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TESSARİNELER İLE HOMOTETİK HAREKETLERE E_2^4 YARI- ÖKLİD UZAYINDA YENİ BİR YAKLAŞIM

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ÖZET

Bu çalışmada, 4 boyutlu yarı Öklid uzayında tessarinesleri kullanarak, Hamilton operatörlerine benzer bir matris verdik ve çeşitli cebirsel özelliklerini tanımladık. Daha sonra bu hareketin homotetik hareket olabilmesi ispatlandı. Bir parametreli homotetik hareket için, pol noktaları , pol eğrileri ve hız merkezleri hakkında bazı teoremler tanımladık. Sonunda, her t anında, bir M_{i_3} hiperyüzeyi üzerinde eğrilerin türevleri ve r' inci dereceden regular eğriler tarafından tanımlanan hareketin sadece $(r-1)$' inci derecen bir hız merkezine sahip olduğu bulundu.

Tessarinesler ile verilen konudaki yöntemden dolayı, çalışma homotetik hareket hakkında bilinmeyen cebirsel özellikleri ve bazı formulleri , gerçekleri ve özellikleri veriyor.

Anahtar kelimeler: *Tessarineler, Homotetik hareketler, Pol eğrileri, Hiperyüzey.*

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A NEW APPROACH TO HOMOTHETIC MOTIONS WITH TESSARINES IN SEMI-EUCLIDEAN SPACE E_2^4

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ABSTRACT

In this study, by using tessarines in 4-dimension semi-Euclidean space, we describe a variety of algebraic properties and give a matrix that is similar to Hamilton operators and we show that the hypersurfaces are obtained and a new motion is defined in E_2^4. Then, this motion is proven to be homothetic motion. For this one parameter homothetic motion, we defined some theorems about velocities, pole points, and pole curves. Finally, It is found that this motion defined by the regular curve of order r on the hypersurface M_{i_3}, at every t- instant, has only one acceleration centre of order $(r-1)$.

Due to the way in which the matter is given with tessarines, the study gives some formulas, facts and properties about homothetic motion and variety of algebraic properties which are not generally known.

Keywords: *Tessarines, Homothetic motions, Pole curves, Hypersurface.*

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1. INTRODUCTION

First time, James Cockle defined the tessarines in 1848, using more modern notation for complex numbers as a successor to complex numbers and algebra similar to the quaternions. The tessarines are coincided with 4 -dimensional vector space R^4 over real numbers. Cockle used tessarines to isolate the hyperbolic cosine series and the hyperbolic sine series in the exponential series. He also showed how zero divisors arise in tessarines, inspiring him to use the term "impossibles." The tessarines are now best known for their subalgebra of real tessarines $t = w + yj$ also called split−complex numbers, which express the parametrization of the unit hyperbola [1-5].

Homothetic motion is a general form of Euclidean motion. It is crucial that homothetic motions are regular motions. These motions have been studied in kinematic and differential geometry in recent years. In 4-dimensional semi-Euclidean space, a one-parameter homothetic motion of a rigid body is generated analytically by

$$Y = h(t)A(t)X_0(t) + C(t) \tag{1}$$

in which X_0 and Y correspond the position vectors of the same point with respect to the rectangular coordinate frames of the moving space K_0 and the fixed space K, respectively. At the inital time $t = t_0$ we suppose that the coordinate system in K_0 and K are coincident. A is an orthonormal $n \times n$ matrix that satisfies the property $A^T \varepsilon A = \varepsilon$, C is a translation vector and g is the homothetic scale of the motion. Also g, A and C are continuously differentiable function of C^∞ class of a real parameter t. It is showed that the Hamilton motions are the homothetic motions in 4-dimensional Euclidean space and at (E^8) with Bicomplex Numbers C_3, respectively, [6-9].

In this study, we define a variety of algebraic properties and give a matrix that is similar to Hamilton operators. By using tessarines product and addition rules we define the hypersurface and a new motion in E_2^4. Then, this motion is proven to be homothetic motion. For this one parameter homothetic motion, we define some theorems about velocities, pole points and pole curves. Finally, It is found that this motion defined by the regular curve of order r on the hypersurface M_3 at every $t-$ instant, has only one acceleration centre of order $(r-1)$.

