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FINANCIAL APPLICATIONS OF STABLE DISTRIBUTIONS: IMPLICATIONS ON TURKISH STOCK MARKET

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Hulya Basegmez[1], Elif Cekici[2]
[1]Marmara University, Faculty of Business Administration, Istanbul, Turkey. hulyabasegmez@gmail.com, hulya.basegmez@marmara.edu.tr
[2]Marmara University, Faculty of Business Administration, Istanbul, Turkey. ecekici@marmara.edu.tr

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ABSTRACT

Purpose- The aim of this study is modelled by examining the trading volumes of the tourism companies located in the high-risk tourism sector and traded in BIST. This modelling will gain point of view for the tourism firms as well as make an important contribution to the decision making of investors who want to invest in this sector.
Methodology- The study is conducted for a sample of 2803 daily trading volumes over the period 01.01.2007-28.09.2017. Then, it is used daily returns rather than daily trading volume data because it provides the ability to measure investment performance independently of the scale used. Daily return data is modelled with stable distributions used with increasing interest in many application areas and that are well-suited to financial asset returns. Parameter estimates is made by using quantiles method which is one of the most known estimation methods.
Findings- By means of the Chi-square test and graphs, it is seen that normal distribution was not suitable for trading volume data. Stable distribution parameters for the log-returns data are estimated according to the quantiles method and obtained the stable parameters α, β, γ and δ. Stable density function is obtained using the MATLAB STBL command according to estimated parameters.
Conclusion- Estimated parameter values indicate that stable distributions can be used as a suitable model for modelling the transaction volume data of analysed index. It has been concluded that it is more appropriate to use the scale parameter of the stable distribution instead of the standard deviation as the risk measure.

Keywords: BIST Tourism, stable distributions, parameter estimation.
JEL Codes: C13, C46, G00

1. INTRODUCTION

The tourism sector is a sector that contributes very much to the national economy. The development of such a sector contributes to meet the foreign currency needs, to create employment and to bring foreign investors to the country. Nature events, internal and external politics, economic relations of the country, socio-cultural differentiations, service quality, environmental factors and many other factors affect the sector (Inskeep,E., 1991; Özdemir, M. A., and Kervankıran, İ., 2013). As a result of these circumstances, the trading volumes of tourism companies which are located in this sector with high risk and traded in the BIST are examined and modelled. This modelling will gain a point of view for tourism firms as well as make an important contribution to decision making of investors who want to invest in this sector. When the trading volume data were analysed, it was observed that normal distribution was not suitable for these data. In this study, daily trading volumes of XTRZM index traded in BIST will be tried to be modelled with Stable Paretian distributions. In the second part of the work, stability and stable distributions are discussed and parametrization of stable rules is mentioned. In the third part, trading volumes belong to the tourism sector are modelled with stable distributions. In the last section, the output of the model is given.

2. STABLE DISTRIBUTIONS

Stable distributions – also called α −stable, Stable Paretian or Levy Stable - are rich class that allow the features like heavy tail and skewness of events and probability distributions that occur as a result of many small effects. This class was introduced by Paul Levy (1925) in his investigations of behaviour of sums of independent random variables. These distributions were then proposed as an alternative to normal distribution by Mandelbrot (1963) and Fama, & Roll (1968) because of enabled features like heavy tail and skewness of financial time series. Press (1972) also suggested the use of stable distributions in the process of creating a model for probability distributions related to price changes of instruments. Zolotarev (1986) has also contributed greatly to this area. Stable distributions are widely used distributions in many systems with different properties, especially in economics. This distribution has a stability characteristic and a class feature in addition to providing a very good fit to the empirical data. If sums of independent, identically distributed random variables have a limit distribution, the limit distribution will be a member of the stable class (Fama, E.F. and Roll,R., 1968). The Central Limit Theorem explains the growth of interest in stable distributions as data models, especially in the economy. The theorem expresses the sum of the random variables with finite variance approximates a Gaussian random variable (Çekici, E., 2003). If the assumption that the variance is finite is removed, with an appropriate scaling, the one and only distribution for the sum of independent identically distributed random variables is stable distributions. Normal distribution is a special case with finite variance of stable distributions. Infinite variance can be characterized by the tail distribution of probability distributions. If it is $P(|x| > k) > 1/k^2$ for large k values in any given $f(x)$ distribution, variance is infinite. When the total probability in the tails is greater than $1/k^2$, the distribution is called as heavy tail (Önalan, Ö., 2010).

2.1. Definitions

One of the most important properties of Normal or Gaussian random variables is that the sum of two normal random variables is again a normal random variable. X is stable random variable if there exist some positive c and some $d \in R$ for any positive constants a and b with

$$aX_1 + b\,X_2 \overset{d}{=} c\,X + d \tag{1}$$

where X is normal random variable, X_1 and X_2 are independent copies of X. The symbol $\overset{d}{=}$ means equality in distribution, that is, it indicates that both sides of the distribution have the same probability law (Nolan, 2016). A random variable is symmetric stable if it is stable and symmetrically distributed around 0, e.g. $X \overset{d}{=} -X$ (Nolan, 2016).

