

# Nussbaum Gain for Adaptive Fuzzy Global Non Singular Sliding Mode Power System Stabilizer

E. Nechadi, M. N. Harmas, N. Essounbouli and A. Hamzaoui

**Abstract**—Power systems stability is enhanced through a novel stabilizer developed around a non singular adaptive fuzzy terminal integral sliding mode approach using the Nussbaum function applied to a nonlinear model of a single machine power system connected to an infinite bus via a double transmission lines subjected to severe faults. Nussbaum gain is used to avoid the problem of controllability of the system. Stability is insured through Lyapunov synthesis. Severe operating conditions are used in a simulation study to test the validity of the proposed method, indicating better performance and satisfactory transient dynamic behavior.

**Index Terms**—Power system stabilizer, adaptive fuzzy global sliding mode; Nussbaum function, Lyapunov stability.

## I. INTRODUCTION

THESE power systems are complex nonlinear systems that often exhibit low frequency oscillations due to insufficient damping caused by adverse operating conditions which can lead to a devastating loss of synchronism [1].

Power system stabilizers are used to suppress these oscillations and improve the overall stability [1-4]. The computation of the fixed parameters of these stabilizers is usually based on the linearized model of the power system around a nominal operating point [5-7]. The operating condition often change as a result of load variation and/or major disturbances, making the dynamic behaviour of the power system different, thus requiring new adjustment of stabilizer parameters for if the latter are kept fixed, controlled power system performance is greatly degraded [7].

Conventional stabilizers, consisting of cascade connected lead-lag compensators derived from a linear model representing the power system at a certain operating point, have long been used to damp oscillations regardless of the varying loading conditions or disturbances [8-12]. However, a lot of research about the design of power system stabilizers has been conducted, using a wide range of strategies, such as sliding controller [13,14], adaptive controller [15-16], adaptive fuzzy controllers [17,18] and a comparison of some approaches to designing power system stabilizers has been presented in [19,20].

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One of these possible methods is the application of adaptive fuzzy sliding controller. Remarkable research effort has been done in the last decade putting forward intelligent fuzzy logic based power system stabilizer as well as optimality in adapting to changing operating conditions as in [21-23].

However, this linear model based control strategies often fail to provide satisfactory results over a wide range of operating conditions besides during severe disturbances, PSS action may actually cause the generator under its control to lose synchronism in an attempt to control its excitation field. In [24] the authors applied the Nussbaum gain with the conventional sliding mode control but the results are unsatisfied.

This paper introduces briefly in the next section the terminal sliding mode control approach used, followed by the second section in which adaptive fuzzy technique is tackled. In third section the design of the non singular adaptive fuzzy terminal sliding mode stabilizer using a Nussbaum gain is undertaken and stability issue addressed. The power system model is presented in the ensuing section followed by simulation and a presentation of results for different operating conditions.

## II. GLOBAL SLIDING MODE CONTROL

Consider a SISO nonlinear system described by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x,t) + g(x,t)u \end{cases} \quad (1)$$

where  $x = [x_1 \ x_2]^T \in R^2$  is the state vector,  $u \in R$  is the input,  $f(x,t)$  and  $g(x,t)$  are the unknown functions nonlinear.

Then,  $f(x,t)$  and  $g(x,t)$  can be written as:

$$f = f_0 + \Delta f \quad (2)$$

$$g = g_0 + \Delta g \quad (3)$$

where  $f_0$ ,  $g_0$  are the nominal functions,  $\Delta f$  and  $\Delta g$  are uncertainties satisfy the conditions:

$$|\Delta f| \leq F \quad (4)$$

$$\frac{g}{g_0} \leq G \quad (5)$$

where  $F$  and  $G$  are positive.

The terminal sliding switching surface as follows:

$$S = \int_0^t x_1^\lambda(\tau) d\tau + \alpha x_1 + \beta x_2 \quad (6)$$

where  $\alpha, \beta$  and  $\lambda$  are the constants positive.

