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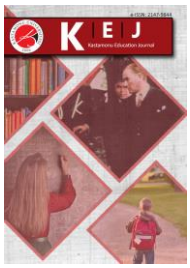

| Research Article / Araştırma Makalesi |

A Content Analysis of the Numbers and Operations Learning Area-themed Articles Published in Turkey Related to Their Topic Trends and Results

Türkiye'de Yayımlanan Sayılar ve İşlemler Öğrenme Alanı Temalı Makalelerinin Konu Eğilimlerine ve Ulaşılan Sonuçlarına İlişkin İçerik Analizi [1]

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Keywords
1. Arithmetic Operations
2. Content Analysis
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Abstract

*Purpose:*This research aims to explore the level/case determination and experimental, related to any variable, articles, which have been conducted on the numbers and operations learning area in mathematics education and published in the educational journals in Turkey, within the scope of their topic trends and results.

Design/Methodology/Approach: Within the scope of the research designed with the descriptive content analysis method, 301 articles were determined and the obtained results were analysed with the content analysis method.

Findings: The result is as follows: In the articles, which were focused on the level/situation determination, mainly the skill articles such as various misconceptions/ errors/ difficulties/ difficulties in fractions/operations with fractions and natural numbers/operations with natural numbers or problem posing skills, mental calculations/ arithmetic operations skills were examined. In the articles on problem posing related to fractions, it was determined that the conceptual difficulties were common and the students overgeneralised about the concept and operations of fractions. In addition, it was observed that the participant had low skill levels of problem posing and problem-solving related to the operations with fractions and natural numbers. In the articles, whose study focus was experimental, mostly the achievement articles in the form of mathematics/academic/student/learning, number/rhythmic counting, and problem-solving success with operations, for fractions and natural numbers, were discussed. When the results of the articles conducted in this scope were analysed, it was determined that the independent variables in the research increased the achievement or performance of the mentioned dependent variables.

Highlights: It was mostly focused on the articles aimed at determining the level or situation in the form of misconceptions/errors/difficulties about fractions and natural numbers in the reviewed articles. Considering this, further studies will be conducted, and academic studies examining the experimental, relational and predictive situations to eliminate misconceptions/errors or difficulties can be carried out within the scope of this subject.

Öz

Çalışmanın amacı: Türkiye'de, matematik eğitiminde, sayılar ve işlemler öğrenme alanında yapılan ve Türkiye'deki eğitim dergilerinde yayımlanan herhangi bir değişkene yönelik düzey/durumun belirleme ile deneysel odaklı makaleleri; konu eğilimleri ve sonuçları kapsamında incelemektir.
Materyal ve Yöntem: Betimsel içerik analizi yöntemi temel alınarak tasarlanan araştırma kapsamında 301 makale belirlenmiş ve ulaşılan veriler içerik analizi yöntemiyle analiz edilmiştir.
Bulgular: Çalışma odağı düzey/durum belirlemeye yönelik olan makalelerde ağırlıklı olarak kesirler/kesirlerle işlemler ve doğal sayılar/doğal sayılarla işlemlere yönelik çeşitli kavram yanılgısı/hata/zorluk/güçlükler veya problem kurma becerisi, zihinsel hesaplama/aritmetik işlem becerisi şeklindeki beceri makaleleri ele alınmıştır. Kesirlere yönelik problem kurma makalelerinde daha çok kavramsal boyutta güçlükler yaşandığı, kesir kavramına ve işlemlerine yönelik öğrencilerin aşırı genelleme yaptıkları belirlenmiştir. Bununla birlikte katılımcıların kesirler ve doğal sayılarla işlemlere yönelik problem kurma ve problem çözme beceri düzeylerinin düşük olduğu görülmüştür. Çalışma odağı deneysel nitelikte olan makalelerde ise daha çok kesirler ve doğal sayılara yönelik matematik/akademik/öğrenci/öğrenme, sayı/ritmik sayma ve işlem ile problem çözme başarısı şeklindeki başarı makaleleri ele alınmıştır. Bu kapsamda yürütülen makalelerin sonuçları analiz edildiğinde ise; genel olarak araştırmalarda yer alan bağımsız değişkenlerin belirtilen bağımlı değişkenlerdeki başarıyı veya performansı artırdığı belirlenmiştir.
Önemli Vurgular: İncelenen makalelerde daha çok kesirler ve doğal sayılarla ilgili yaşanan kavram yanılgısı/hata/güçlük şeklinde düzeyi veya durumu belirlemeye yönelik makalelerde yoğunlaşıldığı buradan hareketle de yapılacak olan yeni çalışmalarda bu konu kapsamında yanılgı/hata veya güçlüklerin giderilmesine yönelik deneysel, ilişkisel ve yordama durumlarının ele alındığı akademik çalışmalara yer verilebilir.

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INTRODUCTION

Individuals having competencies integrated with the knowledge and skills that they will need in every field is one of the priorities of today's education system. In the Mathematics Curriculum, which consists of various competencies such as digital competency, learning to learn, communication in the mother language, communication in foreign languages etc, "Mathematical competency" is explained as the individuals' ability to use and ask for mathematical thinking skills built on a solid arithmetic basis (The Ministry of National Education [MoNE], 2018). The necessity of prioritising the numbers that form the theme of mathematics teaching emerges as a significant factor for individuals with these competencies, especially mathematical competence, to complete mathematics education, including abstract and high-level skills (Baki,2008; Christou & Vosniadou, 2012; Güler, 2017; Vlassis, 2004). Besides, it is stated that the integration of number development with other areas of the curriculum is another remarkable point with the expression of the NTCM (2000, p. 79) as "A student who works with numbers should be able to associate his/her work with other mathematics subjects. For instance, operational fluency (handling the operations correctly and quickly)… both develops students and can improve students' knowledge of data, knowledge of patterns that support the development of rhythmic counting, and algebraic thinking, shape and space, and the number that helps students develop estimation skills" (Van De Walle, 2013).

This importance and necessity that the numbers have also made them come to the forefront in the curricula and studies published from the past to the present. Thus, when the curricula developed by the MoNE between 2005 and 2018 were analysed, it was determined that the most outcomes and class time were given to the Numbers and Operations learning area (İlhan & Aslaner, 2019; MEB, 2005, 2009, 2013, 2015, 2018). The Numbers and Operations learning area consists of the various sub-learning areas such as numbers, sets of numbers and operations with them, relations between numbers, fractions, ratios and proportions, etc. (MoNE, 2018). Within the scope of all these sub-learning areas, it was noticed that there were many studies on various topics such as natural numbers and operations with natural numbers (Albayrak et al., 2019; Ercive & Narlı, 2019; Paydar & Doğan, 2021; Tuluk & Akyüz, 2019), fractions and operations with fractions (Altıparmak & Palabıyık, 2019; Topçu & Gürefe 2020; Özer et al., 2020), decimal notation (Işık et al., 2012; Karataş et al., 2021), percentages (Erdem et al., 2018; Yapıcı & Altay, 2017), multipliers and multiples (Karakuş & Yeşilpınar, 2018), sets (Biber & Tuna, 2016; Yücesan, 2011), integers and operations with integers (Berkant & Yaren, 2020; Bozkurt & Polat, 2011; Erdem et al., 2015; Kiraz & Işık, 2020; Şengül & Zengin, 2015), rational numbers and operations with rational numbers (Altun & Çelik, 2018; Gürbüz & Birgin, 2008; Macit & Nacar, 2019; Yenilmez & Yıldız, 2018), ratio and proportion (Deveci, 2021; Güler & Didiş Kabar, 2017; Şengül & Erdoğan, 2017), exponential expressions (Güzel & Yılmaz, 2020; İymen & Duatepe Paksu, 2015; İymen İkizoğlu & Duatepe Paksu, 2016) ve köklü ifadeler (Aksu et al., 2013; Aydoğdu, 2020; Toluk Uçar, 2015).

Handling the general tendencies related to a subject in specific terms is regarded as significant even in terms of shedding light on further studies in addition to seeing how the studies tend to be (Cohen et al., 2007; Erdem, 2011). Considering this, it is thought that investigating the knowledge of the participants about the trends and current situations of the studies related to their fields with content analysis will contribute much to the literature (Falkingham and Reeves, 1998). In this scope, it was noticed that there are some studies (Arı & Demir, 2020; Kutluca et al., 2018; Ulutaş & Ubuz, 2008; Yaşar & Papatğa, 2015; Yıldız Altan et al., 2021) in the literature investigating the general trends related to mathematics education. In addition, it has been determined that there are studies that investigate the trends related to a specific topic such as metacognition (Baş and Özturan Sağırlı, 2017; Kandal and Baş, 2022), technology-supported education (Bayram, 2019; Tatar et al., 2013; Kutluca et al, 2016), mathematical model and modelling (Albayrak & Çiltaş, 2017; Birgin & Öztürk, 2021; Yenilmez & Yıldız, 2019), mathematics anxiety (Toptaş & Gazel, 2018), realistic mathematics education (Tabak, 2019), problem-solving (Coşkun & Soylu, 2021) and problem themed (Özturan Sağırlı & Baş, 2020).

Within the light of these reviewed studies, no studies have been encountered related to determining the level/situation in the numbers and operations learning area in mathematics education and a study in which experimentally handled research was analysed and evaluated. In light of all these expressions, with this study, it was aimed to present the trends and results of the topics of the articles to researchers and educators as a whole by examining the articles in which the level/situation for the learning of numbers and operations in Turkey was determined and experimentally handled, and to contribute to mathematics education by forming the basis for new studies to be conducted in this scope. Considering this purpose, the following research questions were asked:

1. What are the subject trends and results of the level/situation determination articles published in education journals in Turkey for preschool, primary and secondary school levels with the theme of numbers and operations learning area in mathematics education in Turkey?

2.What are the subject trends and results of the experimental articles published in education journals in Turkey for preschool, primary and secondary school levels with the theme of numbers and operations learning area in mathematics education in Turkey?

METHOD

It was aimed to review the articles on the numbers and operations sub-learning theme, which have been conducted on mathematics education and published in Turkey in terms of their topic trends and results. In this context, the study was designed based on a descriptive content analysis method considering its topic and purpose. Descriptive content analysis, which is among the content analysis types, in which a general trend is specified by investigating and arranging the quantitative and qualitative research, put forth separately, thus provides a significant reference to further studies in the literature (Cohen et al., 2007; Çalık & Sözbilir, 2014; Selçuk et al., 2014).

Data Collection

During the data collection process, firstly the faculties of education journals, social sciences institution journals in Turkey, and 124 private institutions and organisations' journals, which can be viewed online, in Turkey were determined. 1617 volumes and 5021 issues were examined in total by including all the issues of 2020 of these journals in the process. In the second part of the data collection process, each issue was handled separately. The articles were analysed according to the sub-learning areas and functions that take place in the curriculum with the keywords as numbers, natural numbers, integers, four operations and/or skills, arithmetic, arithmetic operations, ratio-proportion, sets, factors and multiples, proportional reasoning, fractions, operations with fractions, rational numbers, operations with rational numbers, exponential expressions and square roots. In addition, the articles which include the words such as verbal problems, problem-solving and problem-posing were examined according to the outcomes of the numbers and operations learning area. In this stage, the articles with foreign samples were excluded from the research. In the third stage of the data collection process, it was aimed to classify the articles by considering the numbers and operations learning and sub-learning areas in the scope of the 2013 preschool teaching curricula and 2018 primary and secondary school Mathematics Course Curriculum. For this purpose, it was observed that there were articles covering the secondary education program among the recorded articles. Since the secondary and high school functions were within the scope of a different learning area and the numbers and algebra in the high school curriculum were together in a learning area, the articles which were related to the high school curriculum were excluded from the research. At the end of this research, a total of 301 articles were recorded and prepared for data analysis.