2. TESSARINES

A tessarine w is an expression of the for

$$w = w_1 + w_2\, i_1 + w_3 i_2 + w_4 i_3 \tag{2}$$

where w_1, w_2, w_3 and w_4 are real numbers and the imaginary units i_1, i_2 and i_3 are governed by the rules:

$$\begin{gathered} i_1^2 = -1,\ i_2^2 = +1,\ i_3^2 = -1 \\ i_1 i_2 = i_2 i_1 = i_3,\ \ i_1 i_3 = i_3 i_1 = -i_2,\ \ i_2 i_3 = i_3 i_2 = i_1 \end{gathered} \tag{3}$$

here it is easy to see that the multiplication of two tessarine is commutative. It is also convenient to write the set of tessarines as

$$T = \{w \mid w = w_1 + w_2\, i_1 + w_3 i_2 + w_4 i_3,\ \ w_{1-4} \in R\}$$

Definition 1. (Conjugations of Tessarines) : Conjugation plays an important role both for algebraic and geometric properties for tessarines, In that case, there are different conjugations according to the imaginary units i_1, i_2 and i_3 for tessarines as follows:

$$\begin{aligned} w^* &= (w_1 - w_2 i_1) + i_2(w_3 - w_4 i_1) \\ w^* &= (w_1 + w_2 i_1) - i_2(w_3 + w_4 i_1) \\ w^* &= (w_1 - w_2 i_1) - i_2(w_3 - w_4 i_1) \end{aligned}$$

where,

1. $ww^* = w_1^2 + w_2^2 + w_3^2 + w_4^2 + 2i_2(w_1 w_3 + w_2 w_4)$
2. $ww^* = w_1^2 - w_2^2 - w_3^2 + w_4^2 + 2i_1(w_1 w_2 - w_3 w_4)$
3. $ww^* = w_1^2 + w_2^2 - w_3^2 - w_4^2 + 2i_3(w_1 w_4 - w_2 w_3)$.

The multiplication of a tessarine $w = w_1 + w_2\, i_1 + w_3 i_2 + w_4 i_3$ by a real scalar μ is defined as

$$\mu w = \mu w_1 + \mu w_2\, i_1 + \mu w_3 i_2 + \mu w_4 i_3.$$

Definition 2. (Product of Tessarines) : Define the product in T by

$$wu = uw = (w_1 + w_2 i_1 + w_3 i_2 + w_4 i_3)(u_1 + u_2 i_1 + u_3 i_2 + u_4 i_3)$$
$$= (w_1 u_1 - w_2 u_2 + w_3 u_3 - w_4 u_4) + i_1(w_1 u_2 + w_2 u_1 + w_3 u_4 + w_4 u_3)$$
$$+ i_2(w_1 u_3 - w_2 u_4 + w_3 u_1 - w_4 u_2) + i_3(w_1 u_4 + w_2 u_3 + w_3 u_2 + w_4 u_1)$$

It is easy to see that the product of two tessarine is commutative. Since the tessarines product is associative, commutative and it distributes over vector addition, T is a real algebra with tessarines product. According to the imaginary units i_1, i_2 and i_3, by considering the product and addition rules of tessarines and the conjugates of the tessarines to be able to define norms, let us consider the hypersurfaces M_1, M_2 and M_3 as follows,

$$M_1 = \{ w \mid w = w_1 + w_2 i_1 + w_3 i_2 + w_4 i_3, \quad w_1 w_3 + w_2 w_4 = 0 \}$$
$$M_2 = \{ w \mid w = w_1 + w_2 i_1 + w_3 i_2 + w_4 i_3, \quad w_1 w_2 - w_3 w_4 = 0 \}$$
$$M_3 = \{ w \mid w = w_1 + w_2 i_1 + w_3 i_2 + w_4 i_3, \quad w_1 w_4 - w_2 w_3 = 0 \}$$

Definition 3. (Norms of Tessarines) : Norms on M_1, M_2 and M_3 hypersurfaces are defined as following

$$\|w\| = w_1^2 + w_2^2 + w_3^2 + w_4^2$$
$$\|w\| = w_1^2 - w_2^2 - w_3^2 + w_4^2$$
$$\|w\| = w_1^2 + w_2^2 - w_3^2 - w_4^2.$$

The system T is a commutative algebra. It is referred as the tessarines algebra and shown with T, briefly one of the bases of this algebra is $\{1, i_1, i_2, i_3\}$ and the dimension is 4. By using equations (2) and (3), we can give this representation to show a mapping into 4x4 matrices (It is possible to give the production T similar to Hamilton operators which has defined [6-9]).

