



Is Cabri 3D Effective for the Teaching of Special Planes in Analytic Geometry?

Erhan Ertekin¹

¹Necmettin Erbakan University, Ahmet Keleşoğlu Education Faculty, Konya, TURKEY

ABSTRACT

This study aims to investigate the effects of teaching Analytical Geometry using the software Cabri 3D on teacher trainees' ability to write the equation of a given special plane, to identify the normal vector of the plane, to draw its graph and to write the equation of a plane presented graphically. The software was used in order to allow students to visually observe the geometric meanings of the variables on the plane equation and to improve students' geometric as well as algebraic comprehension. The research was conducted during the spring term of the 2008-2009 academic year with the participation of 78 students registered at Primary School Mathematics Education Department of a state university in Turkey. The experimental and control groups consisted of 26 and 52 students, respectively. The researcher ran the study, which took place over four lessons. Data collection tool was developed by the researcher and included 14 test items. Six questions tested the ability to write the equation of a given special plane, to identify the normal vector of this plane and to draw its graph. Eight questions asked students to identify the equations of the planes presented graphically. The results indicated that students in the experimental group were significantly more successful than the ones in the control group in terms of identifying the equations of the special planes and their normal vectors and drawing their graphs.

Keywords:

Cabri 3D, special plane, visualization

© 2014 IJESIM. All rights reserved

Article History:

Received 28.02.2014 Received in revised form 15.04.2014 Accepted 13.05.2014 Available online 01.06.2014

Introduction

Geometry is a branch of mathematics which studies point, line, plane, planar figures, space, spatial figures and the relationships between these as well as the measures of geometric figures such as length, angle, area and volume (Baykul, 2002) and which has several sub-themes. One of these themes is analytic geometry, which studies the concepts such as point, line, plane, curve and area algebraically (Sezginman & Abacı, 1985). Descartes and others united algebra and geometry, and since their time analysis proper and geometry have continued their development hand in hand. In fact, analytic geometry may be defined as the theory of analysis geometrically interpreted (Young, 1909). Analytic Geometry can be conceptually studied in two main categories of planar and spatial analytic geometry. Students' three-dimensional thinking skills are fore-grounded in spatial analytic geometry and as evidenced by experience many secondary school students face imagination difficulties in analytic geometry questions of three-dimensional space (Schumann,

¹ Corresponding author's address: Necmettin Erbakan University, Ahmet Keleşoğlu Education Faculty, Konya, TURKEY

Telephone:

Fax:

e-mail: eertekin@konya.edu.tr

<http://dx.doi.org/10.17278/ijesim.2014.01.003>

2003). Likewise, it is possible to say that university students also find analytic geometry difficult; especially more so when three-dimensional thinking skills are required.

Three-dimensional thinking has special significance in spatial reasoning. Three-dimensional spatial skills involve the ability to interpret the figurative knowledge of three-dimensional objects and to visualise these objects (McClintock, Jiang and July, 2002). According to Bishop (1983, p.184) IFI (*interpreting figural information*) refers to “understanding the visual representations and spatial vocabulary used in geometric work, graphs, charts, and diagrams of all types whereas the more dynamic VP (visual processing) “involves visualization and the translation of abstract relationships and nonfigural information into visual terms as well as “the manipulation and transformation of visual representations and visual imagery (cited in Presmeg, 2008). Visual processing is related to different visual representations of mathematical concepts and the transitions among these. Moreover, transitions among different representations of mathematical concepts are important in learning mathematics and problem solving (Lesh, Post, & Behr, 1987). These reveal the importance of spatial reasoning in learning mathematics and problem solving.

Spatial reasoning and geometry are closely related due to the significance of three-dimensional thinking for spatial reasoning and most mathematics educators seem to include spatial reasoning as part of the geometry curriculum (Clements & Battista, 1992). Research (such as Ünal, Jakubowski & Corey, 2009) indicates that students with low spatial skills experience more difficulties in learning geometry. Hence, this underscores the role of three-dimensional thinking skills in learning geometry.

