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Identifying influential individuals in social networks: An example of a location-based online social network

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Abstract

The landscape of information access has evolved significantly with the advent of search engines, social media platforms, and the widespread internet usage. These developments have fostered a global communication network, resulting in intricate connections between individuals. Online social networks have emerged as key facilitators of social interaction, expediting the exchange of information and playing a pivotal role in content dissemination. Within these networks, certain individuals, termed as Key Players, wield considerable influence, profoundly impacting information diffusion. Thus, identifying the most influential individuals within complex network structures comes across as a crucial challenge. In this study, the data obtained from a complex dataset was analyzed with appropriate statistical methods. Within this context, modularity and eigenvector centrality metrics have been used to identify nodes for the first activation aiming to maximize influence in social networks. Visualization and analysis of the dataset are conducted using Gephi software, providing insights into the dynamics of the social network structure and facilitating the identification of key players. As a result of the data analysis, the top five most influential users were identified and the impact of these users on the network was presented with a graphical representation. This study contributes to both the theoretical understanding and practical applications of influence maximization in social networks.

1. Introduction

The processes of accessing information have undergone significant changes from past to present. In ancient times, information was transmitted orally and through written texts. During the Middle Ages, books began to be produced in manuscripts, but access to information remained limited. The Renaissance period, marked by the invention of the printing press, accelerated the production of books and provided greater access to information. With the Industrial Revolution, books and newspapers reached a wider audience, and with the rise of mass media such as radio and television in the mid-20th century, the dissemination of information became much easier. However, the emergence of the internet at the beginning of the digital age with the widespread use of computers revolutionized access to information (Öztürk, 2017). The internet has made it possible to access information instantly and comprehensively. Search engines and social media platforms have created a global communication network between individuals by making it easier to access and share information on every topic (Akkaya, 2021). Mobile technologies have strengthened connectivity by making it possible to access information from anywhere, anytime. Social networking has given individuals the opportunity to come together and communicate in a virtual environment regardless of time and space (Acar et al., 2014; Saçan and Eren, 2021). These developments have led to curiosity and research on how online social networks work.

Online social networks are complex network structures that facilitate users' social interactions through information sharing, photo and video sharing, and status updates (Kadoić et al., 2018). Users adopt online social networks as effective tools to connect with friends, business partners, colleagues, family or people in different corners of the world. While there are many different types of social relationships in real life, these networks often only include certain types of relationships, such as friendship or following, and people's complex social

relationships are often referred to by terms such as "friend" or "follower". In these environments, represented in Figure 1, while we know whether a relationship between two nodes exist, the nature and depth of this relationship often remain unclear (Boyd and Ellison, 2007; Sever et al., 2017). Various analysis methods are needed to resolve this ambiguity.

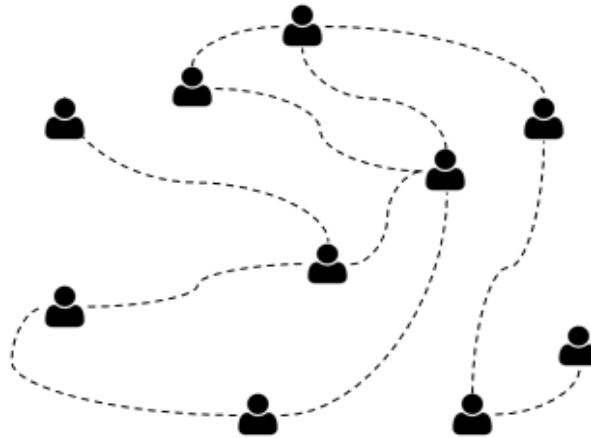


Figure 1. A simple example of a social network.

The process of obtaining information with data from social media platforms is called Social Network Analysis (SNA). SNA is a technique used to visualize and investigate the structures and relationships of a social media network. Its main purpose is to provide researchers with the opportunity to make inferences about individuals or groups and understand their social media behavior by examining the structure of the social media network (Freeman, 2004). Basically, a social media network is a collection of actors and the interactions between them. Understanding the mechanisms of influence spread can lead to optimized strategies that effectively target key players within these networks. In marketing, for instance, identifying and leveraging influential nodes can enhance the reach and impact of campaigns, thereby improving return on investment. In the context of public health, accurately identifying key players can facilitate the dissemination of crucial health information, potentially leading to improved public health outcomes (Zhan et al., 2019). Owing to social media analytics, we can analyze groups and actors in social media networks to understand the early behaviors to be performed by groups and actors. At this stage, various models are needed to simulate the interactions and behaviors of individuals in a computer environment (Jalayer et al., 2018).