$$\varphi : w = w_1 + w_2 i_1 + w_3 i_2 + w_4 i_3 \in T \dashrightarrow \varphi(w) = \begin{bmatrix} w_1 & -w_2 & w_3 & -w_4 \\ w_2 & w_1 & w_4 & w_3 \\ w_3 & -w_4 & w_1 & -w_2 \\ w_4 & w_3 & w_2 & w_1 \end{bmatrix},$$

T is algebraically isomorphic to the matrix algebra

$$\xi = \left\{ \begin{bmatrix} w_1 & -w_2 & w_3 & -w_4 \\ w_2 & w_1 & w_4 & w_3 \\ w_3 & -w_4 & w_1 & -w_2 \\ w_4 & w_3 & w_2 & w_1 \end{bmatrix} \mid (w_1, w_2, w_3, w_4) \in R \right\}$$

and $\varphi(w)$ is a faithful real matrix representation of ξ. Moreover, $\forall\, w, u \in T$ and $\forall\, \gamma \in R$, we obtain

$$\varphi(w+u) = \varphi(w) + \varphi(u),$$
$$\varphi(\gamma w) = \gamma\varphi(w),$$
$$\varphi(wu) = \varphi(w)\varphi(u).$$

Definition 4. E^n with the metric tensor

$$< w, v >= -\sum_{k}^{v} w_k v_k + \sum_{j=v+1}^{n} w_j v_j \quad \dots\dots.. w, v \in E^n, \;\; 0 \le v \le n$$

is called semi-Euclidean space and is defined by E_v^n where v is called the index of the metric. The resulting semi-Euclidean space E_v^n is reduced to E^n if $v = 0$. For n, E_1^n is called Minkowski n space,if $n = 4$, it is the simplest example of a relativistic space time.

Definition 5. Let E_1^n be a semi-Euclidean space furnished with a metric tensor $<,>$ A vector v to E_1^n is called spacelike if $< v, v >> 0$ or $v = 0$, null (a light vector) if

$< v, v >= 0$ or timelike if $< v, v >< 0$.

In the case when $0 \le v \le n$, the signature matrix ε is the diagonal matrix $[\delta_{ij}\varepsilon_j]$ whose diagonal entries are $\varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_v = -1$ and $\varepsilon_v = \varepsilon_{v+1} = \cdots = \varepsilon_n = 1$. Hence

$$\varepsilon = \begin{bmatrix} -I_n & 0 \\ 0 & I_{n-v} \end{bmatrix}.$$

Definition 6. The set of all linear isometries $E_v^n \longrightarrow E_v^n$ is the same as the set $O(v\,;n)$ of all matrices $A\epsilon GL(n,R)$ preserving the scalar product

$$< \; w\,, v > = \varepsilon wv; \quad w, v\epsilon E_v^n$$

The group $O(v,n)$ is denoted by $O_v(n)$. Hence

$$O_v(n) = \{A\epsilon GL(n,R) : < Aw, Av > =< w, v > ;\ w, v\epsilon E_v^n\}$$

$$SO_v(n) = \{A\epsilon O_v(n): detA = 1\}.$$

The following conditions of an nxn matrix are equivalent

(i) $A\epsilon O_v(n)$

(ii) $A^T = \varepsilon A^{T-1}\varepsilon$

(iii) The columns [rows] of A form an orthonormal basis for E_v^n (first v vectors timelike)

(iv) A carries one (hence every) orthonormal basis for E_v^n to an orthonormal basis.

The matrix A is called a real semi-orthogonal matrix [10].

3. HAMILTON MOTIONS WITH TESSARINES IN SEMI-EUCLIDEAN SPACE E_2^4

Denote a hypersurface M_3 and a unit sphere S_2^3, respectively, by considering the product and addition rules of tessarines and one of the conjugates of the tessarines according to the imaginary unit i_3 as following,

$$M_3 = \{w \;\mid w = w_1 + w_2\, i_1 + w_3 i_2 + w_4 i_3,\ w_1 w_4 - w_2\, w_3 = 0\},$$

$$S_2^3 = \{w \;\mid w_1^2 + w_2^2 - w_3^2 - w_4^2 = 1\},$$

$$K = \{w \;\mid w_1^2 + w_2^2 - w_3^2 - w_4^2 = 0\}$$

be *a null cone* in E_2^4.