Two random variables X and Y are said to be of the same type if there exist constants $a > 0$ and $b \in R$ with $X \overset{d}{=} a\,Y + b$ (Nolan, 2016). Non degenerate X is stable if and only if for all $n > 1$, there exist constants $c_n > 0$ and $d_n \in R$ such that

$$X_1 + \cdots + X_n \overset{d}{=} c_n X + d_n \tag{2}$$

Where $X_1, X_2, \dots, X_n$ are independent , identical copies of X (Zolotarev, 1986).

A random variable X is stable if and only if $X \overset{d}{=} a\,Z + b$, Z is a random variable with characteristic function

$$E\exp(iuZ) = \begin{cases} \exp\left(-|u|^{\alpha}\left[1 - i\,\beta\tan\frac{\pi\alpha}{2}\,(sign\,u)\right]\right), & \alpha \neq 1 \\ \exp\left(-|u|\,\left[1 + i\,\beta\frac{2}{\pi}\,(sign\,u)\log|u|\right]\right), & \alpha = 1 \end{cases} \tag{3}$$

where $0 < \alpha \leq 2,\ -1 \leq \beta \leq 1,\ a \neq 0,\ b \in R$ (Nolan, 2016).

2.2. Parametrizations of Stable Laws

The stable distributions requires four parameters to describe: an index of stability $\alpha \in (0,2]$, a skewness parameter $\beta \in [-1,1]$, a scale parameter $\gamma \geq 0$ and a location parameter $\delta \in R$. The parameter α called the tail index, tail exponent or characteristic exponent. The tail exponent α determines the rate at which the tails of the distribution taper off. When $\alpha = 2$, the distribution is a normal distribution. When $\alpha < 2$, the variance is infinite. As the value of the α parameter decreases, the peak gets higher, the region flanking the peak get lower and the tails get heavier. When $\beta = 0$, the distributon is symmetric around μ. When $\beta = 1$, the distribution is totally skewed to the right. When $\beta > 0$, the distribution is skewed to the right. Similarly when $\beta < 0$, the distribution is skewed to the left by reflection (Fama, E. F., 1965). As α approaches 2, β loses its effect and the distribution approaches the Gaussian distribution regardless of β. The parameter δ determines the width and the parameter δ determines the shift of the mode of the density. Generally $\gamma > 0$, although $\gamma = 0$ will sometimes be used to denote a degenerate distribution concerned at δ when it simplifies the statement of a result. Note that for any β>0, the mode goes to $+\infty$ as $\alpha \uparrow 1$, to $+\infty$ as $\alpha \downarrow 1$, and stays near 0 for $\alpha = 1$ (Nolan, J., 1998).

The family of stable distributions have relatively heavier distribution tails (except the case when $\alpha = 2$, the Gaussian distributions). They belong to the family of heavy-tailed distributions. The tail heaviness of stable distributions can be measured by using their stable index (Fan, Z., 2006). There is a large number parametrizations in the literature for stable rules and this situation leads to complexity. The variety of the parametrizations result from the historical evolution and the analysis of many problems using the specialized forms of stable distributions.

In most of recent literature, the notation $S_\alpha(\sigma,\beta,\mu)$ is used for the class of stable laws. However we will use a modified notation of the form $S(\alpha,\beta,\gamma,\delta;k)$ due to three reasons. First reason, the usual notation determine α as different and fixed. In statistical applications, all four parameters $(\alpha,\beta,\gamma,\delta)$ are unknown and need to be estimated for these parameters. Second reason, the scale parameter is not the standard deviation (even Gaussian situation) and the location parameter is not generally the mean. Then we use γ for the scale instead of σ and use δ for the location instead of μ. Third reason, there should be a clear distinction between the different parametrizations; the k integer does that (Nolan, 2016). It is possible to change tail thickness and skewness in stable distributions. There are three cases. Closed form expression of probability density functions for stable distributions except for Normal, Cauchy and Levy distribution does not exist. But the most concrete way to describe all possible of stable distributions can be expressed through the characteristic function.

Example 1. Normal or Gaussian distributions. , $X \sim N(\mu,\sigma^2)$ if it has a density

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \qquad -\infty < x < \infty.$$

Gaussian distribution is stable with parameters $\alpha = 2$ and $\beta = 0$.

Example 2. Cauchy distributions. $X \sim Caushy(\gamma,\delta)$ if it has density

$$f(x) = \frac{1}{\pi} \frac{\gamma}{\gamma^2 + (x-\delta)^2}, \qquad -\infty < x < \infty.$$

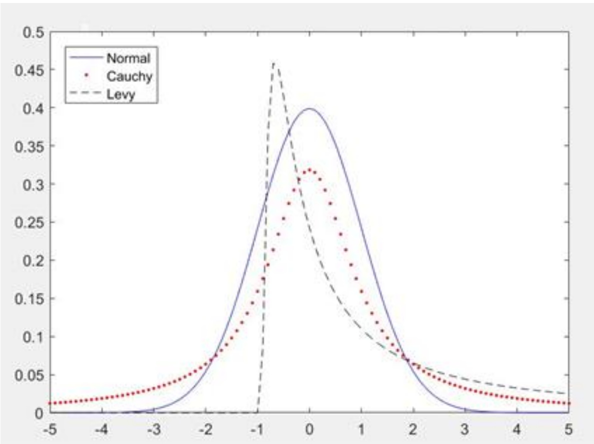
Cauchy distribution is stable with parameters $\alpha = 1$ and $\beta = 0$.

