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Parametric and Non-Parametric Reliability Analysis of The Propeller Unit of an Aircraft Fleet

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Abstract

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Preventive maintenance is performed to sustain the safe and reliable operation of industrial equipments. In order to plan preventive maintenance or evaluate the existing maintenance plan, the failure behavior of the system must be modeled. The failure behavior of a repairable system is modeled utilizing counting processes. In this study failure behavior of propellers belonging to a small aircraft fleet is modeled. First non-parametric estimate of population mean cumulative function (MCF) is obtained. MCF helps discovering the special features of the maintenance data. The parametric model selection depends on the result of the trend analysis of the time between failures. In the second part of the study trend analysis is performed on propeller maintenance data. Based on the trend analysis two prospect parametric models are selected. Reliability measures are estimated using both models and results are compared to evaluate the existing preventive maintenance plan.

Keywords: repairable system reliability, MCF, counting process, reliability metrics

1. Introduction and Background

Increasingly complex systems are being produced to meet today's technological needs. For these systems to fulfill their functions, all the parts, devices, and components that make up the system must work properly. The loss of function of even a small part of the system can have a negative impact on the operation of the entire system. However, as systems are used, the components that make up the systems wear out over time. This reduces the efficiency of the systems and leads to their failure after a certain period.

Maintenance is an important element in ensuring the reliable working of systems throughout their life cycle. Maintenance schedules and types are determined by the system's needs, equipment's nature and condition, and other factors. Insufficient, excessive, or incorrect maintenance can cause malfunctions, which affects the usability of the systems and causes the system to lose performance. System repair costs may increase significantly due to possible secondary failures. An effective maintenance plan must be developed and implemented to keep a system in good working condition. In addition, existing maintenance data should be analyzed over time and the existing maintenance plan should be updated to meet the system's needs. Maintenance data are classified as recurring event data. These data are analyzed using counting processes from stochastic models. Counting processes are models of the occurrence of events over time and are used in systems with recurring events [1, 2].

In the literature, Ye, et al. [3] developed the reliability evaluation framework for hard disk drive (HDD) based on the non-homogeneous Poisson process (NHPP) and illustrated the framework on real data from the HDD tests. Traditional methods for accelerated life test (ALT) data analysis cannot fit the time-to-failure data well. A multi-country production system operational status can be characterized using a task reliability model based on product quality status, as proposed by Yang, et al. [4]. Majumdar [5] proposed a failure model for a repairable hydraulic excavator system, which is modeled by a NHPP with a time-dependent log-linear hazard rate function, and the failure modes of the system, which are at very high risk, are identified by failure mode and effect analysis (FMEA) and appropriate corrective measures are discussed. Huang, et al. [6] proposed the NHPP to model the degradation in the system. The proposed model considers the virtual age concept and uses the production yield rate as a condition variable for the optimal preventive maintenance (PM) framework, then three different case-based PM strategies are proposed for the system. Ali [7] developed intuitive, practically interpreted, and adapted process monitoring strategies to monitor time-between-events (TBE) online and used a power-law NHPP model to develop TBE schedules. Cahoon, et al. [8] discussed the background of reliability growth models. They presented two models based on the Poisson process and competing risks. They discussed how these models can be extended to a Bayesian framework. Li, et al. [9] developed an improved four-parameter NHPP model and presented a meta-action reliability model for machine tools. Said and Taghipour [10] developed the likelihood function corresponding to the failures and preventive maintenance of a fleet of trucks in the mining industry and optimized the parameters of the failure process with some meta-heuristics. The Kijima virtual age models [11] discussed by Jack [12] and the failure density adjustment model for NC machine tools were used by Guo, et al. [13] as part of their imperfect PM model. Van and Bérenguer [14] assumed that deterioration behavior is a Gamma stochastic process and proposed a state-based maintenance policy for deteriorating production systems. Kahle [15] discussed the Kijima models [11] applied to system virtual age and deterioration.

