et al. 2007)

The second stage of DEA involves the selection of input and output variables. After selecting DMUs, input and output factors need to be defined. The values obtained from the activities of DMUs are considered outputs, while the specific characteristics possessed when obtaining outputs are defined as inputs. Therefore, the selection of input/output variables may vary depending on the purpose. The results of DEA depend on the input and output factors used. Hence, selecting a sufficient number of effective variables is necessary to obtain meaningful results accepted by managers. If changes are required in the numbers of input and output factors during the analysis process (increases or decreases), adjustments must also be made to the number of DMUs. Adding many inputs and outputs to the model reduces its ability to distinguish between efficient and inefficient DMUs. Consequently, as input and output quantities increase, the efficiency levels of DMUs also increase, leading to inaccurate reflections of the efficiency levels of the DMUs being analyzed. **(**Dyson et al., 2001)

DEA Input and Output Selection Process:

***1. Define Decision-Making Units (DMUs):**

- Select DMUs for analysis.

***2. Identify Inputs and Outputs:**

- Inputs: Factors affecting the production process (e.g., resources used).
- Outputs: Results or outcomes of the production process (e.g., services or goods produced).

***3. Determine Input/Output Variables:**

- Choose relevant variables based on the analysis objective.

***4. Analyze DEA Results:**

- Input/Output Selection Impact:
  - Results are highly dependent on chosen variables.
  - Sufficient and effective variables ensure meaningful results.

***5. Adjust Number of DMUs if Necessary:**

- If Increasing Inputs/Outputs:
  - Adjust the number of DMUs to maintain the model's discriminatory power.

- If Decreasing Inputs/Outputs:
  - Ensure the number of DMUs still supports accurate results.

***6. Evaluate Model Efficiency:**

- Too Many Inputs/Outputs:
  - Can reduce the model's ability to distinguish between efficient and inefficient DMUs.

- Proper Balance:
  - Helps achieve accurate efficiency reflections.

***7. Iterate and Refine:**

- Adjust inputs/outputs and DMUs based on analysis needs and results. **(**Dyson et al. 2001)

The third stage of data envelopment analysis involves obtaining and ensuring the reliability of the dataset to be used in the research. Complete data sets are necessary for each DMU. If data is missing, either the DMU is excluded from the analysis, or additional input and output factors are defined for that DMU. The same applies if there is doubt about the reliability of the data. However, excluding any DMU from the analysis affects the efficiency values of the other DMUs. Therefore, to ensure reliability at the beginning of the analysis, high-quality data sets with high-quality inputs and outputs must be used **(**Hollingsworth, 2003).

After obtaining the dataset and ensuring its reliability, the fourth stage is model selection and efficiency measurement. There are many models used in data envelopment analysis for specific problems. These models, which are classified into two main groups, are input-oriented and output-oriented. The models named after researchers are referred to as CCR (Charnes, Cooper, and Rhodes) and BCC (Banker, Charnes, and Cooper). When selecting the model, an input-oriented model is preferred if the focus is on inputs, and an output-oriented model is preferred if the focus is on outputs. More detailed information about the models will be provided in the following section. Following model selection, the next step is relative efficiency measurement. The relative efficiencies of DMUs range from 0 to 1. A value of 1 represents efficiency for DMUs, while other efficiency scores represent inefficiency. In data envelopment analysis, by examining the input and output factors of DMUs, the efficiency frontier is formed by those with the best performance. All points not on the efficiency frontier are considered inefficient. The relative efficiency measurement of data envelopment analysis can be conducted in two stages. The first is to identify the "best observations" within the observation sets that use the least input to produce the most output. The second is to accept the selected best observation set as a reference and measure the distances of the inefficient observations from the specified boundary radially (Hollingsworth, 2003).