Example 3. Levy distributions. $X \sim Levy(\gamma,\delta)$ if it has density

$$f(x) = \sqrt{\frac{\gamma}{2\pi}} \frac{1}{(x-\delta)^{\frac{3}{2}}} \exp\left(-\frac{\gamma}{2(x-\delta)}\right), \qquad \delta < x < \infty.$$

Levy distribution is stable with parameters $\alpha = 1/2$ and $\beta = 1$.

Figure 1: Graphs of probability density functions of special stable distributions



Normal and Cauchy distributions are symmetric. Main difference between these distributions is that Cauchy distribution has heavier tail than the others. Figure 1 shows a plot of these tree densities. A random variable X is $S(\alpha,\beta,\gamma,\delta;0)$ if

$$X \overset{d}{=} \begin{cases} \gamma\left(Z - \beta\tan\frac{\pi\alpha}{2}\right) + \delta & , \quad \alpha \neq 1 \\ \gamma Z + \delta & , \quad \alpha = 1 \end{cases} \tag{4}$$

where $Z = Z(\alpha,\beta)$ is given by Eq. (3). X has characteristic function

$$E\exp(iuX) = \begin{cases} \exp\left(-\gamma^{\alpha}|u|^{\alpha}\left[1 + i\,\beta\left(\tan\frac{\pi\alpha}{2}\right)(sign\,u)\,(|\gamma u|^{1-\alpha} - 1)\right] + i\delta u\right) & , \ \alpha \neq 1 \\ \exp\left(-\gamma|u|\ \left[1 + i\,\beta\frac{2}{\pi}\ (sign\,u)\log(\gamma|u|)\right] + i\delta u\right) & , \ \alpha = 1. \end{cases} \tag{5}$$

When the distribution is standardized, that is, scale $\gamma = 1$ and location $\delta = 0$, the symbol $S(\alpha,\beta;0)$ will be used as an abbreviation for $S(\alpha,\beta,1,0;0)$ (Nolan, 2016).

A random variable X is $S(\alpha,\beta,\gamma,\delta;1)$ if

$$X \overset{d}{=} \begin{cases} \gamma Z + \delta & , \ \alpha \neq 1 \\ \gamma Z + \left(\delta + \beta\frac{2}{\pi}\ \gamma\ \log\gamma\right) & , \ \alpha = 1 \end{cases} \tag{6}$$

Where $Z = Z(\alpha,\beta)$ is given by Eq. (3). X has characteristic function

$$E\exp(iuX) = \begin{cases} \exp\left(-\gamma^{\alpha}|u|^{\alpha}\left[1 - i\,\beta\left(\tan\frac{\pi\alpha}{2}\right)(sign\,u)\right] + i\delta u\right), & \alpha \neq 1 \\ \exp\left(-\gamma|u|\ \left[1 + i\,\beta\frac{2}{\pi}\ (sign\,u)\log|u|\right] + i\delta u\right) \ , & \alpha = 1 \end{cases} \tag{7}$$

When the distribution is standardized, that is, scale $\gamma = 1$ and location $\delta = 0$, the symbol $S(\alpha,\beta;1)$ will be used as an abbreviation for $S(\alpha,\beta,1,0;1)$ (Nolan, 2016).

2.3. Densities and Distribution Functions

Let $\{X_t : t \geq 0\}$ denote a Levy process. Levy process $\{X_t\}$ is called semistable if there are $a > 1$, $b > 0$ and $c \in R$ such that $\{X_{at}\} \overset{d}{=} \{bX_t + ct\}$ (or $\alpha > 1$ and $b > 0$ such that $\{X_{at}\} \overset{d}{=} \{bX_t\}$). In the case of stable processes, for any nontrivial semi-stable process, the index $\alpha \in (0,2]$ is determined; this is the number such that, for any a and b standing in the relation above, $b = a^{1/\alpha}$. The notion of semi-stable distributions was introduced by Paul Levy in 1937. If $0 < \alpha < 2$, then the class of semi-stable processes is strictly larger than the class of stable processes (Barndorff-Nielsen, O.E., Mikosh,T. and Resnik, S., 2012).

Explicit formulas for general stable densities do not exist. But lots are known about their theoretical properties. The most valid way to describe all possible stable distributions is by way of the characteristic function or Fourier transform. Stable distributions can be characterized by different forms. The complex valued function

$$\phi(u) = E\exp(iuX) \tag{8}$$

is called the characteristic function (c.f.) of a real random variable X. Here, u is some real valued variable.