Preventive maintenance is performed to sustain the safe and reliable operation of industrial equipment. To plan preventive maintenance or evaluate the existing maintenance plan, the failure behavior of the system must be modeled. The failure behavior of a repairable system is modeled utilizing counting processes. In this paper, the failure behavior of propellers belonging to a small aircraft fleet is modeled. First, the non-parametric estimate of population mean cumulative function (MCF) is obtained. MCF helps to discover the special features of the maintenance data. The parametric model selection depends on the result of the trend analysis of the time between failures. In the second part of the study trend analysis is performed on propeller maintenance data. Based on the trend analysis NHPP and Kijima II models are selected as prospect models. Reliability measures are estimated using both models and results are compared to evaluate the existing preventive maintenance plan.

The remainder of this paper is structured as follows: Section 2 reviews NHPP and Kijima II models. The problem and data definitions are given in Section 3. Reliability analysis of propellers is performed in Section 4. In section 5, results of the proposed models are presented. Finally, the conclusion is given in Section 6.

2. Methodology

Counting processes are stochastic models, defined as the occurrence of events over time. These events are thought of as points on the time axis, usually the time between events is neither independent nor identically distributed [16, 17]. When events are failures of a system, counting processes can be categorized based on the quality of repairs as homogeneous Poisson processes (HPP), renewal processes (RP), non-homogeneous Poisson processes (NHPP), and imperfect repair processes (IRP). The classification of the counting processes according to repair type is shown in Figure 1.

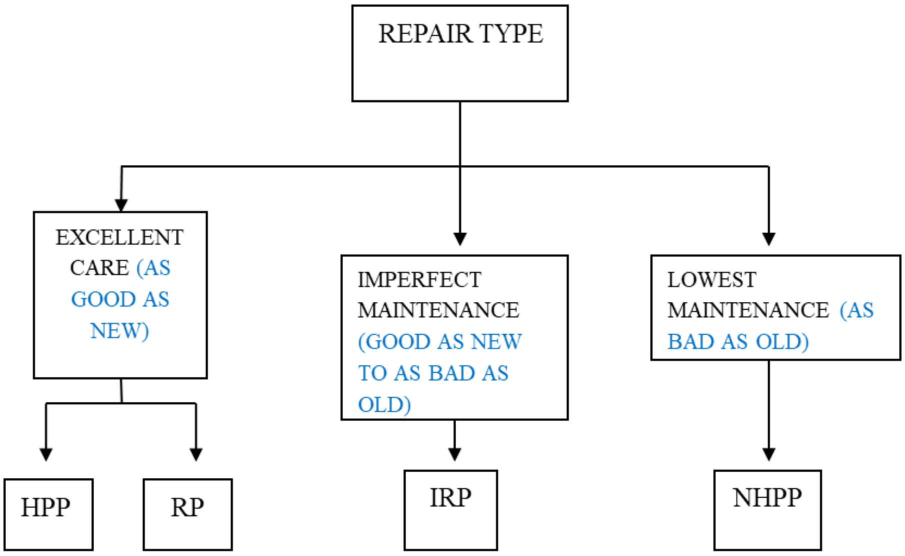

Figure 1: The classification of the counting process [18].

In this study, NHPP and IRP models are used. The NHPP is also called a non-stationary Poisson process [19]. The basic assumption of the NHPP model is that in the event of a failure, the system is repaired to the state it was in immediately before the failure, i.e. the minimal repair or as-bad-as-olds [18, 20]. This assumption is justifiable for repairable systems, such as an engine, since typically only a few components of the system is repaired at a time, restoring it to its pre-failure state [21].

Some applications of the NHPP model in reliability; trend analysis based on failure occurrence, optimal replacement analysis, statistical warranty claim prediction. As an example of these studies; in the study conducted by Rausand and Hoyland [18], Asfaw and Lindqvist [22], Nelson [23], Tang, et al. [24], the NHPP model was used in the problem of trend analysis based on fault occurrences. Sheu, et al. [25], Srivastava and Mondal [26] discussed the NHPP model optimal replacement problems. Kaminskiy and Krivtsov [27], Majeske [28] used the model to analyze the statistical warranty claim prediction.

