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Conceptualizing Pre-Service Mathematics Teachers' Approaches towards Responding to Availability of Tools and Resources in the Context of Knowledge Quartet*

Matematik Öğretmeni Adaylarının Araç ve Kaynakların Erişilebilirliğine Yanıt Vermeye Yönelik Yaklaşımlarının Dörtlü Bilgi Modeli Bağlamında Kavramsallaştırılması

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ABSTRACT: The purpose of this study is to conceptualize the approaches of secondary mathematics pre-service teachers towards contingencies encountered during the teaching process within the context of responding to availability of tools and resources. In order to achieve the goal of the study based on the grounded theory, the nine secondary mathematics pre-service teachers' lessons were observed, and recorded using a video camera, and semi-structured interviews were performed. The data collection process ended after 54 lesson hours of observation, that is, when code saturation was reached. As a result, four sub-codes in the context of responding to availability of tools and resources were identified within the approaches of participants in relation to contingent moments. It is thought that conceptualizing the approaches of participants towards unplanned situations encountered during lessons in real class environment would prove to be beneficial in teacher training.

Keywords: Contingency, grounded theory, knowledge quartet, pre-service mathematics teachers, responding to availability of tools and resources.

ÖZ: Bu çalışmanın amacı, matematik öğretmeni adaylarının öğretim sürecinde karşılaştıkları beklenmeyen olaylara yönelik yaklaşımlarını, araç ve kaynakların erişilebilirliğine yanıt verme bağlamında kavramsallaştırmaktır. Araştırmanın katılımcıları dokuz matematik öğretmeni adayıdır. Gömülü teoriye dayalı çalışmanın amacına ulaşabilmesi için katılımcıların dersleri gözlemlenmiş, video kamera kullanılarak kayıt altına alınmış ve öğretmen adayları ile yarı-yapılandırılmış görüşmeler yapılmıştır. Kod doygunluğuna ulaşıldığında veri toplama işlemi tamamlanmıştır. Böylece katılımcılar tarafından yürütülen toplam 54 saatlik ders incelenmiştir. Sonuç olarak katılımcıların beklenmeyen olaylara ilişkin yaklaşımları içerisinde araç ve kaynakların erişilebilirliğine yanıt verme bağlamında dört alt kod belirlenmiştir. Katılımcıların gerçek sınıf ortamındaki derslerinde ortaya çıkan önceden planlanmamış durumlara ilişkin yaklaşımlarının kavramsallaştırılmasının öğretmen eğitiminde yarar sağlayacağı düşünülmektedir.

Anahtar kelimeler: Beklenmeyen olaylar bilgisi, gömülü teori, dörtlü bilgi modeli, matematik öğretmeni adayı, araçların ve kaynakların erişilebilirliğine yanıt verme.

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Pedagogical Content Knowledge (PCK) is one of the seven knowledge bases that teachers should have (Shulman, 1987). According to Shulman, PCK relies mostly on a teacher's capacity to transform the subject matter knowledge into other forms which can enable students to understand the subject. Examining the studies regarding PCK shows that several frameworks have been presented (Bukova-Güzel, 2010). One of these frameworks is the "Knowledge Quartet (KQ)" which allows mathematics pre-service teachers to be evaluated and developed during their instruction (Rowland et al., 2009). The KQ centers on teachers' observable mathematics-related knowledge as it manifests in situations within the mathematics classroom (Carlsen et al., 2023).

Knowledge Quartet is constituted of four units named foundation, transformation, connection, and contingency, and the codes of these units (Rowland et al., 2005). Foundation deal with beliefs and propositional knowledge related to mathematics and mathematics pedagogy (Rowland, 2005). Transformation involves choosing examples and procedures that help create concepts, making demonstrations that require the teacher to transform his or her own knowledge so that learners can understand it better (Turner, 2005). Connection refers to the teacher's ability to sequence the content and associating the content between mathematics and other subject areas (Fitzmaurice et al., 2021). Contingency, the unit in the focus of the research, deals with situations that cannot be planned before the lesson (Rowland et al., 2009; Thwaites et al., 2005; Turner, 2005). The idea that while most situations in the classroom can be planned, some cannot, prompted the researchers to generate this unit (Rowland et al., 2009). For this reason, contingency refers to the knowledge that a teacher employs in order to address unexpected strategies, questions, or comments from learners (Kgothego & Westaway, 2023). Contingency also covers how teachers respond to unplanned instances in a lesson, which often tests their ability to "think on their feet" (Mutlu & Duatepe Paksu, 2022; Petrou, 2010; Rowland et al., 2005).