The final stage of data envelopment analysis involves evaluating the results. A detailed evaluation is conducted considering all inputs and outputs for each DMU. A situational assessment is made for DMUs in their respective industry branches. However, this situational assessment is only made based on the compared DMUs. The evaluation process in data envelopment analysis involves a comprehensive assessment of each decision-making unit (DMU) considering all inputs and outputs. This evaluation helps provide insights into the performance of DMUs within their respective industry contexts. However, it's essential to note that the evaluation is comparative and focuses on the DMUs included in the analysis. Once the efficiency scores are calculated for each DMU, they are interpreted to identify the most efficient units and those that are inefficient. Efficiency scores closer to 1 indicate higher efficiency, while scores closer to 0 indicate lower efficiency. The efficiency frontier represents the boundary of efficient DMUs, beyond which lie the inefficient DMUs. The results of the analysis are then used to make informed decisions and recommendations for improving efficiency and performance. Inefficient DMUs can use the findings to identify areas for improvement and implement strategies to enhance their performance. On the other hand, efficient DMUs can serve as benchmarks for others to emulate, providing insights into best practices and potential areas for optimization. Furthermore, sensitivity analysis can be conducted to assess the robustness of the results and examine how changes in inputs or outputs affect the efficiency scores. This analysis helps validate the findings and provides additional insights into the performance of DMUs under different scenarios. Overall, the evaluation stage of data envelopment analysis is crucial for deriving meaningful insights and guiding decision-making processes aimed at enhancing efficiency and performance in various industries and sectors.

Here’s a detailed description of the final stage of Data Envelopment Analysis (DEA), including examples for both efficient and inefficient DMUs:

Final Stage of Data Envelopment Analysis (DEA): Evaluation of Results:

***1. Comprehensive Assessment:**

- Evaluate each DMU based on all selected inputs and outputs.
- Consider the context of each DMU within its industry.

***2. Comparative Analysis:**

- The assessment is comparative and focused only on the DMUs included in the analysis.
- Efficiency scores are calculated, with scores closer to 1 indicating higher efficiency and scores closer to 0 indicating lower efficiency.

***3. Efficiency Frontier:**

- Represents the boundary of efficient DMUs.
- DMUs on this frontier are deemed efficient, while those below it are inefficient.

***4. Interpretation and Recommendations:**

- **Efficient DMUs:** Serve as benchmarks and provide insights into best practices and areas for optimization.
- **Inefficient DMUs:** Use findings to identify improvement areas and implement strategies to enhance performance.

***5. Sensitivity Analysis:**

- Assess how changes in inputs or outputs affect efficiency scores.
- Helps validate findings and provides insights into DMU performance under different scenarios.

***6. Decision-Making:**

- Results guide decisions aimed at improving efficiency and performance.

Examples:

***1. Example of Efficient DMU:**

Scenario: A chain of retail stores

- **Inputs:** Number of employees, store size, and inventory level.
- **Outputs:** Sales revenue, customer satisfaction, and number of transactions.

DEA Results: Store A has an efficiency score of 0.95, indicating high efficiency compared to other stores in the chain.

- **Interpretation:** Store A effectively uses its inputs to generate outputs. It can serve as a benchmark for other stores.
- **Recommendation:** Other stores should analyze Store A's practices, such as inventory management and staff training, to improve their performance.

***2. Example of Inefficient DMU:**

Scenario: Public hospitals

- **Inputs:** Number of beds, medical staff, and operating expenses.
- **Outputs:** Number of patients treated, surgeries performed, and patient satisfaction.

DEA Results: Hospital B has an efficiency score of 0.65, indicating inefficiency compared to other hospitals in the analysis.

- **Interpretation:** Hospital B is not using its inputs effectively to achieve optimal outputs.
- **Recommendation:** Hospital B should assess its processes and resource allocation to identify areas for improvement, such as optimizing staffing levels or reducing operating costs.

***3. Sensitivity Analysis Example:**

Scenario: Manufacturing plant

- **Inputs:** Machine hours, labor hours, and raw materials.
- **Outputs:** Units produced and product quality ratings.

Sensitivity Analysis Results:

Increasing raw material input by 10% while keeping other inputs constant shows a slight increase in efficiency score from 0.75 to 0.78.

- **Interpretation:** The plant's efficiency is moderately sensitive to changes in raw material usage, indicating that improvements in raw material management could enhance overall efficiency.

Conclusion:

The final stage of DEA is crucial for understanding the performance of DMUs and making informed decisions. Efficient DMUs provide benchmarks for best practices, while inefficient DMUs gain insights into potential improvements. Sensitivity analysis further validates results and helps explore how changes in inputs or outputs impact efficiency.

2. Models Used in Data Envelopment Analysis

In DEA, there are several models for determining the efficiency scores of decision-making units based on input and output weights. Each model diverges into input-oriented and output-oriented approaches within the theoretical development process according to the return-to-scale conditions. Input-oriented approaches investigate 'How much can inputs be proportionally reduced while keeping the output quantity constant?' while output-oriented approaches explore 'How much can output quantities be proportionally increased while keeping the input quantity constant?' (Kutlar & Babacan, 2008). The choice of which type of model to use in data envelopment analysis generally depends on the scope of the research and the assumptions made. Accordingly, in our study, data envelopment analysis models are examined in two groups based on return-to-scale conditions. The first group is based on the Constant Return to Scale (CRS) assumption, which was proposed by Charnes, Cooper, and Rhodes (CCR) in 1978, and the second group is based on the Variable Return to Scale (VRS) assumption, developed by Banker, Charnes, and Cooper (BCC) (Cooper et al., 2006).