This study presents a workable methodology for influence maximization in social networks using advanced metrics such as modularity and eigenvector centrality. These metrics are instrumental in discerning the structural and positional significance of nodes, allowing for a more precise identification of key players within the network. Modularity is a valuable metric for detecting community structures within networks. By identifying densely connected subgroups, it provides insights into the functional and structural organization of the network (Chen et al., 2013). Eigenvector centrality, on the other hand, extends the concept of node influence by considering not just the immediate connections of a node, but also the influence of its neighbors (O'Malley and Marsden, 2008). This holistic approach ensures that nodes with broader, more impactful connections are identified as key players.

Gephi software was used for the visualization and analysis of the data set considered. Gephi's advanced visualization capabilities enable a comprehensive examination of the network's structure, facilitating the identification of influential nodes (Bastian et al., 2009). By integrating modularity and eigenvector centrality metrics, this study not only contributes to the theoretical framework of influence maximization in social networks, but also provides a practical method to optimize influence maximization in practice. The rest of this paper is organized as follows. Section 2 introduces the methodology briefly. Problem definition and computational study are given in Section 3 and 4. Section 5 contains the results obtained. Finally, conclusions are presented in Section 6.

2. A Brief Literature Review

Several studies have investigated methods to tackle the IM problem and have addressed the challenges of identifying influential users. Domingos and Richardson (2001) initially treated the influence maximization (IM)

problem as an algorithmic challenge. They modeled the influence of customers on each other, which they defined as nodes in a social network, as a Markov random field. In subsequent work (Richardson and Domingos, 2002), they used linear modeling and weighted influence coefficients to optimize viral marketing strategies by independently calculating each customer's probability of purchase. Kempe, Kleiberg, and Tardos (2003) were the pioneers in formulating the problem as a discrete optimization problem. Despite numerous studies addressing the IM problem, there remains a significant research challenge in exploring aspects that aid in identifying influential users, as seen in the work of Zhang, Guo, Yang and Wu (2023). Additionally, as seen in De Salve's study, estimating user impact remains the focus (De Salve et al., 2021). Understanding the dynamics of influential users is crucial, given the profound impact that messages propagated through social networks can have on contemporary society.

Some research suggests that criteria derived from network structure can be used to select the most effective users. Research by Lü et al., (2016) and Das et al. (2018) focuses on metrics applied to rank each node in terms of its influence, known as centrality measures. These measures, derived from the network structure, inform the selection of the most influential users. Kuikka (2024) proposed a method for community detection based on the search for local maxima. Kuikka defined two types of centrality measures: out-centrality, which measures the influence of a node on other nodes in the network, and in-centrality, which measures the influence of a node on other nodes in the network, based on the influence-spread matrix, and used Gephi software to visualize communities in the Facebook network. This method, which detects overlapping and hierarchical communities using influence propagation matrix and probability matrix, has proven its accuracy and efficiency in various networks. We mentioned that online social networks play a highly influential role in information diffusion. Influential nodes can be selected to ensure that all parts of a network are covered. Research has been conducted for information diffusion modeling in many areas such as popular topic detection and identifying influential diffusers. In this context, Li et al. (2018) present a review of solutions to the IM problem from an algorithmic perspective based on diffusion models that simulate the diffusion process. Studies that focus on specific elements of the problem, such as propagation models or simulation of the propagation process, but not on the whole network (Jaouadi and Romdhane, 2019; Guille et al., 2013), do not provide a comprehensive review of various methods for IM.