Control law enabling satisfaction of the attraction phase condition (7) and the equivalent control to maintain state trajectories on the sliding surface is typically given by (8) assuming  $g$  is non-singular.

$$S\dot{S} < 0 \quad (7)$$

**Theorem 1:** For the nonlinear system (1), if we choose the following control law:

$$u_{GSMC} = -g^{-1} \left( \frac{1}{\beta} x_1^\lambda + \frac{\alpha}{\beta} x_2 + f + \frac{k^*}{\beta} \text{sign}(S) \right) \quad (8)$$

where  $k^*$  indicates the control gain with the sliding function, then the system is stable.

**Proof:**

Choosing the Lyapunov function candidate to be

$$V = \frac{1}{2} S^T S \quad (9)$$

Therefore

$$\dot{V} = S^T \dot{S}$$

$$\dot{V} = S^T [x_1^\lambda + \alpha x_2 + \beta \dot{x}_2]$$

$$\dot{V} \leq -k^* |S|$$

thus:  $\dot{V} \leq 0$ .

### III. ADAPTIVE FUZZY GLOBAL SLIDING MODE CONTROL

In this section, the procedure to construct an adaptive fuzzy gain to the terminal sliding mode controller.  $\hat{k}$  is the approximation of the  $k^*$ , using the singleton fuzzifier, product fuzzy inference and center gravity defuzzifier, the inferred output is:

$$\hat{k}(x, \theta_k) = \xi^T(x) \theta_k \quad (10)$$

where  $\theta_k = [\theta_{1k}, \theta_{2k}, \dots, \theta_{mk}]$  is the vector of parameters,  $\xi = [\xi_1, \xi_2, \dots, \xi_m]^T$  is the vector of fuzzy basis functions.

The minimum approximation error is:

$$\varepsilon_k = k^* - \xi^T(x) \theta_k^* \quad (11)$$

where  $\theta_k^*$  is the optimal approximation parameter and  $\tilde{\theta}_k = \theta_k - \theta_k^*$ .

**Theorem 2:** For the nonlinear system (1), if we choose the following control law:

$$u_{AFGSMC} = -g_0^{-1} \left( \frac{1}{\beta} x_1^\lambda + \frac{\alpha}{\beta} x_2 + f_0 + \frac{\hat{k}}{\beta} \text{sign}(S) \right) \quad (12)$$

and if,

$$k^* \geq \frac{1}{G} \eta + \left| \frac{\beta F}{G} + \left( \frac{1-G}{G} \right) (x_1^\lambda + \alpha x_2 + \beta f_0) \right| + |\varepsilon_k| \quad (13)$$

and choose the adaptation law:

$$\dot{\theta}_k = \gamma G |S| \xi^T(x) \quad (14)$$

then the system is stable.

**Proof:**

Choosing the Lyapunov function candidate to be

$$V = \frac{1}{2} S^T S + \frac{1}{2\gamma} \tilde{\theta}_k^T \tilde{\theta}_k \quad (15)$$

Therefore

$$\dot{V} = S^T \dot{S} + \frac{1}{\gamma} \tilde{\theta}_k^T \dot{\theta}_k$$

$$\dot{V} = S^T \left[ (x_1^\lambda + \alpha x_2 + \beta f) \right.$$

$$\left. - \beta \left( \frac{g}{g_0} \right) \left( \frac{1}{\beta} x_1^\lambda + \frac{\alpha}{\beta} x_2 + f_0 + \frac{\hat{k}}{\beta} \text{sgn}(S) \right) \right] + \frac{1}{\gamma} \tilde{\theta}_k^T \dot{\theta}_k$$

$$\leq -\eta |S| + S \left( (1-G)(x_1^\lambda + \alpha x_2 + \beta f) + \beta F \right) + G \hat{k} |S| + \frac{1}{\gamma} \tilde{\theta}_k^T \dot{\theta}_k$$

$$\leq -\eta |S|.$$

### IV. NON SINGULAR ADAPTIVE FUZZY GLOBAL SLIDING MODE CONTROL

**Definition**

A function is called a Nussbaum-type function if it has the following properties:

$$\limsup_{y \rightarrow \infty} \frac{1}{y} \int_0^y N(\zeta) d\zeta = +\infty \quad (16)$$

$$\liminf_{y \rightarrow \infty} \frac{1}{y} \int_0^y N(\zeta) d\zeta = -\infty \quad (17)$$