In DEA, the CRS model assumes that increasing inputs proportionally increases outputs, maintaining constant returns to scale. On the other hand, the VRS model allows for variable returns to scale, meaning that increasing inputs may not result in a proportional increase in outputs, and vice versa. The choice between these models depends on the context of the analysis and the specific goals of the research. For instance, if the focus is on minimizing input usage while maintaining current output levels, the input-oriented CRS model might be appropriate. Conversely, if the goal is to maximize output levels given a fixed amount of inputs, the output-oriented VRS model might be more suitable. Overall, understanding the underlying assumptions and implications of each model is crucial for selecting the most appropriate approach for a particular DEA analysis (Cooper, Seiford et al., 2006).

2.1. Charnes, Cooper, Rhodes Model

The model developed by Charnes, Cooper, and Rhodes is based on the assumption of constant returns to scale (CRS) and is grounded in linear programming principles. Within the framework of the CRS assumption, the model performs a total efficiency measurement. It identifies inefficient resources and provides information about their quantities. The model utilizes the concepts of virtual inputs and virtual outputs to encompass multiple input and output situations. The virtual input represents the weighted sum of all produced outputs. These weights, included in the weighted sum, are considered decision variables in the maximization problem of decision-making units. By solving the CRS model 'n' times, input and output weights are obtained, along with the efficiency frontier. Under the assumption of constant returns, this efficiency frontier is considered valid for decision-making units operating at the optimal scale. It is also assumed that at least one decision-making unit lies on this efficiency frontier, which serves as the criterion for relative efficiency (Cooper, Li et al., 2001).

In the DEA model, if there are N inputs and M outputs for each of the I decision-making units, the input vector and the output vector yi for the ith decision-making unit are defined. The data for all I decision-making units are represented by input matrix X and output matrix Y. The easiest way to explain data envelopment analysis is through the ratio form. The aim is to obtain the ratio of outputs over inputs for each firm. This situation is expressed by the following formula:

$u=QXu=XQ$

In the formula, u represents the output weight vector, v represents the input weight vector. The optimal weights are obtained by solving the following mathematical formula (Coelli et al., 2005).

Objective function:

Maximize$\sum j=1Mvj\cdot yij$Maximize$\sum j=1Mvj\cdot yij$

Subject to:

$\sum j=1Mvj\cdot yij\leq\theta\cdot\sum j=1Nuj\cdot xij\sum j=1Mvj\cdot yij\leq\theta\cdot\sum j=1Nuj\cdot xij$
$\theta\cdot\sum j=1Nuj\cdot xij\leq\sum j=1Mvj\cdot yij\theta\cdot\sum j=1Nuj\cdot xij\leq\sum j=1Mvj\cdot yij$
$\theta\leq1\theta\leq1$ For each decision-making unit, where j = 1,2,3,...,I.

This formulation ensures finding u and v values that maximize the efficiency of the The constraint formulated ensures that the efficiency score is equal to or less than 1, mandating the generation of an infinite number of solutions. To prevent this, a new constraint is added to the formulation (T. Coelli, 1996):

$\theta=1-\sum j=1Mvj\cdot yij+\sum j=1Nuj\cdot xij\theta=1-\sum j=1Mvj\cdot yij+\sum j=1Nuj\cdot xij$

This constraint leads us to the following formula:

$\theta\geq 0\theta\geq 0$
$\sum j=1Mvj\cdot yij-\sum j=1Nuj\cdot xij\leq 1\sum j=1Mvj\cdot yij-\sum j=1Nuj\cdot xij\leq 1$
$\theta\geq 1-\sum j=1Mvj\cdot yij+\sum j=1Nuj\cdot xij\theta\geq 1-\sum j=1Mvj\cdot yij+\sum j=1Nuj\cdot xij$
$\theta\leq 1\theta\leq 1$