The literature review also reveals that there are studies that propose various methods and classifications. Arora et al. (2017) examined how IM techniques cope with different propagation models, datasets and parameters. The results show that there is no single best technique for IM, but some of them provide two key features. Yang and Pei (2019) provide a comprehensive review of impact analysis in evolving networks, categorizing the research into five primary tasks. The initial three tasks focus on bringing to light influential nodes when network's evolution is fully known, while the remaining two tasks address network evolution detection for effective influence analysis when the network evolution is not fully accessible. Bian et al. (2019) discusses the current status and future trends in identifying top- k nodes in social networks. It highlights significant progress in research on top- k influential nodes compared to top- k significant nodes. Banerjee and others (2020) present a comprehensive literature review and classification scheme on this topic. Taking a similar approach, Günneç, Raghavan and Zhang (2020) focus on the case where all active neighbors of a node have equal influence on the node and it is desired to activate the entire network. They design a branch-and-cut approach on random graphs for this case, which they call the Least Cost Influence Problem. Kazemzadeh et al. (2023) recently developed the Influence Maximization Based on Community structure (IMBC) algorithm to optimize influence maximization by addressing challenges in time efficiency and seed node selection through optimal pruning and scoring adjustment, focusing on nodes with high Rich-Club coefficients. Experimental results demonstrate IMBC outperforming recent algorithms in influence spread and runtime efficiency, highlighting its significance as a recent advancement in the field. Pattanayak et al. (2024) combine degree centrality and betweenness within community-based strategies to enhance social network influence. Their Community Diversified Seed Selection approach, validated on benchmarks and real-world networks like Facebook, outperforms traditional centrality methods in spreading rate, execution time, and complexity, making it ideal for large-scale applications (Pattanayak et al., 2024).

Table 1. Summary of the previous researches

Author(s)	Year	Methodology	Application Area/Dataset(s)
Domingos & Richardson	2001	Linear Modeling	Viral marketing
Richardson & Domingos	2002	Linear Modeling	Viral marketing

Author(s)	Year	Methodology	Application Area/Dataset(s)
Kempe et al.	2003	Greedy Algorithm	A collaboration graph obtained from co-authorships in physics publications
Guille et al.	2013	Taxonomy	N/A
Lü et al.	2016	Centrality Measures	A directed short-message communication network
Arora et al.	2017	Propagation Models, Datasets, and Parameters	Real (large) graphs from arXiv, SNAP repositories and a snapshot of the Twitter network
Das et al.	2018	Centrality Measures	N/A
Li et al.	2018	IM techniques based on their algorithmic designs and Taxonomy	N/A
Yang & Pei	2019	Influence Models for Static Networks	Conducted a comprehensive review of impact analysis in evolving networks
Bian et al.	2019	Reviewed and Classified	Discussed the current status and future trends in identifying top- k nodes in social networks
Jaouadi & Romdhane	2019	Reviewed Propagation Models and Simulation	On the influence maximization problem in social networks
Banerjee et al.	2020	Classification	Presented a comprehensive literature review and classification scheme
Günneç et al.	2020	Branch-and-Cut Approach	16 real-world graphs and simulated graphs
De Salve et al.	2021	Prediction Algorithms	Facebook groups dataset
Zhang et al.	2023	Sandwich Approximation	Netscience, HepTh and SNAP datasets
Kazemzadeh et al.	2023	IMBC Algorithm	The datasets include Caida, Cond mat, Douban, PGP, and Route views
Pattanayak et al.	2024	The Community Diversified Seed Selection	With the data received from Facebook
Kuikka	2024	Influence-Spreading Matrix and Centrality Measures	Zachary's Karate Club network, the American football games network, the Dolphin social network, and two moderate-size networks, the Facebook social circles network and the Government Facebook network
This study	2024	Centrality Measures	Brightkite from SNAP

3. Methodology

3.1. Influence Maximization Problem

In social networks, certain individuals may exert greater influence due to their social status, charismatic characteristics, and other factors. Identifying these individuals is crucial for efficiently disseminating information within the network or tracing the origin of dispersed content. This identification process forms the cornerstone of influence maximization (IM) problems. As its core, the IM problem involves a type of influencer detection problem. In IM, the goal is to influence the maximum number of individuals by identifying individuals to be selected as key players (Gursoy and Gunneç, 2018; Tong et al., 2016). A small group of these individuals has the potential to significantly impact a large portion of the network once they are activated (i.e., adopt a specific idea,

product, etc.). The IM problem seeks to identify a set of k active individuals (where k is a positive integer) within a given network, under various influence diffusion models (Kempe et al. 2003; Morone et al., 2015).