Let us define the following parametrized curve,

$$w: I \subset R \longrightarrow M_3 \subset E_2^4 \text{ given by}$$

$$w(t) = |w_1 + w_2\, i_1 + w_3 i_2 + w_4 i_3| \text{ for every } t \in I\,.$$

We suppose that the curve $w(t)$ is differentiable regular curve of order$r$. Let position vector of the curve be timelike. Let the curve be a unit velocity timelike curve ($< w, v >> -1$). The operator Γ similar to the Hamilton operator, corresponding to $w(t)$ is defined by the following matrix:

$$\Gamma = \Gamma\big(w(t)\big) = \left\{ \begin{bmatrix} w_1 & -w_2 & w_3 & -w_4 \\ w_2 & w_1 & w_4 & w_3 \\ w_3 & -w_4 & w_1 & -w_2 \\ w_4 & w_3 & w_2 & w_1 \end{bmatrix} \mid (w_1, w_2, w_3, w_4) \in R \right\}.$$

Theorem 1. The Hamilton motion determined by equation (1) in semi-Euclidean space E_2^4 is a homothetic motion.

Proof. Let $\|w'(t)\| = 1$, $w(t)$ be a unit velocity curve. If $w(t)$ does not pass through the orijin and $w(t)$, the above matrix can be represent as $\Gamma = g\xi$ where $\xi = \frac{\Gamma}{g}$,

$$\Gamma = g \begin{bmatrix} \frac{w_1}{g} & \frac{-w_2}{g} & \frac{w_3}{g} & \frac{-w_4}{g} \\ \frac{w_2}{g} & \frac{w_1}{g} & \frac{w_4}{g} & \frac{w_3}{g} \\ \frac{w_3}{g} & \frac{-w_4}{g} & \frac{w_1}{g} & \frac{-w_2}{g} \\ \frac{w_4}{g} & \frac{w_3}{g} & \frac{w_2}{g} & \frac{w_1}{g} \end{bmatrix} \qquad (4)$$

and

$$g: I \subset R \longrightarrow R$$

$$t \longrightarrow w(t) = \sqrt{|w_1^2 + w_2^2 - w_3^2 - w_4^2|}.$$

As the position of the curve are defined by using tessarines is timelike,

$w_1^2 + w_2^2 - w_3^2 - w_4^2 > 0$. In the equation (3), we find $\xi\varepsilon\xi^T = \xi^T\varepsilon\xi = I_4$

and $\det\xi = 1$, where

$$\varepsilon = \begin{bmatrix} -I_2 & 0 \\ 0 & I_2 \end{bmatrix}$$

Thus Γ is a homothetic matrix. Since $\Gamma = g\xi$ is a homothetic matrix determines a homothetic motion.

Theorem 2. Let $w(t) \in S_2^3 \cap M_3$. In equation $\Gamma(\mathrm{t}) = g(t)\xi(\mathrm{t})$, $\xi(\mathrm{t})$ is a scalar matrix then , ξ matrix is a semi-orthogonal matrix "the matrix ξ is $SO(4\,;\,2)$ ".

Proof. If $w(t) \in S_2^3$, where $w_1^2 + w_2^2 - w_3^2 - w_4^2 = 1$. Using equation (4), in equation $\Gamma(\mathrm{t}) = g(t)\xi(\mathrm{t})$, we have $\Gamma^{-1} = \varepsilon\Gamma\varepsilon$ and $\det\xi = 1$.

Theorem 3. In equation $\Gamma(\mathrm{t}) = g(t)\xi(\mathrm{t})$, the matrix ξ in E_2^4 is semi-orthogonal matrix.

Proof. Since $(t) \in M_3$, $w(t) \notin K$ and $w_1 w_4 - w_2\, w_3 = 0$.

In equation $\Gamma(\mathrm{t}) = g(t)\xi(\mathrm{t})$. The matrix ξ has been shown by $\xi^T\varepsilon\xi = \varepsilon$. Let the signature matrix be given as

$$\varepsilon = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

where, the matrix ξ is semiorthogonal matrix and $\det\xi$=1.

Theorem 4. Let $w(t)$ be a unit velocity curve and $w'(t) \in M_3$ then the derivation operator Γ' of $\Gamma = g\xi$ is real semi-orthogonal matrix in E_2^4.

Proof. Since $w(t)$ is a unit velocity curve, $w_1^2 + w_2^2 - w_3^2 - w_4^2 = 1$ and $w'(t) \in M_3$, then $w_1 w_4 - w_2\, w_3 = 0$. Thus, $\Gamma'\varepsilon(\Gamma^T)' = (\Gamma^T)'\varepsilon\Gamma'$ and $\det\Gamma'$=1.