Rowland et al. (2011) suggested that the consideration of contingency including its possible triggers and reasons of such triggers had an important but, yet unrecognized, place in mathematics teacher education. Mathematics teachers may not predict contingent moments before they happen as well as pre-service teachers. Contingency consists of four codes named as deviation from lesson agenda, teacher insight, responding to students' ideas, and responding to the (un)availability of tools and resources (Thwaites et al., 2011). In the study we focused on responding to the (un)availability of tools and resources.

Responding to the (Un)Availability of Tools and Resources

The code of responding to the (un)availability of tools and resources is related to the tools and resources teachers use to concretize, in particular, abstract concepts (Rowland et al., 2015). Such tools, resources and materials may be the main tool of the lesson plan, or they may be included in the lesson as an opportunity although they were not in the plan initially (Rowland et al., 2011). Cohen et al. (2003) stated that accessing more resources for teaching does not cause learning. The teacher can benefit from the tools and resources available in the classroom or that come to mind during the lesson by integrating these materials into the lesson, even if they are not included in the lesson plan.In fact, this unexpected availability of a resource prompted a change in pedagogical strategy (Thwaites et al., 2011). The teacher's ability is important in the use of tools and resources for effective teaching. Researchers report that schools and teachers with the same resources do different things, with results for learning because of the uses of resources (Cohen et al., 2003).

Educational resources, conventionally conceived, refer to money or the things that money buys, including books, buildings, libraries, teachers' formal qualifications, and more (Cohen et al., 2003). The tools and resources that teachers can make use of during the lesson can be either analog or digital (Rowland et al., 2015). For example, the teacher can add a software specific to algebra-geometry to the lecturing, as well as add a material such as a hundred chart that crosses his or her mind during lecturing (Kula, 2014).

When the literature was reviewed, it was seen that the descriptions of sub-codes and triggers of the Contingency unit, and the comprehensive descriptions and examples were limited (Rowland et al., 2011). For example, the fact that there isn't any detailed information on the content and triggers of even the most cited code of the Contingency unit, "responding to student's ideas" code, is one of the factors that led to the designing of this study. For this reason, it is important to determine the contingent moments and triggers that lead to "responding to the (un)availability of tools and resources" code in the classroom, which there is not much information about it. However, it is aimed to give more information about the code by determining which sub-codes are included under this code.

With this study, for the first time in Türkiye, the approaches of pre-service mathematics teachers towards unexpected situations they encounter in their teaching processes are conceptualized within the framework of Knowledge Quartet. With the conceptual framework revealed, teachers, and especially novice teachers, can have the opportunity to review their own approaches to responding to the (un)availability of tools and resources. It is also thought that in-service and pre-service mathematics teachers can be guided about what approaches are appropriate for more effective teaching and more supportive of understanding, and what interventions can be made when contingent events occur.

The study was planned to be conducted with student teachers because Peterson and Leatham (2009) states that experienced teachers and teacher educators can often recognize important mathematical moments during the lesson and use them to support student's learning, but especially novice teachers cannot recognize these moments or act professionally (as cited in Stockero & Van Zoest, 2013). It is important to determine the approaches during such situations because inexperienced or pre-service teachers may not be able to decide how they will respond to an unexpected situation or may have difficulties while handling these situations. It is seen as situations that can take time for student teachers who do not have the experience (Rowland et al., 2011). For this reason, it is important to determine how pre-service mathematics teachers approach contingent moments that they may encounter during their lessons. Therefore, the research problem was determined as "How to conceptualize pre-service mathematics teachers' approaches to unexpected situations that require them to responding to the availability of tools and resources that can be used in their teaching processes?" In accordance with research problem, the purpose of this study is to understand the approaches of pre-service mathematics teachers who are facing controversy during the teaching process within the context of responding to availability of tools and resources.

Method

This was a grounded theory study which aimed to conceptualize the pre-service mathematics teachers' responding to availability of tools and resources. Grounded theory emerges especially when there is no satisfactory theory on a certain subject and the subject to be researched is not sufficiently understood (Charmaz, 2006; Punch, 2005; Strauss & Corbin, 1998). Since the sub-codes and their triggers of respond the (un)availability of the tools and resources are not clear enough, the grounded theory was intended to be used. In this context, we aimed to conceptualize the sub-codes and their triggers. As a result of the data analysis, four sub-codes in the context of responding to availability of tools and resources were identified within the approaches of pre-service teachers in relation to contingent moments. Additionally, triggers that cause the surfacing of each sub-code were determined and it was attempted to provide more detailed information regarding the sub-codes.

Participants

The participants in the research were nine senior pre-service secondary mathematics teachers. The participants had almost completed the last semester in the program. The participants took the courses which prepare them to be mathematics teachers such as Calculus, Analytic Geometry, Differential Equations, Algebra, Mathematical Modeling, History of Mathematics, Mathematical Applications with Computers, Mathematical Thinking, etc. In addition to these courses, they also took courses such as Introduction to Educational Sciences, Curriculum Development, Assessment and Evaluation, Classroom Management, etc. Apart from this, there were courses related to school-based placement.

