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Portfolio Selection Analysis with a Fermatean Fuzzy-type AHP

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Abstract

This study aims to tackle decision-making problems on interval-valued Fermatean fuzzy sets; the current research proposed an approach based on the AHP method. The interval-valued Fermatean fuzzy set is a mathematical framework used in decision-making and modelling scenarios that involve uncertainty and imprecision. The interval-valued Fermatean fuzzy set extends traditional fuzzy sets by incorporating an additional layer of flexibility and expressiveness, particularly in cases where precise membership degrees are difficult to assign. The AHP method makes the problem more understandable by dividing it into a hierarchy of targets, criteria, sub-criteria, and alternatives, comparing and prioritizing options, and checking consistency. Multi-attribute decision-making algorithms are well-suited for portfolio selection problems. Complex subjective preferences and diversified financial indices affect investment decisions within the multi-attribute decision-making paradigm. For the investment portfolio selection problem, an algorithm implementation based on an interval-valued Fermatean fuzzy set is chosen. The S&P 500 companies are examined. Ten criteria are established for choosing investment portfolios. The investment portfolios were selected using a multi-attribute decision-making method based on interval-valued Fermatean fuzzy sets. The algorithm based on interval-valued Fermatean fuzzy sets and the portfolio decision-making process using these criteria is suitable for choosing the right options. A model that illustrates how choices about investment portfolios should be made using this procedure was created using a grounded theory methodology. The results show that efficient decision-making methods for investment portfolios create a portfolio mindset and assist in concentrating selection efforts on the appropriate projects. Additionally, it enables extremely flexible decision-making within the investment portfolio. These findings offer managers an evidence-based method for making decisions about their portfolios.

Keywords Portfolio selection · Interval-valued Fermatean Fuzzy Set · MADM · AHP

Jel Codes C63, G11, F21

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Portfolio Selection Analysis with a Fermatean Fuzzy-type AHP

The contemporary portfolio theory is a helpful method for selecting investments to maximize overall returns while retaining a reasonable level of risk. An investment portfolio is built to optimize expected return given a specific level of risk using this mathematical framework. Modern portfolio theory offers that instead of concentrating just on an investment's inherent risk and return characteristics, one should also consider how the pooled investment affects the risk and return of the overall portfolio. As stated differently, an investor can assemble a portfolio that includes diverse assets that will generate more significant returns without entailing greater risk. Selecting a portfolio is the unifying process in Modern Portfolio Theory. However, there is considerable debate on the best approach. Markowitz is credited with developing most of the modern portfolio theory. He proposed that the best way to choose assets for each portfolio would be to use quadratic programming to create a collection of efficient portfolios. According to Markowitz, an efficient portfolio comprises investments that produce the best returns for a given risk. High levels of uncertainty and risk, or a lack of relevant information, impair the decision maker's ability to conduct an appropriate analysis.

Numerous models, including the multi-period mean-variance model, the continuous-time model, the anticipated utility model, the dynamic asset allocation model, and the behavioural mean-variance model, have been created since Markowitz (1952) established the well-known mean-variance model in portfolio selection. One of the drawbacks of using variance to quantify risk, according to Markowitz (1959), is that the variance calculations consider both high and low returns. Furthermore, investors should view less-than-anticipated returns as undesired returns. As a result, variance is only a good risk indicator when the underlying return distribution is symmetrical with second-order moments. Otherwise, strong returns will be viewed as risk by variance-based models. The renowned mean-variance model of Markowitz (1952), which quantifies the variability of returns below the mean, is the most significant of the numerous models created to address this issue. Many studies have employed the mean-variance model because it aligns with investors' sense of risk. Uncertainty has always been the biggest issue for decision-makers who have looked for solutions. Numerous fuzzy set (FS) models have been created to describe uncertainty.

Selecting or optimising the optimal portfolio among portfolio alternatives is crucial for decision-makers in today's competitive landscape. Under restricted resources and limits, these choices or optimizations are made based on several criteria. The issue of multi-attribute decision-making (MADM) in portfolio selection arises from this circumstance. The inability of decision-makers to make wise choices among options when faced with inaccurate or inadequate information is another challenge when choosing a portfolio. Therefore, using the FS theory can be a promising way to deal with the different uncertainties that come with portfolio selection. Fuzzy methods in portfolio selection models have drawn more attention recently.

How people reason and form ideas in real-world situations is the field of study of academic disciplines such as psychology, philosophy, cognitive science, and artificial intelligence. Various statistical and mathematical models usually describe these processes; therefore, decision-making (DM) becomes essential. Behaviour management is the choice of behaviour types an individual or organisation needs to work on to accomplish a specific goal. Research proposes that while many judgments in daily life may be made without conscious thought, complicated and meaningful decisions require a more significant amount of thought and effort. When there are well-defined and limited options in a separate scenario, multi-attribute decision-making (MADM) is employed. The quantity of solutions for the MADM problems is set. MADM techniques are frequently used while making decisions, including rating, comparing, and selecting possibilities. These methods are commonly selected because they enable quick DM without requiring intricate package software or mathematical computations. The MADM approach only manages to accomplish one objective. The goal is to resolve the choice issue in the best possible way—least expensively and most favourably.

Uncertainty is the state in which various outcomes of a particular occurrence are feasible but their probability is unknown. As a result, understanding uncertainty is essential to the DM process. Understanding the probability that events will occur in reality requires time and effort. As a result, there is some degree of ambiguity during the whole DM procedure. Fuzzy logic theory provides a strong foundation for logical reasoning in imprecise and unclear data (Zadeh, 1965). Fuzzy logic theory has allowed computers to interpret human language and use human knowledge. It switches from using numerical expressions to using symbols at this point. Fuzzy sets (FS) are symbolic expressions of this type. Choice variables are a part of FSs, much like the probability states. FSs are generated when an objective membership degree(MD) is assigned to each alternative instead of the associated probability values.

Analysing various factors, such as market indications and financial performance, is necessary when investing in mutual funds. When choosing a portfolio, the Markowitz model considers risk and return. However, choosing a mutual fund is a MADM challenge because it involves several funds and criteria. Various MADM techniques have been employed to select portfolios.

When choosing a portfolio, decision-makers need to consider the most effective criteria and how they interact with one another. IVFFS is often applied in MADM problems with imprecise or uncertain decision criteria. It helps handle subjective judgments by representing membership and non-membership degrees as intervals. When dealing with datasets with inherent uncertainty or incomplete information, IVFFS provides a framework to model and reason with interval-valued degrees of membership and non-membership. IVFFS is suitable for systems that rely on human input, such as expert systems or human-centric decision models. Humans often provide uncertain or interval-based assessments rather than precise values. In situations involving multiple stakeholders or experts, IVFFS can aggregate diverse opinions by considering interval-valued degrees of agreement and disagreement. IVFFS can model systems with ambiguous or vague operational parameters, such as robotics or process control. IVFFS is used in applications requiring robust handling of ambiguous or overlapping patterns, such as image processing or data clustering. IVFFS supports decision-making in sustainability and environmental management, where data uncertainty is prevalent.