If the density $f(x)$ exists, Eq.(8) is the Fourier transform of that density and defined by,

$$\phi(u) = E\exp(iuX) = \int_{-\infty}^{\infty}\exp(iux)\ f(x)dx. \tag{9}$$

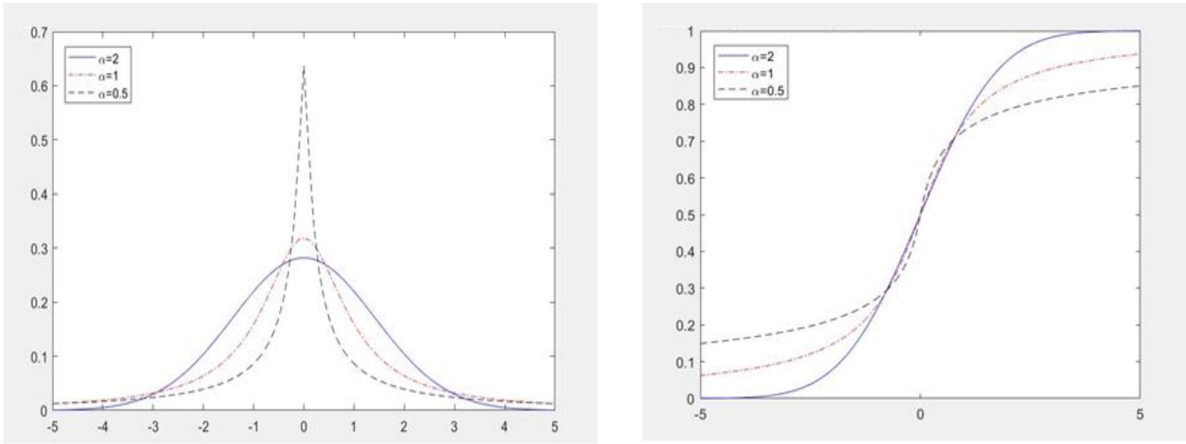
Thus, the inverse Fourier transform

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(-iux)\ \phi(u)\ du \tag{10}$$

allows us reconstruct the probability density function of a distribution from a known characteristic function (Uchaikin & Zolotarev, 1999).

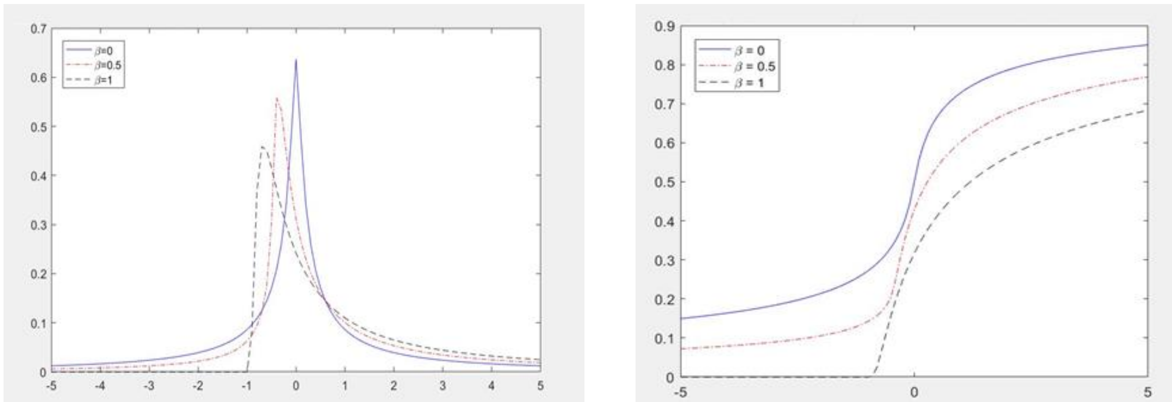
It is necessary to distinguish between probability density function (pdf) and cumulative distribution function (cdf) in different parametrizations. $f(x|\alpha,\beta,\gamma,\delta;k)$ and $F(x|\alpha,\beta,\gamma,\delta;k)$ denote the probability density function and the cumulative distribution function of a $S(\alpha,\beta,\gamma,\delta;k)$ distribution, respectively. Figure 2(a) and 2(b) show plot of densities and cumulative distribution functions for different alpha values. The plots in Figure 3(a) and 3(b) also show the densities and cumulative distribution functions for different beta values.

Figure 2: Stable Densities (a) and Cumulative Distribution Functions (b) for $\alpha = 0.5$, $\alpha = 1$ and $\alpha = 2$.



(a) (b)

Figure 3: Stable Densities (a) and Cumulative Distribution Functions (b) for $\beta = 0$, $\beta = 0.5$ ve $\beta = 1$.