The study was conducted with volunteer pre-service teachers who encountered a contingent moment within the six lesson hours taught within the scope of the Teaching Practice. The participants of the study were determined using theoretical sampling (Patton, 2002), one of the purposeful sampling methods. Theoretical sample means the sample intended to be used throughout the development of the theory (Glaser, 1978; Glaser & Strauss, 1967). In grounded theory, while the initial sample is used to collect the first data in a study, the theoretical sample directs the researcher on where to go and where to collect data (Charmaz, 2006; Neill, 2007).

When creating the initial sampling, it was taken into account that the participants were attending the Teaching Practice at Anatolian High School. The reason why Qualified High School was chosen was the idea that the students would be more successful and would be able to analyze the lessons more because they came within the scope of the exam they took after secondary school, and that the participants would be able to encounter more contingent events. Additionally, one participant taught at the 10th grade land two participants taught at the 11th grade. Thus, it was aimed to examine the participants' approaches to unexpected situations that may arise in teaching a lesson at different grades and at the same grade level but with different subjects and different participants. After the first data collection process was conducted with three participants, it was observed that the categories began to de created.

At this stage, it was wondered how contingent moments might occur in different ways in a different type of school and what approaches pre-service teachers would take, and it was thought that the participants who went to Vocational High School could contribute to the conceptualization. For this reason, in the continuation of the study, those who continued their teaching practice in Vocational High School were selected as participants. Pre-service teachers who taught subjects that were not included in the initial sample were similarly included in the study at different grade levels. The sub-codes of the data obtained after the observed lessons of the three participants were shaped and their final form was given. However, since it was desired to verify whether there were new sub-codes that might emerge, it was desired to work at different grade levels and on different subjects with participants from both types of high schools. Information about the participants' pseudonym chose by themselves, school type, class, and subject is given in Table 1.

Table 1

Participants' Information

Participants	School Type	Class	Subject
Seyfi	Anatolian High School	10	Trigonometry
Salim	Anatolian High School	11	Matrix, determinant, and linear equation systems
Aysun	Anatolian High School	11	Binomial expansion-probability
Ufuk	Vocational High School	9	Rational numbers
Gülbin	Vocational High School	10	Midsector-height-area of triangular region
Cumhur	Vocational High School	11	Series
Efe	Vocational High School	9	Radical expressions
Sercan	Anatolian High School	10	Trigonometry
Halil	Anatolian High School	12	Derivative

Before conducting their lectures, the participants prepared lesson plans. They conducted their lessons for students in different grade levels. Different grade levels were chosen because of the idea that more diverse categories may emerge. It is also thought that if pre-service teachers teach different course topics and different grade levels, it would be factor in reaching rich categories.

Data Collection Procedure

Participants were asked to prepare a detailed six-hour lesson plan before their teaching. They were given a lesson planning template. The planning template includes the activities and roles of the teachers and their students, the allocated time, the reason for the selection of the activity, and so on, and the sources from which they are used and for what purpose. In light of this information, it is aimed to determine whether the situations that occurred during the lesson are unexpected, whether the participant has changed the plan or not, and what is the approach to the unexpected situation. Participants carried out their teaching in the schools in the direction of the lesson plans. The participants' lessons were recorded using a video camera. They were observed during their teaching and field notes were taken regarding contingent moments. In the field notes the contingent moment and the triggers to cause this contingent moment were noted. It is aimed to save time by focusing on the relevant moments of the video recording in the coding work. The field notes were used in the initial coding and thus it was decided whether the data reached the saturation. After the categories reached saturation, data collection was terminated. Thus, a total of 54-hour courses of nine pre-service teachers were observed. In order to prevent data loss, lessons were videotaped. While transcription was in process, data familiarity was gained. Transcriptions also provided convenience in the second coding. The semi-structured interviews were performed concerning the contingent moments.

Data Analysis

Initial coding began with field notes taken in class. During data collection process, two researchers performed coding after each lesson and when the code saturation achieved, the data collection process was completed. Thus, a total of 54 hours of lessons by 9 pre-service teachers was examined within the context of the approach towards contingent moments. After the data collection process, video recordings of each pre-service teacher were transcribed and prepared for the second coding. A more detailed coding process has been started with transcript documents. Thus, the data were examined both more thoroughly and the first coding was determined whether there were any cases that were not noticed. With the second coding, the analysis was verified. Then two researchers conceptualized the data. While generating sub-codes, the transcriptions of each lesson were reviewed many times. In the grounded theory process, data analysis began with initial coding, continued with focus coding and then theoretical coding (Charmaz, 2006). An attempt was made to find an answer to how the sub-codes that emerged through theoretical coding could be related to each other while creating the theory. Data analysis continued until no new subcodes emerged. Thus, the sub-codes and their triggers presented in the results were reached. The audio records of the interviews were also transcribed verbatim, and these transcriptions were used to support the findings.