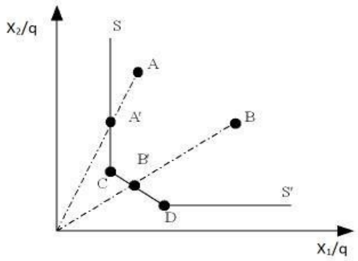
To specify a different linear programming model, a change from u and v to μ and v is made. This form is known as the multiplier form of linear programming model. Using binary linear programming, the equivalent envelope form of this problem is obtained with the following formula (Ramanathan, 2003):

$\theta=\sum i=1I\lambda i\cdot(1-\sum j=1Mvj\cdot yij+\sum j=1Nuj\cdot xij)\theta=\sum i=1I\lambda i\cdot(1-\sum j=1Mvj\cdot yij+\sum j=1Nuj\cdot xij),\ \lambda i\geq 0\lambda i\geq 0$
$\sum i=1I\lambda i=1\sum i=1I\lambda i=1$

This formula represents the numerical value of θ, expressing the vector λ. The envelope form created contains several constraints compared to the multiplier form, thus generally being the preferred solution form (N+M<I+1). Here, the value of θ is obtained from the efficiency score of the i-th firm. According to Farrell, $\theta \leq 1$. Hence, it represents the technically efficient firm on the boundary. The linear programming form must be solved for each firm. Therefore, the value of θ is obtained separately for each firm (T. Coelli, 1996).

Data envelopment analysis, as a non-parametric frontier analysis, may pose some challenges in efficiency measurement. The problem encountered here stems from the fact that the piecewise linear boundaries run parallel to the axes. This situation is mostly observed in non-parametric methods, whereas it rarely occurs in parametric methods. To explain this problem, Figure 1 will be utilized.

Figure 1: Efficiency measurement and input slack (Coelli, 1996)



To illustrate the problem in the figure, efficient firms C and D are compared with inefficient firms A and B. For the inefficient firms A and B, the efficiency ratios are measured as OA'/OA and OB'/OB, respectively. In the figure, it is investigated whether the input amounts used by the input surplus firms A and B can be reduced while maintaining the same output levels. This situation is known as input slack in the literature. Similarly, the output slack situation arises when there are more inputs or multiple outputs. Some authors within data envelopment analysis suggest considering and reporting both Farrell's measurement and non-zero input and output slacks for a more accurate technical efficiency indicator. If:

$\sum j=1Mvj\cdot yij-\sum j=1Nuj\cdot xij=0\sum j=1Mvj\cdot yij-\sum j=1Nuj\cdot xij=0$

Then, the output slack for the i-th firm will be 0. Similarly:

$\sum j=1Nuj\cdot xij-\sum j=1Mvj\cdot yij=0\sum j=1Nuj\cdot xij-\sum j=1Mvj\cdot yij=0$

Then, the input slack will be 0. This situation will be elaborated on in detail regarding the input-oriented CRR efficiency (T. J. Coelli, Rao et al., 2005).

2.2. Input-oriented CRR model

The input-oriented CRR model investigates how much the input quantity needs to be reduced to reach the existing output level without changing the output quantity. In a data envelopment analysis, if there are n decision-making units with m inputs and s outputs, the model aims to maximize the output/input ratio for the jth decision-making unit, which can be expressed as a fractional form of the VZA model (Charnes et al., 1978):

$\sum r=1 s vkr \cdot yrj \sum i=1 m uki \cdot xij \sum i=1 m uki \cdot xij \sum r=1 s vkr \cdot yrj$

Here, $vkrvkr$ and $ukiuki$ represent the weights given by the kth decision-making unit for the rth output and ith input, respectively (Charnes, Cooper et al., 1978).

Constraint:

$j=1,\ldots,nj=1,\ldots,n$
$vkr\geq 0,uki\geq 0vkr\geq 0,uki\geq 0$

The constraint added in Equation 2.17 ensures that the efficiency score of each decision-making unit does not exceed 1. The additional constraints in Equations 2.18 and 2.19 prevent negative input and output weights. Symbols used in the equations:

- kk: Number of decision-making units
- $vkrvkr$: Weight given to the rth output by the decision-making units
- $ukiuki$: Weight given to the ith input by the decision-making units
- $yrjyrj$: Output produced by the jth decision-making units
- $xijxij$: Input used by the jth decision-making units
- nn: Number of decision-making units (Cvetkoska, 2011).