Myers and Alpet (1968) created the AHP, and Saaty refined it as a viable instrument for solving DM issues in 1977. AHP is an MCDM method that can evaluate quantitative and qualitative criteria in DM, including group or individual preferences, experiences, intuitions, knowledge, judgments, and thoughts in the decision process, and solve complex problems by considering them in a hierarchical structure. In DM, the decision-maker might include objective and subjective thinking. As a result, this circumstance allows decision-makers to identify their DM mechanisms. In AHP, there are at least three levels of hierarchy. The purpose is at the top of the hierarchy. A sub-level comprises the main criterion and any sub-criteria that fall within the main criterion, if any. There are DM possibilities at the bottom level. The number of criteria and each criterion's definition must be accurate for pairwise comparisons to be consistent. The criteria ought to be grouped by their shared traits. AHP can be used for different types of criteria. It is an excellent technique for collaborative DM. The flexibility of the outcome can be examined thanks to the sensitivity analysis. Due to the subjectivity in creating pairwise comparisons and hierarchy matrices, skilled and knowledgeable individuals are required.

Objectives

Investors have various possibilities thanks to the market's abundance of investment tools. These investment tools can be used to construct several different portfolio alternatives. The key consideration here is how investors will select between these portfolios. In 1952, Harry Markowitz described the Modern Portfolio Theory's tenets, which were created in finance literature against the Traditional Portfolio to solve the portfolio selection problem. In addition to ignoring the relationship between the securities included in the portfolio and failing to adequately address numerical approaches in investment instrument selection, traditional portfolio theory is based solely on the principles of securities selection and over-diversification by sector. Modern portfolio theory addresses The diversification strategy quantitatively, which is supported by statistical tools. When risk is not considered, a portfolio based solely on the predicted return is not ideal.

Accordingly, the securities to be included in the portfolio must have a negative correlation with one another to reduce risk through diversification. Put another way; diversification will no longer have the same risk-reducing impact if the returns of the investment instruments in the portfolio move in the same direction and with the same intensity. This is where the most important distinction between the traditional and modern portfolios becomes apparent. Currently, contemporary portfolio theory aims to either maximise return at a given risk level or reduce risk at a given return level. In order to choose the best portfolio, it is first important to identify the efficient portfolios that may be made from all of the market's securities and to ascertain their efficient limitations. The primary benefit of contemporary portfolio theory is its ability to demonstrate how investment instruments with varying weights in the portfolio can produce the same expected return at a reduced risk level. According to the finance literature, high risk should be taken for a high projected return. Even if this is typically the case, it is important to demonstrate the risk-return ratio that the generated portfolio will yield, the appropriate weighting, and how various combinations of the same portfolio at the same risk level might yield higher returns. In any event, there will be a limit to the projected return and the amount of risk that may be taken. Eugene Fama proposed this limit in the Efficient Markets Hypothesis. According to the relevant hypothesis, high-risk assets must be added to the portfolio at a specific rate to achieve better returns than the efficient frontier.

Despite its effectiveness, the mean-variance model developed by Markowitz in 1952 does not adequately address the financial applications of today. Large fund managers today may make mistakes because of their limitations. This model does not entirely account for transaction costs, sector-related concerns, regulatory standards, and floor and ceiling practices in stock transactions and limits such short selling in portfolio fund selection. As a result, fund managers, practitioners, and regulators are starting to take notice of multi-objective portfolio optimisation models that consider several additional restrictions, including transaction costs.

Our groundbreaking study tackles Interval-valued Fermatean FSs (IVFHFSs) for multi-objective portfolio selection and successfully overcomes several potential fund management limitations. In order to create a thorough portfolio, it also incorporates several limits.

Necessity

Some researchers have used the mean-half variance concept in fuzzy environments since fuzzy theories work so well. IVFFS is one of the fuzzy theories that can better capture decision-makers' hesitancies and represent unclear information with various values. Using various fuzzy theories and approaches, numerous studies have examined portfolio models to capture investors' risk attitudes. Investors can naturally select portfolios with high membership degrees under the score and low MD variability when evaluating them using utility criteria.

IVFFS meet data representation requirements for investment portfolio selection. Rarely does an expert tend to identify the problem in specific settings. Uncertainty may arise from a lack of confidence in communicating investment portfolio selection criteria, or imperfection leads to doubt about the value of a variable, a decision to be made, or a conclusion to be drawn for the actual criterion. Multiple factors, such as incomplete knowledge, stochasticity, or acquisition errors, could lead to uncertainty. As a result, the IVFFS approach would contain indeterminate traits and behaviours, as well as unpredictable situations, implying that indeterminacy plays a role. When a decision-maker must weigh several factors when selecting options, the issue becomes one MADM that can be resolved with relevant tools. IVFFSs and AHP techniques are appropriate tools in this regard. These make sense for the following reasons, which can be explained:

1. It can evaluate portfolios based on various criteria;
2. It is capable of accounting for the interrelationships between the DM criteria;
3. It can use language to convey the decisions made by decision-makers.

Based on these thoughts, we defined the study's research question as follows: Can IVFFS approaches with financial indicators assist in choosing an investment portfolio that outperforms the market benchmark?

Originality

A method called MADM is used to prioritize the problem of portfolio selection. This work's fuzzy methodology catches the false information that separates decision-makers' assessments. Portfolio selection has been extensively studied in various domains, including machine learning, artificial intelligence, and traditional and quantitative finance. The general aim of portfolio selection is to devote money to a group of assets to achieve specific long-term goals. As with other investment consulting problems, portfolio selection is affected by many direct and indirect factors. In this sense, academics, managers, investors, and practitioners have faced challenges in identifying, evaluating, and choosing criteria for assessing and choosing portfolios. This study's numerical application intends to create a portfolio selection model to assist account holders who intend to make the correct decisions and demonstrate the model's uses in financial markets. According to the IVFF-AHP process, a method has been proposed for MADM to provide planners with more reliable options. Because of its pliable build, the proposed approach is a valuable instrument that may be implemented for various complex decision problems with several competing criteria. Using MD and ND, FFSs, as opposed to IFS and PFS, more effectively convey the ambiguity of inaccurate information. The expert team is expected to assess the needs and weigh possible options regarding the requirements. Based on experts' opinions, the IVFF-AHP was used to calculate the weights of the evaluation criteria.

Calculating statistical parameters requires much data, which is a drawback of portfolio models. However, non-statistical methods have been devised as time series data might not be available for some new financial products, like stocks. Numerous studies in the literature have used data-based methodologies, including experimental research, genetic algorithms, artificial neural networks, and stochastic programming. Even though the studies only require a modest quantity of quantitative data, data are necessary to create models and determine the best portfolio among the preferences in the aforementioned ways. As a result, this study introduced and employed qualitative data from experts or decision-makers to identify the best possible portfolio.