For the counting process $N_i(\,t\,), t \geq\ 0$ to be NHPP with the rate function $w_i(t)$, must satisfy the following conditions; $N_i(0) = 0$, $N_i(\,t\,), t \geq\ 0$ is an independent increment and it is not possible for more than 1 failure to occur in at the same time. For the NHPP, the probability mass function of the number of events occurring at time $(0, t]$ for subpopulation i is according to a Poisson distribution.

$$P(N_i(t) = n) = \frac{W_i(t)^n e^{-W_i(t)}}{n!} \tag{1}$$

In Equation 1, $N_i(t)$ is the random variable representing the number of events at time $(0, t]$ for subpopulation i. $W_i(t)$; is the expected value known as the mean cumulative function (MCF) for subpopulation i at time $(0, t]$ expressed in Equation 2.

$$E[N_i(t)] = W_i(T) = \int_0^T w_i(t)\,dt, \tag{2}$$

where $w_i(t)$ is rate of occurrence of failures (ROCOF) at time t for unit i.

In order to calculate the ROCOF for the NHPP model, a parametric model is needed. The power-law was first discussed by Duane [29], and later broadened by Crow [30].The power–law NHPP is used to model failure times that occur at an increasing, decreasing and constant rate. When the rate constant NHPP becomes HPP. It is often used for the reliability of repairable systems or complex systems.

$$W_i(T) = \int_0^T \left(\frac{1}{\lambda_i}\right)^{\beta_i} t^{\beta_i - 1} dt = \left(\frac{T}{\lambda_i}\right)^{\beta_i}, \tag{3}$$

where β_i is the shape parameter and λ_i is scale parameter. The power-law NHPP model is highly sensitive to β_i. Accurate calculation of β_i is crucial. This calculation is challenging for noisy data sets and/or subpopulations with few data points. Therefore, in such cases, a power law NHPP model with a common shape parameter is proposed [20]. According to the value of β_i, the following situations are observed; if $0 < \beta_i < 1$, the time between failures increases, if $\beta_i = 1$, the NHPP model is reduced to the HPP model, and if $\beta_i > 1$, the time between failures decreases.

The general renewal process (GRP) is an IRP model. In particular, GRP models model the failure behavior of a given system and enable an understanding of the effects of repairs on the age of the system. The GRP is an appropriate model when the state of the system after repair is between "as good as new" and "as bad as new". In the study by Crocker [31], the failures were analyzed by special forms of GRP Kijima model I and Kijima model II. It was revealed that the system would not be as good as new after repair. The models developed in Kijima [11] study proposed these two models, which deal with a general assumption regarding the repair situation of the GRP [32].

Kijima model I assumes that the repair after system failure is based only on the removal of the last damage, while Kijima model II assumes that the repair after failure removes the damage accumulated up to the present time and all wear and tear.

$$R(x) = P(X > x), \tag{4}$$

where $R(x)$ is the reliability or survivor function, distribution function $F(x) = 1 - R(x)$, densitiy function $f(x) = -dR(x)/dx$.

$$r(x) = f(x)/R(x), \tag{5}$$

where r(x) is the hazard rate and $H(x) = \int_0^x r(u)du$ is the cumulative hazard function. In the event of a system failure, a repair is performed, and the time between the $(i-1)$th failure and the (i)th failure is the system uptime, expressed as X_i, and the distribution of X_i depends on the value of V_{i-1}, which is the virtual age of the system after the $(i-1)$th repair [33].