Data Analysis

A content analysis method was used in the reached data analysis in the study. The obtained data are presented to the reader by processing unnoticed concepts and themes in the content analysis more deeply with descriptive analysis (Yıldırım & Şimşek, 2018, p. 242). The data analysis process of the experimental and level/situation articles handled related to the numbers and operations learning area consisted of two parts: the analysis of topic trends and reached results. In the first stage of the data analysis related to the articles' topic trends, 301 reached articles were coded according to the sub-learning areas that take place in two different levels as the primary and secondary school within the scope of numbers and operations learning area primarily in the secondary school curriculum. Based on the template prepared by Baş and Özturan Sağırlı (2017), the articles were examined according to their methods within the scope of two study focuses: experimental and level/situation. In the last stage of the data analysis related to the topic trends, the articles that take place within the scope of each study focus were analysed within the scope of the determined categories as achievement, skill, misconception, misconception/complexity/difficulty/error, behaviour, persistence, perception, affective features, opinion/thought/thought process, strategy/method, competence, perception/image/schema/representation type, teaching/procedure quality, readiness level and others. During the data analysis process related to the categories constituted within the scope of the study focuses, the features which have also been mentioned above such as achievement, skill, misconception etc. were taken into consideration. For instance, the articles which were reviewed in terms of the achievement level and situation in the "Operations with Natural Numbers" sub-learning area were examined by determining various sub-headings related to achievement such as problem-solving achievement, problem posing achievement, number sense performance and number detection achievement and handled within the scope of the category of achievement. Throughout this analysis process, a draft list was created as the categories under each study focus would be the main title and the codes under the categories would be the sub-titles. The codes under the categories were stated in the finding section. The created study focus and category list are presented in Figure 1.

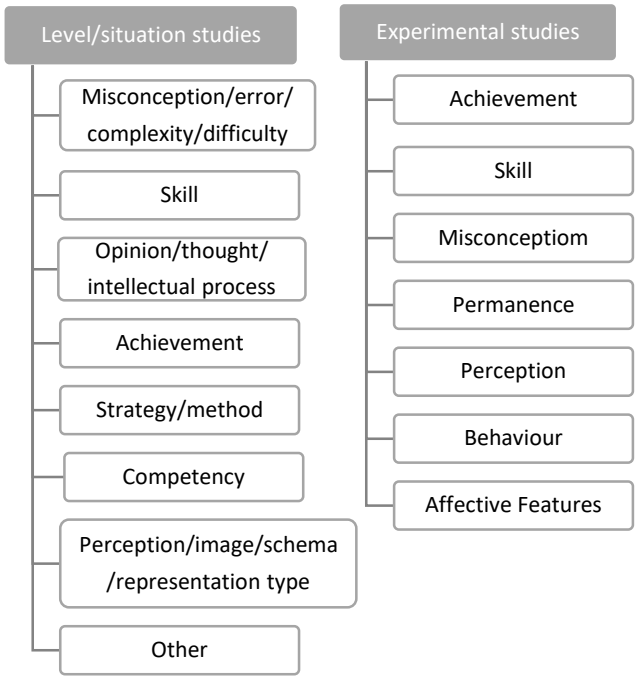


Figure 1. Study focus and category list used in the data analysis process

The analysis process which was performed by using the study focus and category list in Figure 1, was carried out as follows.

1. In the process, among the expressions that take place in the purpose, problem and sub-problems of the articles, only those related to the numbers and operations learning area were subjected to the analysis. For instance, the article, carried out experimentally, with the heading of "The Effect of Mathematics Teaching with Caricatures on the Students' Academic Achievements and Attitudes", was evaluated in the category of achievement since the effect of teaching with caricatures within the scope of Multipliers and Multiples sub-learning area on the academic achievements of the student in the learning area of numbers and operations was examined. The part of attitude towards mathematics was excluded from the analysis since it lay beyond the scope of the numbers and operations learning area.

2. During the analysis process, sometimes the same article topics were analysed under different categories. For instance, an article whose basic aim was to "investigate the explanations of primary and secondary school mathematics teachers' misconceptions and their views on the underlying causes" was coded under two different categories as misconceptions and opinions.

3. During the process, the analysis was made by considering the sub-problems rather than the aims of the articles. For instance, when the article, whose purpose was to "investigate the effect of number sense-based teaching on the students' self-competencies and performances for mathematics" was analysed by considering its aims, the sub-problems of the study were taken into consideration to clarify the situation mentioned with the "self-competency or performance related to mathematics." In this context, from the sub-problems of the article as "Is there a significant difference between number sense, mathematics self-efficacy, number sense self-efficacy, the realisation of mathematics in daily life, problem-solving achievement and mathematics achievement of sixth-grade students who have or have not undergone a number sense-based teaching process?", the problems that included the self-competencies related to number sense, problem-solving achievement and mathematics achievement were analysed.

4. In the analysis of the articles, codes and categories were created by considering how the concepts were used in the articles. For instance, the article named " Intuitive development of primary school students in the concepts of integer", is realised to be examined within the scope of both categories of achievement and skill at first sight. However, when the content of the article was reviewed, it was noticed that the primary school students' intuitive mathematics skills for integers, ordering in integers, and adding and subtracting in integers were studied and in this scope, it was understood that it was necessary to analyse the article in the skill category within the scope of the integers and operations in integers sub-learning and the coding was performed in this direction.

5. During the analyses, it was realised that each article was analysed within the scope of its topic and sometimes an article was examined within the scope of more than one sub-learning. In this context, the code and category analysis related to different sub-learning areas within the scope of the same article was performed. For instance, in the article named "the approaches of pre-service mathematics teachers to the possible student mistakes about exponential and radical numbers", the pre-service teachers' approaches to errors that students may have in terms of both exponential and radical numbers, as well as their opinions and solution suggestions about the causes of errors were analysed. The article was analysed with the categories as misconception/error/problem/difficulty and image/perception/opinion/thought/schema and by coding separately both to the exponential expressions and square root expressions sub-learning areas.

In the stage that constitutes the second part of the data analysis process, the articles analysed according to their methods related to the numbers and operations learning area were examined according to their results. In this stage, to analyse the articles within the context of their results, while the results of the qualitative studies such as case, opinion, perception, strategy/method related to the level/situation articles within the scope of the study focus determined in the topic part were coded by taking into consideration directly, the results of the experimental articles were coded as there is/no difference, there is/no effect. Accordingly, whether the studies referring to the same results were encountered in similar studies or not was determined first. When the studies with the same result existed, they were presented first. For instance, in the studies in which the effect of class level on the students' number perception, estimation and problem-solving achievement were investigated, and when it was seen the class level affected these three different achievement significantly, this situation was presented as "It was observed that the class level significantly affects these three separate achievements, and with the increase in class levels, student performances towards these concepts also increase." In addition, it was paid attention to the content of the subject part of the articles when the findings related to this section were expressed.

In the process of collecting, examining and analyzing data regarding each sub-problem in the study, we worked with an academician who is an expert in the field of qualitative methods, and arrangements were made within the scope of the processes in line with their feedback and suggestions.For instance, in the S316 coded research, whose method was not mentioned, the data related to the secondary school fifth graders' problem-posing skills were subjected to descriptive analysis after collecting with the help of problem-solving and posing activity papers. By considering the topic, problem, data collection tools and data analysis, it was analysed with the case study code in the qualitative category by reaching a consensus.

In the analyses, the coding and analysis process was repeated after waiting 8 weeks by the researcher for the reliability of the coding. The fit rate of the obtained results and codings to the earlier coding was found as 93%. The parts which were coded separately were clarified by taking the professional's opinion and the findings were made ready to report.

FINDINGS

The articles examined in the field of numbers and operations were analysed within the scope of the two study focuses such as level/situation determination and experimental and of all the sub-learning areas. Because the high number of sub-learning fields made the presentation of the findings difficult, the findings were presented as follows in codes and categories of the study focus. In addition, to reflect the purposes of the researchers in findings as much as possible, the terms they included were tried to be presented as they were.

Findings related to the topic trends and results of the level/situation determination-focused studies

Within the context or a part of the 212 (70%) articles which were examined, the level or situation of a feature in the numbers and operations learning area was tried to be determined. The codes and categories used in the articles examined in this scope are presented in Figure 2.

Misconception/error/complexity/difficulty category	Achievement category
Problem posing (14)	Problem solving achievement (8)
Pointing on number line (7)	Problem posing achievement (6)
Problem solving (7)	Number sense/feeling performance (4)
Application for operations (6)	Forming/reinforcing knowledge achievement(3)
Order (6)	Performance of using knowledge and skill operationally/conceptually (2)
Fraction-rational number-integer relationship (3)	Modelling/model using performance (2)
Modelling verbal problems (3)	Academic achievement (2)
Reading-writing numbers/digit value (2)	Performance for multiplication and division (1)
Part-whole relationship (2)	Number perception achievement (1)
Skill category	Strategy/method category
Problem posing skill (10)	Problem solving/posing strategies (9)

Mental calculation/arithmetic operation skill (5)	Mental calculation/estimation strategies (6)
Mathematical realisation/discrimination (4)	Proportional reasoning strategies (4)
Problem solving skill (3)	(in fraction/decimal fractions) İnformal strategies for multiplication and division (2)
Proportional reasoning skill (3)	(in sorting with fractions) Reference point strategies (1)
Mathematical modelling skill (3)	Adidactic teaching method (1)
Mathematical association/relational thinking (2)	Competence category
Ability to switch between representations (2)	Pedagogical knowledge and context knowledge competency (14)
Number sense skill (2)	Competency to identify and correct misconception/difficulty/error (7)
Problem classification skill (1)	Problem posing competency (2)
Skill of using in daily life (1)	Perception/image/schema/representation type category
Metacognition skill (1)	Perception for number and number sets/comprehension(5)
Abstraction skill (1)	Perception for set concept/ schema (3)
Opinion/thought/intellectual process category	Fraction/rational number concept image (3)
Thoughts/opinions/intellectual processes for teaching/method/activity/evaluation process (11)	Division operation (in rational numbers) / perception of process choice (in verbal problems) (2)
Opinion/point of view for problem posing-solving achievement/type (7)	Problem solving/rational-irrational representation type (2)
Opinions about misconception/difficulty(3)	Sense of infinity (1)
Opinions for mathematical modelling/using (3)	Category of other
Thoughts/opinions for the concept of infinity (2)	Verbal problem type (2)
Opinion for caunting and the concept of number (1)	Fraction model (1)

Figure 2. Codes and categories handled in level/situation articles

As presented in Figure 2, it is seen that the articles examined related to level/situation determination in the numbers and operations learning area consisted of 8 categories as misconception (53), skill (37), opinion/thought/intellectual process (28), achievement (27), strategy/method (25), competency (22), perception/image/schema/representation type (16) and other (3); and each category consists of different codes. Each of the codes and categories created in the articles was examined under different heading.

1.Misconception/error/complexity/difficulty category: It was determined that the feature which was studied most was misconception. The sub-learning areas that the articles take place in the scope of the category are the operations with fractions (16), fractions (11), operations with natural numbers (7), addition in natural numbers (3), subtraction in natural numbers (3), multiplication in natural numbers (3), division in natural numbers (2), natural numbers (2), decimal notation (2), exponential expressions (1), square root expressions (1), integers (1) and operations with integers (1). The content of the category and the results of the articles under the category are explained below separately under each code in the category.