Through out this paper the even Nussbaum function:

$$N(\zeta) = \exp(\zeta^2) \cos((\pi/2)\zeta) \quad (18)$$

is employed and  $\zeta$  is a variable determined later.

In this section, the fuzzy logic model is expressed as the following form:

$$\hat{f}(x) = \theta_f \xi^T(x) \quad (19)$$

approximates the unknown system function  $f(x)$  with the approximation error  $\delta_f$ , such that:

$$\delta_f = f(x) - \xi^T(x) \theta_f \quad (20)$$

where  $\theta_f = [\theta_{f1}, \theta_{f2}, \dots, \theta_{fn}]$  is the vector of parameters,  $\xi = [\xi_1, \xi_2, \dots, \xi_m]^T$  is the vector of fuzzy basis functions.

V. ADAPTIVE FUZZY GLOBAL SLIDING MODE CONTROL USING NUSSBAUM FUNCTION

In previous research on indirect adaptive fuzzy method, the controller with  $1/\hat{g}(x)$  can be singular because it cannot be guaranteed that  $\hat{g}(x)$  is not equal to zero at any moment where  $\hat{g}(x)$  denotes the approximation of  $g(x)$ . A Nussbaum function is used to avoid appearance of the singularity problem and should satisfy the following condition:

$$gN(\zeta) \leq \chi \tag{21}$$

**Theorem 3:** For the nonlinear system (1), if we choose the following control law:

$$u_{NTSMC} = -N(\zeta) \left( \frac{1}{\beta} x_1^\lambda + \frac{\alpha}{\beta} x_2 + \hat{f} + \frac{\hat{k}}{\beta} \text{sign}(S) \right) \tag{22}$$

and if

$$k^* \geq \frac{1}{\chi} \eta + \left| \frac{\beta F}{\chi} + \left( \frac{1-\chi}{\chi} \right) (x_1^\lambda + \alpha x_2) \right| + |\varepsilon_k| \tag{23}$$

with

$$\dot{\zeta} = x_1^\lambda + \alpha x_2 \tag{24}$$

and choose the adaptation law:

$$\dot{\theta}_k = \gamma_2 \chi |S| \xi^T(x) \tag{25}$$

$$\dot{\theta}_f = -\gamma_1 \beta (1-\chi) S \xi^T(x) \tag{26}$$

then the system is stable.

PROOF:

Choosing the Lyapunov function candidate to be

$$V = \frac{1}{2} S^T S + \frac{1}{2\gamma_1} \theta_f^T \theta_f + \frac{1}{2\gamma_2} \tilde{\theta}_k^T \tilde{\theta}_k \tag{27}$$

Therefore;

$$\begin{aligned} \dot{V} &= S^T \dot{S} + \frac{1}{\gamma_1} \theta_f^T \dot{\theta}_f + \frac{1}{\gamma_2} \tilde{\theta}_k^T \dot{\tilde{\theta}}_k \\ &= S^T \left[ (1-gN(\zeta)) (x_1^\lambda + \alpha x_2 + \beta \hat{f}) + \beta \delta_f - gN(\zeta) \hat{k} \text{sign}(S) \right] \\ &\quad + \frac{1}{\gamma_1} \theta_f^T \dot{\theta}_f + \frac{1}{\gamma_2} \tilde{\theta}_k^T \dot{\tilde{\theta}}_k \\ &\leq -\eta |S| + S(1-\chi) \beta \xi^T(x) \theta_f + \frac{1}{\gamma_1} \dot{\theta}_f^T \theta_f - \chi \xi^T(x) |S| \tilde{\theta}_k \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{\gamma_2} \tilde{\theta}_k^T \dot{\tilde{\theta}}_k \\ &\leq -\eta |S|. \end{aligned}$$