Although most objects around us are three-dimensional, a two-dimensional representation of a three-dimensional image causes several problems. In education, most of us deal with representations of three-dimensional structures on two-dimensional surfaces such as on the board or on paper. Yet, it is important to note that a significant number of students have problems in three-dimensional drawings and visualising three-dimensional structures (Thalman, 1983). Thus, Delice et al. (2009) identified class teacher trainees’ difficulties in drawing the open forms of three-dimensional basic geometric objects based on their closed representations and likewise, drawing the closed representations of these objects based on their open representations. Drawings of a three-dimensional object on a two dimensional flat paper are either incomplete and cause optical illusions and different perceptions or it is impossible to see the configurations of objects from different angles in a single drawing due to the static environment, even if they are perfect drawings (Baki, Kösa & Karakuş, 2008). Similarly, Schumann (2003) stated that drawings on the board and on hand-made transparencies do not provide opportunities to develop sufficient spatial-geometric comprehension of spatial analytic geometry questions. Therefore, the accessibility of tools, which offer visualisation opportunities, is essential. These opportunities would, in particular, allow students to see three-dimensional structures from different angles, open it when they want and to rotate it.

Mathematics is a subject that has diagrams, tables, spatial arrangements of signifiers such as symbols, and other inscriptions as essential components (Presmeg, 2006) and thus during the recent years visualisation has become widely accepted in doing and learning mathematics. Visualization is taken to include processes of constructing and transforming both visual mental imagery and all of the inscriptions of a spatial nature that may be implicated in doing mathematics (Presmeg, 1997b). On the other hand, visualisation is a key component not only of relational purposes but also of reasoning, problem solving and even proving (Arcavi, 2003). Usiskin (1987) stated that one of the four components of geometry is visualization, drawing and construction of figures cited in (Clements & Battista, 1992).

Tools to encourage the use of visualization and the deployment of a range of forms of representation are becoming widely available mainly via the use of computers. Some understanding of their educational significance and mode of operation is gradually emerging (Gilbert, 2005). Moreover, the availability of computers and graphical calculators which can be used as tools for visualisation and students’ use of many mathematical concepts in an interactive environment which foregrounds their experiences highlighted the use of these tools in mathematics education. Cabri2D-3D and GeoGebra are some of the dynamic geometry software products that can be used in geometry education. Dynamic geometry software products, which have been increasing in number and quality during the recent years, are being used in mathematics education as a significant tool to enable students to discover mathematical concepts and the relationships among these concepts, as well as to explain and model the concepts. Dynamic geometry software products

aim to develop visualisation, discovery and mathematical ideas. These products contribute to learning not only via visualisation but also by allowing students experience (Köse & Özdaş, 2009). Several studies revealed that students' difficulties in geometry could be overcome and students can achieve higher levels of comprehension of geometric concepts by the use of these products (Tutak & Birgin, 2008; Güven & Karataş, 2002, Hollebrands, 2003). Hence, the use of such software products in the classroom is important and teachers have crucial responsibility in their application. However, it might be difficult for in-service teachers to know about and implement these programmes on their own. That is why education faculties, which train teachers, should provide relevant education to the teachers who wish to support teaching with computers (Baki, 1996).

Among dynamic geometry software products, Cabri is perhaps the most widely researched product. Cabri Geometry is the first dynamic geometry software, and like other dynamic geometry software, is a micro-world that allows exploration of several ways to the solution of a problem and discovery of concepts and relationships. The 3D version of Cabri could facilitate, in particular, the teaching of three-dimensional concepts of geometry. Cabri Geometry 3D is the first three-dimensional dynamic geometry system which even in version 1.0 meets many of the requirements of a tool for three-dimensional synthetic geometry for dynamic treatment, similar to the options offered by 2D dynamic geometry systems (Laborde & Bainville, 2004). Accascina & Rogaro (2006), explained the existence of subsidiary tools such as models, concrete tools and diagrams which can assist proper construction of concept images of three-dimensional objects and stated that the software Cabri 3D is a potentially important tool for the development of visual teaching of solid geometry.

Cabri 3D allows the user to construct and manipulate solid geometry objects in three dimensions via a 2D interface. By using Cabri 3D, three-dimensional objects such as prism, pyramids, cylinder and cone can be constructed, rotated and seen from a certain aspect on the screen and also prisms can be opened on the screen. Prisms and half plane can be intersected and thus, new three-dimensional objects may be formed. It is a practical tool for solid analytic geometry, too. Lines, vectors, planes and conics can be constructed on the screen and seen from different viewpoints by rotating screen with a simple dragging. Vectorial operations such as vector sum, cross product and dot product can be performed. Coordinates of a point or vector, equations of a line, plane or sphere in space can be represented on the screen (Kösa & Karakuş, 2010).

In education faculties, one of the courses where the aforementioned software products could be used is analytic geometry. These software products are potentially supportive tools for teacher trainees both to learn the programme and to construct, shift and rotate relevant concepts in an interactive environment, as well as to overcome difficulties in observing equations of the concepts algebraically. One of the concepts which teacher trainees find challenging in analytic geometry and the difficulties in which, as the authors believe, could be overcome by technological support is special plane equations. It is possible to construct this type of planes and observe their equations using Cabri 3D.