(a) (b)

Density function can be organized as a polynomial function with infinite term. But in this case, the number of terms is infinite causes problems in using the maximum likelihood method. Thus, a standard parametrized integral expression of the density given by (Zolotarev, 1986)

$$f(x|\alpha,\beta,\gamma,\delta) = \frac{1}{\pi\gamma}\int_0^\infty \exp(-u^\alpha)\cos\left(u\left(\frac{x-\delta}{\gamma}\right) - \beta\, u^\alpha \tan\left(\frac{\pi\alpha}{2}\right)\right)du. \tag{11}$$

2.4. Tail Probabilities, Moments and Quantiles

When $\alpha = 2$, the normal distribution has easily understandable asymtotic tail properties. Information about the tails of non-Gaussian ($\alpha < 2$) stable laws will be given. In the $\alpha < 2$ case, the tails of stable distributions are asymtotically fit to Pareto law. Let $X \sim S(\alpha,\beta,\gamma,\delta;0)$ with $0 < \alpha < 2$ and $-1 < \beta \leq 1$. Then as $x \to \infty$,

$$P(X > x) \sim \gamma^\alpha\, c_\alpha\, (1+\beta)x^{-\alpha} \tag{12}$$

$$P(X < -x) \sim \gamma^\alpha\, c_\alpha\, (1-\beta)x^{-\alpha} \tag{13}$$

where $c_\alpha = \sin\left(\frac{\pi\alpha}{2}\right)\Gamma(\alpha)/\pi$. When $\alpha > 1$, the mean of distribution exists and $E(X) = \mu$. In general, a stable random variable has the pth moment if and only if $0 < p < \alpha$, e.g. $E|X|^p < \infty$.

Stable distributed random variables have finite fractional absolute moments only of order less than the stable index, α say. The traditional statistical methods cannot be used effectively in estimating the parameters of a stable distribution, since densities of stable random variables do not exist closed form except for a few special case (Fan, Z., 2006). However, various methods have been proposed for parameter estimation of stable distribution in the literature. The most known of these methods include: Hill estimator (Hill, 1975), quantiles method (Fama and Roll, 1971; McCulloch, 1986), the logarithmic moments method (Kuruoğlu,E., 2001), the empirical characteristics method (Yang, 2012), and the Maximum likelihood method (Nolan, 2001). We will investigate their accuracy in the following. The most commonly used of these method is the quantiles method.

The quantiles method

The quantiles method was pioneered by Fama and Roll (1971) but was much more appreciated through McCulloch (1986) after its extension to include asymmetric distributions and for $\alpha \in [0.6, 2]$ cases unlike the former approach that restricts it to $\alpha \geq 1$ (Kateregga, M., Mataramvura, S. and Taylor, D., 2017). Although the method proposed by Fama and Roll is simpler, it leads to bias in the estimation of α and γ.

Suppose we have n independent drawings x_i from the stable distribution $S(x; \alpha, \beta, \gamma, \delta)$, whose parameters are to be estimated. Let x_p be the pth quantile of population so that $S(x_p; \alpha, \beta, \gamma, \delta) = p$ and $\hat{x}_p$ be the corresponding sample quantile, then $\hat{x}_p$ is a consistent estimator of x_p. In this case, the estimatorss α and β are given by $\hat{\alpha} = \psi_1(\hat{v}_\alpha, \hat{v}_\beta)$ ve $\hat{\beta} = \psi_2(\hat{v}_\alpha, \hat{v}_\beta)$ where

$$\hat{v}_\alpha = \frac{\hat{x}_{0.95} - \hat{x}_{0.05}}{\hat{x}_{0.75} - \hat{x}_{0.25}}, \qquad \hat{v}_\beta = \frac{\hat{x}_{0.95} + \hat{x}_{0.05} - 2\,\hat{x}_{0.05}}{\hat{x}_{0.95} - \hat{x}_{0.05}}.$$

These indexes in independent of both γ and δ. The values of functions $\psi_1(\hat{v}_\alpha, \hat{v}_\beta)$ and $\psi_2(\hat{v}_\alpha, \hat{v}_\beta)$ given in Table III and IV in McCulloch (1986) by linear interpolation. The scale parameter is given by

$$\hat{\gamma} = \frac{\hat{x}_{0.75} - \hat{x}_{0.25}}{\psi_3(\hat{\alpha}, \hat{\beta})}$$

where $\psi_3(\hat{\alpha}, \hat{\beta})$ is given by Table V in McCulloch (1986). A consistent estimator of γ is obtained through interpolation. Finally, the location parameter δ is is estimated through a new parameter defined by

$$\zeta = \begin{cases} \delta + \gamma \tan\left(\frac{\pi\,\alpha}{2}\right), & \alpha \neq 1 \\ \delta, & \alpha = 1. \end{cases}$$

Further, the parameter ζ is estimated by $\hat{\zeta} = \hat{x}_{0.5} + \hat{\gamma}\ \psi_5(\hat{\alpha}, \hat{\beta})$ where is obtained from Table VII (McCulloch, 1986) by linear interpolation. The consistent estimator of location parameter is given by

$$\hat{\delta} = \hat{\zeta} - \hat{\beta}\ \hat{\gamma}\ \tan\left(\frac{\pi\,\alpha}{2}\right).$$

2.5. Simulation of Stable Variables

Difficulties of simulating sequence of stable results from the fact that analytic expressions for the inverse F^{-1} of the cumulative distribution function do not exist. However, the algorithm for constructing a standard stable random variable $X \sim S_\alpha(1, \beta, 0)$ is the following (Weron, 1996):