Theorem 5. In semi-Euclidean space E_2^4, Hamilton motion determined by the derivation operator is a regular motion and it is independent of g.

Proof. This motion is regular as detΓ'=1 also, the value of detΓ' is independentof g.

4. POLE POINTS AND POLE CURVES OF THE MOTION WITH TESSARINES IN SEMI-EUCLIDEAN SPACE E_2^4

To find the pole points in semi-Euclidean space E_2^4 we have to solve the equation

$$\Gamma' X_0 + C' = 0. \tag{5}$$

Any solution of equation (5) is a pole point of motion at that instant in K_0. Because, by Theorem 4, we have det Γ' =1. Hence the equation (4.1) has only one solution, i.e.

$$X_0 = (-\Gamma')^{-1}(C)$$

at every t-instant. In this case the following theorem can be given.

Theorem 6. If $w(t)$ is a unit velocity curve and $w'(t) \in M_3$, then the pole point corresponding to each t-instant in K_0 is the rotation by $(-\Gamma')^{-1}$ of the speed vektor (C') of the translation vector at that moment.

Proof. As the matrix Γ' is semi-orthogonal, the matrix $(\Gamma')^{-1}$ is orthogonal too. Thus, it makes a rotation.

5. ACCELARATION CENTRES OF ORDER $(r-1)$ OF THE MOTION WITH TESSARINES IN SEMI-EUCLIDEAN SPACE E_2^4

Definition 7. The set of the zeros of sliding acceleration of order r is called the acceleration centre of order $(r-1)$.

In order to find the acceleration centre of order $(r-1)$, by using definition 7, we have to find the solutions of the equation

$$\Gamma^{(r)} X_0 + C^{(r)} = 0 \tag{6}$$

where

$$\Gamma^{(r)} = \frac{d^r\Gamma}{dt^r} \text{ and } C^{(r)} = \frac{d^r C}{dt^r}.$$

Let w be a regular curve of order r and $w^{(r)} \in M_3$. Then we have

$$w_1^{(r)} w_4^{(r)} - w_2^{(r)} w_3^{(r)} = 0.$$

Thus,

$$\left|\left(w_1^{(r)}\right)^2 + \left(w_2^{(r)}\right)^2 - \left(w_3^{(r)}\right)^2 - \left(w_4^{(r)}\right)^2\right| \neq 0.$$

Also,we have

$$\det\Gamma^{(r)} = \left(w_1^{(r)}\right)^2 + \left(w_2^{(r)}\right)^2 - \left(w_3^{(r)}\right)^2 - \left(w_4^{(r)}\right)^2.$$

Then $\det\Gamma^{(r)}$. Therefor matrix $\Gamma^{(r)}$ has an inverse and by equation (6), the acceleration centre of order $(r-1)$ at every t $-$instant, is

$$X_0 = [\Gamma^{(r)}]^{-1}[-C^{(r)}].$$

Example 1. Let $w: I \subset R \to M_3 \subset E_2^4$ be a curve given by

$$t \to w(t) = \frac{1}{\sqrt{2}}(cht, -cht, sht, sht, sht).$$

Note that $w(t) \in S_2^3$ and since $\|w(t)\| = 1$, then $w(t)$ is a unit velocity curve. Moreover, $w(t) \in M_3, w'(t) \in M_3, \ldots, w^{(r)}(t) \in M_3$. Thus $w(t)$ satisfies all conditions of the above theorems.

Example 2. $: I \subset R \to M_3 \subset E_2^4$ is defined by $w(t) = (\sinh t, t, \cosh t, \sqrt{3}t)$ for every

$t \in I$. Let $C(0, t, 0, 0)$. Because $w(t) = (\sinh t, t, \cosh t, \sqrt{3}t)$ does not pass through the origin, the matrix Γ can be represented as

$$\Gamma = \Gamma(w(t)) = \sqrt{2t^2+1}\begin{bmatrix} \frac{\sinh t}{\sqrt{2t^2+1}} & \frac{-t}{\sqrt{2t^2+1}} & \frac{\cosh t}{\sqrt{2t^2+1}} & \frac{-\sqrt{3}t}{\sqrt{2t^2+1}} \\ \frac{t}{\sqrt{2t^2+1}} & \frac{\sinh t}{\sqrt{2t^2+1}} & \frac{\sqrt{3}t}{\sqrt{2t^2+1}} & \frac{\cosh t}{\sqrt{2t^2+1}} \\ \frac{\cosh t}{\sqrt{2t^2+1}} & \frac{-\sqrt{3}t}{\sqrt{2t^2+1}} & \frac{\sinh t}{\sqrt{2t^2+1}} & \frac{-t}{\sqrt{2t^2+1}} \\ \frac{\sqrt{3}t}{\sqrt{2t^2+1}} & \frac{\cosh t}{\sqrt{2t^2+1}} & \frac{t}{\sqrt{2t^2+1}} & \frac{\sinh t}{\sqrt{2t^2+1}} \end{bmatrix}$$