Although the concept of planes is essentially a two-dimensional concept, its equation can be algebraically identified in relation to the components of its normal vector in three-dimensional space. The concept of planes can also require high spatial skills for high level problems about the concept, except when the components of its normal vector are provided operationally and its equation is asked for. The general plane equation is $ax+by+cz+d=0$ and $(a,b,c,d \in \mathfrak{R})$, the coefficients of a , b and c are components of the normal vector of the plane. When a , b , c and d have the following values, these planes are called special planes;

- if $d \neq 0$ and only one of the coefficients of a , b , c is zero, then the plane is parallel to one of the axes,
- if $d \neq 0$ and any two of the coefficients of a , b , c are zero, then the plane is parallel to one of the coordinate axes,
- if $d=0$ and one of the coefficients of a , b , c are zero, then the plane passes through one of coordinate axes,
- if $d=0$ and all of the coefficients of a , b , c are different than zero, then the plane passes through the origin but does not contain any of the coordinate axes (Sezginman & Abacı, 1985).

Considering the third item, if the component of the normal vector is $(1,1,0)$, then the plane equation would be $x+y=0$, which is nothing but the line equation in two-dimensional space. Yet, during one of the mathematics education courses I teach, when I asked teacher trainees whether this equation could be a plane equation, their reply was "it is not a plane equation". This response and their doubtful looks made me think that teaching these concepts only algebraically perhaps develops a unidirectional perspective and that students know only certain operational information about the concepts.

Conceptually, changing these coefficients means rotating the plane in space and changing the constant d means that the plane is shifted in space. These two facts, in addition to three-dimensional thinking, are crucial in writing the equation of a plane in general form that is presented graphically or in drawing a plane given as equation; two objectives which are the topics of this study. In the traditional classroom, students may find it hard to visualise the changes that would occur on a plane when given coefficients and the constant are altered. Hence, Kösa et al. (2008) stated that space geometry is difficult to teach by traditional tools such as pen and paper in the traditional classroom and those procedures carried out using three-dimensional static diagrams drawn on plane would both complicate learners' perception of the relationships between the objects and decrease their interest in the topic.

There are few studies in the area and based on the reasons stated above, the present study aims to investigate the effects of teaching Analytic Geometry using the software Cabri 3D on Primary Mathematics teacher trainees' ability to write the equation of a given special plane, to identify the normal vector of this plane, draw its graph and to write the equation of a plane presented graphically.

Method

Research Design

This research has an experimental design, because it aims to investigate the effects of using a dynamic geometry software product on Primary Mathematics teacher trainees' ability to write the equation of a given special plane, to identify the normal vector of this plane, draw its graph and to write the equation of a plane presented graphically. Teacher trainees were not given a pre-test at the beginning of the study, thus it was a controlled post-test experimental design. In grouping the teacher trainees into experimental and control groups at the beginning of the research, their mid-term Analytic Geometry scores were used as the basis to form equivalent groups. Three classes in Year 3 registered at Selçuk University Ahmet Keleşoğlu Education Faculty Primary mathematics education were compared and the analysis indicated that the classes were similar in terms of success. Thus, two of these classes were assigned as the control group and the third one as the experimental group.

The study was conducted in the second term of 2008-2009 academic year during the course of four lessons. The lessons were taught by the researchers in the experimental group in a technologically supported classroom, while in the control groups they were taught by the tutors of the course using traditional teaching methods. Traditional teaching methods here mean tutors' provision of the definition and properties of the concepts and solving example problems. In the experimental group, for the teaching of the special plane equations, Cabri 3D programme was dynamically used only for visualisation purposes benefiting from the opportunities provided by Cabri 3D such as dragging and rotating the geometric object.

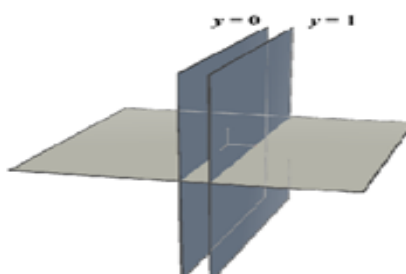
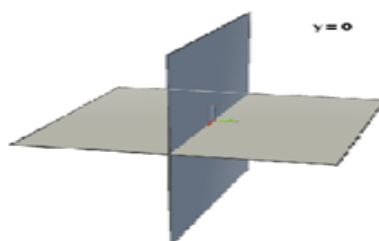
Procedures

Teacher trainees were taught the theme of special plane equations on the eleventh week of the second term of 2008-2009 academic year by projecting the images obtained from Cabri 3D on data show during the course of four hours under researchers' control highlighting visualisation opportunities of the programme. Teaching was based on constructivists principles where teacher trainees were given activity sheets to write down plane equations and draw the planes supported by in-class discussions. These procedures are presented in Table 1.