- Generate a random variable *V* uniformly distributed on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and an independent exponential random variable *W* with mean 1.
- For $\alpha \neq 1$ compute,

$$X = S_{\alpha,\beta}\,\frac{\sin\left(\alpha\left(V + B_{\alpha,\beta}\right)\right)}{(\cos(V))^{\frac{1}{\alpha}}}\left[\frac{\cos\left(V - \alpha\left(V + B_{\alpha,\beta}\right)\right)}{W}\right]^{(1-\alpha)/\alpha} \tag{14}$$

where

$$B_{\alpha,\beta} = \frac{\arctan\left(\beta \tan\frac{\pi\alpha}{2}\right)}{\alpha},$$

$$S_{\alpha,\beta} = \left(1 + \beta^2 \tan^2\left(\frac{\pi\alpha}{2}\right)\right)^{\frac{1}{2\alpha}}.$$

- For $\alpha = 1$ compute,

$$X = \frac{2}{\pi}\left(\left(\frac{\pi}{2} + \beta V\right)\tan V - \beta \ln\left(\frac{\frac{\pi}{2} W \cos V}{\frac{\pi}{2} + \beta V}\right)\right) \quad (15)$$

Then, given the formulas for simulation of a standard stable random variable, we can easily simulate a stable random variable for all admissible values of the parameters α, β, γ and δ using the following property: if $X \sim S_\alpha(1, \beta, 0)$, then

$$Y = \begin{cases} \gamma X + \delta & , \quad \alpha \neq 1 \\ \gamma X + \frac{2}{\pi} \beta \gamma \ln \gamma + \delta \ , & \quad \alpha = 1 \end{cases} \quad (16)$$

is $S(\alpha, \beta, \gamma, \delta; k)$ (Borak, Hardle and Weron, 2005).

3. THE APPLICATION OF STABLE LAWS

In this part of the study, daily trading volumes between 01.01.2007-28.09.2017 (2804 days) of XTRZM index traded in BIST were used. Data set used in this study was obtained from the Bloomberg database. Analysed tourism companies in this study are traded in the BIST with the codes "AVTUR, MAALT, MARTI, METUR, NTTUR, TEKTU, UTPYA".

In this study, we worked with daily returns rather than daily trading volume data because it provides the ability to measure investment performance independently of the scale used. Let Y_t be daily trading volume of index and S_t be the value of a financial asset or portfolio at time t. Then, return values for $[t, t+1]$ is given by (Önalan, Ö., 2010):

$$X_{t+1} = S_{t+1} - S_t = \ln Y_{t+1} - \ln Y_t.$$

Figure 4 show the daily returns above stated period of XTRZM indexes. Table 1 also show descriptive statistics of daily returns related to stock.

Figure 4: Graph of Daily Returns of XTRZM Index for the Period 01.01.2007-28.09.2017

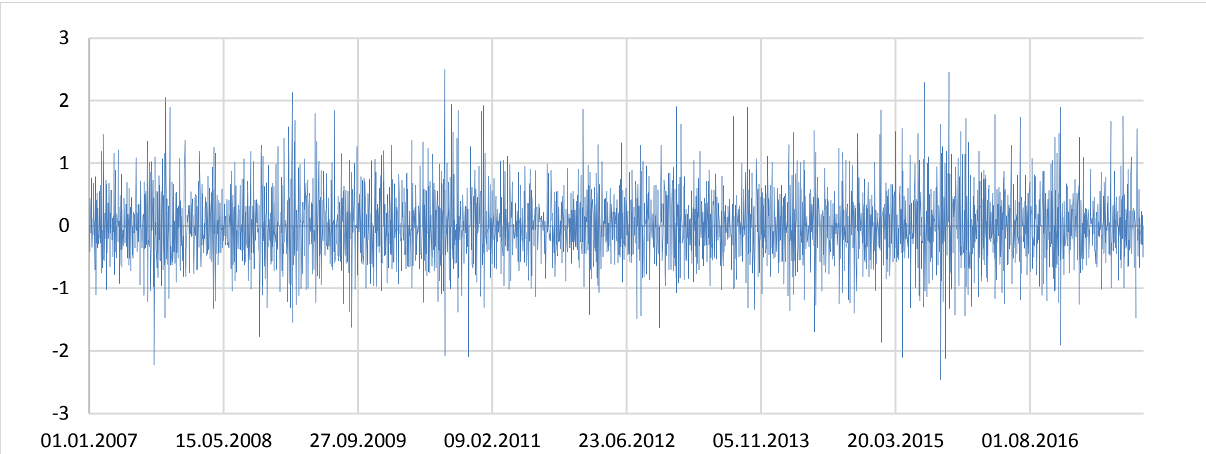


Table 1: Descriptive Statistics Related to XTRZM Index's Daily Returns

	Mean	Std. deviation	Skewness	Kurtosis	Min	Max
XTRZM	0.0003434	0.567	0.233	1.182	-2.462	2.498

Firstly, BIST Tourism index's daily return is considered and the suitability of these returns for normal distribution is examined. Chi-square test (goodness of fit) is used to perform to determine if normal distribution of observed data was appropriate. First of all, the time series of XTRZM index is organised as a frequency series. Then, Chi-square value obtained from observed values and the calculated values is found as $\chi^2_{cal} = 2802{,}92$. Hypotheses about the distribution of daily returns are given

H_0 : The distribution of daily returns is suitable for normal distribution,

H_1: The distribution of daily returns is not suitable for normal distribution.