1.1. When the results of the articles handled with code Problem Posing (14) are analyzed, it is generally seen that the conceptual dimension of division is ignored in dividing fractions. There is more difficulty at the conceptual level in posing problems for dividing fractions than in posing problems for multiplication. It was determined that there were deficiencies in attributing meaning to, multiplication in whole number fractions, dividing two fractions and fraction numbers. Besides, it was determined that there were difficulties arising from reasons such as conceptual difficulties, teachers' habits in the teaching process, difficulties in language, lack of data in general in the focus of the errors in problem posing studies related to the addition and subtraction in

fractions. In addition, it was noticed that except for the problem-posing activities of symbolic expressions, the error, which was seen to ignore the part-whole relationship, continued in problem posing activities for open-ended verbal stories. Moreover, it was determined that the problems experienced in the problem posing studies that necessitate four operations in natural numbers originated from the logic error, deficiencies in using of language, posing inconvenient problems and using incomplete data.

1.2.When the results of the articles examined with the pointing on the number line (7) code, it was determined that the errors or difficulties encountered in fractions were due to the reasons such as moving along the integers on the number line as much as the number in the denominator and taking the number in the denominator, ignoring the minus in negative rational numbers and showing its place on the number line like a positive number, and drawing a number line containing only natural numbers. It was determined that in pointing decimals in the number line the students had the misconceptions such as ignoring the comma, treating the part of the number after the comma as an integer, starting to count intervals on the number line from the wrong place.

1.3. When the results of the articles examined with the problem solving (7) code were analysed, it was determined that the students had difficulties in using real life situations in problems that require comparing and ordering in fractions and concrete object representations in problem solving. On the other hand, it was observed in 2 articles that the mistakes made by pre-service teachers and secondary school students while solving non-routine problems involving ordinal numbers are generally +/-1 error type (the problem result for ordinal numbers is calculated as incorrectly as +1 or -1) and that the pre-service mathematics teachers made ± 1 error type mostly in type I problems (problems that can be solved by modelling correctly by adding or subtracting two numbers). It was found that the primary school students had difficulties in solving the arithmetic verbal problems that required multiplication and division, they usually determined their operation preferences according to the keywords in the problem, generalised the rules of addition and subtraction instead of multiplication and division, and had difficulty in understanding the rule of multiplying by 0 and 1, continuing the operation without shifting digits. Moreover, it was observed in an article that the secondary school mathematics teachers had problems in application of the operation priority rule in problem solving.

1.4. When the results of the articles examined with the ordering (6) code, in the articles on fractions and decimal notation, it was determined the students made various errors as they thought that the fraction with a large denominator was large, that the students who wanted to equate the numerator and denominator only expanded the numerator or denominator, considered the number of fractions as a whole and sorted according to the total size of the numerator and denominator, or sorting the numerator and denominator among themselves. Besides, it was determined that the errors in the ordering for decimal notation were caused by the students' ignoring the comma and perceiving the part after the comma as an integer.

1.5. When the results of the articles with the code of application related to operations (addition (7), subtraction (6)and multiplication(6) operation) were analysed, it was observed that the most important learning difficulties of the students in the subjects related to addition-subtraction and multiplication in fractions were to think about the numerators and denominators of the fractions separately and to apply the rules they had learned about fractions to later subjects (for example, adapting the rule of addition to multiplication). In addition, it was determined in an article that the teachers had difficulties in teaching what negative integer means, subtraction of integers, and using counting stamps. In an article related to the expressions with exponents and square roots, the reasons for making mathematical mistakes were that teachers had mathematical concept errors, did not talk about the meaning of definitions and did not encourage students to ask questions, as well as students' unwillingness towards the lesson.

1.6. When the results of the articles examined with the fraction-rational number-integer relationship (3) code were analysed, it was determined that the students had misconceptions as dividing a number by 0 (thinking of 0 as an absorber) or division of 0 by a number (in the case that the number is not a rational number), scope for the set of numbers (thinking that rational numbers do not include integers and natural numbers), and the fraction line necessity (a number must have a fractional line or numerator and denominator for it to be a rational number).

1.7. When the results of the articles examined with the modelling verbal problems (3) code were analysed, it was determined that the students had problems in modelling verbal problems, discriminating the necessary information in problem solving and setting correct relationship in forming the given information in general. In addition, it was observed that the students focused only on the numeric data in problem solving and had difficulties in drawing correct diagram in the process.

1.8. When the results of the articles examined with the Reading-writing numbers/digit value (2) code were analysed, it was determined that the students had difficulties in reading and writing numbers, grouping numbers and writing groups according to place values after grouping numbers.

1.9. When the results of the articles examined with the part-whole relationship (2) code were analysed, it was determined that the students had misconceptions as symbolising a whole that is not divided into equal parts with a fractional expression, or that fractional expressions have equal magnitude without considering the reference size.

2.Skill category: The sub-learning areas of the articles in this category are the operations with natural numbers (11), operations with fractions (10), ratio and proportion (7), fractions (4), addition operation in natural numbers (4), subtraction operation in natural numbers (3), multiplication operation in natural numbers (3), natural numbers (2), division operation in natural numbers (1), rates (1) and sets (1). The content of the category and the results of the articles in the category were explained separately under each code which takes place in the category.

2.1. When the results of the articles examined with the problem posing skill (10) code were analysed, it was determined that the participants' basic operation, operations with fractions and problem posing skill levels for sets skills were low in general. It was also found in an article, in which the participants' problem posing skill related to the four operation was high, that the students were able to set verbal story problems mainly related to addition and subtraction. In addition, it was noticed that in an article both the teachers and the pre-service teachers mostly succeeded in posing problems such as combining / combining with result unknown / separation with change unknown / separation with result unknown / without change unknown.

2.2. When the results of the articles examined with the mental calculation/arithmetic operation skill (5) code were analysed, it was determined that the readiness levels of the primary school students related to the arithmetic operation skills were sufficient; however, the cardinal value principle, which is one of the counting principles, was less developed compared with the other principles in preschool children. Besides, it was encountered in an articles carried out with adults that the individuals who got and did not get formal education had similar mental calculation skills in addition and subtraction, and partially different in multiplication and division.

2.3. When the results of the articles examined with the mathematical realisation/discrimination skill (4) code were analysed, it was expressed in the articles conducted with pre-service teachers on operations with fractions and fractions that the pre-service teachers' skills in identifying and discriminating students' knowledge, learning strategies and misconceptions were high. It was referred in an article related to the topic of ratio-proportion that the students could not discriminate the additive and multiplicative ratio-proportion problems.

2.4. When the results of the articles examined with the mathematical modelling skill (3) code were analysed, it was observed that the pre-service teachers had not used the mathematical models related to fractions regularly and their model using levels decreased in time.

2.5. When the results of the articles examined with the problem solving skill (3) code were analysed, it was found that the participants' problem solving skill levels were low in general. As the students' class levels increased, their transitions from arithmetic solutions to algebraic solutions (although it was positive, this change and development was low) were low; they had difficulty in understanding the problems without solutions rather than the non-routine problems.

2.6. When the results of the articles examined with the proprotional reasoning skill (3) code were analysed, it was determined that while the students were able to solve the problems related to the ratio-proportion problems, they were not able to explain these problems and their skills of recognising inversely proportional relationships were lower than the skills of recognising directly proportional relationships.

2.7. When the results of the articles examined with the mathematical association/relational thinking skill (2) code were analysed, it was determined that the students mathematical association skills were low in an article conducted related to the topic of ratio-proportion. On the other hand, in a different article carried out with the secondary school students related to the operations with natural numbers, it was determined that the students were able to make the associations between the operations of addition-subtraction, addition-multiplication and multiplication-division by using commutative, distributive and associative properties.

2.8. When the results of the articles examined with the transition between representatives skill (2) code were analysed, it was observed that the students were more successful in using different representatives in addition/subtraction operations in fractions. In addition, it was found that the students were more successful in transitions between numeric-numeric, model-model and numeric-model transitions even in both operation types more than the other transitions.

2.9. When the results of the articles examined with the number sense skill (2) code were analysed, it was determined that the students' number sense was quite low and they tended to use rule-oriented solutions instead of using number sense in solving their problems about percentages in general.

2.10. When the results of the articles examined with the problem classification skill (1) code were analysed, it was observed that the students' problem classification skills related to the proportional reasoning problems were at a moderate level.

2.11. When the results of the articles examined with the skill of using in daily life (1) code were analysed, it was observed that the students, who did not apply to the primary school mathematics related to addition and multiplication in natural numbers in daily life, continued as if they used the materials in their solutions even in the situations in which they did not have materials (paper-pencil). In addition, it was determined that those, who preferred using the mathematics given in school in daily life practices, made calculations with easier and more proper solutions by grouping the given.

2.12. When the results of the articles examined with the metacognition skills (1) code were analysed, it was determined that the metacognitive skills that the students used in the solutions of the problems related to fractions most were orderly as the monitoring, estimating and planning skills.

2.13. When the results of the articles examined with the abstraction skill (1) code were analysed, it was determined that the students used the knowledge that they generated within the scope of the first problem in the next problem and they were able to abstract the linear relationship information correctly at a certain level in the process of creating linear relationship information for ratio-proportion.

3.Opinion/thought/intellectual process category: The sub-leraning areas of the articles which have been conducted within the scope of this category are; fractions (7), operations with fractions (6), operations with natural numbers (3), operations with integers (3), natural numbers (2), rational numbers (2), operations with rational numbers (2), multipliers and multiples (2), addition operation with natural numbers (1), subtraction with natural numbers (1), multiplication operation with natural numbers (1), division operation with natural numbers (1), decimal notation (1), rates (1), ratio and proportion (1), square root expressions (1) and exponential expressions (1). The content of the categories and the results of the articles under the categories are explained separately under each code in the relevant category.

3.1. When the results of the articles examined with the thoughts/opinions/intellectual processes for teaching/method/activity/evaluation process (11) code were analysed, it was determined that the participants claimed positive opinions for the applied teaching model/method/activity. It was detected that in a school in which the university model was used increased the interest towards mathematics related to the fractions, similar activities developed within the scope of the Babil counting system might be an effective mean of teaching, in the subject of rates studied with the information exchange technique, a majority of the students had positive thoughts related to the IET (Information Exchange Technique). In addition, the students expressed related to the learning environment based on the conceptual change approach that the courses became more entertaining, they understood the subject better and enabled to learning by doing in some parts of the course. The students' intellectual processes during the stages were investigated in an article in which the project work related to the fractions and decimal fractions. It was stated in the project that the factors such as acting together, using time well, producing products and presenting the product that is offered to individuals by working with a group are also crucial in addition to the intellectual process for the students.

3.2. When the results of the articles examined with the Opinion/point of view for problem posing-solving achievement/type (7) code were analysed, it was stated that, in the articles related to four operations in natural numbers and addition and subtraction operations in fractions, the problem posing enabled to the students' better comprehension and structuring a subject, operations and problem cognitively. At the same time, it was determined that the detudents developed positive attitude towards the course. In addition, it was noticed that problem posing had a significent place in teachers' self-evaluations and they exhibited positive point of view towards problem posing. Besides, it was stated in the articles that the majority of teachers took the children's mental developments into account in the verbal problem sorts presented to them. It was claimed by the teachers that some verbal problem types were not included as they were not suitable to the children's ages, characteristics of development and the functions in the curriculum.

3.3. When the results of the articles examined with the opinions about misconception/difficulty (3) code were analysed, it was observed that the teachers stated that the students had difficulties in some subjects and had false learnings in general. These subjects which were mentioned were expressed as presenting and modelling fractions on the number line, understanding the concept of fractions, four operations on fractions, comparing rational numbers and four operations on rational numbers. In this context, they claimed that the order they followed and the models they used while teaching the subjects were different. In addition, the pre-service teachers stated that teachers play a more active role in the formation of false learnings in students in an article on square root expressions.