Table 1. Course programme

Course Content	
1st lesson	Introduction to the programme Cabri 3D.
2nd lesson	Provision of the general formula of the plane equation and using the software Cabri 3D, constructing and rotating various planes and allowing teacher trainees discover how changes in coefficients of a , b , c and d of the equation $ax+by+cz+d=0$ affects the position of the plane in space and related guesswork.
3rd lesson	Visualising the position of a plane in space when $d \neq 0$, coefficient a is zero and coefficients b and c are different than zero; asking teacher trainees to draw the images of a plane in separate cases where coefficient b and coefficient c are zero; checking their drawings using Cabri 3D. Visualising the position of a plane in space when $d \neq 0$, and coefficients a and b are zero; asking teacher trainees to draw the images of a plane when coefficients a , c and b , c are zero; checking their drawings using Cabri 3D.
4th lesson	Visualising the position of a plane in space when $d=0$, and only one of the coefficients a , b , c is zero; asking teacher trainees to draw the images of the plane in separate cases where coefficient b and coefficient c are zero; checking their drawings using Cabri 3D. Asking teacher trainees to draw the image of a plane when $d=0$ and each of the coefficients of a , b , c are different than zero; checking the drawings using Cabri 3D following a whole class discussion.

It is important to note here that during the class activities it was not possible to input the algebraic equation of the plane into the software Cabri 3D and obtain its image. Hence, the researcher drew appropriate planes by using the programme and then showed the algebraic equations of the planes in Cabri. An example of these steps is as follows.



validity of the test, two tutors who had taught Analytic Geometry previously were consulted. They stated that the test items were able to measure the identified objectives.

Data Analysis

Independent samples t-tests were run in order to test the equivalence of the experimental and control groups and to test the significance of the difference between the post-test scores in terms of being able to write the identified plane equation, to identify the normal vector of the same plane, to draw the plane and to write the equation of the given plane.

Results and Findings

In this section, data analysis results for writing the given special plane equation, identifying the normal vector of the plane, drawing the graph of the plane and writing the equation of the plane presented graphically are presented and interpreted respectively.

Writing the Equation of a Given Plane

Independent samples t-test results conducted at the beginning of the study in order to test the equivalence of experimental and control groups in terms of students' mid-term exam results are presented in Table 2.

Table 2. t-test results for the mean scores of the experimental and control groups prior to the study

		\bar{x}	Sd	t	p
Experimental	6	68.85	27.25	0,519	0.605
Control	2	65.19	30.29		

According to the results presented in Table 2, mid-term exam results of the experimental and control groups were 68.85 and 65.19 respectively. The significance of the difference between the mean scores was tested using an independent samples t-test. The t value of 0.519 was not significant at 0,05 level. This suggested that at the beginning of the study, the experimental and control groups were not significantly different in terms of success, in other words the groups were equivalent.

Another independent samples t-test was conducted in order to identify whether there was a significant difference in terms of mean scores between the teacher trainees in the experimental group and the teacher trainees in the control group for writing the equation of a given special plane, identifying the normal vector of the same plane, drawing its graph and writing the equation of a plane presented graphically. The t-test results are given in table 3.

Table 3. t-test results for the mean score differences of the experimental and control groups

			\bar{x}	Sd	t	p
Writing the Equation of the Given Special Plane	Experimental	6	3.23	2.30	3.3	0.01
	Control	2	1.75	1.60		
Identifying the Normal Vector of the Special Plane	Experimental	6	3.23	2.25	3.8	0.00
	Control	2	1.48	1.70		
Drawing the Graph of the Special Plane	Experimental	6	3.50	1.80	2.1	0.04
	Control	2	2.62	1.76		
Writing the Equation of the Special Plane Presented Graphically	Experimental	6	2.81	2.71	3.2	0.00
	Control	2	1.12	1.93		

As presented in Table 3, for writing the equation of a given special plane, the mean score of the experimental group students was 3.23, and of the control group students was 1.75; for identifying the normal

vector of the given special plane, the mean score of the experimental group students was 3.23, and of the control group students was 1.48; for drawing the graph of the given special plane, the mean score of the experimental group students was 3.50, and of the control group students was 2.65; for writing the equation of a special plane presented graphically, the mean score of the experimental group students was 2.81, and of the control group students was 1.12. Independent groups t-test results indicate significant differences between the mean scores of the experimental and control groups at level 0,05 for all four dimensions. Given that the items that measured the objectives of drawing the graph of a given special plane and writing the equation of a special plane presented graphically were items that involved transition between the algebraic form to the geometric form and vice versa, the software Cabri 3D could be argued to create a significant difference for the experimental group in terms of transition between different representations of mathematical knowledge. In other words, the software Cabri 3D made a difference for the experimental group students in interpreting figural knowledge (IFI) and visual processing (VP).