$$P(X_i \geq x \setminus V_{i-1} = v) = \frac{R(x+v)}{R(x)}, \tag{5}$$

$$V_i = \gamma(V_{i-1}, X_i), \qquad i \geq 0, V_0 = 0 \tag{6}$$

where γ is the repair function. Situations by virtual age can be summarized as follows: if $V_i = 0,\ i \geq 0$, replacement ("as good as new") for The RP or HPP, if $V_i = V_{i-1} + X_i,\ i \geq 0$, minimal repair ("as bad as old") for The NHPP, if $V_i = V_{i-1} + \alpha_i X_i,\ i \geq 0$ where α_i is a random variable and $0 \leq \alpha_i \leq 1$, Kijima model I, and $V_i = \alpha_i(V_{i-1} + X_i),\ i \geq 0$ and $0 \leq \alpha_i \leq 1$, Kijima model II. Maintenance models are constrained by the fact that repairs are either "good as new" or "minimum repair", so the general repair models Kijima model 1 and Kijima model 2 are more suitable as they provide flexibility in modeling the degree of repair between the two extremes of repair [33]. When the virtual age V_i is increased considerably, it will have an infinite failure rate and the uptimes will stochastically decrease towards a boundary value of zero. For this reason, Kijima model I cannot be used in a repair strategy that aims to maintain a constant long-term expected time between failures in the system, instead Kijima model II is a more appropriate approach.

3. Description of the Problem

Propellers are the parts that use the energy generated by the engines to accelerate the air mass so that the aircraft can move through the air. Propeller maintenance data for a fleet of aircraft is analyzed using counting processes and suitable reliability measures are estimated to evaluate the existing maintenance plan. The fleet consists of 34 aircraft and the failed propeller is removed from the aircraft and replaced with another propeller from the stock. The repaired propeller is sent to stock room. The propellers are regularly checked and overhauled as part of preventive maintenance. The propeller is tracked in the system using the serial number. We have maintenance data for a total of 72 propellers. The maintenance data used in the study was compiled from various sources. An example of the data can be found in Table 1. The table shows the accumulated flight hours between removals of the propeller *P01*. From this data the accumulated flight hours between failures are calculated for the reliability analysis.

Table 1: Maintenance data

Serial Number	Reason	Flight Time Between Disassemblies
P01	Overhaul	1118.0
P01	Failure	1909.7
P01	Failure	2422.2
P01	Failure	2481.3

Propellers suffered different numbers of failures. This information is summarized in Table 2. For example, 22 propellers had no failures and 25 had only one failure. The total number of failures for the whole population of propellers is 96 failures.

Table 2: The Number of Propellers that have Suffered a Given Number of Failures

Number of Failures	Propeller Units
0	22
1	25
2	13
3	7
4	1

All propellers are from the same manufacturer, and their ages and operating conditions are the same, forming a homogeneous population. Therefore, maintenance data of all propellers are polled. Reliability analysis is performed on the pooled data and reliability metrics are calculated for the fleet.

4. Methodology and Case Study

Two modeling approaches are proposed to analyze the propeller unit data and the systems are modeled separately for each modeling approach. The propeller units are not considered as good as new after overhauls so the cumulative flight hours between failures are taken as a basis. Figure 2 below presents the event plot of propeller units. Cross marks indicate the accumulated flight hours that failures took place.