Influence maximization is an important issue, in social network researches. The IM problem intends to identify the most influential individuals, prompting extensive research into criteria defining influence and the development of corresponding algorithms. Key evaluation metrics for social influence include Degree, Proximity, Eigenvector, Katz and Betweenness Centralities (Kempe et al. 2003; Peng et al., 2018).

Degree Centrality is computed as the number of adjacent edges of a node. Betweenness Centrality is determined by the percentage of shortest paths between two nodes that pass through a given node. Proximity Centrality represents the average shortest path from all other nodes to a node. Katz Centrality refers to the sum of the degrees of a given node's neighbors. The concept of Katz Centrality posits that if a node has influential neighbors, then this node is also influential. Eigenvector Centrality is similar in basic idea and computation to Katz Centrality (Peng et al., 2018). It performs well if the graph is strongly connected and eigenvector centrality depends on the quality as well as the number of links, such that a node with a small number of high-quality links contributes more than a node with a large number of lower-quality links (Codal and Coşkun, 2016). However, since actual directed graphs often lack a large connected component, this poses difficulties in practical applications (Temizsoy et al., 2017).

3.2. Eigenvector Centrality

Eigenvector centrality is a centrality metric used in network analysis and is used to assess the importance of a node. This metric relates the importance of a node to other important nodes that have a direct connection to that node. That is, the importance of a node depends on the number of other important nodes to which it is directly connected and their level of importance (Kadoić et al., 2018). Eigenvector centrality considers the nature of the links when determining the importance of a node. This metric is based on a mathematical model where each node is represented by an eigenvector. The eigenvector centrality of a node is based on a weighted sum of the eigenvector centralities of other nodes to which that node is connected (O'Malley and Marsden, 2008; Gürsakal, 2009).

When the score for a node i is defined and the neighborhood matrix (A_{ij}) represents the links in the network, the centrality score is proportional to the total score of all interconnected nodes. M_i is the set of nodes to which node i is connected, N is the total number of nodes, and λ is the eigenvector coefficient for the actor. The mathematical representation of the eigenvector centrality of node i is as follows:

$$x_i = \frac{1}{\lambda} \sum_{j \in M(i)} x_j = \frac{1}{\lambda} \sum_{j=1}^N A_{ij} x_j \quad (1)$$

Eigenvector centrality can often more accurately reflect the importance of a node because this metric takes into account the influence of other important nodes that are connected to the node (Gürsakal, 2009). This allows the importance of a node to be determined not only if it has a large number of connections, but also by considering how important these connections are.

Eigenvector centrality is used in many fields such as social network analysis, communication network models, internetwork analysis and many others. This metric is a useful tool for identifying important players in the network and understanding critical nodes in the network structure.

4. Implementation

4.1. Problem Definition and Characteristics of the Data Set Used

This study employs the SNAP dataset, which was developed by Stanford University in 2004 as part of their research on social network analysis.

Within the SNAP datasets, the Brightkite dataset, which pertains to a location-based online social network ("Stanford large network dataset collection," n.d.), was utilized. The friendship network consists of 58,228 nodes and 214,078 edges. The dataset encompasses a total of 4,491,143 check-ins made by users between April 2008 and October 2010 (Stanford University, 2023).

4.2. Modularity Analysis of the Data Set

Social networks are complex networks that represent relationships between individuals or organizations. Modularity analysis is a technique used to elucidate the relationships between subgroups or modules within these networks. Modularity is a metric that measures the strength of division of a network into modules, with a high modularity indicating dense connections between nodes within modules but sparse connections between nodes in different modules (Blondel et al, 2008; Chen et al., 2013).

For the purpose of modularity analysis, the data set must be imported into the Gephi program, a software designed for the visualization and analysis of large network graphs. The dataset is imported via a CSV (Comma-Separated Values) file, selected from the import screen to preview the data. Figure 2 illustrates that 58,228 nodes and 214,078 edges are successfully imported into the program. The initial visualization of these imported data in Gephi is depicted in Figure 3.