VI. POWER SYSTEM MODEL

The power system model considered in this paper is a nonlinear model representing a synchronous machine connected to an infinite bus via a double circuit transmission line. The power system schematic diagram including turbine, transformer, automatic voltage regulator and PSS is shown in Fig.1 [25-26].

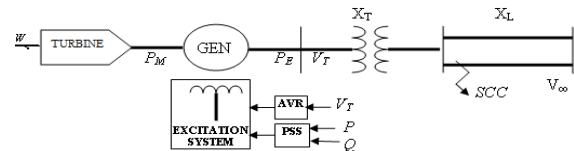


Fig.1 One-line SMIB diagram with AVR and PSS.

A fourth order classic representation is:

$$\begin{aligned} \dot{\delta} &= \omega_0 (\omega - \omega_{ref}) \\ \dot{\omega} &= \frac{1}{M} (P_m - P_e) \\ \dot{e}'_q &= \frac{1}{T'_{d0}} (e_{fd} - e_q) \\ \dot{e}_{fd} &= \frac{1}{T_A} (K_A (E_{ref} - V_t + u) - e_{fd}) \end{aligned} \tag{28}$$

A nonlinear representation the machine during a transient period after a major disturbance has occurred in the system is follows:

$$\begin{cases} \dot{z}_1 = \frac{1}{M} z_2 \\ \dot{z}_2 = f(z, t) + g(z, t)u \end{cases} \tag{29}$$

where the state variable are expressed as:  $z = [\Delta\omega \ \Delta P]$ , with  $\Delta\omega$  is speed deviation,  $\Delta P = P_m - P_e$  is the accelerating power and  $M$  is inertia moment coefficient.

The fact that the governor time constant is large compared to the time constants of the synchronous machine and its exciter, so that during the first few seconds after the occurrence of a severe disturbance the governor function can be ignored. Therefore the mechanical input power is constant during the transient integral, say less than 5 seconds after the disturbance has occurred [19].

Then, (29) can be written in form of system (1), if you make a change of variable as follows:

$$\begin{cases} x_1 = z_1 \\ x_2 = \frac{1}{M} z_2 \end{cases} \tag{30}$$

The parameters of the single machine infinite bus system are as follows:

$$x_e = 0.2 p.u., x_q = 0.36 p.u., x_d = 1.86 p.u., x'_d = 0.25 p.u., D = 0, T'_{d0} = 6s, H = 4s, T_A = 0.05, K_A = 50, V = 1 p.u., P_0 = 0.9 p.u., Q_0 = 0.3 p.u.$$

### VII. SIMULATION

The soundness of the proposed PSS was tested and performance as well as robustness tests were conducted and compared to a classic CPSS [13] confirming, through computer simulations, good transient behaviour with the proposed control despite severe operating conditions illustrated by the following case studies. Five fuzzy sets for each input are sufficient for the PSS to be designed.

The fuzzy sets for inputs  $\Delta\omega$  and  $\Delta P$  are defined according to the membership functions shown in Fig.2.1 and Fig.2.2. The initial value of the  $\theta_f, \theta_k$  is chosen to be zero and the 0.09 respectively.