Research Gap

Professionals assess several investment options based on various criteria, which can be regarded as a MADM issue when evaluating investment portfolio selection. It is crucial to employ several techniques and choose the most effective outcome to overcome the challenges posed by the evaluation environment's complexity and ambiguity, professional psychological conduct, and cognitive uncertainty. The model developed using AHP based on IVFHFS is anticipated to have less uncertainty.

Contribution

This paper presents an MADM model consisting of an AHP based on IVFFSs. The new methodology selects the best options to include in an investment portfolio by evaluating alternatives based on several financial variables. A methodology based on IVFHFSs has been devised to rank the options for investment portfolio selection according to the research's methodological part. A MADM problem has been examined in the selection problem. Furthermore, comparative analyses were performed to confirm the veracity of the recommended options and methodology.

This article's primary goals are to:

- Create a new MADM method in an IVFHF environment;
- Use the AHP technique according to the IVFHFS framework in the new technique;
- Apply the proposed approach to study investment portfolio selection;
- Test the effectiveness and benefit of the new model through a comparative analysis.
- The superiority of the model and its implications for investment portfolio selection are given.

Literature

Every day, increasingly money is put into mutual funds. According to contemporary portfolio theory, the goal of the approach is to build a portfolio that maximises expected return given a specific amount of market risk and minimises risk for risk-averse investors given a specified level of expected return. Markowitz (1952) completed this ground-breaking research. Markowitz (1959) conducted a second excellent portfolio selection study, employing a two-stage method. Goldfarb and Iyengar (2003) developed a robust portfolio selection method to fully combat the optimal portfolio's susceptibility to statistical and modelling errors in anticipating relevant market factors.

Making a selection becomes more challenging when choosing an asset because it requires carefully analysing several factors. Therefore, some academics have chosen assets for investment using a range of MADM strategies. Joshi and Kumar (2014) and Vetschera and De Almeida (2012) introduced the PROMETHEE and TOPSIS techniques in the portfolio selection process. Biswas et al. (2019) used the Data Envelopment Analysis technique to select relatively superior portfolios from a portfolio collection. Ultimately, the portfolios were sorted by five-year annualised return, net asset value, information ratio, beta, R-squared, Jensen's alpha, Sharpe ratio, and Sortino ratio. In this case, the MABAC approach was used. When choosing portfolios for DM, several research papers (Karmakar, Dutta & Biswas, 2018; Ogryczak, 2000; Ronyastra, Gunatra & Ciptomulyono, 2015) employed a multi-criteria method. The capacity to consider various elements and options leads to the adoption of MADM methods in portfolio selection. The hybrid strategy that uses the ELECTRE method of building a course portfolio was created by (de Araujo Costa et al., 2022; Mellem et al., 2022). The fuzzy-AHP technique (Gupta et al., 2014) has been applied to derive each asset's ethical performance score based on investor preferences. Based on investor ratings of the financial criteria, a fuzzy multi-criteria DM algorithm was used to determine each asset's financial quality score.

An MD's ambiguity and vagueness were shown using the idea of a fuzzy set(FS), as put out by Zadeh (1965). By associating an element's membership degree (ND) with an item, Atanassov's (1986) intuitionistic FS (IFS) provides a more comprehensive explanation of the assessment data. Yager (2013) introduced the Pythagorean FS(PFS) concept to extend the range of MD and ND such that $MD^2 + ND^2 \leq 1$, considering the IFS weakness previously discussed. Therefore, PFS offers professionals more evaluation opportunities to express their opinions on various objectives. The complexity of the DM framework increases the difficulties specialists have in providing reliable evaluation data. By including the cubic total of MD and ND, the Fermatean FS(FFS) was the first to broaden the reach of information statements. As a result, FFS handles ambiguous choice problems more effectively and practically than IFS and PFS. According to Senepati and Yager (2020), the FFS was developed. Owing to its benefits in clarifying information and offering professionals more options, scholars have pushed for the development of numerous DM systems to address real-world DM and assessment issues. Research on FS, IFS, and PFS is extensive and includes studies (Akram & Naz, 2018; Akram, Shareef & Al-Kenani, 2024; Garg, 2019; Garg, Majumder & Nath, 2022; Garg et al. 2024; Kirisci, 2019; Kirisci 2020; Yager & Abbasov, 2013). Studies on FFS took their place in the literature in a short time(Akram et al., 2022; Kirisci, 2022a, 2022b, 2023, 2024a, 2024b; Senapati & Yager, 2019a, 2019b, 2020; Simsek & Kirisci, 2023).

Preliminaries

The IVFFS is a mathematical framework used in DM and modelling scenarios that involve uncertainty and imprecision. The IVFFS extends traditional FSs by incorporating an additional layer of flexibility and expressiveness, particularly in cases where precise MDs are difficult to assign. It is beneficial in the following instances: IVFFS is often applied in MCDM problems with imprecise or uncertain decision criteria. It helps handle subjective judgments by representing MD and ND as intervals. IVFFS provides a framework to model and reason with interval-valued MD and ND when dealing with datasets with inherent uncertainty or incomplete information. IVFFS is suitable for systems that rely on human input, such as expert systems or human-centric decision models. Humans often provide uncertain or interval-based assessments rather than precise values. In situations involving multiple stakeholders or experts, IVFFS can aggregate diverse opinions by considering interval-valued degrees of agreement and disagreement. IVFFS can model systems with ambiguous or vague operational parameters, such as robotics or process control. IVFFS is used in applications requiring robust handling of ambiguous or overlapping patterns, such as image processing or data clustering. IVFFS supports DM in sustainability and environmental management, where data uncertainty is prevalent.

The reasons for choosing IVFFS over other fuzzy models can be summarised as follows: IVFFS provides a more flexible structure by allowing MD, ND, and hesitation degrees to be expressed as intervals. Better suited for cases where precise values for membership are unavailable. It is useful in real-world problems where uncertainty is an intrinsic characteristic. When decision-makers have incomplete information or doubt the accuracy of information from different sources, IVFFS provides a more flexible model. It is especially preferred when the uncertainty in the evaluations of experts needs to be expressed in intervals. IVFFS offers a more flexible representation for each criterion when many criteria need to be evaluated. Working with interval values produces more realistic results when human judgments or preferences are uncertain. If information is uncertain due to the time-varying nature of a particular system, intervals can better reflect the variable conditions. IVFFS is used in fuzzy logic-based artificial intelligence systems, intensive learning, or data analysis processes to model uncertain data. In systems that work with incomplete data, uncertainty can be expressed with interval values. IVFFS applies when uncertainties need to be expressed comprehensively in project management, financial analysis, or security risk assessment.

By combining the advantages of FFSs (which allow more extensive ranges for MD and ND than IFSs) and interval-valued representations, IVFFS provides a robust mathematical framework for dealing with complex uncertainty. This method allows DEs to express and cope with uncertainties in the broader spectrum, thus enabling more robust and reliable decisions.

Let X be a non-empty set.