where

$$g : I \subset R \rightarrow R$$
$$t \rightarrow g(t) = \|w(t)\| = \sqrt{|-(2t^2+1)|}.$$

We find $\xi^T \varepsilon \xi \varepsilon = I_4$ and detξ=1 and $\Gamma' \in SO(4;2)$. In this case, in equation (4), the motion is given by

$$Y = \sqrt{2t^2+1}\begin{bmatrix} \frac{sinht}{\sqrt{2t^2+1}} & \frac{-t}{\sqrt{2t^2+1}} & \frac{cosht}{\sqrt{2t^2+1}} & \frac{-\sqrt{3}t}{\sqrt{2t^2+1}} \\ \frac{t}{\sqrt{2t^2+1}} & \frac{sinht}{\sqrt{2t^2+1}} & \frac{\sqrt{3}t}{\sqrt{2t^2+1}} & \frac{cosht}{\sqrt{2t^2+1}} \\ \frac{cosht}{\sqrt{2t^2+1}} & \frac{-\sqrt{3}t}{\sqrt{2t^2+1}} & \frac{sinht}{\sqrt{2t^2+1}} & \frac{-t}{\sqrt{2t^2+1}} \\ \frac{\sqrt{3}t}{\sqrt{2t^2+1}} & \frac{cosht}{\sqrt{2t^2+1}} & \frac{t}{\sqrt{2t^2+1}} & \frac{sinht}{\sqrt{2t^2+1}} \end{bmatrix} X_0 + \begin{bmatrix} 0 \\ t \\ 0 \\ 0 \end{bmatrix}.$$

Hence geometrical path of pole points in the Hamilton motion is determined by above equation as

$$X_0 = \begin{bmatrix} -1 \\ -cosht \\ -\sqrt{3} \\ -sinht \end{bmatrix}.$$

6. CONCLUSION

Using the product and addition rules of tessarines and one of the conjugates of the tessarines

the hypersurface and a new motion are defined in E_2^4. Then, this new motion is proven to be homothetic motion. It is found that this new motion defined by the regular curve of order r on the hypersurface M_3 at every t - instant, has only one acceleration centre of order $(r-1)$.

REFERENCES

[1] J. Cockle, *On Certain Functions Resembling Quaternions and on a New Imaginary in Algebra*, Philosophical magazine, London-Dublin-Edinburgh, 1848.

[2] J. Cockle, *On a New Imaginary in Algebra Philosophical magazine*, series3, London-Dublin-Edinburgh, 34, pp. 37--47, 1849.

[3] J. Cockle, *On the Symbols of Algebra and on the Theory of Tessarines*, 34, pp. 406-410, Philosophical magazine, series3, London-Dublin-Edinburgh, 1849.

[4] J. Cockle, *On Impossible Equations, on Impossible Quantities and on Tessarines*, ,Philosophical magazine, London-Dublin-Edinburgh, 1850.

[5] J. Cockle, *On the True Amplitude of a Tessarine*, Philosophical magazine, London-Dublin-Edinburgh, 1850.

[6] Y. Yaylı , *Homothetic Motions at E4*. Mech. Mach. Theory., 27 (3), 303-305, 1992.

[7] F. Babadağ, *Homothetic Motions And Bicomplex Numbers, Algebras, Groups And Geometrıes,* Vol. 26, Number 4,193-201, 2009.

[8] F. Babadağ, Y. Yaylı and N. Ekmekci, *Homothetic Motions at* (E^8) *with Bicomplex Numbers* (C_3), Int. J. Contemp. Math. Sciences, Vol. 4, no. 33, 1619 - 1626, 2009.

[9] F. Babadağ, *The Real Matrices forms of the Bicomplex Numbers and Homothetic Exponential motions*, Journal of Advances in Mathematics, ISSN 2347-1921, Vol 8, No. 1, 1401 - 1406, 2014.

[10] B. O'Neill, *Semi-Riemannian geometry*, Academic Press, New York, 1983.