Degree of freedom is equal to 6, since number of class is equal to 9 and parameter number of estimated is equal to 2. Then, $\chi^2_{0.05,6} = 12{,}592$. In this case, H_0 is rejected. Rejecting the assumption of normal distribution of daily returns supports the hypothesis that a stable distribution is suitable for this data. In addition to these information, Figure 6 show a normal Q-Q plot for the daily returns.

Figure 5: Histogram for Daily Returns of XTRZM Index

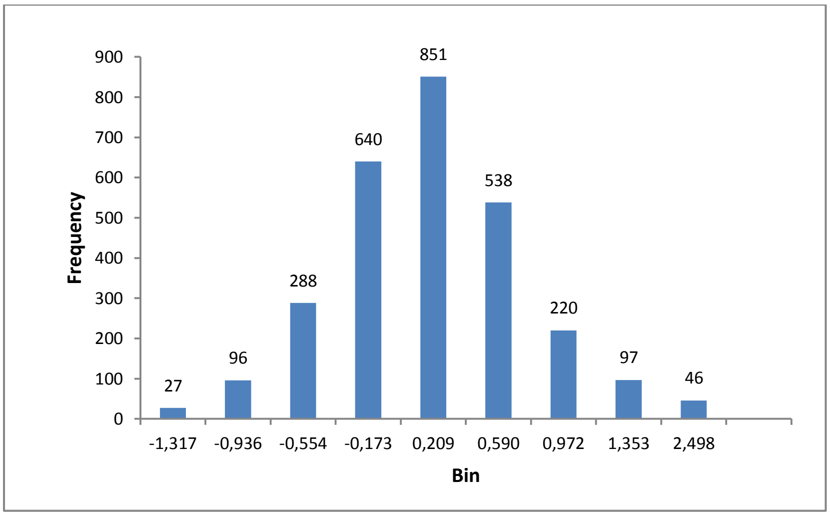
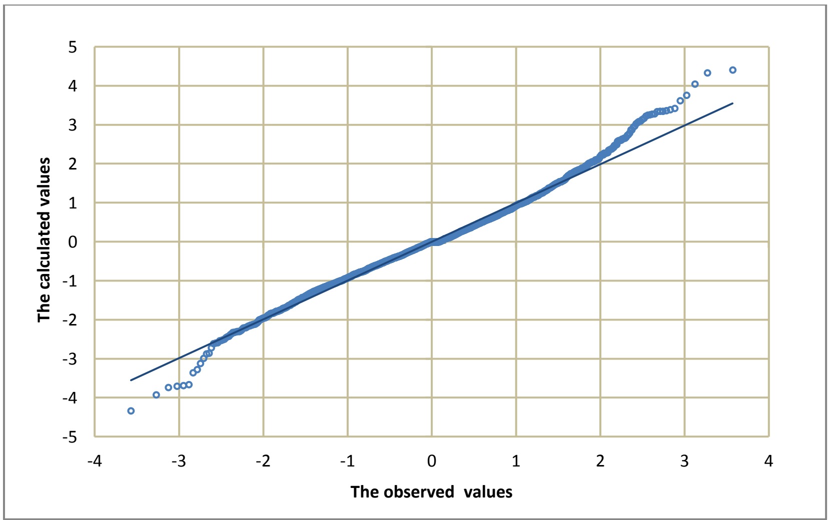


Figure 6: A Normal Q-Q Plot for the Daily Returns



Stable distribution parameters for the log-returns data are estimated according to McCulloch method and obtained the stable parameters in Table 2. $\hat{\alpha} = 1.21$ is a value in the preferred range for stable distribution.

Table 2: Stable Distribution Parameters Estimated from the Log-Returns Data

Index\ Parameter	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$
XTRZM	1.21	1	0.0006	0.0038

Figure 7: Comparison of Standardized Actual Returns and Simulated Standard Stable Returns