3.4. When the results of the articles examined with the opinions for mathematical modelling/using (3) code were analysed, all of the teachers claimed that the modellings related to the natural numbers and integers enabled the subjects to be understood better, permanent and eye-catching by visualising them. However, they expressed that model using was time consuming and not suitable for every function. In addition, especially the teachers stated that modelling with counting stamps in four operation in integers could be used as a concretising and complementary material, but it was not an adequate material.

3.5. When the results of the articles examined with the thoughts/opinions for the concept of infinity (2) code were analysed, it was realised in two articles conducted with the students that most of the students claimed that the rational numbers were infinite and some of them tended to explain the infinity of rational numbers with the concepts of limitlessness and continuity. When the students' thoughts about the infinity of open intervals were considered, it was referred that a mojority of students expressed that there was an infinite element in the given open interval, but they could not explain the reason for this, and some of them associated the concept of countability with the infinity of this interval. Besides, when the students' intuitive thinking about comparing infinite sets was examined, it was realised that they thought that the infinite sets could also be equivalent based on the equivalence on finite sets.

3.6. When the result of the article examined with the opinion for caunting and the concept of number (1) code was analysed, it was emphasised by the pre-service teachers that the number and duration of the activities conducted related to the subject should be increased.

4.Category of achievement: The sub-learning areas of the articles under this category are operations with natural numbers (8), operations with fractions (8), addition/subtraction/multiplication/division operations with natural numbers (3), exponential expressions (3), ratio and proportion (3), decimal notation (3), integers (3), operations in/with rational numbers (2), multipliers and multiples (1) and operations with integers (1). The content of the category and the results of the articles under the category are explained below by analysing separately under each code.

4.1.When the results of the articles examined with the problem solving achievement (8) code were analysed, the participants' achievement levels were examined in six articles related to the operations with natural numbers, in two articles related to the proportional and non-proportional problems. In general, it was observed that while the pre-service teachers and the secondary school students could solve the routine problems, their achievement levels were low in solving the nonroutine problems. In addition, while it was observed that the students were more successful in solving the verbal problems in which a coherent language was used, on the other hand, that they were not able to distinguish the proportional and non-proportional problem situations was found in an article.

4.2. When the results of the articles examined with the problem posing achievement (6) code were analysed, it was found that the students were able to pose routine problems related to different meaning which four operations include in general in problem posing articles related to four operations. In an article conducted with the mathematics teachers related to the operations with fractions, it was realised that the teachers were able to pose routine problems at the knowledge level more. Moreover, in an article carried out with the fifth graders, it was detected that the students' problem posing levels related to the addition and

4.3. When the results of the articles examined with the number sense/feeling performance (4) code were analysed, it was observed that the performance of the students and pre-service teachers was generally low in articles investigating the number senses relevant to the exponential expressions, decimal notation, fractions and rational numbers. In addition, it was determined that the participants used the written calculation method instead of estimation and interpretation in solving the problems.

4.4. When the results of the articles examined with the forming/reinforcing knowledge achievement (3) code were analysed, it was observed that students were able to form the concept of GCD-LCM and integer, and they were able to both create and use the knowledge of proportionality constant in three articles conducted within the framework of cognitive actions in the form of recognition, use, creation and reinforcement (RBC+C Abstraction Model).

4.5. When the results of the articles examined with the performance of using knowledge and skill operationally/conceptually (2) code were analysed, it was determiend that the students showed higher performance in addition to compound and integer fractions compared with the multiplication and subtraction operation. It was stated that the performances differed according to the fraction types and they showed the lowest performance in integer fractions in the questions in which the concept of fraction and its types were examined. In addition, it was noticed that the students could not made the transitions between the steps properly in the operations with multiple steps.

4.6.When the results of the articles examined with the modelling/model using performance (2) code were analysed, it was found that both the teachers and the students were more successful in the modelling with counting stamps of the addition operation with integers compared to the subtraction, multiplication and division operations. Similarly, it was determined that while pre-service teachers were successful in using all models (area, region and length models) in addition and subtraction operations with fractions, they could not demonstrate the same achievement in multiplication and division operations.

4.7. When the results of the articles examined with the Academic achievement (2) code were analysed, it was stated in the article in which the students' academic achievements were explored within the scope of the Kassel Project Algebra Test (KaPAT) scores, the students' academic achievement levels were quite low. In addition, in another article in which the students' academic achievements were investigated in the subject of ratio-proportion, it was seen that the students had a moderate level of achievement.

4.8. When the result of the article examined with the performance related to multiplication and division operation (1) code was analysed, it was observed that most of the pre-service teachers correctly determined the location of the comma used in multiplication and division operations with decimal notations.

4.9. When the result of the article examined with the performance related to number perception achievement (1) code was analysed, it was determined that the number perception levels of the students increased in parallel with the increase in the secondary school grade level, but the achievement level of the students was low in general.

5.Strategy/method/teaching category: The sub-learning areas of the articles within the scope of this category are the operations with natural numbers (7), ratio and proportion (5), addition operation with natural numbers (4), subtraction operation with natural numbers (3), multiplication/division operation with natural numbers (2), fractions (2), decimal notation (2), multipliers and multiples (2) and operations with integers (1). The content of the category and the results of the articles under the category are discussed separately under each code in the category and explained below.

5.1. When the results of the articles examined with the problem solving/posing strategies (9) code were analysed, it was noticed that the students and pre-service teachers created few and informal solutions (counting based solutions or solutions with traces of counting) even in the problems with small numbers in solving the nonroutine problems including ordinal numbers. The answers in the studies that included addition and subtraction, one after the other or side by side were coded as the formal strategy. It was found that the table and list making strategies were used, in addition to the operational solutions, in the solution of the daily life problems, the problem-solving strategies used by students at different stages determined by non-routine and DTT (Didactic Teaching Theory) were handled more systematically after the activity. It was determined that the problem solving strategies discussed were the strategies of reasoning, trial and error, pattern search, systematic list making, and problem simplification strategies. It was emphasised that the 6 year-old-group students made models by commonly using the ready materials in problem solving. In addition, since the primary school students posed flexible problems which necessitates addition-

subtraction operation easier and they were faster, it was observed that they preferred to use numbers that are 10 and multiples of 10, and that they solved their own problems correctly. In an article, it was found that the teachers could use number sense strategies at a moderate level while solving number sense problems, and in the number sense article for pre-service teachers, it was concluded that the pre-service teachers had low number perception and they tried to solve problems with more rule-based methods.

5.2. When the results of the articles examined with the mental calculation/estimation strategies (6) code were analysed, it was determined that, among the operational estimation strategies, rounding, grouping, using matching numbers, rearranging, using front-end numbers, and algorithmic computing strategies were used. In the mental calculations for addition and subtraction, it was observed that the students mostly used the strategy of "add by dividing into tens and ones", and the strategy of "dividing numbers by referencing them" the least. In subtraction operation, it was found that the students used the strategy of “decrement by ten” at most, and at least “subtraction tens and units”.

5.3. When the results of the articles examined with the proportional reasoning strategies (4) code were analysed, it was determined that the strategies which were generally used by the students were equivalent fraction, unit ratio, part-part or part-whole, increment, change multiplier, ratio table and in and out multiplication algorithm. Among these determined strategies, it was seen that the inside out algorithm was preferred the most; and the 8th grade students used more various strategies than the 6th and 7th grade students.

5.4. When the results of the articles examined with the (in fraction/decimal fractions) informal strategies for multiplication and division (2) code were analysed, it was determined the pre-service teachers orderly used the methods of fraction conversion, memorisation and exponential expression within the scope of the rule-based strategies in the multiplication operations with decimal notations. In addition, it was observed that orderly the pre-post, reference and rounding methods were preferred within the scope of the computational estimation-based strategies. It was determined in the division operations with decimal notations that the pre-service teachers used the rule-based strategies of converting to fractions, expanding fractions, memorisation and exponential methods, respectively, and they preferred pre-post and reference methods, respectively, from the computational estimation-based strategies.

5.5. When the result of the article examined with the (in sorting with fractions) reference point strategies (1) code was analysed, it was observed that the students generally drew shapes or number lines, equalised denominators, and sometimes turned to decimal fractions while comparing or sorting fractions. Besides, it was found that the use of reference points in the form of half, whole and closeness to zero was very little in comparing fractions.

5.6. When the result of the article examined with the adidactive teaching method (1) code was analysed, it was observed that the basic conditions of the adidactic environment (DDT) can be met, and an environment in which the concept of ratio and proportion was discovered by the students themselves with the stages of responsibility, action, verification and institutionalisation in this method design.

6. Competency category: The sub-learning areas of the articles within the scope of this category are fractions (7), operations with fractions (5), operations with natural numbers (3), rational numbers (3), division operation with natural numbers (2), decimal notation (2), natural numbers (1), multiplication operation in natural numbers (1), sets (1), operations with integers (1), operations with rational numbers (1), ratio and proportion (1) and square root expressions (1). The content of the category and the articles’ results under the category are discussed separately under each code in the category and explained below.

6.1. When the results of the articles examined with the pedagogical knowledge and content knowledge competency (14) code were analysed, it was determined in 5 articles conducted with teachers and pre-service teachers on fractions that most of the teachers and pre-service teachers did not have the sufficient knowledge level about the different meanings of fractions, models and operations with fractions. Even in natural numbers which is the basic number set, the pre-service teachers had difficulty in giving a proper definition, applied to highly abstract explanations in the topic of division with zero; however, it was concluded that they had insufficient content knowledges from their abstract explanations. Similarly, it was noticed that the participants were insufficient in understanding and explaining the mathematical meaning of the algorithm of the division operation related to the digit concept. It was claimed in an article that the teachers wrote wrong or deficient definitions in general related to the universal and infinite sets, and in an article, mostly the pre-service mathematics teachers studying at the first class in university were insufficient in showing the rational and irrational numbers on number line. In addition, an article in which pre-service science and mathematics teachers were able to conceptualize rational numbers at a higher level than pre-service primary school teachers.

6.2. When the results of the articles examined with the competency to identify and correct misconception/difficulty/error (7) code were analysed, it was concluded that the pre-service teachers were insufficient to identify the wrong information and misconceptions of the students in finding the appropriate model for dividing the given fraction into two, dividing it into parts and examining the effect of dividing on fractions, whether zero is an integer and when determining integers, information about number sets and questions about square root expressions. In addition, it was detected that that they could partially see the misconceptions of students about showing fractions and decimals on the number line. Moreover, it was stated in an article conducted with the primary school and mathematics teachers that most of the teachers could not fully explain the reasons for various pedagogical misconceptions (zero added to the number after the comma has no effect and adding zero to the number when multiplying by 10), especially about multiplying the number by 10. In this scope, it was found in another article which was

examined that teachers could determine student understandings about the concept of zero and interpret the reasons for these understandings from different perspectives.

6.3. When the results of the articles examined with the problem posing competency (2) code were analysed, it was determined that the pre-service teachers were able to create problems related to ratio-proportion and four operations in natural numbers from the rearrangement questions. It was determined that they used the value substitution and context techniques mostly in the problems created. Besides, it was concluded that the pre-service teachers did not have the same competency in the semi-structured and free problem posing situations within the scope of the same subject.

7. Perception/image/representation type/schema category: The sub-learning areas of the articles under this category are square root expressions (4), natural numbers (3), sets (2), rational numbers (2), addition/subtraction/multiplication and division operation with natural numbers (1). The content of the category and the articles' results under the category are discussed separately under each code in the category and explained below.