To solve this model based on linear programming principles, Cooper et al. transformed it in 1962. As a result of the transformation, the objective function of the model is expressed as:

Maximizeθ Maximizeθ

Constraints:

$\theta \cdot \sum i=1 m \sum r=1 s vkr \cdot yrj \leq \sum i=1 m \sum r=1 s uki \cdot xij \theta \cdot \sum i=1 m \sum r=1 s vkr \cdot yrj \leq \sum i=1 m \sum r=1 s uki \cdot xij$
$vkr\geq 0,uki\geq 0vkr\geq 0,uki\geq 0$
$\theta\leq 1\theta\leq 1$
$\theta\geq 0\theta\geq 0$

The input oriented CRR model, developed under the assumption of constant returns to scale, measures relative total efficiency. Data envelopment analysis models can be expressed in primal and dual forms, like linear programming models. The dual model is preferred as it provides more important managerial insights compared to the primal model. The dual form of the input-oriented CRR model is shown as follows (Ramanathan, 2003):

Minimize$\sum i=1 m \sum j=1 n \mu ij - \sum r=1 s \sum j=1 n \lambda rj$Minimize$\sum i=1 m \sum j=1 n \mu ij - \sum r=1 s \sum j=1 n \lambda rj$

Subject to: $\mu ij\geq 0,\lambda rj\geq 0\mu ij\geq 0,\lambda rj\geq 0$

$\sum i=1 m \mu ij \cdot xij - \sum r=1 s \lambda rj \cdot yrj \geq 0 \sum i=1 m \mu ij \cdot xij - \sum r=1 s \lambda rj \cdot yrj \geq 0$
$\sum r=1 s \lambda rj \cdot yrj - \sum i=1 m \mu ij \cdot xij = 0 \sum r=1 s \lambda rj \cdot yrj - \sum i=1 m \mu ij \cdot xij = 0$
$\sum j=1 n \mu ij \cdot xij - \sum j=1 n \lambda rj \cdot yrj = 1 \sum j=1 n \mu ij \cdot xij - \sum j=1 n \lambda rj \cdot yrj = 1$

The elements of the "max-slack" solution for the input-oriented CRR model are defined as $\theta*\theta*$, $\lambda*\lambda*$, $s-s-$, $s+s+$. If the "max-slack" solution is $s-=0s-=0$ and $s+=0s+=0$, it is called a "zero-slack" solution. In this case, $\theta*=1\theta*=1$, and since there is no surplus in input quantity or shortage in output quantity, the decision-making units are efficient. Therefore, if $\theta*<1\theta*<1$ and $s-s-$ or $s+s+$ slack is not equal to zero, the decision-making units are inefficient. It is necessary for $\theta*\theta*$ and the slacks to be equal to zero for the efficiency of decision-making units. Solutions and transformations performed for the input-oriented CRR model can also be applied to the output-oriented CRR model (Cooper et al., 2001).

2.3. Banker, Charnes, and Cooper Model

The assumption of constant returns to scale (CRS) is considered realistic only when all decision-making units operate at the optimal scale. However, imperfect competition conditions, financial constraints, government intervention, and similar situations may prevent decision-making units from operating at the optimal scale. In contrast to this, Banker, Charnes, and Cooper propose an extended version of the CRS model based on the assumption of Variable Returns to Scale (VRS). The linear solution of the VRS model, obtained by adding the convexity constraint to the linear solution of the CRS linear programming model, is shown below.

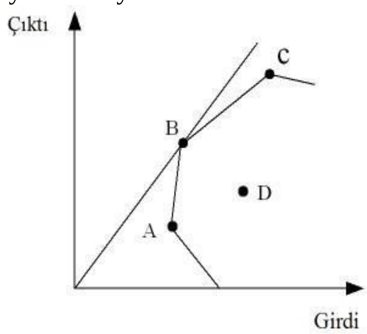
Objective function:
MaximizeθMaximizeθ

Constraints:
$\theta\cdot\sum i=1m\sum r=1svkr\cdot yrj\leq\sum i=1m\sum r=1suki\cdot xij\theta\cdot\sum i=1m\sum r=1svkr\cdot yrj\leq\sum i=1m\sum r=1suki\cdot xij$
$vkr\geq0,uki\geq0vkr\geq0,uki\geq0$
$\theta\leq1,\theta\geq0\theta\leq1,\theta\geq0$

Similar to the CRS model, the solution of the VRS model also occurs in two stages. In the first stage, θ is minimized. In the second stage, input surpluses and output shortages are maximized to satisfy the optimal decision value equality. Considering the variable returns to scale, the VRS model provides efficiency scores as technical efficiency and scale efficiency, unlike the CRS model. This model, which divides the efficiency of decision-making units into scale efficiency and technical efficiency, defines whether the inefficiency of inefficient decision-making units arises from scale inefficiency or operational inefficiency (T. Coelli, 1996). Additionally, by adding this constraint to the model, the types of returns to scale for decision-making units can be determined. Accordingly, if the sum of the calculated λ for the jth decision-making unit is greater than one, the decision-making unit represents decreasing returns to scale; if it is less than one, increasing returns to scale; and if it is equal to zero, constant returns to scale are represented.