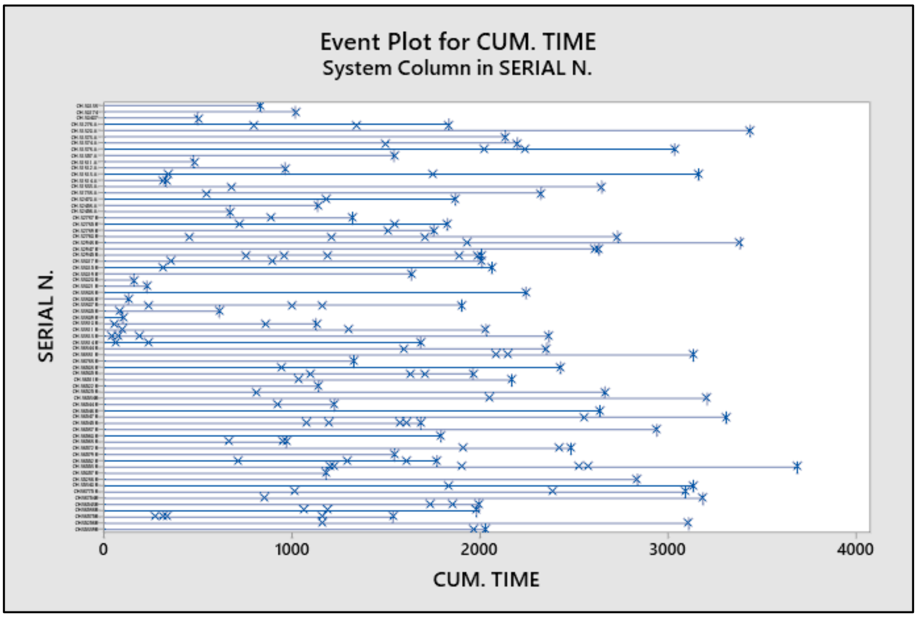

Figure 2: Event Plot for Cumulative Time

4.1. Trend Test

The general trend of the data was first determined using a non-parametric model. In the non-parametric model, the MCF is estimated over time from the data without assuming a model. MCF is a plot of the average cumulative number of failures per unit, over time. The MCF is estimated for each time point at which a failure occurs and averaged over the number of units observed at that time point (number of units at risk) [23].

The shape of the MCF gives information on the shape of the ROCOF. If MCF is linear, ROCOF is constant in time and there is no trend in the time between failures. If MCF is convex, ROCOF is an increasing function in time and the time between failures decreases with time. If MCF is concave, ROCOF is a decreasing function in time and the time between failures increases over time. In these two cases, ROCOF has a monotonic increase or decrease. When the MCF changes shape, the ROCOF is also not monotonic. The MCF plot for propeller units is given in Figure 3 below. The slight convex nature of the figure shows that the propeller deteriorates more often as it ages. This is an expected result as the propeller is a mechanical system [34].

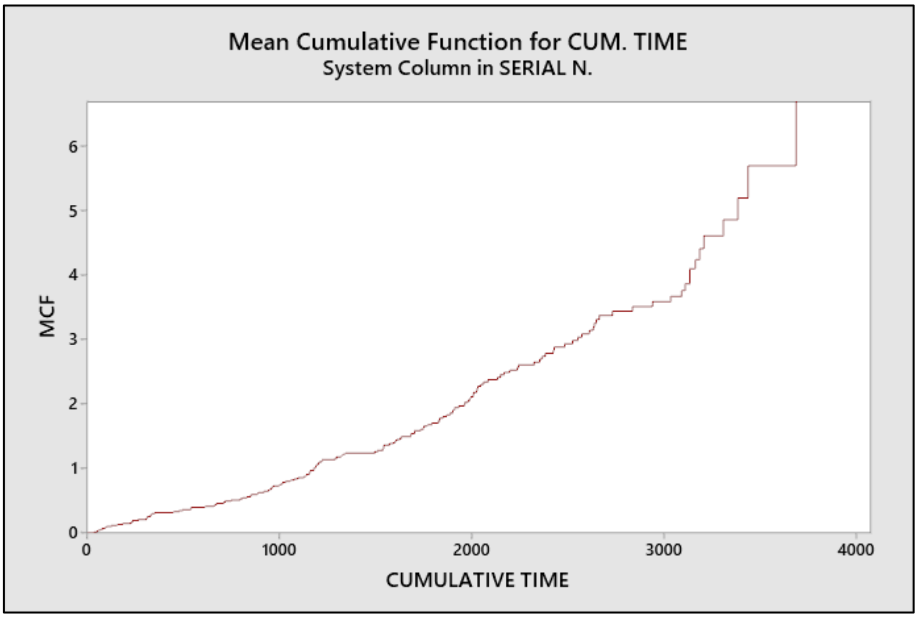

Figure 3: Mean Cumulative Function for Propeller Units

MCF values for some flight hours are given in Table 3. As an example, the third row of the table can be explained as follows. It is estimated that an average of 2.032 failures per unit will occur in 2133.2 flight hours. The standard error of this prediction is 0.225 and the 95% confidence interval (CI) is calculated as 1.635-2.525.