7.1. When the results of the articles examined with the Perception for number and number sets/comprehension (natural number, rational number, irrational number, real number) (5) code were analysed, it was determined that most of the participants in the articles carried out the pre-service teachers made overgeneralisation with limited number of samples (such as √2, √3, e and π), tried to explain the irrational number concept, "even the rooted numbers are irrational." In additon, it was detected that both the students, who could make proper explanations related to the rational and irrational numbers and those who could not, had smattering knowledge about the cardinalities of these number sets. In addition, it was observed that the pre-service teachers used the types of representation they associated with these numbers according to their past experiences rather than definitions and formal information while determining any given real number as a rational or irrational number. In 2 articles conducted with the students, it was observed that the students had perceptions as there is a very close relationship between the students' number concepts and the way they are written, the numbers in which only the square root sign is used are irrational, the numbers written with a fraction line are rational, or the numbers written using commas are decimal numbers. Moreover, it was realised that the students regarded only the positive ones as number among the integers; the rate of perceiving negative, rational and irrational numbers as number decreased.

7.2. When the results of the articles examined with the perception/schema related to set concept) (3) code were analysed, the relationship between mathematical world and daily life of the pre-service teachers related to the set concept and representation was taken into consideration. It was realised that the pre-service teachers did not show the achievement they showed in the definition of the set, they emphasised the shape surrounding the set rather than the way the elements were expressed in the set representation, and they had misconceptions related to the purpose of using the shapes that symbolise the set.

7.3. When the results of the articles examined with the fraction/rational number concept image (3) code were analysed, it was claimed in two articles carried out with the pre-service teachers and students that the students generally had the part-whole, division and numerator-denominator concept images; the pre-service teachers mostly used the part-whole concept image. In an article conducted within the scope of the rational numbers, it was observed that the students followed the explanations in books more and they used the interpretation of part-whole.

7.4. When the results of the articles examined with the Division operation (in rational numbers) / perception of process choice (in verbal problems) (2) code were analysed, most of the students claimed that they did not look for meaning in the operations that they did the division process based on the algorithm or they attributed wrong meaning to the operations. In addition, when they were asked to do division operation with figure, it was observed that most of the students could not associate teh concept of fraction with division operation. In addition, it was determined that students who generally chose the operation correctly in problems without keywords perceived the keyword addition as subtraction; in problems with keywords and the subtraction keyword as addition.

7.5. When the results of the articles examined with the Problem solving/rational-irrational representation type (2) code were analysed, the students generally used the representations as picture (drawing and figure), symbolic and colloquial representations in solving the problems related to putting in order and comparing. In the article on the representation forms of rational and irrational numbers, it was claimed that pre-service teachers showed rational numbers mostly in fraction form, did not use different representations much, and expressed irrational numbers in decimal form.

7.6. When the result of the article examined with the sense of infinity (1) code was analysed it was determined that the concept of sense of infinity was expressed by the students as "without end", "continuing" and "endless" and the students perceived this concept of a structure with no end.

8.Category of other: This category consists of two codes as the verbal problem type (2) and fraction model (1). The articles in the verbal problem type code discussed the verbal problem types offered to the students by teacher or activity book. One of the articles consisted of the verbal problems types presented by teachers or educational materials in the field of natural numbers sub-learning for preschool students; an the other consisted of the verbal problems types presented to primary school students for four operations within the scope of the sub-learning area of addition/subtraction/multiplication/division with natural numbers. In the code of fraction model, the fraction models which were preferred most, are explained in the explanations of the teachers about the concept of fraction. The results of the articles under the category are discussed under the code in the category and explained below.

8.1.When the results of the articles examined with the verbal problem type (2) code were analysed, the article carried out with the preschool students indicated that both teachers and mathematics activity books offered the result-unknown problem types to children and ignored the other problem types. In the article, in which the verbal problems types that classroom teachers present to students for addition, subtraction, multiplication and division were examined, it was stated that there were situations in which the amount of results is not known from the joining and separation problems related to addition and subtraction, the part-part-whole problems were the situations in which the part is not known, and the comparison problems are the types of problems for the situations which the difference is not known. Moreover, the problem types which was presented most related to multiplicaiton and division were the co-group problems, repeated addition, fair sharing, division of measurement and repeated subtraction, and multiplication problems from comparison problems

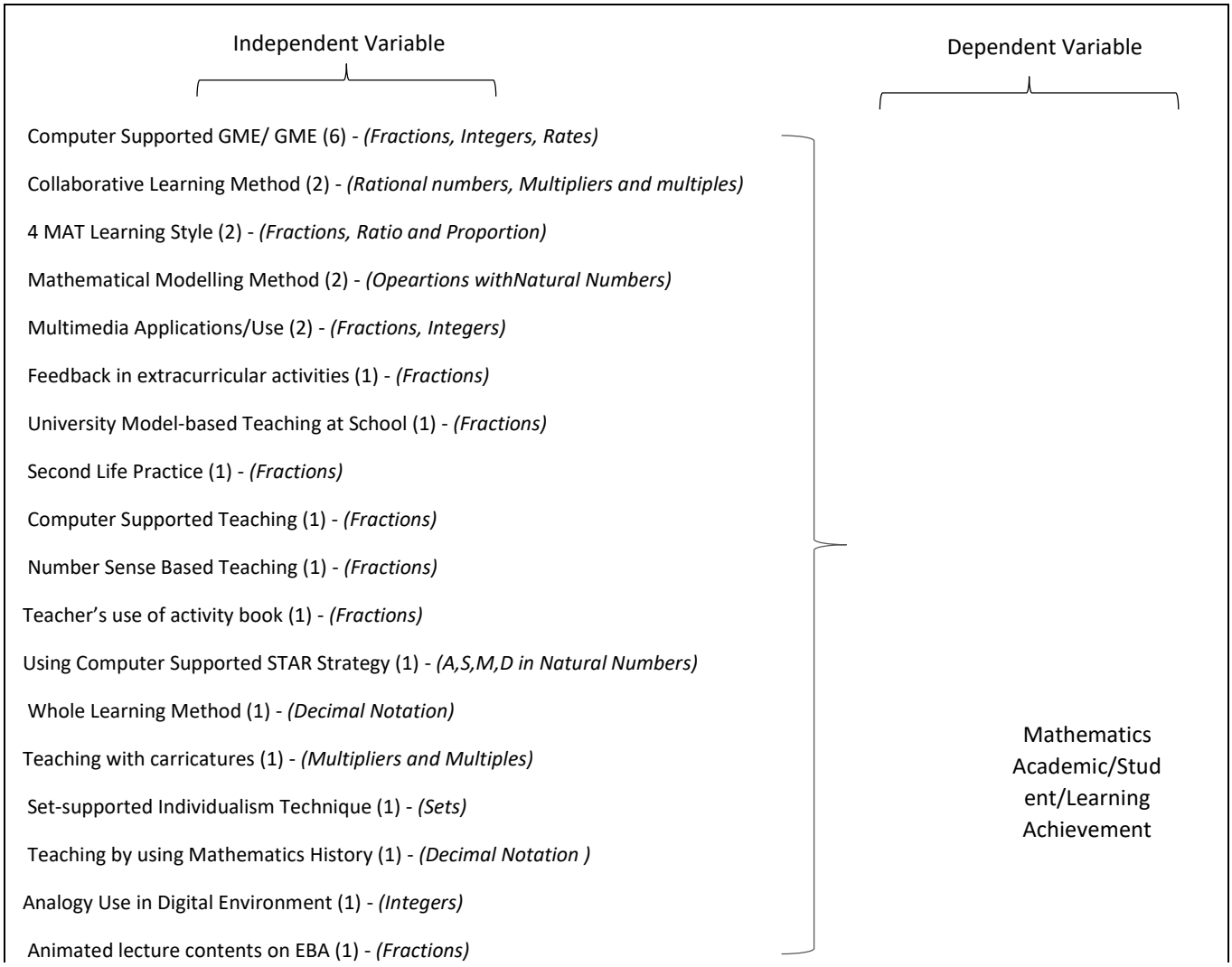
8.2. When the result of the article examined with the fraction model (1) code was analysed, it was determined that the set, area and length model were the most preferred models by the teachers and the model selection in teaching fractions were different between the infrastructures of the fractions.

Findings related to the subject trends and results of the experimental focused articles

It was determined that 62 (20%) of the reviewed articles within the framework of the numbers and operations learning area were experimental. The articles reviewed in this scope consist of seven categories as achievement (36), skill (12), misconception (3) behaviour (2), permanency (3), perception (3) and affective features (3), and different codes under these categories. Each of the code and category used in experimental articles was analysed under different heading and are presented below.

1. Achievement category: In the articles conducted experimentally, the feature which was studied most as the dependent variable was achievement. The sub-learning areas of the articles in this category are addition operation in natural numbers (4), subtraction operation in natural numbers (4), operations with fractions (4), multiplication operations with natural numbers (2), natural numbers (3), operations with natural numbers (3), fractions (3), division operation with natural numbers (2) and decimal notation (1).

The findings related to the dependent and independent variables used in the articles under the category of achievement are presented in Figure 3.



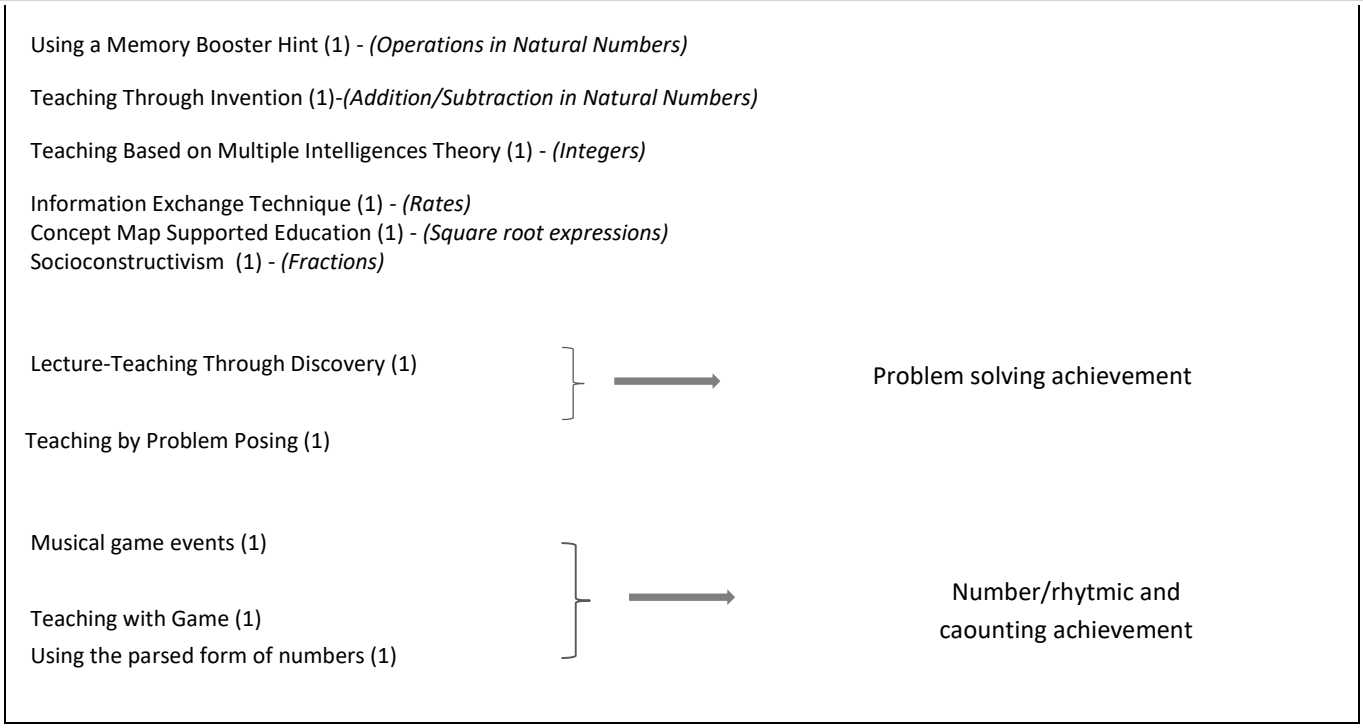


Figure 3. Dependent and İndependent Variables in Achievement Articles

As it is presented in Figure 3, it was determined that mathematics/academic/student/ learning achievement was examined as the dependent variable in plenty of experimental achievement articles (86%).