The set

$$A = \{(x, \zeta_A(x), \eta_A(x)) : x \in X\} \tag{1}$$

Is called an IFS, where the functions $\zeta_A, \eta_A : X \to [0,1]$ define the MD and ND of an element to the sets A. ($0 \leq \zeta_A(x) + \eta_A(x) \leq 1$, for $\forall x \in X$).

The hesitancy degree $\theta_A(x) = 1 - \zeta_A(x) - \eta_A(x)$.

The set

$$B = \{(x, \zeta_B(x), \eta_B(x)) : x \in X\} \tag{2}$$

Is called a PFS, where the functions $\zeta_B, \eta_B : X \to [0,1]$ defined the MD and ND of an element to the sets B ($0 \leq \zeta_B^2(x) + \eta_B^2(x) \leq 1$, for $\forall x \in X$).

The hesitancy degree $\theta_B(x) = \sqrt{1 - (\zeta_B^2(x) + \eta_B^2(x))}$.

The set

$$C = \{(x, \zeta_C(x), \eta_C(x)) : x \in X\} \tag{3}$$

Is called FFS, where the functions $\zeta_C, \eta_C : X \to [0,1]$ defined the MD and ND of an element to the sets C ($0 \leq \zeta_C^3(x) + \eta_C^3(x) \leq 1$, for $\forall x \in X$).

The hesitancy degree $\theta_C(x) = \sqrt[3]{1 - (\zeta_C^3(x) + \eta_C^3(x))}$.

The set

$$D = \{(x, \zeta_D(x), \eta_D(x)) : x \in X\} \tag{4}$$

Is called IVFFS, where the functions $\zeta_D, \eta_D \subseteq [0,1]$ define the MD and ND of an element to the sets D.

For each $x \in X, \zeta_D(x)$ and $\eta_D(x)$ are closed intervals, and their lower and upper bounds are denoted by $\zeta_D^L(x), \zeta_D^U(x), \eta_D^L(x), \eta_D^U(x)$, respectively. Therefore, D can also be expressed as follows:

$$\zeta_D(x) = \left[\zeta_D^L(x), \zeta_D^U(x)\right] \subseteq [0,1]; \eta_D(x) = \left[\eta_D^L(x), \eta_D^U(x)\right] \subseteq [0,1] \tag{5}$$

where the expression is subject to the condition $0 \leq \left(\zeta_D^U(x)\right)^3 + \left(\eta_D^U(x)\right)^3 \leq 1$.

For each $x \in X$, $\theta_D(x) = \left[\theta_D^L(x), \theta_D^U(x)\right]$ is called the hesitancy degree in IVFFSs, where

$$\theta_D^L(x) = \sqrt[3]{1 - \left[\left(\zeta_D^L(x)\right)^3 + \left(\eta_D^L(x)\right)^3\right]}; \theta_D^U(x) = \sqrt[3]{1 - \left[\left(\zeta_D^U(x)\right)^3 + \left(\eta_D^U(x)\right)^3\right]}. \tag{6}$$

Consider the three IVFFNs $D = \left(\left[\zeta_D^L(x), \zeta_D^U(x)\right], \left[\eta_D^L(x), \eta_D^U(x)\right]\right)$, $D_1 = \left(\left[\zeta_{D_1}^L(x), \zeta_{D_1}^U(x)\right], \left[\eta_{D_1}^L(x), \eta_{D_2}^U(x)\right]\right)$, $D_2 = \left(\left[\zeta_{D_2}^L(x), \zeta_{D_2}^U(x)\right], \left[\eta_{D_2}^L(x), \eta_{D_2}^U(x)\right]\right)$. Then,

$$D_1 \boxplus D_2 = \left(\left[\begin{array}{c} \sqrt[3]{\left(\zeta_{D_1}^L\right)^3 + \left(\zeta_{D_2}^L\right)^3 - \left(\zeta_{D_1}^L\right)^3 . \left(\zeta_{D_2}^L\right)^3} \\ \sqrt[3]{\left(\zeta_{D_1}^U\right)^3 + \left(\zeta_{D_2}^U\right)^3 - \left(\zeta_{D_1}^U\right)^3 . \left(\zeta_{D_2}^U\right)^3} \end{array} \right], \left[\eta_{D_1}^L \cdot \eta_{D_2}^l, \eta_{D_1}^u \cdot \eta_{D_2}^U\right] \right), \tag{7}$$

$$D_1 \boxtimes D_2 = \left(\left[\zeta_{D_1}^L \cdot \zeta_{D_2}^l, \zeta_{D_1}^u \cdot \zeta_{D_2}^U\right], \left[\begin{array}{c} \sqrt[3]{\left(\eta_{D_1}^L\right)^3 + \left(\eta_{D_2}^L\right)^3 - \left(\eta_{D_1}^L\right)^3 . \left(\eta_{D_2}^L\right)^3} \\ \sqrt[3]{\left(\eta_{D_1}^U\right)^3 + \left(\eta_{D_2}^U\right)^3 - \left(\eta_{D_1}^U\right)^3 . \left(\eta_{D_2}^U\right)^3} \end{array} \right], \right), \tag{8}$$

$$\lambda D = \left(\left[\sqrt[3]{1 - \left(1 - (\zeta_D^L)^3\right)^\lambda}, \sqrt[3]{1 - \left(1 - (\zeta_D^U)^3\right)^\lambda} \right], \left[\left(\eta_D^L\right)^\lambda, \left(\eta_D^U\right)^\lambda \right] \right), \tag{9}$$

$$D^\lambda = \left(\left[\left(\zeta_D^L\right)^\lambda, \left(\zeta_D^U\right)^\lambda \right], \left[\sqrt[3]{1 - \left(1 - (\eta_D^L)^3\right)^\lambda}, \sqrt[3]{1 - \left(1 - (\eta_D^U)^3\right)^\lambda} \right] \right). \tag{10}$$

The IVFF weighted average operator is a mapping $IVFFWA : D^n \to D$, where

$$IVFFWA(D_1, D_2, ..., D_n) = (\left[\sqrt[3]{\left(1 - \prod_{i=1}^n \left(1 - \left(\zeta_{D_i}^L\right)^3\right)^{\omega_i}\right)}, \sqrt[3]{\left(1 - \prod_{i=1}^n \left(1 - \left(\zeta_{D_i}^U\right)^3\right)^{\omega_i}\right)} \right] \tag{11}$$

$$\left[\prod_{i=1}^n \left(\eta_{D_i}^L\right)^{\omega_i}, \prod_{i=1}^n \left(\eta_{D_i}^U\right)^{\omega_i} \right]). \tag{12}$$

The IVFF-weighted geometric operator is a mapping $IVFFWG : D^n \to D$, where

$$IVFFWG(D_1, D_2, ..., D_n) = (\left[\prod_{i=1}^n \left(\zeta_{D_i}^L\right)^{\omega_i}, \prod_{i=1}^n \left(\zeta_{D_i}^U\right)^{\omega_i} \right] \tag{13}$$

$$\left[\sqrt[3]{\left(1 - \prod_{i=1}^n \left(1 - \left(\eta_{D_i}^L\right)^3\right)^{\omega_i}\right)}, \sqrt[3]{\left(1 - \prod_{i=1}^n \left(1 - \left(\eta_{D_i}^U\right)^3\right)^{\omega_i}\right)} \right]). \tag{14}$$

Portfolio Selection

Let ω_j be the vector set that is used to specify the criterion's weights. The linguistic terms and their corresponding IVFF values are shown in Table 1.