Another difference between the CRS and VRS models is that in the VRS model, the decision-making unit k's objective function includes the free slack variable vv, which represents the weights related to output. With this free slack variable and constraints, the model transforms from a linear structure to a convex structure (Figure 2).

Figure 2: Efficiency boundary for VRS and CRS models (Coelli, 1996)



In Figure 2, the efficiency boundary for CRS and VRS models is examined with decision-making units A, B, C, and D. For the CRS model, the efficiency boundary is a straight line connecting the origin to point B, while for the VRS model, it is a piecewise convex structure containing points A, B, and C. Due to this convex structure, the VRS model exhibits the property of variable returns to scale. Therefore, in the figure, the AB line segment represents increasing returns to scale, point B represents constant returns to scale, and the BC line segment represents decreasing returns to scale. Additionally, compared to the CRS model where only point B is efficient, in the VRS model, points A, B, and C are efficient. The VRS model can also be examined both input-oriented and output-oriented (Algın, 2014).

2.4. Input-oriented BCC model

The mathematical representation of the input-oriented primal and dual forms of the model developed by Banker, Charnes, and Cooper under the assumption of variable returns to scale is shown below.

Primal form of the input-oriented BCC model:

MaximizeθMaximizeθ

Subject to:Subject to:

$\sum i=1m\sum r=1svkr\cdot yrj\leq\theta\cdot\sum i=1m\sum r=1suki\cdot xij\sum i=1m\sum r=1svkr\cdot yrj\leq\theta\cdot\sum i=1m\sum r=1suki\cdot xij$
$vkr\geq0,uki\geq0vkr\geq0,uki\geq0$
$\theta\leq1,\theta\geq0\theta\leq1,\theta\geq0$

Dual form of the input-oriented BCC model:
Minimize$\sum i=1m\sum j=1n\mu ij-\sum r=1s\sum j=1n\lambda rj$Minimize$\sum i=1m\sum j=1n\mu ij-\sum r=1s\sum j=1n\lambda rj$

Subject to:Subject to:

$\sum i=1m\mu ij\cdot xij-\sum r=1s\lambda rj\cdot yrj=0\sum i=1m\mu ij\cdot xij-\sum r=1s\lambda rj\cdot yrj=0$
$\sum r=1s\lambda rj\cdot yrj-\sum i=1m\mu ij\cdot xij=1\sum r=1s\lambda rj\cdot yrj-\sum i=1m\mu ij\cdot xij=1$
$\mu ij\geq0,\lambda rj\geq0\mu ij\geq0,\lambda rj\geq0$

For decision-making units to be relatively technically efficient, the value of the objective function in the primal model should be $\theta\theta$, and in the dual model, it should be $1/\theta1/\theta$. For inefficiency cases, in the primal model, $\theta<1\theta<1$, and in the dual model, $\theta>1\theta>1$ (Behdioğlu & Özcan, 2009).

3. Conclusion

DEA is an important tool used to evaluate the efficiency of decision-making units. The Charnes, Cooper, and Rhodes (CCR) model measures the overall efficiency of decision-making units based on the assumption of constant returns to scale. This model provides insights by identifying inefficient resources. On the other hand, the model developed by Banker, Charnes, and Cooper (BCC) based on the assumption of variable returns to scale has added a new dimension to data envelopment analysis. The BCC model offers a more flexible approach in evaluating the efficiency of decision-making units. Through its primal and dual forms, the input-oriented BCC model provides information regarding the optimal use of inputs by decision-making units. These models are essential tools for analyzing and improving the efficiency of decision-making units. Maximizing the objective function in the primal form or minimizing it in the dual form helps enhance the technical efficiency of decision-making units. Additionally, the solutions obtained from these models can determine the returns to scale, thus providing a better understanding of the activity levels of decision-making units. In conclusion, data envelopment analysis models can assist businesses in utilizing their resources more effectively and improving their performance. These models can be used to identify the strengths and weaknesses of decision-making units and contribute to the strategic decision-making processes of businesses.

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