This is followed orderly by number/rhythmic counting and operation achievement (8%) and problem-solving achievement (6%). The results of the articles under this category were analysed separately under each code in the category and are explained below.

1.1.When the results of the articles examined with the mathematics Academic/Student/Learning Achievement (34) code were analysed, it was determined that the independent variables in the articles increased the achievement or the performance in the dependent variables mentioned in the codes. In two articles in which the effect of education with caricatures in multipliers and multiples subjects and IET (Information Exchange Technique) used in the course in the subject of rates, it was found that no significant difference emerged between the academic achievements of the students in the experimental and control groups.

1.2. When the results of two articles examined with the problem-solving achievement (2) code were analysed, no significant difference was found between the two methods in terms of routine problems in the article examining the effect of direct expression and discovery teaching on square root expressions on problem-solving achievement. Besides, it was concluded that a significant difference was encountered in favour of teaching through discovery in terms of the achievement of solving nonroutine problems. The article examined the scope of decimal notation, it was found that the problem posed by teaching affected the problem-solving achievement of the students with different number perception levels.

1.3. When the results of the articles examined with the number/rhythmic counting and operation achievement (3) code were analysed, it was observed that the achievement of the preschool students in the experimental group in which the number and operation concept teaching was given with musical activities increased. similarly, in another article, in which the education with games was applied, the rhythmic counting and performance related to four operations of the primary school students increased. Moreover, it was determined that the achievement of primary school students in addition and subtraction increased in the teaching made by using the solved form of numbers.

2 Skill Category: The sub-learning areas of the articles in this category are operations with natural numbers (7), subtraction with natural numbers (6), natural numbers (3), operations with natural numbers (3), multiplication operation with natural numbers (2), division operation with natural numbers (2), fractions (2), integers (2), ratio and proportion (2) and rates (1).

Findings related to the dependent and independent variables applied under the skill category are presented in Figure 4.

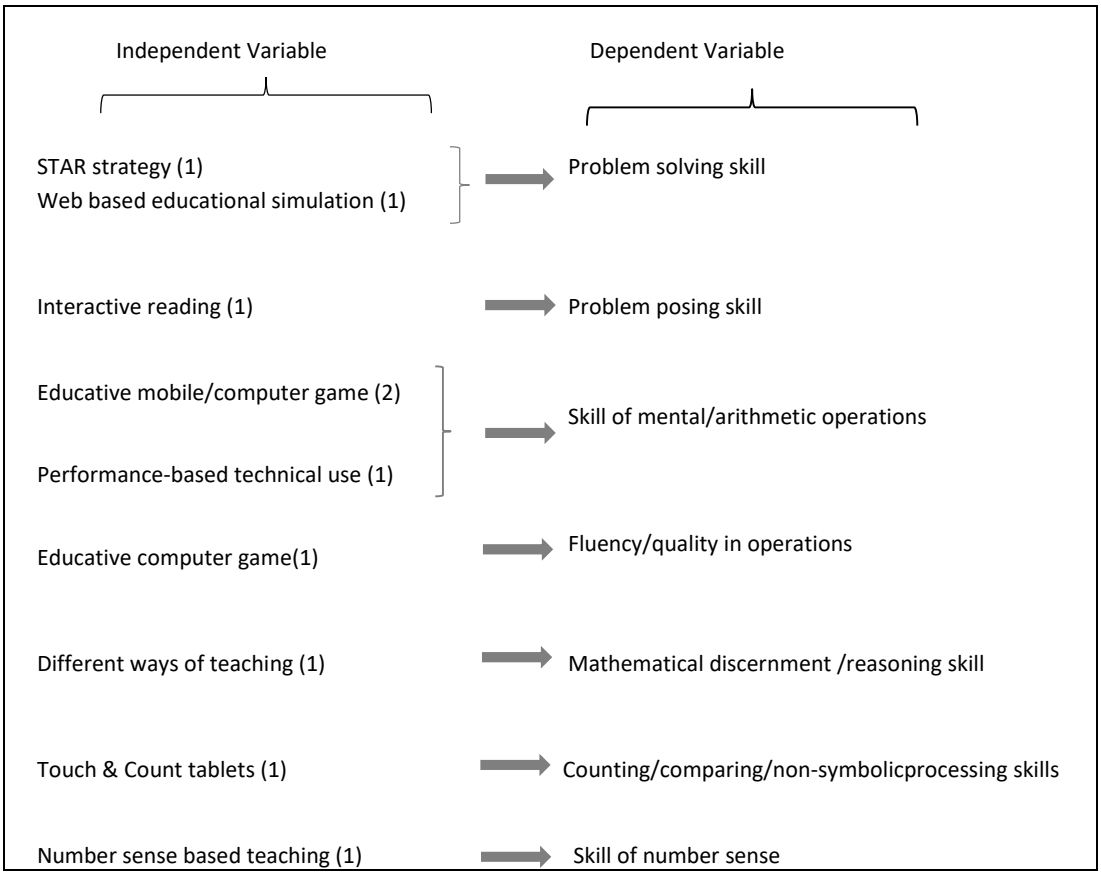


Figure 4. Dependent and Independent Variables in Skill Articles

As it is presented in Figure 4, it is observed as the dependent variable in the experimental skill articles that mostly problem posing/solving and the skills of mental/arithmetic operation were examined. The results of the articles under this category are explained separately below under each code in the category.

2.1. When the results of the articles examined with the problem-posing/solving skill (3) code were analysed, it was expressed that the students were able to make sense of and solve verbal problems for four operations with ratio and proportion, respectively in the educational environment in which Web-Based Educational Simulations and the STAR strategy in computer-aided mathematics lessons were used. In addition, in an article, it was concluded that interactive reading creates a significant difference in the problem-posing skills of the students for the four operations.

2.2. When the results of the articles examined with the skill of mental/arithmetic operations (3) code were analysed, it was determined in one of the two articles, in which the effects of mathematical computer and mobile games on mathematics education were investigated, that these games developed the students' skills of mental operations, in another article that it did not affect the students' basic arithmetic operations skills. In an article, with the educational materials used, it was observed that preschool children could easily perform mathematical skills (arithmetic processing skills) such as finding the number of objects in a group and comparing two groups in terms of scarcity and abundance.

2.3. When the results of the articles examined with the fluency/quality in operations (2) code were analysed, it was stated that the use of performance-based techniques increased the fluency in the addition process of the students, and the tendency of the students who played the educative computer game to perform complex addition operations increased.

2.4. When the results of the articles examined with the mathematical discernment/reasoning skill (2) code were analysed, it was determined in an article that, it was seen that children were able to perform different reasoning strategies with the educational materials used. It was observed that the students' mathematical discernment level increased significantly in the collaborative learning environments in which different learning environments were explored.

2.5. When the results of the articles examined with the counting/comparing/ non-symbolic processing skills (1) code were analysed, it was observed that the touch & count tablets had a positive effect on the counting, comparing and sorting skills of preschool children between 48-60 months.

2.6. When the result of the article examined with the number sense skill (1) code was analysed, it was claimed that the number sense-based teaching process had a positive effect on the number sense skills of the students.

3. Misconception category: the sub-learning areas of the articles under this category are fractions (1), operations with fractions (1), rational numbers (1), operations with rational numbers (1) and square root expressions (1). Findings related to the dependent and independent variables used in the articles under the category of misconception are presented in Figure 5.

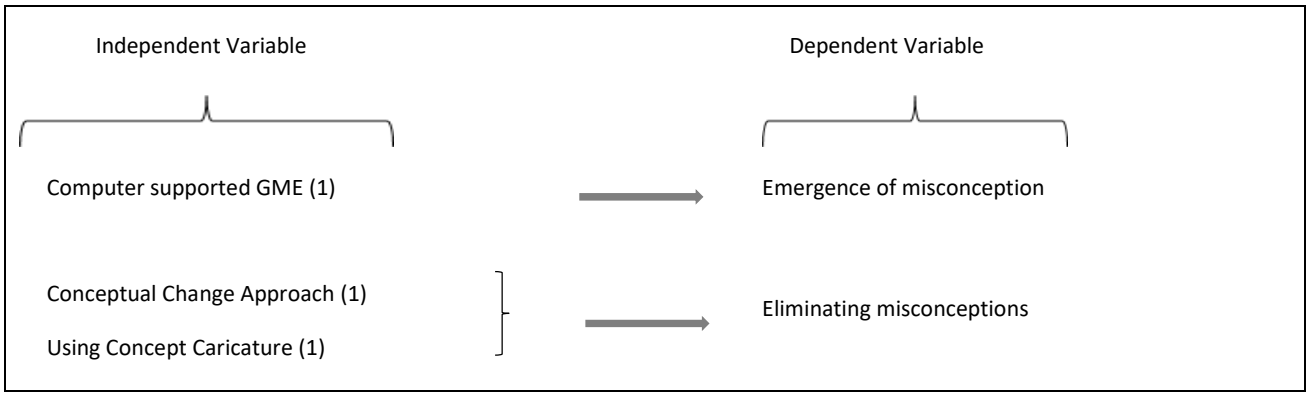


Figure 5. Dependent and Independent Variables in the Articles of Misconception

As it is presented in Figure 5, it is seen that eliminating the misconception variable is the most examined variable as the dependent variable in the experimental misconception articles. The results of the articles under this category are explained separately below under each code in the category.

3.1. When the result of the article examined with the emergence of misconception (1) code was analysed, misconceptions were found in some of the students in both groups after the article in which the computer-assisted Realistic Mathematics Education Approach for operations with fractions and fractions was discussed. However, it was observed that the students in the experimental group had less misconception than the other group.

3.2. When the results of the articles examined with the eliminating misconception (2) code were analysed, it was realised that the conceptual change approach and concept caricature used in the articles were effective in eliminating students' misconceptions about/ with rational numbers/operations and square root expressions, respectively.

4. The category of permanence: This category consists of articles investigating the effects of variables such as teaching method/model and strategy on the learning permanence of participants within the scope of different sub-learning domains. This category consists of 1 code as learning permanency/permanency level (3). The sub-learning areas of the articles under this category are fractions (1), decimal notations (1) and ratio and proportion (1).

Findings related to the dependent and independent variables used in the articles reviewed under the category of permanency are presented in Figure 6.

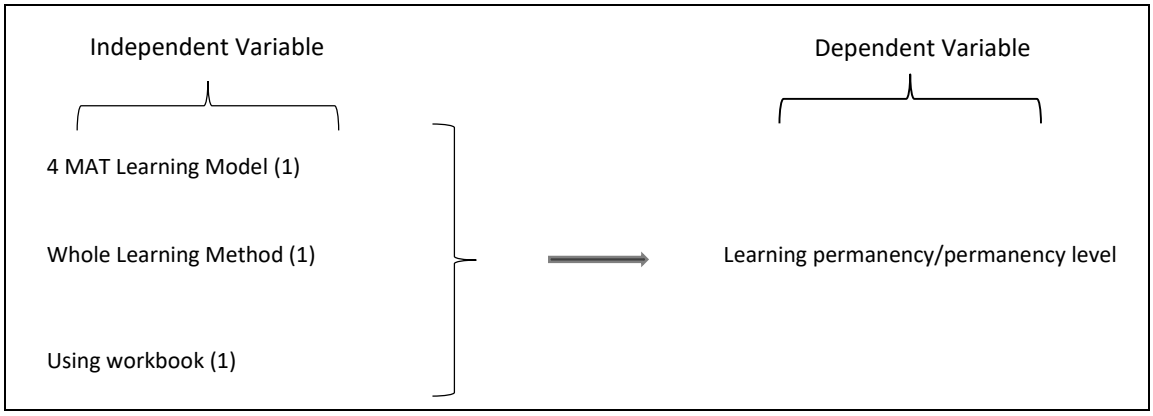


Figure 6. Dependent and Independent Variables ın Permanency Articles

As it is presented in Figure 6, there are the variables as 4 MAT Learning Model, Whole Learning Method and workbook use by teachers are the independent variables in the articles which are experimental and conducted to investigate the permanency levels of the participants. The results of the articles under this category are explained below by applying the code under the category.