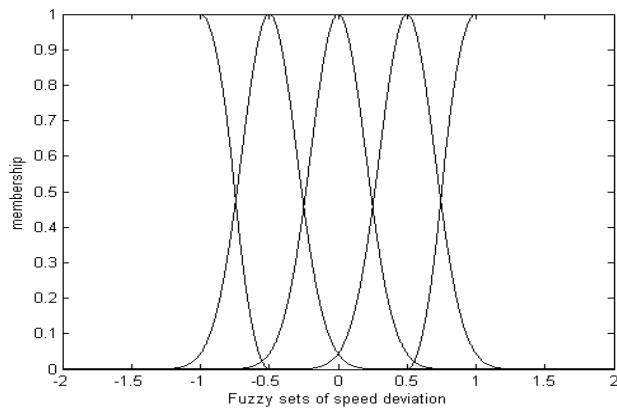


Fig.2.1 Fuzzy sets for input  $\Delta\omega$ .

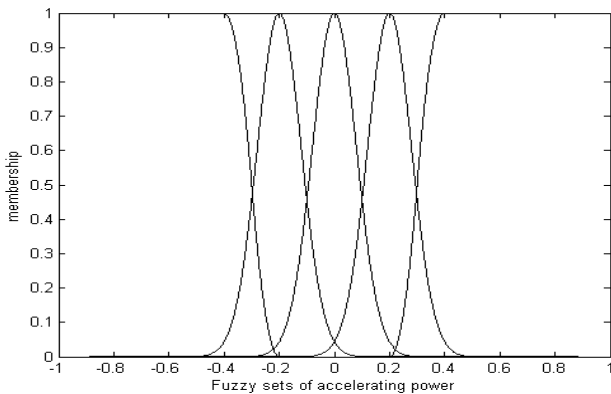


Fig.2.2 Fuzzy sets for input  $\Delta P$ .

*Case 1:* First the simulation results for normal load condition are shown in Fig.3 with PSS calculated on proposed control. Performances of the proposed PSS are clearly superior while a greater control effort is solicited.

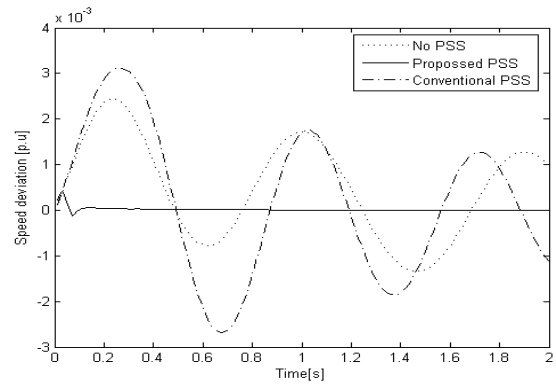


Fig.3.1 Speed deviation.

*Case 2:* Operating conditions change abruptly from light to heavy load condition, i.e.  $Q$  is changed from 0.3 p.u. to 0.8 p.u and  $x_e = 0.45 p.u.$  The simulation results in Fig.4 show a better transient performance for the proposed control.

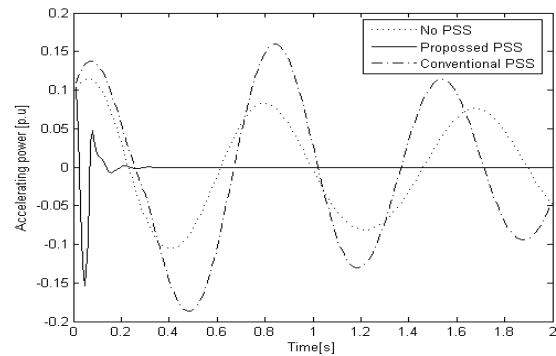


Fig.3.2 Accelerating power.

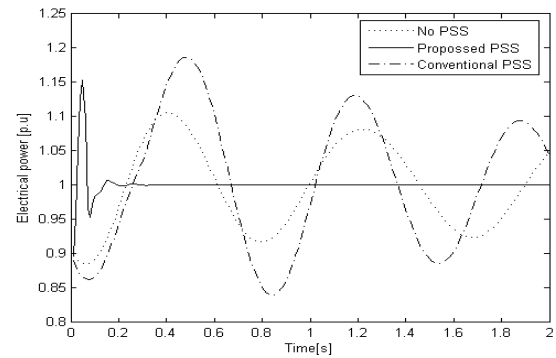


Fig.3.3 Electrical power.