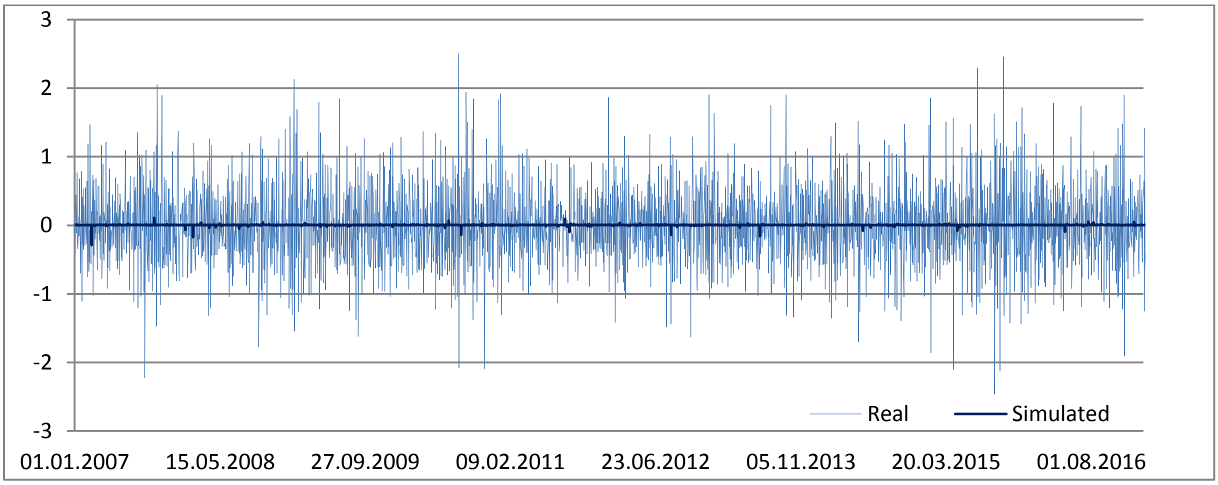


Figure 8 shows the stable density function obtained by using the MATLAB STBL command proposed by Veillette (2014) according to estimated parameters for $\hat{\alpha} = 1.21$, $\hat{\beta} = 1$, $\hat{\gamma} = 0.0006$ and $\hat{\delta} = 0.0038$. Cumulative distribution function is also given Figure 9. Here, ıt should also be noted that Figure 8 shows a large similarity to the graph of the Levy distribution shown in Fig.1.

Figure 8: The Stable Density Function for $\boldsymbol{\alpha = 1.21}$, $\boldsymbol{\beta = 1}$, $\boldsymbol{\gamma = 0.0006}$ and $\boldsymbol{\delta = 0.0038}$.

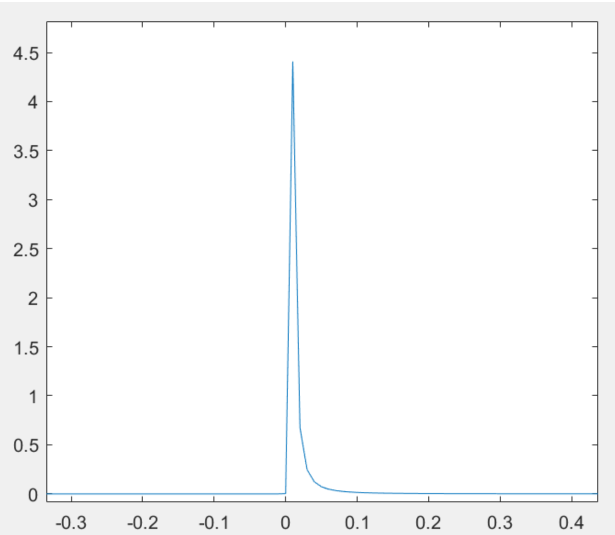
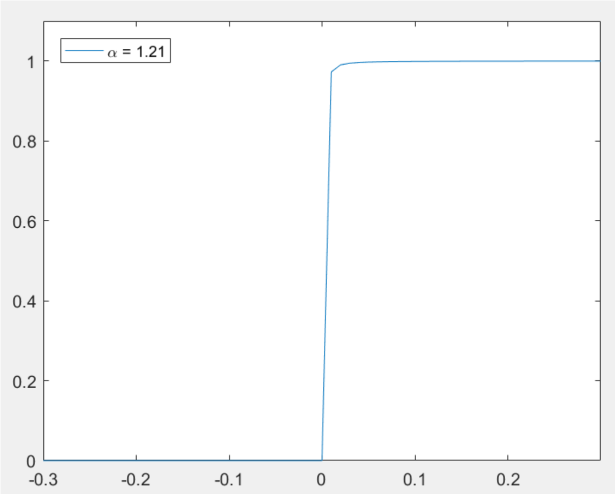


Figure 9: The Stable Cumulative Distribution Function $\alpha = 1.21$, $\beta = 1$, $\gamma = 0.0006$ and $\delta = 0.0038$.



4. CONCLUSION

In this study, it is investigated whether the trading volume data of XTRZM index traded in BIST are suitable for stable distributions. The XTRZM index traded in BIST is examined and the suitability of these volumes for normal distribution is searched. By means of the Chi-square test and graphs, it was seen that normal distribution was not suitable for these volumes. Using the 2804 daily trading volumes data, the parameters of stable distribution are estimated as $\alpha = 1.21$, $\beta = 1$, $\gamma = 0.0006$ and $\delta = 0.0038$. As a result, estimated parameter values show that Stable Paretian distributions can be used as an appropriate model for modelling trading volume data. It has been concluded that it is more appropriate to use the scale parameter of the stable distribution instead of the standard deviation as the risk measure.

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