4.1. When the results of the articles examined with the learning permanency/permanency level (3) code were analysed, it was determined that the teaching carried out within the scope of ratio and proportion based on 4MAT learning style model ensured the permanence of learning. In another article in which the whole learning method was used, it was detected that the permanence level of decimal notation in the experimental group was higher than that of the control group. Besides, it was determined that the auxiliary books used in the teaching of fractions are more effective than the teaching with the textbook.

5. Perception category: The sub-learning areas of the articles within the scope of this category are natural numbers (2), operations with integers (2), integers (1) and decimal notation (1).

Findings related to the dependent and independent variables used in the articles reviewed under the category of perception are presented in Figure 7.

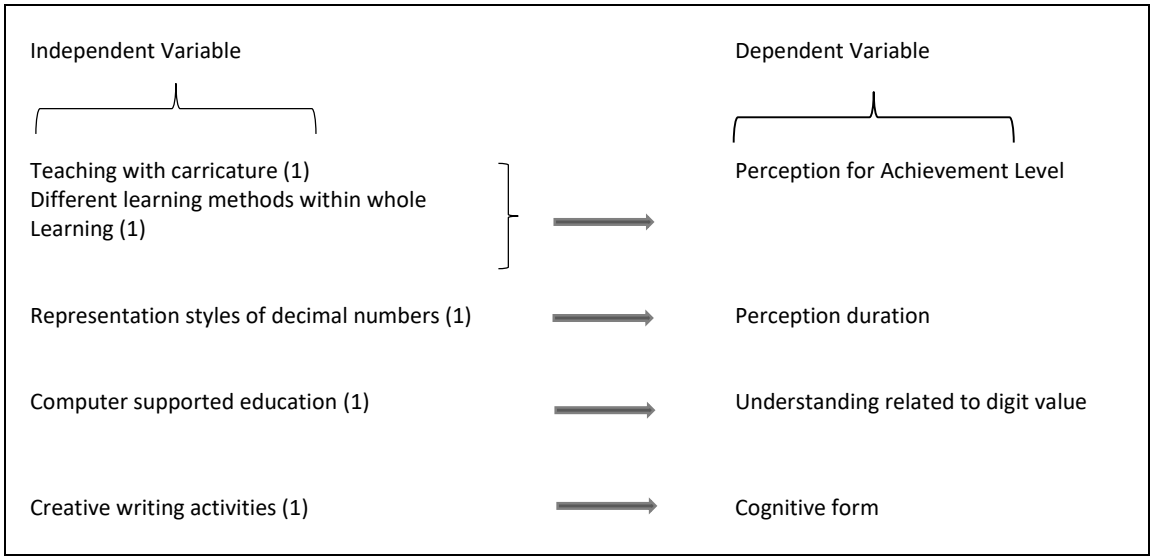


Figure 7. Dependent and Independent Variables in Perception Articles

As it is presented in Figure 7, there are the variables such as teaching with caricatures, different teaching methods (teaching by the invention, dramatization, question and answer, teaching with games and puzzles) within the scope of whole learning, computer supported education, creative writing activities and decimal notation representation styles as the independent variable in the descriptive perception articles. The results of the articles under this category are explained below by exploring separately under each code.

5.1. When the results of the articles examined with the perception related to achievement level (2) code were analysed, it was observed that teaching with different teaching methods (invention method, dramatisation, question-answer, teaching with games and puzzles) in line with the whole learning principles for integers and operations with integers had a positive effect on the students' perceived achievement levels in mathematics. Similarly, it was claimed that teaching with caricatures increased students' perceived mathematics achievement levels for integers and operations.

5.2. When the result of the article examined with the perception of cognitive structure in numbers (1) code was analysed, it was determined that creative writing activities did not create a significant difference between the pre-and post-scores determined by considering the mathematical concepts and relationships related to numbers in the mind maps of pre-service mathematics teachers.

5.3. When the result of the article examined with the understanding related to digit value (1) code was analysed, it was determined that instruction including computer-assisted educational material improved the digit value comprehension of primary school 3rd-grade students.

5.4. When the result of the article examined with the perception duration (1) code was analysed, it was found that the different representations of decimal numbers (table and graphic) were more effective in determining decimal numbers and detecting the largest of these numbers in the shortest time, showing these numbers with graphics.

6. Affective feature category: The sub-learning areas under the articles within the scope of this category are fractions (2), decimal notation (1), integers (1), rates (1) and ratio and proportion (1).

Findings related to the dependent and independent variables used in the articles reviewed under the affective features are presented in Figure 8.

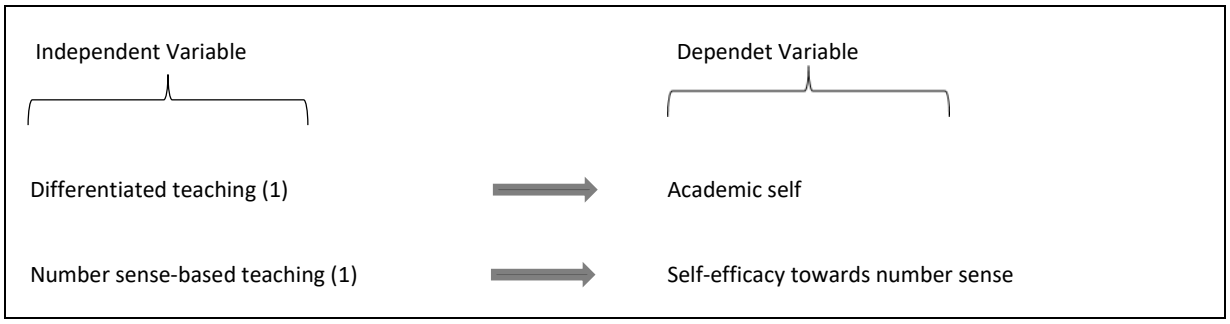


Figure 8. Dependent and Independent Variables ın Affective Feature Articles

As it is presented in Figure 8, there are variables in the form of differentiated and number sense-based teaching as independent variables in the articles examining the experimental effective features. the results of the articles under this category are explained below by examining separately under each code in the category.

6.1. When the result of the article examined with the self-efficacy towards number sense (1) code was analysed, it was claimed that the number sense-based teaching did not create a significant difference in students' self-efficacy towards number sense.

6.2. When the result of the article examined with the academic self (1) code was analysed, it was concluded that the differentiated teaching program for gifted and talented students increased the academic self-esteem of the students within the scope of fractions.

7. Category of behaviour: Within the scope of this category, u kategori kapsamındaki makalenin yer aldığı alt öğrenme alanları ise doğal sayılar (1), Doğal sayılarla toplama işlemi (1), doğal sayılarla çıkarma işlemi (1), doğal sayılarla çarpma işlemi (1), doğal sayılarla bölme işlemi (1) ve kümeler (1) şeklindedir. Bu kategori altında yer alan makalenin sonucu kategori içinde yer alan kod altında ele alınarak aşağıda açıklanmıştır.

7.1. When the result of the article examined with the achievement of target behaviour (1) code was analysed, it was observed that the level of students' achievement of target behaviours for sets and numbers in the program was higher in the group in which the learning-teaching environment was organised by the researcher (the teacher's plans, activity and game cards, in the learning environment where the follow-up tests were prepared by the researcher) than in the other group (the learning environment planned by the teacher).

DISCUSSION AND RECOMMENDATIONS

In this section of the research, the results reached related to the subject trends and results of the articles on experimental and level/situation determination conducted in the numbers and operations learning area in Turkey are presented within the scope of the research problem. In addition, the prominent results of the research are summarised in this section in order not to repeat the findings section.

It was determined that a great number of articles reviewed within the scope of the numbers and operations learning area, consisted of the level/situation determination articles. That the number of level/situation determination articles which have been reviewed being high, can be interpreted as the researchers may have had difficulty in providing a valid environment or adequate criteria for other research methods. The features discussed in the articles published in this context are, in the order most commonly studied, misconception, skill, opinion/thought/intellectual process, success, strategy/method, competence and perception/image/schema/representation type.. The sub-features discussed under these features and the results of the articles made in this context can be summarised as follows within the scope of sub-learning areas:

It was determined that the most studied feature in the level/situation determination articles is a misconception and the sub-learning areas studied under this feature were orderly as fractions/operations with fractions and natural numbers/operations with natural numbers, the articles conducted on the other sub-learning areas were few. The topic trends in these sub-learning areas were problem-posing, problem-solving, pointing on the number line, ordering and applications related to four operations. This is orderly followed by the fraction-rational number-integer relations, modelling verbal problems, reading-writing numbers/digit value and misconception/error/challenges/difficulties the students, pre-service teachers, and teachers experienced related top art-whole relations. It was expected situation that misconceptions came forth in the obtained results to provide meaningful learning in fractions and other subjects (Soylu, 2008), to develop problem-solving skills and to learn advanced subjects such as algebra (Işık & Kar, 2012) in the light of the literature examined. That is, this situation is also supported by foreign literature stating that students' thinking style, reasoning and problem-solving skills in mathematics education are focused on as well as mistakes and misconceptions (Heinze, 2005; Henningsen & Stein, 1997).

When the results of the articles conducted for misconception were analysed, it was observed that ,in problem-posing articles about operations with fractions/fractions, there were conceptual difficulties (Işık, 2011; Işık & Kar, 2012), in problem-posing articles about operations with natural numbers/natural numbers, posing problems for operations other than the desired four operations, not being able to answer, problems such as using incomplete data, using decimal numbers instead of natural numbers, writing exercises and posing problems for different subjects were experienced during problem posing (Arıkan and Ünal, 2013; Kılıç, 2013). In the problem-solving articles, it was determined that the students experienced difficulties in solving problems that require comparison and ordering in fractions and ordinal number problems for four operations. Besides, it was observed that students had various misconceptions and difficulties in ordering fractions, pointing them on the number line, and operating on fractions by over-generalising and specialising (Soylu & Soylu, 2005; Biber et al., 2013).

It was determined that the second most studied feature in the level/situation determination articles was the skill and the most studied sub-learning areas in skill articles were the operations with natural numbers and operations with fractions. The sub-studied features which were studied most in the skill articles were determined to be problem posing, mental calculation/arithmetic, mathematical noticing/discriminating, modelling, proportional reasoning and problem-solving skills. It can be claimed that problem-posing is characterised as a skill related to problem-solving and is seen as an important skill as well as problem-solving (Kojima et al., 2015), and it also plays a significant role in improving academic achievement and mathematical comprehension (Solórzano, 2015).