Trustworthiness

Lincoln and Guba (1985) state that credibility, transferability, dependability, and confirmability criteria will be considered in evaluating qualitative research. In order to increase credibility, extracts taken from course transcripts, observation notes and interviews were included while presenting the findings. Also, to increase the transferability of the study, the method, participants, data collection tools and data analysis method were explained in detail. In the study, dependability was tried to be provided by giving detailed information about the data collection process, the created sub-codes, their triggers, and the basic features of the sub-codes. Confirmability was attempted to be ensured by analyzing the data by two researchers and finalizing the sub-codes and triggers by reaching a consensus.

Ethical Procedures

Ethics committee permission was obtained from Dokuz Eylül University The Institute of Educational Science Ethics Board for the study's implementation (08.05.2013, No: 12018877/604.01.02./879849).

Results

When the approaches of pre-service mathematics teachers towards contingent moments were analyzed in which they had to respond to the (un)availability of the tools and resources that may be used in the teaching process, the following sub-codes emerged:

- Demonstrating,
- Using a classroom object,
- Using mathematical software,
- Changing tools-resources.

In the following sections, each sub-code is explained and the triggers that lead to the emergence of these sub-codes are included. Extracts from some participants' lecturing that are thought to best explain each subcodes and its triggers presented, and field notes are also included. Additionally, observation notes are sometimes used, and transcripts of interviews are sometimes quoted to make findings more understandable.

Demonstrating

The demonstrating sub-code is related to situations in which pre-service teachers concretize abstract concepts. By asking students for help or doing it themselves, student teachers realize demonstrating situations that their pupils cannot conceive.

Triggers that motivate pre-service teachers for demonstrating:

- that they are easily accessible,
- ease in reflecting instantaneous changes,
- thought that it would help to achieve the outcome and
- it can provide concretization.

In his first lesson, Hasan conducted his teaching process concerning the learning goals: "The student determines the maximum or minimum point that should be found within the range with the help of the second derivative. The student explains the concepts of concave, convex, and inflection point on the graph of a function." Hasan stated that he would associate the function with distance, the first derivative function with speed, and the second derivative function with acceleration. Hasan's student had difficulty understanding the relationship between the increase in acceleration the change in speed and distance. The lesson regarding this difficulty Hasan's discourse and the actions he has taken are as follows:

Hasan:	*Now I started from position 0. I'm in an accelerated movement. I speed up, since my acceleration is positive it increases my speed. So now my speed was one, I've added an acceleration, and it became three, I started to go faster. I'm starting to go a lot more. The distance I got ...*
Student:	*Is increasing.*
Hasan:	*Is it increasing? What was it at first; I got 1, then 3, then 5. Okay? So, the distance I covered is increased when my acceleration was positive, right?*
Student:	*Yes.*
Hasan:	*Okay, we don't have a problem here. Well, what about the distance I covered when my acceleration is positive, how does it increase? I started to go faster at every turn. The distance increased faster. Increased increasingly. In other words, when my acceleration was positive, it leads to a convexity. Okay?*
Student:	*How was it decreasing increasingly?*

Hasan: *Now.*

Student: *How was it decreasing increasingly?*

Hasan: *Okay. I increased my acceleration, and then I reduced it. What happened to my speed? It is decreasing, but I can still cover distance, because my speed is not 0. My acceleration is now negative. I covered the maximum distance I could. (Demonstrating these by walking)*

Student: *Are you going back?*

Hasan: *I am going back. My acceleration is now negative, my speed is going backwards. What happened when my acceleration went to negative? I've covered the maximum distance, I went back. I took the maximum distance and went back.*


(Drawing this with his hand.)

Hasan: *Because my acceleration is negative. I've covered the maximum distance, I went back. So, what is it? There is a maximum point. Okay? So, when my acceleration is negative, I covered the distance I could, and I had to go back because my speed was not negative anymore. So, if the acceleration is negative, there is a maximum point. Okay?*

Student: *Okay.*

Hasan: *It was negative, I was going backwards. My acceleration was negative. My acceleration went to positive, reduced my speed, and reduced it to 0. The turning point is 0, where my acceleration made my speed 0. Then I went to the positive. Because my acceleration is positive, my speed went up and I went back the last point I could go. (Demonstrating these by walking)*

Student: *I hit the bottom.*

Hasan: *Okay, we can say that. It's also the minimum point. I went backwards, the speed increased, and the minimum point occurred.*

(Drawing this with his hand.)