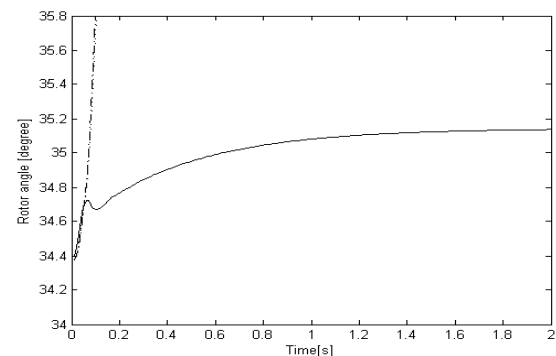


Fig.3.4 Rotor angle.

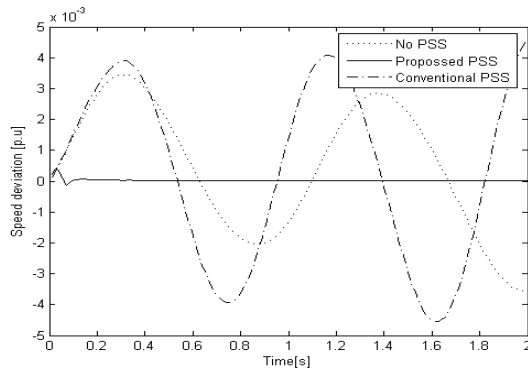


Fig.4.1 Speed deviation in heavy reactive power case.

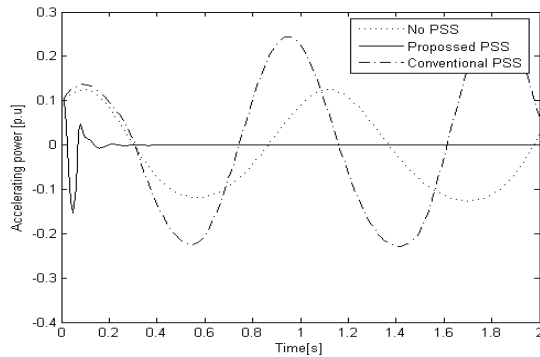


Fig.4.2 Accelerating power in heavy reactive power case.

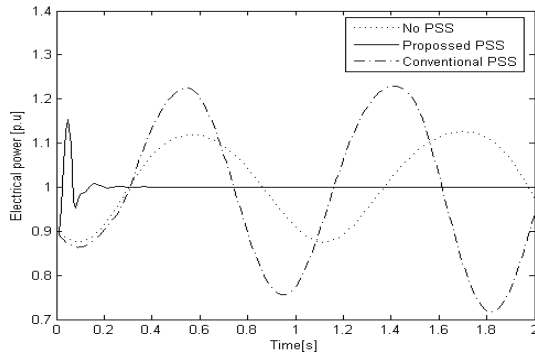


Fig.4.3 Electrical power in heavy reactive power case.

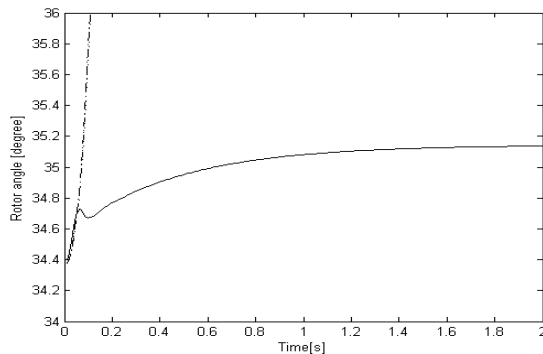


Fig.4.4 Rotor angle in heavy reactive power case.