Method

Establish the problem's objective, decision-making alternatives, and available options. The linguistic terms and their related interval-valued fuzzy numbers are shown in Table 2.

The consistency ratio is defined as

$$CRT = \frac{CIX}{RIX} \tag{15}$$

where $CIX = \frac{\delta_{\max}}{n-1}$, RIX is the consistency index and $\delta_{\max}$ is the random index and principal eigenvalue for CRT, respectively.

Step 1: Establish the criteria and options before building the hierarchical structure.

Step 2: According to the expert judgments in Table 2, create the pairwise comparison matrix $Z = \left(z_{ij}\right)_{m \times m}$. For $z_{ij} = \left(\left[\zeta_{ij}^L, \zeta_{ij}^U\right], \left[\eta_{ij}^L, \eta_{ij}^U\right]\right)$,

$$Z = \begin{bmatrix} z_{11} z_{12} z_{21} \ z_{22} \ \cdots z_{1m} \cdots \ z_{2m} \\ \vdots\vdots z_{m1} \ z_{m2} \ \ddots\vdots\cdots \ z_{mm} \end{bmatrix} \tag{16}$$

Table 1

Linguistic terms and IVFFN values

Linguistic Terms	IVFFN values			
	m_L	m_U	n_L	n_U
Certainly High Importance(CH)	0.95	1	0	0
Very High Importance(VH)	0.8	0.9	0.1	0.2
High Importance(H)	0.7	0.8	0.2	0.3
Slightly More Importance(SM)	0.6	0.65	0.35	0.4
Equally Importance(EI)	0.5	0.5	0.5	0.5
Slightly Less Importance(SL)	0.35	0.4	0.6	0.65
Low Importance(L)	0.2	0.3	0.7	0.8
Very Low Importance(VL)	0.1	0.2	0.8	0.9
Certainly Low Importance(CL)	0	0	0.95	1

Step 3: Verify each pairwise comparison matrix's (Z) correctness. Here, we compare the sharp numbers acquired after defuzzifying to the IVFFNs listed in Table 2 using Saaty's scale to gauge the consistency of expert opinions. After that, Saaty's method for classical consistency is used.

Step 4: Aggregate the judgments of experts.

The pairwise comparison matrix created by each expert is aggregated using the IVFFWG aggregation algorithm.

$$IVFFWG(z_1, z_2, ..., z_k) = \left(\left[\prod_{k=1}^{K} (\zeta_k^L)^{\omega_k}, \prod_{k=1}^{K} (\zeta_k^U)^{\omega_k} \right], \right. \tag{17}$$

$$\left. \left[\sqrt[3]{\left(1 - \prod_{k=1}^{K} \left(1 - (\eta_k^L)^3\right)^{\omega_k}\right)}, \sqrt[3]{\left(1 - \prod_{k=1}^{K} \left(1 - (\eta_k^U)^3\right)^{\omega_k}\right)} \right] \right). \qquad (2) \tag{18}$$

Step 5: Using Equations (3) and (4), obtain the difference matrix $D = (d_{ij})_{m \times m}$ between the lower and upper points of the MD and ND.

$$d_{ij}^L = (\zeta_{ij}^L)^3 - (\eta_{ij}^U)^3 \qquad (3) \tag{19}$$

$$d_{ij}^L = (\zeta_{ij}^U)^3 - (\eta_{ij}^L)^3 \qquad (4) \tag{20}$$

Step 6: Using Equations (5) and (6), compute the interval multiplicative matrix $S = (s_{ij})_{m \times m}$.

$$s_{ij}^L = \sqrt[3]{1000^{d_{ij}^L}} \qquad (5) \tag{21}$$

$$s_{ij}^U = \sqrt[3]{1000^{d_{ij}^U}} \qquad (6) \tag{22}$$

Step 7: Find the indeterminacy value $T = (t_{ij})_{m \times m}$ of z_{ij} using Equation (7).

$$t_{ij} = 1 - \left(\zeta_{ijU}^3 - \zeta_{ijL}^3\right) - \left(\eta_{ijU}^3 - \eta_{ijL}^3\right) \qquad (7) \tag{23}$$

Step 8: Equation (8) can produce the matrix of un-normalised weights $R = (r_{ij})_{m \times m}$ by multiplying the indeterminacy degrees by the $S = (s_{ij})_{m \times m}$ matrix.

$$r_{ij} = \left(\frac{s_{ij}^L + s_{ij}^U}{2}\right) t_{ij} \qquad (8) \qquad (24)$$

Step 9: Equation (9), when applied, yields the normalised priority weights ω_i.

$$\omega_i = \frac{\sum_{j=1}^{m} r_{ij}}{\sum_{i=1}^{m} \sum_{j=1}^{m} r_{ij}} \qquad (9) \qquad (25)$$

Step 10: Rank the options according to the normalised priority weights you acquired in Step 9.

Algorithm

Algorithm 1 IVFF-AHP

Input: Number of evaluation criteria and pairwise comparison matrices.

Output: Normalised priority weights.

Begin

For j=1; m **do**

1. Input: Pairwise comparison matrix using the Table 1.
2. Convert the linguistic terms into the corresponding IVFFNs.
3. Check the consistency analysis

    For all Z **do**

    CRT using Equation 1.

    End for

4. **If** CRT>0.1

    Return to Step 1.

    Else

    Go to Step 5.

    End if

5. Compute the IVFFWG using Equation 2.

End for

6. Calculate the difference matrix using Equations 3 and 4.
7. Compute the multiplicative matrix using Equations 5,6.
8. Using Equation 7, determine the cij's indeterminacy value.
9. Equation 8 can be used to obtain the un-normalised weight matrix.
10. Determine the normalised priority weights using Equation 9.

11. Rank the options according to the normalised priority weights.

End

The flowchart of Algorithm 1 is given in Figure 1.

Problem Design

Due to the abundance of investment possibilities and the different elements that impact the market, including news, political unpredictability, and economic events, investors are concerned about portfolio optimisation and selection (Wang et al., 2011). Many studies have used various criteria to compile and choose investments because there are many things to consider when establishing an investment portfolio.

A mind map has been developed according to the research to aid in understanding the aspects involved in choosing an investment (Moccellin et al., 2021; Wu et al., 2022). This tool aids in visually organising criteria and problem structure (Hassan & Banerjee, 2021). The variables that affect investment choice are volatility(A), profitability(B), valuation(C), and liquidity(D). These divisions are displayed in Figure 2. Figure 2 evaluates the asset based on fundamental indicators, which evaluate the enterprise's fundamentals by examining the performance of the business and the macroeconomic environment in which the asset is situated (Trzebinski & Majerowska, 2019). This is based on studies (Moccellin et al., 2021; Wu et al., 2022). Volatility, the degree and frequency of value changes in a particular asset over time, is another subject covered in the mind map (De Blassis, 2023). In addition to the criteria, the variety of investment possibilities is a consideration that should be emphasised while choosing a portfolio.