Hasan: *So, when my acceleration becomes positive and I was going backwards, I started to go positively forwards. I had a minimum point. Okay, do we understand?*

When Hasan realized that his students had difficulty in understanding, he started demonstrating by walking in classroom and moving his hands. In this way he tried to make his students understand by concretizing the physical interpretation of the derivative. Hasan walked in the classroom by increasing or decreasing his speed according to his purpose, showed how the graph would look like with his hands. The observational note taken by the researcher regarding this section of Hasan's lecturing is given below:

> While talking about the physical interpretation of the derivative, he conducted a mutual question-and-answer session with his students. He questioned his students about the origins of their thoughts. He tried to figure it out by walking because he realized that his students were experiencing difficulties in understanding accelerated movement. At this point, both he and his students made comments on the increase and the decrease in acceleration. Hasan showed the maximum and minimum points of the graph by drawing with his hand on the air (A Section from Hasan's 1st Lesson Observation Note).

Similarly, Hasan expressed his thoughts in the interview conducted with him after the lesson:

> When I talked about the concepts such as speed, acceleration and distance, they had difficulties with understanding. I realized that I couldn't help them by lecturing on the board; I started walking into the classroom. I speeded up, down and walked forwards and backwards (A Section from Hasan's 1st Lesson Interview)

Using a Classroom Object

The sub-code of using a classroom object is concerned with the inclusion of an element in the classroom environment by the participants in order to ensure that students have a better understanding. Pre-service teachers integrate an object they think will serve for the purpose by correlating it with the subject of the lesson.

Triggers that motivate pre-service teachers for using a classroom object,

- that they are easily accessible,
- establishing a correlation with different disciplines,
- increasing the comprehensibility of the course subject, and
- the thought that it will help students to concretize an idea.

In her first lesson, Gülbin conducted her teaching with the activities she prepared for the learning goal: "The student determines that the medians in triangles intersect at one point and applies this in exercises." Gülbin handed out the activity in Figure 1 to her students which she thought would help students figure out the centroid.

Figure 1

The Activity Prepared by Gülbin for Her First Lesson

Activity 1

Step 1: Prepare a paper in the form of a triangle.

Step 2: Determine the midpoints of each side and mark.

Step 3: Combine the midpoints that you marked with the corresponding corners of the triangle with straight lines.

Step 4: Find the intersection point of the straight lines you drew in third step and mark it. (Let's call this point M)

Step 5: Hold your pencil upright and put the paper onto the nib of the pencil so as to nib touches point M.

Step 6: Did the paper stand in balance? Could different triangles stay on the nib of the pencil? Why?

After she distributed the activity, despite not being included in the lesson plan, she stated that the table, chair, and camera in the classroom stand in balance, as follows:

Gülbin:	*Friends, for example the table and the chair stay in balance. How can they? Look, there is this camera for example, the camera stand stays in balance. Well, how come these objects stand in balance?*
Student:	*With their legs.*
Gülbin:	*With their legs. Well, what if we put their legs in a different position, would they still stand in balance?*
Student:	*Yes.*
Student:	*No.*

Student: *If we center them on their centroid, they would.*
Gülbin: *Your friend said that if we centered then on their centroid, they would have. Anyone who agrees or disagrees? If you agree or disagree, why?*

Gülbin gave examples of table and chair and stated that they stand in balance. When she noticed the camera legs, she included them to her examples. Thus, she tried to capture the attention and interest of her students in order to correlate the concept of centroid with physics course. In addition, she helped them to concretize the concept by relating it to daily life. She created a platform in class for discussing on how to find out the position of a centroid. The observation note taken by the researcher regarding this section of Gülbin's lesson is given below:

> Gülbin distributed the activity to her students. She asked her students how the table and chair stand in balance (not included in the lesson plan). At that moment, the tripod caught her eyes. She said that the camera leg stands in balance. In this way, she tried her students to reach at the concept of centroid (A Section from Gülbin's 1_{st} Lesson Observation Note).

Similarly, Gülbin expressed her thoughts in the interview conducted with her after the lesson:

> After I distributed the activity, I said that the table and chair were standing in balance. At that moment, they crossed my mind, and then camera legs caught my attention. I said that it was also standing in balance. I asked them how it was standing in balance considering whether they can arrive at centroid. They could have remembered it from the physics course. I also thought to make them see the relationship between math and physics. After all, the table, chair, and the camera are before their eyes (A Section from Gülbin's 1^{st} Lesson Interview).