*Case 3:* We now consider the case of the sudden occurrence of importing reactive power causing a change in  $Q$  from the light value to  $-0.3 p.u$  and strong connection ( $x_c = 0.1 p.u$ ). Again the simulation results shown in Fig.5

seem to indicate a good transient behaviour with superior performance due to the proposed PSS.

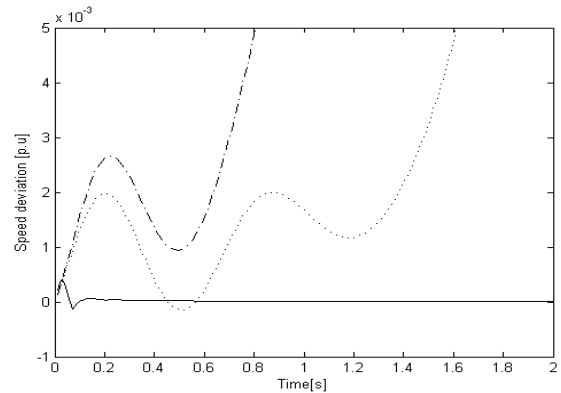


Fig.5.1 Speed deviation for case 3.

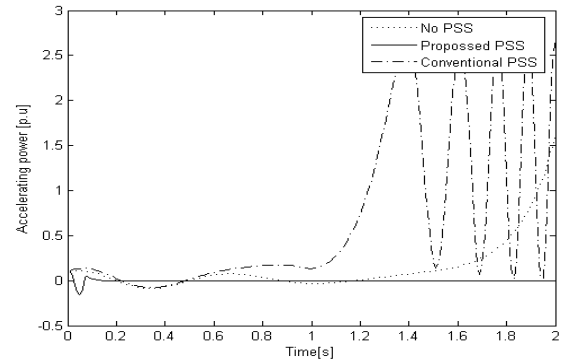


Fig.5.2 Accelerating power for case 3.

### VIII. CONCLUSION

We introduced in this paper, based on the adaptive fuzzy terminal sliding mode controller and the Nussbaum gain, a new non singular power system stabilizer that enhances damping and improves transient dynamics of a single synchronous machine using a nonlinear model of the power system. Different load conditions as well as severe perturbations were used to evaluate the proposed power system stabilizer effectiveness in rapidly reducing oscillations that could lead to loss of synchronism if not treated. Simulation results exhibit superior performance over classical PSS and in absence of PSS.

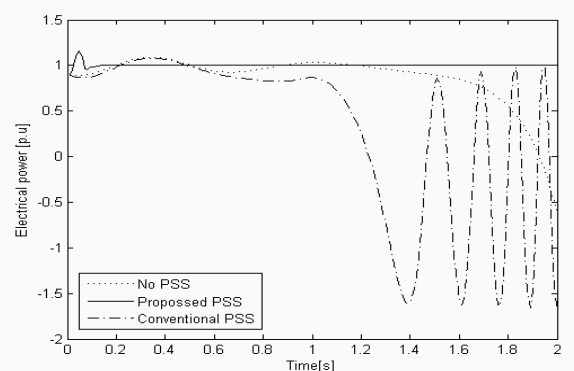


Fig.5.3 Electrical power for case 3.

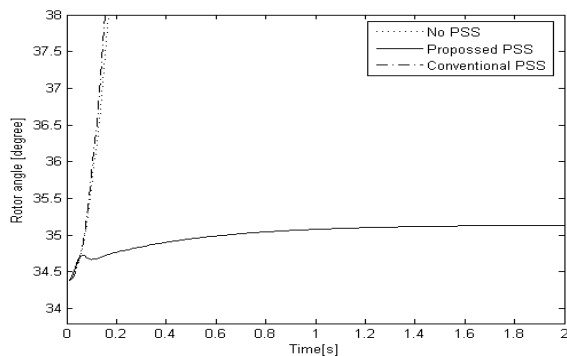


Fig.5.4 Rotor angle for case 3.

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