Table 3: Estimation of MCF

Time	**MCF**	**Std. Error**	**95% Normal CI limit**		**Serial Number**
			Upper	**Lower**	
32	0.014	0.014	0.099	0.002	*P59*
1190.5	1.008	0.135	0.312	0.775	*P11*
2133.2	2.032	0.225	2.525	1.635	*P35*

After determining the general trend of the data with the nonparametric model, a trend test was applied to the time between failures of the propeller units to determine the appropriate parametric model and the trend test results are given in the Table 4.

Table 4: Trend Test Results for Propeller Units

	MIL-Hdbk-189		**Laplace's**		
	TTT-based	**Pooled**	**TTT-based**	**Pooled**	**Anderson-Darling**
Test Statistic	190.83	157.27	1.96	1.59	3.01
P-value	0.036	0.100	0.050	0.112	0.027

DF	234	188

Since the units are homogeneous, the test results based on the total-time-on-test statistic are analyzed. The test results are evaluated for 0,05 significance level. Since the P-values are less than or equal to 0.05, there is a trend. Based on the trend test results and the nonparametric model results a nonstationary counting process model is suitable for modeling the reliability of the propellers. Power law model and Kjima model II are chosen as prospect models.

4.2. Proposed Modeling Approach 1

Parameters of the power law model are estimated by maximum likelihood method. The parameter estimates of the power law model are given in the Table 5. From the table, the shape parameter ($\boldsymbol{\beta}$) is estimated as 1.11 and the scale ($\boldsymbol{\theta}$) parameter is estimated as 1447.51. Although the 95% confidence interval contains values less than 1, the trend test results, and the non-parametric MCF model results support the trend in the time between failures as the propeller unit ages.

Table 5: Estimation of Power Law Model Parameters

Parameter	**Estimate**	**Std. Error**	**95% Normal CI Limit**	
			Lower	**Upper**
Shape ($\boldsymbol{\beta}$)	1.11	0.089	0.952	1.303
Scale ($\boldsymbol{\theta}$)	1447.51	143.707	1191.56	1758.44

The average cumulative number of failures predicted from the model is given in Figure 4. The blue dots in the figure represent the data and the solid line represents the values predicted from the model, thus the figure shows that the model predictions are close to the actual data.

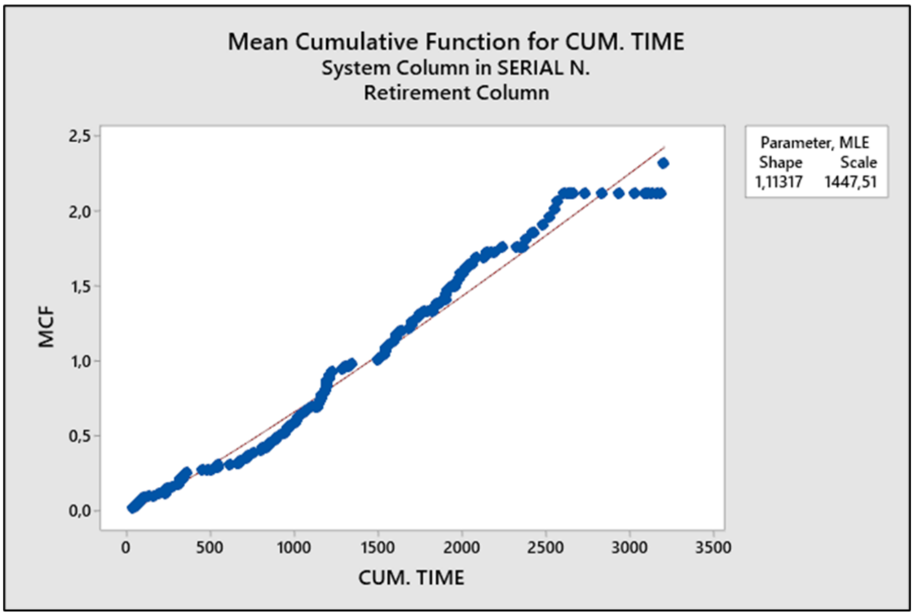

Figure 4: MCF for Cumulative Time

The values of the ROCOF for the power law model at some time points are given in the Table 6. For example, a propeller that has completed 1000 hours of flight time will fail on average $\mathbf{7.38 \times 10^{-4}}$ times in an hour flight. This is a very low failure density. As can be seen from the values in the table, ROCOF increases slowly over time.