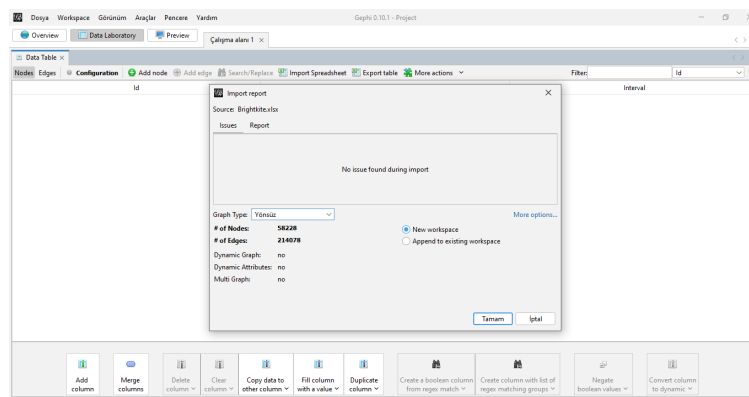


Figure 2. Importing data into the Gephi program.

Prior to commencing the modularity analysis, the "layout" option in the Gephi interface is employed to render the complex image comprehensible. The layout is organized in various ways according to the chosen layout algorithm. This process yields meaningful visuals for analysts and first-time viewers of the analysis results. Gephi provides many layout algorithms by default. Among these options, the "OpenOrd" layout algorithm was selected for its efficiency in visualizing large networks. The OpenOrd algorithm evaluates undirected weighted graphs with a fixed number of iterations controlled through an annealing simulation type programme and aims to better distinguish clusters. Figure 4 displays the graph obtained when the "OpenOrd" layout algorithm is executed with its default settings.

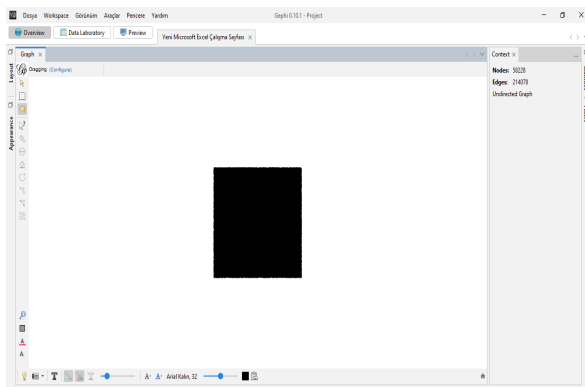


Figure 3. Meaningless and complex transfer image of nodes and edges data.

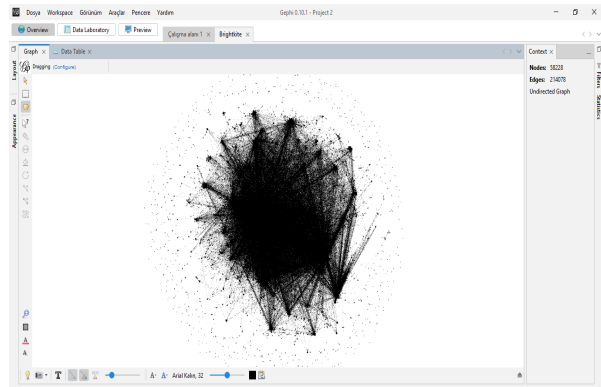


Figure 4. Network view obtained by running the "OpenOrd" layout algorithm.

Following the application of the OpenOrd algorithm, 829 classes were identified through modularity analysis, as shown in Figure 5. The horizontal axis indicates the modularity class and the vertical axis indicates the number of nodes in this class.