In addition to the criteria, the variety of investment possibilities is a consideration that should be emphasised while choosing a portfolio. Companies listed in the S&P 500 were considered for the study. After 68 S&P 500 businesses had their data removed due to anomalies, 432 companies were evaluated. A 36-month observation period was obtained by collecting data from the S&P 500 website between January 1, 2021, and January 1, 2024.

Criteria: A_{11}, Return Variation, A_{12} – Market Uncertainties; B_{11} – Dividend Yield, B_{12} – Return; C_{11} – Indepedness, C_{12} – Ebitda; D_{11}, Current Liquidity, D_{12} – General Liquidity.

These criteria are determined as cost/benefit as follows: EBITDA and volatility return are costs, and the other criteria are benefits.

MADM is a process that builds and addresses judgmental and planning issues, determines the most practical choice based on expert opinions, and preserves given attributes. Professionals usually base their conclusions on various study approaches and theoretical frameworks based on their knowledge of these scenarios. For this reason, MADM is essential to achieving the highest quality model with the highest level of professional agreement. Here, we propose a new MADM method that the IVFFSs have developed.

Figure 1
Flowchart

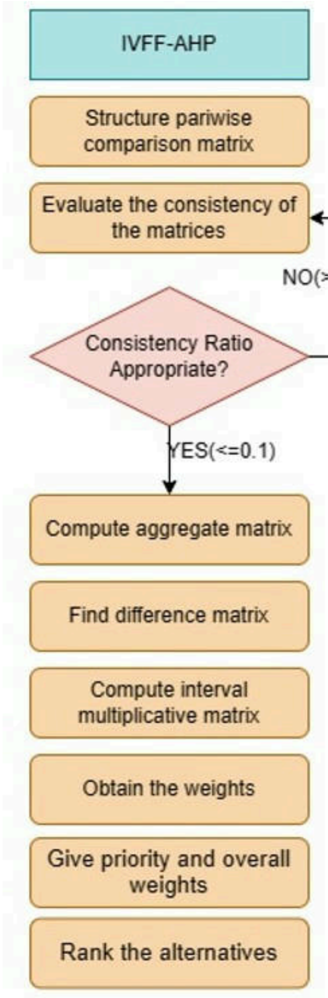

Figure 2
Hierarchy Trees of Portfolio Selection

Computations

Three experts developed paired comparison matrices to evaluate these options and criteria using the language concepts listed in Table 1. The pairwise comparison matrices of language terms for the primary criteria, secondary criteria, and alternates are displayed together with the consistency ratio in Tables 1-6. The consistency ratios of the paired comparison matrices were computed using the linguistic scale and the related numerical values listed in Table 1. The subsequent phases of the created technique are displayed on the significant criterion due to space limitations. Matrix conversion to IVFFNs using the appropriate scale is followed by the IVFFWG operator, which aggregates each expert's opinion according to the linguistic terms in the pairwise comparison. Table 7 displays the total IVFF results for the main criteria. Next, the IVFF-AHP was used to determine the criterion weights and alternatives. Equations (3) and (4) were used to determine the lower and upper values of the MD and ND. The difference matrix between these values is shown in Table 8. To compute the interval multiplicative matrix in Table 9, apply equations (5) and (6). Equation (7) uses the indeterminacy values given by Equation (8) to produce the matrix of weights before normalisation, as shown in Table 10. The final overall criteria weights are shown in Table 11.

Table 2

Pairwise comparison matrix of the main criteria

	Expert1				Expert2				Expert3			
	A	**B**	**C**	**D**	**A**	**B**	**C**	**D**	**A**	**B**	**C**	**D**
A	EI	VH	H	H	EI	H	H	H	EI	VH	VH	VH
B	SL	EI	VH	VH	EI	EI	VH	CH	SL	EI	VH	VH
C	SM	H	EI	H	H	H	EI	VH	SM	SM	EI	H
D	SL	SM	H	EI	SM	H	VH	EI	EI	H	H	EI
CRT	0.035				0.008				0.053			

Table 3

Pairwise comparison matrix of Volatility

	Expert1		Expert2		Expert3	
	A_{11}	A_{12}	A_{11}	A_{12}	A_{11}	A_{12}
A_{11}	EI	SL	EI	L	EI	L
A_{12}	VH	EI	CH	EI	VH	EI
CRT	0.094		0.047		0.049	

Table 4

Pairwise comparison matrix of Profitability

	Expert1		Expert2		Expert3	
	B_{11}	B_{12}	B_{11}	B_{12}	B_{11}	B_{12}
B_{11}	EI	SM	EI	SL	EI	EI
B_{12}	VH	EI	CH	EI	VH	EI
CRT	0.061		0.085		0.39	

Table 5

Pairwise comparison matrix of the valuation

	Expert1		Expert2		Expert3	
	C_{11}	C_{12}	C_{11}	C_{12}	C_{11}	C_{12}
C_{11}	EI	L	EI	SL	EI	SL
C_{12}	H	EI	VH	EI	VH	EI
CRT	0.058		0.042		0.06	

Table 6

Pairwise comparison matrix of the liquidity

	Expert1		Expert2		Expert3	
	D_{11}	D_{12}	D_{11}	D_{12}	D_{11}	D_{12}
D_{11}	EI	H	EI	VH	EI	H
D_{12}	CH	EI	CH	EI	VH	EI
CRT	0.077		0.073		0.088	

Table 7

Aggregated IVFFSs for the main criteria

	A	B	C	D
A	([0.50, 0.50], [0.50,0.50])	([0.21, 0.13], [0.92,0.90])	([0.86, 0.79], [0.22,0.27])	([0.16, 0.19], [0.88,0.85])
B	([0.18, 0.26], [0.89,0.77])	([0.50, 0.50], [0.50,0.5]0)	([0.75, 0.72], [0.38,0.35])	([0.84, 0.80], [0.35,0.30])
C	([0.75, 0.71], [0.32,0.29])	([0.54, 0.48], [0.60,0.63])	([0.50, 0.50], [0.50,0.50])	([0.91, 0.79], [0.22,0.27])
D	([0.82, 0.78], [0.33,0.30])	([0.42, 0.37], [0.65,0.72])	([0.89, 0.83], [0.13,0.24])	([0.50, 0.50], [0.50,0.50])

Table 8

Difference matrix

	A	B	C	D
A	(0.00, 0.00)	(-0.72, -0.78)	(0.62, 0.48)	(-0.61, -0.67)
B	(-0.45, -0.69)	(0.00, 0.00)	(0.38, 0.32)	(0.57, 0.47)
C	(0.40, 0.33)	(-0.10, -0.11)	(0.00, 0.00)	(0.73, 0.48)
D	(0.52, 0.44)	(-0.30, -0.22)	(0.70, 0.57)	(0.00, 0.00)