Table 6: Proposed Model 1 Density Function Values

Time (t) (flight hour)	**Density function, λ (t) (failure/flight hour)**
1000	0.000738
2000	0.000798
3000	0.000835
4000	0.000863

Estimates of the values of the mean time between failures at some time points and the 95% confidence intervals of the estimates are given in the Table 7. For example, for a small-time interval dt starting with 1000 flight hours, the mean time between failures (MTBF) is approximately 1356 hours. As the ROCOF increases over time, MTBF decreases over time as shown in the table.

Table 7: Proposed Model 1 MTBF Values

Time	**MTBF (flight hour)**	**95% Normal CI**
1000	1355.93	1109-1657
2000	1253.62	970-1619
3000	1197.40	874.84-1638.90

4.3. Proposed Modeling Approach 2

When the Kijima II model is applied for all data, the model parameters are estimated by maximum likelihood method and given in the Table 8. The value of the parameter $\boldsymbol{q}$, which indicates the repair quality, is approximately zero. This indicates that the repair quality is very good. After repair, the systems are almost as good as new. This also supports the fact that the confidence interval for $\boldsymbol{\beta}$ parameter of NHPP includes also 1. Therefore, Kijima model is more appropriate for modelling propellers failure behavior.

Table 8: Kijima II Model Parameters

Parameter	**Estimate**
β	0.900173
λ	0.001473
q	0.000015

For the Kijima II model, the values of ROCOF at some time points are given in the Table 9. For example, a propeller that has completed 1000 flight hours will fail an average of $\mathbf{6.87 \times 10^{-4}}$ times in a hour flight. This is a very low failure density.

Table 9: Proposed Model 2 Density Function Values

Time (t) (flight hour)	**Density function, λ (t) (failure/flight hour)**
1000	0.000687
2000	0.000659
3000	0.000844

The values of the mean time between failures at some time points are given in the Table 10. The mean time between failures in 1000 flight hours is 1456.43 hours. A propeller unit with 1000 flight hours (which may have failed and been repaired) has a 0.41 probability of flying for another 1000 hours without failure.

Table 10: Proposed Model 2 MTBF Values

Time (flight hour)	**MTBF (flight hour)**	**95% Normal CI Limit**
1000	1456.43	1156.15-1888.38
2000	1517.47	1177.55-2025.30

The values of the average number of failures per propeller and the average number of propeller unit failures for the fleet at some time points are given in the Table 11. For example, the mean number of failures per propeller unit between 1500-2000 flight hours is 0.315. Since there are 34 aircraft in the fleet and each aircraft has one propeller unit, the average number of propeller unit failures for the fleet between 1500-2000 flight hours is $\mathbf{34 \times 0.315 = 10.71}$.

Table 11: Mean Number of Failures for the Kijima II Model

Time (flight hour)	**Mean Failure ($E[N(t)]$)**	**Mean Number of Failures for the Fleet**
0-1000	0.396	13.464
1000-2000	0.343	11.662
1000-1500	0.326	11.084
1500-2000	0.315	10.71
2000-2500	0.307	10.438

5. Conclusions

In this paper, the maintenance data of propellers are analyzed using parametric and non-parametric models. Analysis results showed that the Kjima II model more appropriate to model the failure behavior of propellers.

The Kjima II model used in the proposed modeling approach 2 suggests that the repair quality is almost as good as new. This shows that the current preventive maintenance plan is effective and the quality of the corrective maintenance is satisfactory.

Contribution of Researchers

All researchers have contributed equally to writing this paper.

Conflicts of Interest

The authors declare no conflict of interest.

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