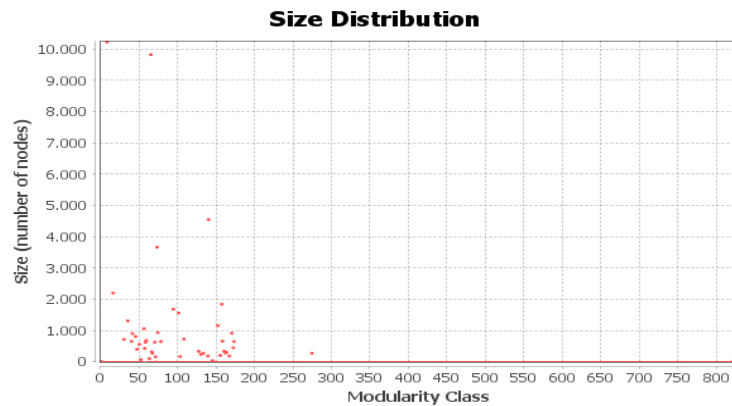


Figure 5. Scatterplot of classes obtained according to modularity analysis.

The number of identified clusters is notably high. When the clusters with the highest number of elements are coloured, the visualization in Figure 6 is produced; however, this alone is insufficient for drawing significant inferences. To derive meaningful insights, eigenvector centrality analysis was subsequently applied based on the available data.

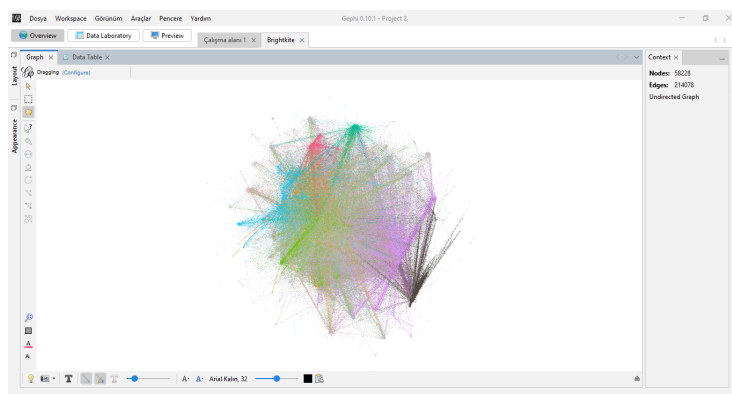


Figure 6. Network view of classes obtained according to modularity analysis.

4.3. Eigenvector Centrality Analysis of the Data Set

Gephi is a graph analysis and visualization software that supports various network analysis metrics, including eigenvector centrality analysis. Eigenvector centrality analysis is a measure used to determine the importance of nodes within a network by considering the influence of other significant nodes to which they are directly connected. Specifically, a node's eigenvector centrality is influenced by the centrality values of the nodes it connects to. A node with a high eigenvector centrality score is closely connected to other important nodes in the network.

The "Eigenvector Centrality" option under "Node Overview" in the "Statistics" section of Gephi was applied to the modularity-analyzed dataset. By utilizing the values obtained from the eigenvector centrality analysis following the modularity analysis during the visualization phase, key players within the network can be visually identified. As illustrated in Figure 7, visualizing the network according to the results of the eigenvector centrality analysis, combined with the prior modularity analysis, allows for the identification of nodes with the highest centrality within each group.

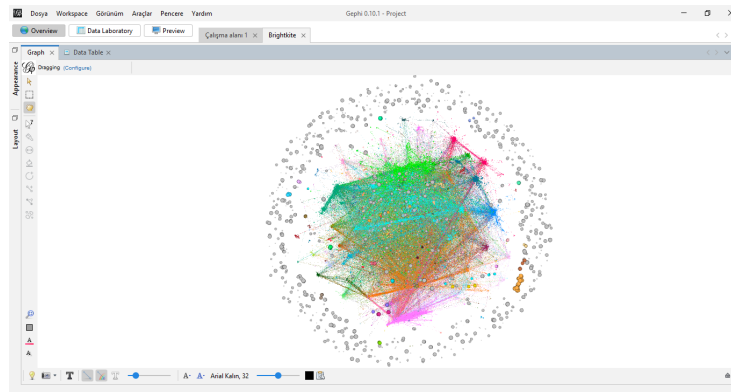


Figure 7. New appearance and coloring of the network according to the values obtained from the modularity and eigenvector centrality analysis.