In her fourth lesson, Gülbin asked her students what they think when they heard "height", with an intention towards learning goal: "The student can calculate the height of a triangle". The students responded Gülbin as following:

Gülbin: *What would you think about height if we're not in geometry lesson? What does height mean?*
Student: *We know height from physics.*
Gülbin: *Well, what if I wasn't going to school, wouldn't I know anything about height? Don't you ever use height out of school?*
Student: *No.*
Gülbin: *Where do you use it, for instance?*
Student: *While measuring.*
Gülbin: *For instance?*
Student: *(cannot be understood)*
Gülbin: *Well, how was those heights measured?*
Student: *With a measuring stick.*
Gülbin: *Yes. How, for instance? If I say the height of the ceiling from the floor, how can be measured? Where do I measure exactly? Or when I measure the height from the floor to the lamp, where do I exactly measure?*
Student: *From the ceiling to the floor.*
Gülbin: *Floor, where?*
Student: *Vertically.*
Gülbin: *Vertically. 90 degrees. Why do I take 90 degrees?*
Student: *As the shortest distance.*

A student stated that they used height in physics classes. Gülbin directed them to express how they used height in their daily lives. She wanted them to tell how they measure the length. When her students could not draw a conclusion that she wanted, Gülbin integrated classroom objects into her lecturing by asking students to state the distance from the floor to the ceiling or from the lamp to the floor. Based upon the lamp in the classroom Gülbin tried to make her students grasp the concept of height. The observation note taken by the researcher regarding this section of Gülbin's lesson is given below:

> Gülbin asked her students what comes to their mind when they heard "height". Her students gave different answers. When she raised the topic to measuring of height, she asked how they measured. Then, she let them deduce perpendicularity by asking how the distance from the lamp on the ceiling to the floor could be measured (A Section from Gülbin's 4th Lesson Observation Note)

Using Mathematical Software

The sub-code of using of mathematical software is related to integration of technology into teaching although this software is not included in the lesson plan. Participants use mathematical software to grab students' attention, as well as ensuring students to concretize the concepts in a shorter period by taking advantage of the dynamism of the software.

Triggers that motivate pre-service teachers for using mathematical software,

- thinking that it will help in generalizing,
- thinking that it will ease making comparisons,
- thinking that students can make choices among alternatives,
- thinking to deal with student difficulties,
- to include different examples,
- to demonstrate.

In his fifth lesson, Seyfi conducted his teaching with the activities in Figure 2 he prepared for the learning goal: "The student can define periods and periodic functions; find periods of and draw graphics of trigonometric functions". findings.

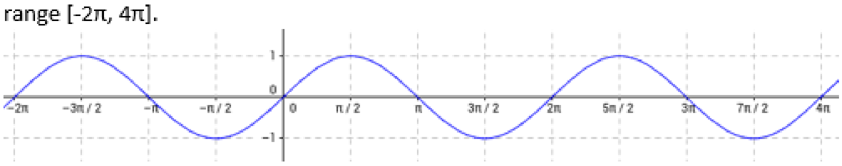
Figure 2

The Activity Prepared by Seyfi for His Fifth Lesson

* Fill in the table below.

x	-2π	$\frac{-3\pi}{2}$	$-\pi$	$\frac{-\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
sinx													

According to the values you find in the table, the graph of f (x) = sinx function is as follows in the range [-2π, 4π].

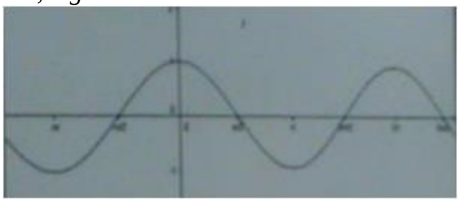


Looking at the graph,
- Could you say the period of the f (x) = sinx?
- How does it make it easier to know the period of the function in the graph drawing?
- Similarly, draw a graph of the function f (x) = cosx and specify the period.

Seyfi's students had difficulties in drawing f(x)=cosx graphic which was the last step of the activity. So, he turned on the computer and included using software to his lecturing and drew the graph of the function by GeoGebra.

Seyfi: *It says "Similarly, draw a graph of the function f(x)=cosx and specify the period." It actually says, find the period of cosx. When I took a glance at the classroom, friends, I couldn't see anyone who could draw the graph completely. I thought we could do this way. Here is the software called GeoGebra. I draw the graph on the software and let's decide together to its period. Besides, we can see what the period would be easier. I'm going to draw now the graphs with the help of the software. It's right before your eyes. Then we'll explore the periods for these functions. Alright? The, we'll try to figure out how these periods are changing, depending on what. What was happening in our x-axis? π becomes $\pi/2$; right? These were the values.*

Geogebra:



Seyfi: *The graph of cosines x, is that correct?*
Student: *Yes.*
Seyfi: *Let's try to say something about the period of our cosines x.*
Student: *(cannot be understood)*
Seyfi: *Then 0 and 2π. Now, let's take little notes. We've already said for the sinus x, it was 2π, right?*
Student: *What happens to π ?*
Seyfi: *Friends, does it repeat one for π ?*
Student: *0 and 2π.*
Seyfi: *Then, what is the period?*
Student: *0 and 2π.*