Table 9

Interval multiplicative matrix

	A	B	C	D
A	(1.00, 1.00)	(0.20, 0.17)	(4.17, 3.02)	(0.25, 0.21)
B	(0.35, 0.20)	(1.00, 1.00)	(2.40, 2.09)	(3.72, 2.95)
C	(2.51, 2.14)	(0.80, 0.78)	(1.00, 1.00)	(5.37, 3.02)
D	(3.31, 0.75)	(0.50, 0.60)	(5.01, 3.72)	(1.00, 1.00)

Table 10
Weights

	A	B	C	D
A	1.0	1.96	1.94	9.32
B	0.52	1.0	1.94	9.32
C	0.52	0.52	1.0	4.55
D	0.09	0.09	0.22	1.0

Table 11
Priority and overall weights

Main Criteria	A		B		C		D	
Weights	0.24		0.27		0.23		0.26	
Sub-criteria	A_{11}	A_{12}	B_{11}	B_{12}	C_{11}	C_{12}	D_{11}	D_{12}
Weihgts	0.34	0.13	0.25	0.1	0.33	0.18	0.29	0.36
Overall	0.09	0.12	0.11	0.15	0.09	0.18	0.16	0.1

Equation 8 was utilised to determine the weights, which are displayed in Table 10, before normalisation. Following the application of the results of all these computations to the sub-criteria, Table 11 displays the ultimate priority weights for the primary and secondary criteria. With a weight of 0.27, the results show that "profitability" needs are the most crucial. With a weight of 0.23, "valuation" is the least significant.

Discussion

Portfolio selection is the assembly of an investment portfolio that maximises a shareholder's expected return while minimising risk. The aim of portfolio selection is to find the asset combination that will provide the maximum return at a given risk level or the lowest risk, given the desired rate of return. Selecting a portfolio is complex and requires a deep understanding of financial markets, risk management strategies, and investment techniques. Many investors seek the assistance of certified financial counsellors or portfolio managers to guide them through the process. The process of selecting the optimal portfolio involves several steps:

Establish investment goals: The investor's financial objectives, investment horizon, and risk tolerance are ascertained in the first step.

Identify asset allocation: Asset allocation is the process of dividing an investment portfolio among several asset classes, such as stocks, bonds, and cash. The ideal asset allocation depends on the investor's financial goals, risk tolerance, and market conditions.

Choose individual investments: The next step in each asset class is to choose individual assets. Examining the characteristics of certain assets, such as their volatility, correlation with other investments, and past returns, is necessary to achieve this.

Portfolio optimisation: After the individual assets have been chosen, the portfolio needs to be adjusted by changing the weights of each investment to achieve the desired risk and return profile. Quantitative methods such as current portfolio theory or mean-variance optimisation can accomplish this.

Last, the portfolio should be periodically evaluated and rebalanced as needed to preserve the desired asset allocation and risk profile.

Developing a long-term investment strategy is one of the most important and challenging tasks that will enable you to invest with confidence. Investment planning does not have to be difficult.

The more stocks available for investment, the more difficult and computationally complex portfolio optimisation becomes. With expert judgments and MADM, the proposed study aims to give investors the best possible outcome when choosing an investment portfolio. Effective roles are chosen from the literature, and the reality of the criteria is considered when making the decision. Taking into account the previously described processes yields the selected result.

Comparison

The data from the example presented in sub-sections 4.3 and 4.4 will be analysed using the techniques found in studies in (Atanassov, & Gargov, 1989) and (Liang, Zhang, & Liu, 2015).

The IFS is generalised in the spirit of regular IVFSs. The IVIFS is the name of the novel idea. IVIFSs (Atanassov, & Gargov, 1989) are a fascinating and valuable tool for problem modelling in practical applications. In artificial intelligence, socioeconomic systems, data analysis, and decision-making, ranking IVIFSs is essential.

In parallel with Atanassov's IVIFS, Liang et al. (2015) created IVPFS, a new extension of PFS, to handle imprecise and uncertain information in real-world group decision-making issues. A novel DM technique based on IVPFNs is presented in this paper to solve MCGDM issues in an IVPF setting.

According to the ranking alternatives in Table 12, B is the most suitable alternative.

Table 12

Ranking of the alternatives

Method	Ranking
IVIFS (Atanassov, & Gargov, 1989)	B > C > D > A
IVPFS (Liang, Zhang, & Liu, 2015)	B > D > C > A
Our Methods	B > D > A > C

Advantages of IVFFSs and Decision-Making Aprroach

The FFS extends the traditional set, which consists of the FS, IFS, and PFS. The degree of satisfaction that members and nonmembers have with the criteria when their total squares are ≤1 distinguishes PFS, one of the most popular extensions. To ensure that the sum of the squares is more than 1, the decision-maker may occasionally provide the MD and ND of a particular characteristic. As a result, the PFS needs to manage this situation more expertly. The FFS theory seeks to address this issue. It is among the most thorough theories that can handle inconsistent, ambiguous, and incomplete information—all commonly encountered in real-world scenarios. Therefore, the Fermatean fuzzy information is more appropriate for DM in fundamental scientific and technical implementations.

Unlike classical fuzzy sets or traditional FFSs, IVFFSs represent MD, ND, and hesitation degrees as intervals rather than precise numbers. This provides a richer and more flexible way to capture uncertainty. The FF framework allows membership and non-membership values whose cubes sum to at most 1, generalising IFSs and PFSs. Using intervals for these values further enhances this expressiveness, accommodating a wider range of uncertainty. By modelling the hesitation degree (the uncertainty between membership and non-membership) as an interval, IVFFSs provide more nuanced, valuable information in real-world scenarios where exact hesitation is challenging to pinpoint. The additional flexibility and expressiveness help DM models better capture experts' opinions, incomplete information, or ambiguous data, leading to more reliable and robust decisions. The additional flexibility and expressiveness help DM models better capture experts' opinions, incomplete information, or ambiguous data, leading to more reliable and robust decisions. IVFFSs generalise many existing fuzzy sets models, such as IFSs, PFSs, and FFSs, making them versatile for various applications.

IVFFSs offer a powerful and flexible tool to model uncertainty with intervals on membership, non-membership, and hesitation degrees under the FF framework, which enhances the modelling capacity and accuracy in fuzzy-based decision-making and information processing.

Limitations

The portfolio selection problem, which aims to distribute capital among financial assets most effectively to maximise return and minimise risk, is one of the fundamental problems in finance. The accuracy and dependability of the data used to create asset allocation is the most significant limitation when selecting a portfolio. Although mean-variance optimisation models use long-term capital market assumptions, there is no guarantee that returns, risks, and correlations will remain stable over the long term. Transaction fees may be incurred when different assets in the portfolio are purchased and sold often. These transaction costs comprise commissions, brokerage fees, and charges associated with trading shares. These costs can lower your investment returns and decrease the profitability of your portfolio. When choosing a portfolio, this scenario could produce deceptive outcomes. In addition, among the constraints that arise when deciding on portfolio selection are:

- High fees and costs.
- The difficulty of consistently outperforming the market.
- An increased risk of underperformance.