Gephi enables detailed analysis results to be obtained for large datasets through filtering. When a detailed filter is applied to the network view in Figure 7, the players significantly influencing the network can be observed. When the filter range is set with a minimum degree value of 155, the resulting view is depicted in Figure 8. Additionally, the "OpenOrd" layout algorithm is executed on this view, followed by labeling, resulting in a much clearer graphical output. As a result of filtering aimed at reducing complexity by removing nodes with weaker connections from the network, it is observed that there are 174 nodes with a degree value of at least 155 (0.3% of all nodes in the network) and 2,569 edges (1.2% of all edges in the network). Depending on the filter range, the desired detailed network view can be obtained.

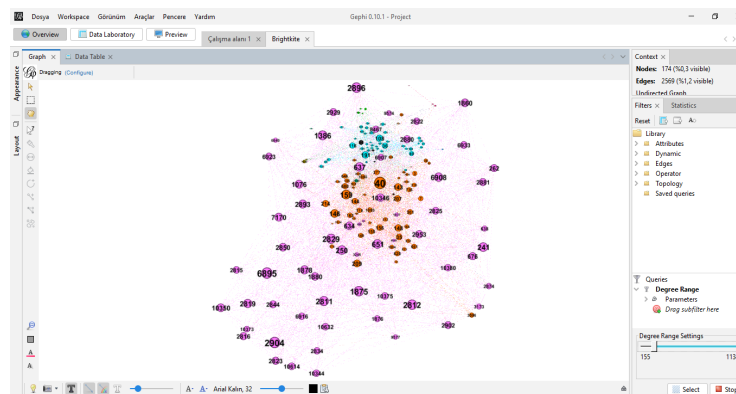


Figure 8. Filtered network view with a node degree of at least 155 nodes.

5. Results

This study employs modularity and eigenvector centrality metrics to identify the nodes that serve as initial influencers, or key players, for maximizing influence in social networks. Visualization and analysis of the dataset were conducted using Gephi software.

By integrating the outcomes of eigenvector centrality analysis with modularity analysis, nodes with the highest influence within each group were identified. Figure 7 and 8 depict the resultant visualization, achieved by coloring or sizing the nodes based on their eigenvector centrality values to illustrate the analysis findings. Figure 9 presents the eigenvector centrality values of the nodes, sorted from highest to lowest influence. Notably, the top 5 nodes with the highest influence degrees are nodes 40, 159, 651, 250, and 634, respectively. Node 40 and 159 belong to the 7th class, whereas the rest are part of the 72nd class. When intending to propagate an effect throughout the network, one or more nodes can be selected from each group based on their eigenvector centrality values.

Id	Label	Interval	Modularity Class	Eigenvector Centrality
40		7	1.0	
150	150	7	0.776245	
651	651	72	0.775019	
250	250	72	0.760911	
634	634	72	0.681106	
2004	2004	72	0.618132	
6995	6995	72	0.617054	
191	191	64	0.58754	
108	108	64	0.578511	
2096	2096	72	0.587231	
1875	1875	72	0.563815	
2812	2812	72	0.558178	
2829	2829	72	0.545069	
2811	2811	72	0.537961	
207	207	7	0.532197	
1386	1386	72	0.531178	
35	35	7	0.529633	
156	156	7	0.516973	
241	241	72	0.515964	
637	637	72	0.508074	
6909	6909	72	0.504023	
116	116	64	0.493226	
2819	2819	72	0.4931	
94	94	64	0.481933	
1878	1878	72	0.480438	

Figure 9. Results of eigenvector centrality analysis.

Figure 10 illustrates the findings from the Connected Components Report, revealing the presence of 547 weakly connected components within the network. This result reflects a significant degree of fragmentation, characterized by numerous subgraphs that remain isolated from one another. Tarjan's Depth-First Search algorithm was employed to effectively identify these components, underscoring its suitability for such network analyses. The observed fragmentation suggests a deficiency in network cohesiveness, which may adversely affect communication and the flow of information across the entire system. A detailed examination of the distribution and sizes of these components could inform strategies aimed at enhancing connectivity. Furthermore, pinpointing critical nodes that can serve as bridges between disconnected components may facilitate improved network integration. These findings highlight the potential need for network restructuring to foster a more interconnected system.