When Seyfi saw that students were having difficulties in drawing graphs, he opened the GeoGebra software and draw the graph of the function although it was not included in the lesson plan. He also asked students to discuss the period of function through the graphic that he had drawn. Thus, the made his students arrive what will be the period of f(x)=cosx function. The observation note taken by the researcher regarding this section of Seyfi's lesson is given below:

> He gave Activity 5 to his students. They talked about the examinations on f (x) = sinx function on the activity paper. He then drew the graph of the f (x) = cosx which is other phase of the activity and gave his students time to find the period. At this stage, he walked around the classroom to see what students were doing. He answered questions of the students. Then he said that there was nobody could exactly draw the graph and he would draw it with software. He said that they would see the periods easier. Using the scrolling property of the program he moved along the x-axis and could show the period of the graph more clearly. He did not mention that he would use the software in the lesson plan (A Section from Seyfi's 5th Lesson Observation Note).

In the later part of the lesson, he has also made use of the GeoGebra software while generalizing about the periods of trigonometric functions. Having seen that the software served his purpose, Seyfi expressed his thoughts in the interview conducted with him after the lesson:

> In the previous activity, we talked about the periods with the examples from daily life. Well, like TV series and so on. Then, we examined another periodic function which I gave the graph of. Now I have also given this activity to distinctively show the trigonometric functions. I walked around the classroom while the students were drawing the graph of cosx, some students were making mistakes. GeoGebra came to my mind, teacher. We were taught in technology course at the university, I turned on the computer thinking that it would be useful. I drew the graph on the software, they liked it a lot. I even used it generalizations in the later part of the lesson. It made it convenient for me too, my teacher. We scrolled; we did (A Section from Seyfi's 5th Lesson Interview).

Changing Tools-Resources

The changing tools-resources sub-code is related to the situations in which student teachers change tools and resources they planned to use. Student teachers remove the tools and resources that they have previously decided to use from their teaching and tend towards different tools and resources for the learning goal process.

Triggers that motivate student teachers for changing tools-resources,

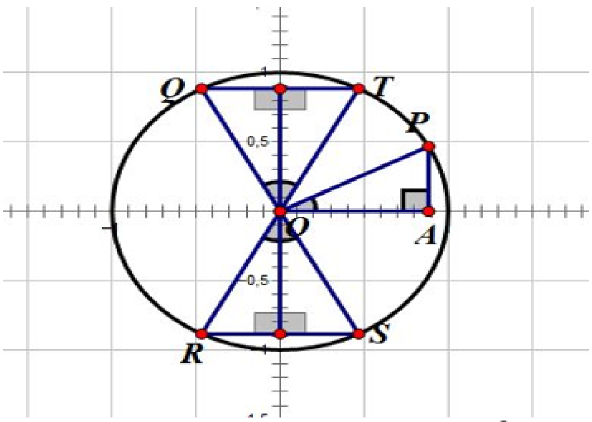
- the tools-resources cause students to get bored,
- thinking that they are not helpful for teaching,
- unable to reach tools-resources.

In his first lesson, Seyfi made use of two different activities for the learning goal: "The student can write trigonometric ratios of kπ / 2 ± α, with k∈Z, in terms of trigonometric ratio of α." In second lesson, Seyfi did not use Activity-3 in Figure 3, which was actually in his plan.

Figure 3

The Activity Prepared by Seyfi for His Second Lesson

Activiy-3:



Using the unit circle above, find the trigonometric ratios of $\frac{\pi}{2}+\alpha$, $\frac{3\pi}{2}+\alpha$, $\frac{3\pi}{2}-\alpha$.

Fill the following tables according to the examples.

x	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$\frac{5\pi}{2}-\alpha$
sinx	cos α		-cos α		
cosx	sin α		-sin α		sin α
tanx			cot α		
cotx					

x	**sinx**	**cosx**	**tanx**	**cotx**
90- α				
90+ α			-cot α	
270- α	-cos α			
270+ α				
450+ α				

The reason that Seyfi did not use this activity is that he observed that his students got bored of the activities he used in his first lesson and that he did not find the activities helpful. Seyfi expressed his thoughts in the interview conducted with him after the lesson:

> I prepared all those activities, but they did not follow in the first lesson. It sounded unfamiliar to them, I guess. So, they got bored. Then I thought that they would get bored and would not follow the lesson, I said to myself that I would then give up the activity and they would follow. We went on from the work sheet (A Section from Seyfi's 2nd Lesson Interview).

Discussion and Conclusion

As a conclusion of the conceptualization of pre-service teachers' approaches to contingent moments they encounter in teaching in the context of responding to the availability of tools and resources; the sub-codes of demonstrating, using a classroom object, using mathematical software, and changing tools-resources were identified. Triggers that cause these sub-codes to occur are also identified in this study.