When the results of the articles conducted on skills were analysed, it was noticed that the participants' (students, teachers and pre-service teachers) problem-posing and problem-solving skills were low in the studies related to operations with fractions, operations with natural numbers and sets (Akçay and Ardıç, 2020; Albayrak et al., 2006). It can be claimed that this situation originated from the reasons such as the difficulties experienced in the conceptual dimension for fractions and natural numbers, teachers' habits in the teaching process, logic errors, poor use of language, posing inappropriate problems and using incomplete data (Kar and Işık, 2015; Kılıç, 2013; Özer et al., 2020). Besides, it was observed in the articles investigating proportional reasoning skills that while solving the problems related to ratio-proportion, the students could not define these concepts and their skills of recognising inversely proportional relationships were lower than the skills of recognising directly proportional relationships (Toluk Uçar and Bozkuş, 2016). It is thought that the proportional reasoning skill is extremely complicated and difficult skill affects this situation (Cramer et al., 1993; Pittalis et al., 2003). In the articles investigating primary school students' arithmetic operation skills, it was realised that the students' readiness levels were sufficient; however, the cardinal value principle, one of the counting principles, was less developed than the other principles in preschool children. Finally, in the modelling articles, it was claimed that the teachers did not use the mathematical models on fractions regularly and their model use levels decreased over time. All these situations are indicators that the teachers did not have enough knowledge about models or did not get modelling education related to the topic at and after the postgraduate level.

The sub-learning areas examined in the articles, in which the features such as opinion/thought/thought process related to determining level/situation, are fractions and operations with fractions sub-learning areas. The sub-features examined within the scope of opinion/thought/ thought process are as thoughts/opinions/ thought processes related to teaching/method/activity/evaluation process, misconceptions/opinions about difficulty, views on mathematical modelling/modelling use, view/perspective on problem posing-solving achievement/type, infinity and counting-number concepts thoughts/opinions.

When the results of the articles conducted within the scope of opinion/thought/thought process were examined, it was noticed that the participants had positive opinions related to teaching/method/activity/evaluation process on fractions, integers and rates subjects (Işık & Kar, 2013; Özdemir & Göktepe Yıldız, 2015; Tanışlı & Sağlam, 2006). This result of the research can be claimed to be supported by various research in the foreign literature (Clark, 2012; Huntley & Flores, 2010; Leikin & Zaslavsky, 1997). Similarly, it was noticed that in addition that the participants claimed positive opinions in the articles examined within the scope of the thoughts for mathematical modelling/model use, opinion/viewpoint for problem posing-solving achievement/type (Çelik & Çiltaş, 2015), there were also articles suggesting that the materials used in the modelling articles were not sufficient (Bozkurt & Polat, 2011). This situation can be an indicator that the contents of the materials used in courses by teachers are limited or they can not go beyond the use of similar materials.

In the articles in which the achievement feature related to level/situation determination was investigated, the sub-learning areas which were studied most were operations with natural numbers and operations with fractions. The sub-features examined in the achievement articles were respectively problem-solving, problem posing and number sense/feeling performance or achievement.

When the results of the articles conducted within the scope of achievement were analysed, it was observed that the participants mostly could solve the routine problems and had low achievement levels in solving the nonroutine problems. It can be thought that this situation originated from the that the non-routine problems cannot be solved with a known method or formula, that they are the problems that require the student to analyse the data carefully, make a creative attempt, and use one or more strategies in its solution (Polya, 1985). Similarly, even in the problem-posing articles, it was noticed that the participants could pose the problems at the level of knowledge at most and they had a low problem posing achievement levels. Besides, it was noticed that the number sense use achievement of the participants related to the exponential and rational numbers was at a low level (Bayram & Duatepe-Paksu, 2014; İymen & Duatepe–Paksu, 2015; Yenilmez & Yıldız, 2018). This situation may be an indicator that the participants preferred rule-based strategies to number-sense strategies in solving the problems. This can be interpreted as the students or pre-service teachers are directed to rule-based memorisation rather than guessing or interpreting the question solutions that will lead to the correct answer in the education given during the process from primary school to undergraduate level.

The most studied sub-learning area in the articles discussing the strategy/method feature for level/situation determination has been determined as operations with natural numbers. The sub-feature examined in the strategy/method articles was determined as the problem-solving/posing strategy.

When the results of the articles related to the problem-solving strategies which came forth within the strategy/method articles were analysed, it was observed that the participants used few informal strategies in the solution of the problems with small numbers (Dinç Artut & Tarım, 2006; Dinç Artut & Tarım, 2009). This result of the research can be claimed to be supported by the research result conducted by Verschaffel et al., (1999). Besides, it was observed that they used table and list-making strategies in addition to the operational solutions in the solutions of daily life problems (Güler & Didiş Kabar, 2017), and they used the strategies respectively as problem simplification, trial and error, systematic list making, pattern search and reasoning in solving the non-routine problems (Gök & Erdoğan, 2015).

It was determined in the articles, in which the competency feature related to level/situation determination was analysed, that the sub-learning areas which were studied most were fractions and operations with fractions. The sub-features examined in the

competence articles were in order of pedagogical knowledge and content knowledge competency, misconception/complexity/difficulty/error determination and elimination competency and problem posing competency. It can be claimed that content knowledge is of great importance in effective mathematics teaching and that this knowledge is given to the students with an appropriate pedagogical approach is among the indispensables of effective teaching in concentrating on pedagogical knowledge and content knowledge competency in articles (Shulman, 1986 and 1987; Manouchehri, 1998)

When the results of the articles, in which the competency feature was examined, it was determined that the participants had not had enough knowledge levels on fractions and operations with fractions (Özkaya & Konyalıoğlu, 2019; Zeybek Şimşek, 2020), natural number and number sets (Ercire & Narlı, 2019), universal and infinite sets (Yazıcı & Albayrak, 2019), and they had knowledge deficiencies in the definition of concepts such as the perception of infinity and the arrangement of rational and irrational numbers on the number line (Çevikbaş & Argün, 2017).

The deficiency experienced in this scope is considered a significant insufficiency in terms of mathematics education. Thus, it is emphasised in the literature that definitions and explanations that play a role in shaping students' concept images have great importance in terms of mathematics education (Vinner, 2002; Edwards & Ward, 2004; Zazkis & Leikin, 2007; Leikin & Zazkis, 2010). Besides, it was noticed that the teachers and pre-service teachers were deficient in terms of determining the students' mistakes in fractions, number sets, operations related to natural numbers and square root expressions or using the appropriate method in eliminating these mistakes (Kubar & Cakiroğlu, 2017; Özkaya et al., 2013; Yavuz Mumcu, 2016), it was determined that they were able to partially identify student misconceptions about representing fractions and decimals on the number line (Girit & Akyüz, 2016), and in an article, that the teachers were able to determine students' understandings about the concept of zero (Bütün & Erdoğan, 2020).

That the teachers or pre-service teachers were insufficient in determining and eliminating the misconceptions that their students had related to number sets, operations related to natural numbers and square root expressions might be an indicator that the participants could not internalise the concepts related to the topic entirely. In addition, while the participants were observed to be successful in creating problems related to ratio, proportion and four operations with natural numbers, they were deficient in semi-structured and free problem-posing situations (Albayrak et al., 2006; Bayazit & Kırnap Dönmez, 2017).

It was determined that the most studied sub-learning area in the articles on the perception feature for the level/situation determination was analysed, was the square root expressions. The sub-features examined most in the perception articles are perception/understanding related to number and number sets (natural number, rational number, irrational number, real number), perception/schema for set concept and the image of fraction/rational number concept.

When the results of the articles in which the perception feature was examined, it was claimed that most of the participants defined the irrational numbers in a limited way and by using generalisations, established a very close relationship between the concepts of number and number and the way numbers are written (Baştürk, 2015), had misperceptions in describing the concept of set, described the concept of infinity as "endless", "continuing" and "unending" (Narlı & Narlı, 2012), they tried to adapt the methods they applied for finite sets to infinite sets as well in comparing the infinite sets (İpek et al., 2009). In parallel with this result of the research, studies were also encountered in the foreign literature (Duval, 1983; Falk et al., 1986). In addition, it was determined that the images of the fraction concept were division, part-whole and numerator-denominator concept images, respectively (Macit and Altay, 2020). This result of the research was also claimed by the studies conducted by Kieren (1993) and Lamon (2007).

It was noticed that the most conducted articles, which were carried out after the articles on determining the level/situation in the field of numbers and operations learning, were the experimental articles (62, 20%). The features, which were studied most in the experimental articles, were determined orderly as achievement and skill. The fact that the concept of achievement came forth in the reviewed articles, it is used as an evaluation tool when making comparisons in national and international exams (Ertane Baş, 2019).

The most studied sub-learning areas in the experimental articles in which the achievement feature was examined were determined as fractions and natural numbers. In addition, the independent variables examined in these articles were the variables as computer-supported teaching, computer-supported realistic mathematics education, number sense-based teaching, multiple teaching environments, cooperative teaching, 4MAT learning style, university model-based teaching at school, EBA-supported animated lectures, mathematical modelling, teaching with games, knowledge exchange technique, computer-supported STAR strategy in education. When the results of the articles conducted in this scope were analysed, it was determined that the independent variables in the studies increased the achievement or performance of the dependent variables (Alkaş Ulusoy, 2020; Altıparmak & Çiftçi, 2018; Aydıntan et al., 2012; Çoruk & Çakır, 2017; Demirdöğen & Kaçar, 201; Eraz & Öksüz, 2015; Ertem & Akbaş, 2019; Yazgan & Altun, 2010; Yiğit & İpek, 2015). It was determined only in two articles, in which the effect of mathematics teaching with caricatures on the students' academic achievement (Güler et al., 2013) and the effect of ICT (Information Change Technique) used in courses in the subject of rates on the students' achievements was studied (Tanışlı & Sağlam, 2006), that whether there was a significant difference between the academic/learning achievement of the experimental and control groups. This result of the research can be claimed to be originated from the limitations of the physical environment in the studies or the level of teachers' proficiency in using these methods. In addition, in the article examining the effect of direct expression and discovery teaching on square root expressions on problem-solving achievement, there was no significant difference between the two methods only in terms of routine problems (Kablan et al., 2019).

It was found that the sub-learning areas, which were studied most in the experimental articles on skill feature, were determined to be the natural numbers and operations with natural numbers. The most examined sub-features in the skill articles studied in this scope were the problem-posing/solving skill and mental/arithmetic (do/explain) skill. The independent variables examined in the experimental articles related to skill were the STAR strategy, games, education materials, Touch&Count number tablets and number sense-based teaching. When the results of the articles were analysed, it was determined that the independent variables in the research increased the skill of the mentioned dependent variables (Aktaş et al., 2018; Erdem & Soylu, 2018; İpek & Malaş, 2013; Karal et al., 2010; Mutlu et al., 2019; Yurtbakan & Aydoğdu İskenderoğlu, 2020). Only in an article, it was seen that the effect of educative computer games on the development of basic arithmetic processing skills was not statistically significant (Kula & Erdem, 2005).

Considering all these research results, the recommendations that are thought to contribute to the literature are presented below.

It was mostly focused on the articles aimed at determining the level or situation in the form of misconceptions/errors/difficulties about fractions and natural numbers in the reviewed articles. Considering this, further studies will be conducted, and academic studies examining the experimental, relational and predictive situations to eliminate misconceptions/errors or difficulties can be carried out within the scope of this subject.

It was noticed in the experimental studies which were reviewed that applications such as the computer-supported GME or GME, 4MAT learning style, STAR strategy and Second Life were realised to increase achievement. Considering this, the number of in-class applications can be increased related to this field.

Declaration of Conflicting Interests

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Statements of publication ethics

We hereby declare that the study has not unethical issues and that research and publication ethics have been observed carefully.

Researchers' contribution rate

The study was conducted and reported with equal collaboration of the researchers.

Ethics Committee Approval Information

Ethics Committee Approval for this research was obtained from the Human Research Committee on Ethics at Erzincan Binali Yıldırım University (Reference number: E-85748827-050.06.04-57538; Date: 28/01/2021).

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