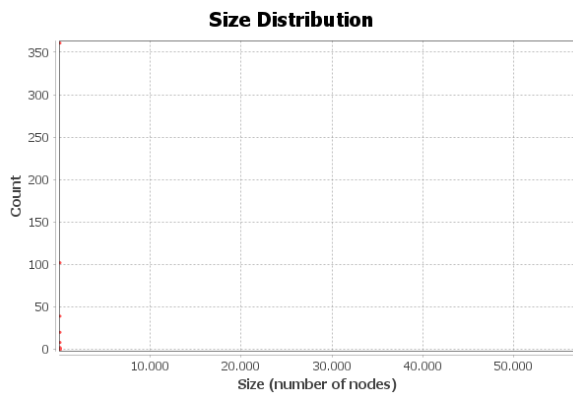


Figure 10. Connected components report.

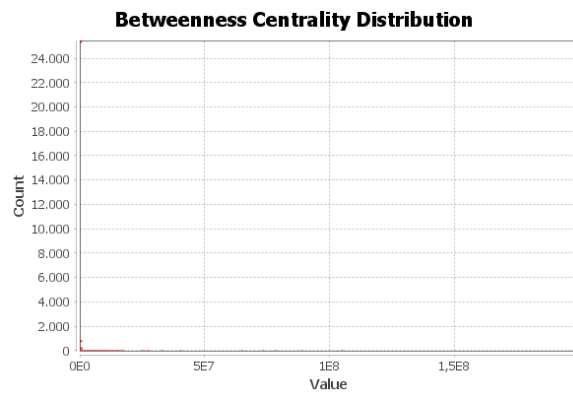


Figure 11. Graph distance report.

The Graph Distance Report, as presented in Figure 11, reveals critical metrics about the network's structure. The network's diameter, which is the longest shortest path between any two nodes, is 18. This high diameter indicates the presence of distant nodes that may require numerous steps to connect. The radius, representing the shortest longest path from a central node to any other node, is 1, highlighting the existence of highly central nodes within the network. An average path length of 7.371 suggests that, on average, nodes are relatively far apart, indicating potential inefficiencies in communication or data transfer across the network. These metrics, derived using Brandes' algorithm for betweenness centrality, provide insights into the network's efficiency and resilience. Understanding these distances can aid in optimizing routes and improving network robustness.

6. Conclusions

This study examines the influence maximization (IM) problem within social networks by employing modularity and eigenvector centrality analyses. The primary objective was to identify key nodes with the highest potential to maximize influence spread across the network. Using the Brightkite dataset from the SNAP collection, the network's structure and key players were visualized and analyzed with the aid of Gephi software.

The modularity analysis effectively segmented the network into distinct communities and revealed the degree of interconnectedness among nodes within these communities. The eigenvector centrality analysis further identified the most influential nodes within these communities, highlighting which nodes could play a pivotal role in network-wide information dissemination. Additionally, the structural analysis of the network unveiled important aspects of its cohesion and efficiency. The identification of 547 weakly connected components underscored the fragmentation within the network and potential barriers to seamless communication. The network's diameter of 18 and average path length of 7.371 suggest that, while the network contains central nodes, it also includes distant nodes that may impede efficient communication.

However, this study has certain limitations. The reliance on a single dataset, specifically the Brightkite dataset, may restrict the generalizability of the findings to other social networks with different structures or user behaviors. The centrality measures and modularity analysis employed may not fully capture the complexity of dynamic or multi-layered networks where relationships evolve over time. The study also does not account for potential external factors that could influence the network's structure or the spread of influence. Therefore, while the findings contribute to the understanding of influence maximization, future research should address these limitations and explore alternative datasets, algorithms, and models to enhance the robustness and applicability of the results. Future studies could investigate the application of similar methodologies to different datasets to validate the findings across diverse social networks. Moreover, exploring the effectiveness of alternative algorithms for influence maximization could provide valuable insights into the robustness of the approach. Additionally, comparative analyses across various social networks may offer deeper insights into the dynamics of social network structures and mechanisms of influence diffusion.

Contribution of Researchers

In this study; Buşra Baytur contributed to the design of the research, data collection, analysis, evaluation of the findings and preparation of the manuscript; Eren Özceylan contributed to the design, monitoring, control, evaluation and preparation of the manuscript.

Conflicts of Interest

The authors declared that there is no conflict of interest.

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