These portfolio selection criteria may impact the overall investment returns. Therefore, before choosing a portfolio, carefully considering fees and possible returns is crucial.

Potential challenges in choosing a portfolio at the decision point: The challenge of balancing the chosen portfolio in terms of crucial factors like risk and completion time, the existence of multiple and frequently conflicting goals, some of which may be qualitative, uncertainty, and risk, the possibility of interdependencies among assets, and the number of applicable portfolios can all influence decisions. In addition to these difficulties, resource restrictions frequently necessitate considering limitations such as money, personnel, and facilities or equipment. Resource limitations are not usually included correctly in the portfolio selection process, which is the primary cause of some portfolios being chosen but not yet finished. When resource constraints are erroneous in a portfolio decision that fails, a choice model that considers resource limitations can help the decision-maker steer clear of such mistakes. When resource supply and consumption diverge, portfolio DM may become more complicated.

One drawback of this model is that most research has shown that securities yields do not always follow the normal distribution. However, the method is challenging to compute, particularly for the multiple-project portfolio problem. This computation is much more intricate.

Despite being a specific type of uncertainty avoidance, risk aversion is typically required because it can be challenging to determine the exact probabilities of a theoretical, time-consuming, multi-variable task such as optimal portfolio selection. This study still has several problems. First, risk and uncertainty are not the same thing. The impacts of risk choice, rather than uncertainty preference, are the main emphasis of this study. One strategy for avoiding uncertainty is risk aversion. This is even more important because it might be challenging to determine the exact likelihood of real-world problems. Beyond the benefits of the proposed IVFFS-based method, its incapacity to generate a thorough assessment of the available options restricts its application in particular DM circumstances.

Furthermore, when there are several criteria and possibilities, developing IVFFSs might become difficult. By contrasting it with other approaches, the study's DM algorithm's superiority is illustrated, and its benefits are articulated. Risk preference was operationalised in this study. However, more thorough analysis techniques must be developed to estimate the likelihood of dangerous features when choosing an investment portfolio.

Implications

Researchers have made several changes to Markowitz's classical framework to improve its practical applicability after seeing how poorly it reflects the dynamics in the actual world. Various approaches have been put out to deal with the problem, particularly regarding decision-making methods. The study's findings demonstrate that an investor's portfolio that receives decision-making information from a panel of experts under uncertain circumstances can be solved using the proposed fuzzy portfolio model.

The study's conclusions have consequences for managers, decision-makers, and practitioners. Defining the criteria, measuring their significance, putting them into practice using IVFFS, and reviewing the outcomes constitute a methodical process for choosing and evaluating portfolios. Significantly, other real-world issues can also be solved using this idea. Experts in the field, such as managers, investors, and engineers, can select the best industrial stock portfolios using the measurements, methodology, and research framework created for this study. After reviewing the research findings, offering a few theoretical and practical conclusions is acceptable. This study's main academic goal was to illustrate the model-development process for the relationships between DM and project portfolio management. Therefore, more research must concentrate on and look into crucial subjects. From a practical standpoint, it is possible to confirm that the profitability and return on investment transactions were the most critical pieces of information the stakeholders relied on when making decisions during the portfolio management process. The objectives are to reduce the risks of investing alternatives while maintaining an all-encompassing viewpoint when choosing a portfolio and reacting promptly and suitably to possibilities and hazards in the ever-changing market environment. Managing and keeping track of many products on the market makes portfolio DM more difficult. This article's research on the effectiveness of DM in portfolio selection can offer a framework for weighing the advantages and disadvantages of each asset, which can aid in choosing investments. For instance, an autocratic leadership style, an excessive dependence on subjective judgement, and a lack of procedures to provide more evidence-based inputs likely result in poor portfolio decisions. Companies may simultaneously focus on developing a portfolio mindset and balancing their portfolio decision process towards evidence-based DM to increase the efficacy of their portfolio decisions and, ideally, their long-term company performance. This could be achieved by encouraging cross-functional cooperation, encouraging critical thinking exercises, and implementing market immersion tactics. By choosing an appropriate portfolio and promptly removing gains that become marginal due to market fluctuations, agility in DM can help optimise returns. Feedback loops may develop in addition to any interactions between the DM processes. The process has been modelled linearly. Conversely, the DM process for a portfolio is continuous.

Fuzzy MADM techniques are recommended for investors and portfolio managers to handle unclear and ambiguous data effectively. By using these cutting-edge strategies, stakeholders can improve the accuracy and robustness of their portfolio selection procedures while managing risks and uncertainties. Its performance is compared to competing models to ensure that the proposed model is robust. To help the financial sector adopt these innovative methods, policymakers should establish rules and guidelines that support the fuzzy MADM models. Financial institutions may be eligible for incentives and training programs to improve their investment strategies and create a more stable and effective market environment if they integrate sophisticated decision-making tools into their operations. The study's conclusions show that to achieve the most significant result, the investment portfolio's choices must be sensible, reliable, and well-balanced. Since the most significant disadvantage of investing is the inability to choose the best solutions, investors worry about profitability. Investors may consequently grow reluctant to choose a portfolio. These studies' recommendations serve as a guide for scholars and businesses alike. Given the outcomes, prospective investors must be capable of selecting a profitable portfolio, making wise choices, and can turn a profit.

Conclusion

The FS, IFS, and PFS are all generalised into the FFS. One of the most popular FS expansions, PFS, requires that the sum of an object's MD and ND squares must equal or be less than one. In certain instances, the decision-maker may indicate the extent of an MD or MD so that the sum of the squares is more significant than 1. As a result, the PFS mishandles this scenario. Among the most comprehensive theories is FFS. It can handle inconsistent, ambiguous, and fragmentary data often found in real-world environments. The Fermatean fuzzy information DM is more appropriate for respectable applications in science and engineering.

Making decisions concerning real-world problems and producing solutions takes time and effort. Therefore, it is critical to reduce uncertainty when choosing the optimal course of action. Effective management of the relationships between the inputs is also necessary for DM to be most beneficial.

Over the long term, value investments in the stock market yield higher returns than cash deposits. This paper proposes an MDM framework for portfolio selection based on the financial performance of enterprises. The following are the algorithms for the MDM approach employing the AHP methodology and IVFFS. The decision model that had been built was put to the test through portfolio selection. Its efficacy was confirmed using comparative and sensitivity analysis. The model is successful in supporting investment decisions, according to the results.

In future studies, it is necessary to discuss whether it is possible to use different parameters found in the literature and the impact on the results when used. The approach presented in this study can also be extended to additional uncertain fields(linguistic term sets, probabilistic linguistic term sets, etc.) in future research. In addition, this method can be studied together with methods such as TOPSIS and the Chouqet integral.

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