Pre-service teachers have shown demonstrating approach with the thought that they are easily accessible, it eases reflecting instantaneous changes, it will help to achieve the outcome, and it can provide concretization. In this way, they demonstrated abstract situations that students could not concretize. On the other hand, participants made use of classroom objects for their teaching with the thought that it would help students to concretize an idea, that they are easily accessible, to establish a correlation with different disciplines, and that it increases the comprehensibility of the course subject. Rowland et al. (2011) stated that, the pre-service teacher included the 1-100 counting cards that he noticed at the moment, in his lecturing although it was not in the lesson plan.

Pre-service teachers have integrated mathematical software into their teaching because they thought that it would help making generalizations or comparisons, that students can make choices among alternatives, to deal with student difficulties, and that they realized the necessity to concretize. Also, Laborde (2001) stated that using digital technologies, such as the dynamic software package GeoGebra, can place significant demands on teachers' mathematical knowledge and hence provide opportunities for making such knowledge visible and available for exploration (as cited in Bretscher, 2019). Corcoran (2013) stated that the students had difficulty because the pre-service teacher conducted the lecturing only verbally and as in the lesson plan. However, while they discuss the concepts of learning and practice of teachers, Ball and Cohen (1999) argue that these concepts include the teacher to size up from moment to moment (cited in Stockero & Van Zoest, 2013). From this point of view, it is important for pre-service teachers to concretize the situations that are not understood during teaching even though they were approached abstractly in the lesson plan.

Pre-service teachers resorted changing the tools-resources with the thought that the tools-resources in the lesson plan cause students to get bored, and that they are not helpful for teaching. By removing some of the tools-resources, the pre-service teachers turned to different tools and resources to make their students comprehend the intended learning goal. In Johnson's (2011) study, when the Smartboard stopped working after a pre-service teacher started her lesson, she could not continue through the presentation she prepared. She could not use the Smartboard, but she had a whiteboard and could share with her students what she remembered from the plan. To address such situations effectively, it is important that pre-service teachers undergo comprehensive preparation.

Pre-service teachers are expected to be informed about different contingent moments. In this respect, it is considered that this study, in which the contingent moments and the possible approaches to these moments that mathematics pre-service teachers who have not yet have the adequate experience would encounter were identified, will provide important contributions to both pre-service teachers and the teacher training process. Compared to experienced teachers, pre-service teachers may have difficulties in noticing the deficiencies in their teaching and overcoming these difficulties since they are not experienced (Rowland et al., 2011). Mason (1998) states that the greatest difference between experienced and novice teachers is "the form and structure of their attention" (cited in Stockero & Van Zoest, 2013). As can be seen in Kula (2011), while some pre-service teachers may overcome some contingent moments, others may not. Similarly, Kgothego and Westaway (2023) revealed that teachers who prepared their lessons through lesson study process did not want to deviate from the lesson plan. Nevertheless, it has been suggested that teachers should exhibit flexibility in the implementation of their lesson plans to better align with the mathematical needs of their students. For this reason, in Special Teaching Methods, Instructional Technologies and Material Development, and Technology Assisted Mathematics Teaching courses, it is important to address the approaches that help understanding students and the approaches that are focused on when and for what the technology can be used in class. In addition, it is suggested that the approaches related to students' thinking in Mathematical Thinking courses and the approaches such as individual observation, examination, evaluation and development of teacher interventions in Teaching Practice courses should be included in the curriculum of existing courses.

Given that pre-service teachers' experiences in real class environment are not adequate, they cannot be expected to have complete knowledge of contingent moments. This, once again, underlines the importance that pre-service teachers should be trained and gain experience in this direction. Mason (1998, 2011) suggests that improving the awareness of teachers may help to increase in their attention to what they should focus on their lecturing (cited in Stockera & Van Zoest, 2013). Informing the students about what difficulties they may face with in which course topic, what misconceptions they may have and what common mistakes they can make (Ryan & Williams, 2007) can also help them to be prepared for surprises and to have insight (Rowland et al., 2011).

It is suggested for the future studies to determine which of the exhibited approaches are beneficial for students to learn better and for an effective mathematics teaching. It is now possible to investigate which of the exhibited approaches to contingent moments effects comprehension and how. This study, which was conducted with pre-service teachers, may also be conducted with experienced teachers. By raising awareness to the identified approaches and examining first and last lessons of pre-service teachers, the effectiveness of such approaches can be researched. The improvement process of the pre-service teachers' approaches to contingent moments can be observed. It will be possible to examine whether similar sub-codes will emerge for different schools, experienced or inexperienced teachers and different countries.

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Statement of Responsibility

Both authors contributed to the whole process such as data collection, initial conceptualization, drafting of the original manuscript, methodology, etc. of the study.

Conflicts of Interest

The authors have declared that there are no potential conflicts of interest regarding the content of the article.

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