The students in the experimental group obtained the same mean scores ($\bar{x}=3.23$) in writing the equation of a given special plane and in identifying the normal vector of the special plane. However, these mean scores of the students in the control group were 1.75 in writing the equation and 1.48 in identifying the normal vector. This could be an indication that the students in the control group were inclined to memorise special plane equations and did not possess the conceptual knowledge, because writing the equation of the special plane is closely related to the identification of the components of the normal vector first. Therefore, a lower mean score of the control group students in identifying the vector compared to their mean scores in writing the equation clearly supports our argument. In other words, teacher trainees in the control group were significantly not able to interpret the variables of the general equation of the plane compared with the teacher trainees in the experimental group.

This is an indication that the use of Cabri 3D dynamic geometry software is more effective for the teaching of special plane equations in the analytic geometry course compared with the traditional teaching method.

Discussion and Conclusion

Some research in the literature indicates that the software Cabri 3D supports visualisation (Güven & Kösa, 2008; Kösa & Karakuş, 2010) and is beneficial in assisting learners' construction of concept images of geometric concepts.

The findings of this research indicated that computer assisted teaching using dynamic geometry software with the experimental group students was more effective than the traditional teaching with the control group in writing the equation of the given special planes, identifying the normal vector of the plane, drawing its graph and in writing the equation of the special planes presented graphically. This could be accepted as an indication that owing to the visualisation opportunities provided by the Cabri 3D dynamic geometry software used in the study, teacher trainees' conceptual images for special planes improved and thus, they were able to imagine the plane in three-dimensional space better. This finding is in line with the study of Accassina & Rogaro (2006) who found that Cabri 3D was beneficial in learners' construction of the concept images of geometric concepts.

The results of this study indicate that Cabri 3D dynamic geometry software is a potentially important tool for the improvement of the visual teaching of three-dimensional geometric concepts (Accassina & Rogaro, 2006). As cited in Christou et al. (2007), Presmeg (1986) and Harel & Sowder (1998) argued that dynamic visualisation could be a powerful tool in better comprehension and acquisition of many mathematical concepts. On the other hand, in the recent years several concerns have been raised on the limitations of the traditional approach in the learning and teaching of geometry, and in the learning and teaching of spatial skills in particular (Christou et al. 2007). The performance of the control group students in the present study clearly represents this concern.

Moreover, the findings of this research also indicated that when special plane equations were taught with the traditional teaching method, teacher trainees were inclined to memorise the concept; while computer supported teaching with the software Cabri 3D contributed to learners' conceptual learning. In fact, the identification of special plane equations, as mentioned previously, is closely related to the identification of the components of the normal vector of the plane, which could be accepted as conceptual knowledge. Moreover, the identification of the components of the normal vector requires three-dimensional thinking skills. Hence, the fact that the mean scores of the experimental group students in identifying the normal vector and the plane equation were equal and were also higher than those of the control group is an indication of consistency and suggests conceptual learning. Yet, the fact that the mean score of the control group students in identifying the equation of the special plane was higher than their mean score in identifying the normal vector of the plane is an indication of inconsistency and implies rote learning of the identification of special plane equations. This signifies the contribution of the software Cabri 3D to three-dimensional thinking skills, and in more general terms to spatial skills. In conclusion, the findings obtained in relation to the use of the software Cabri 3D in the teaching of special planes as part of the space analytic geometry course are in parallel with the findings of Kösa & Karakuş (2010) and both studies indicate benefits of the software Cabri 3D for that specific theme of the course.

Given these conclusions, we suggest the use of the software Cabri 3D for the teaching of special plane equations. Further research can investigate whether similar results would be obtained with the inclusion of high school students in the sample. Moreover, studies on the effectiveness of the software Cabri 3D in other themes of the space analytic geometry course would contribute significantly to mathematics education at university and high school levels. Specifically, future studies could investigate how high school students' success in solving equation systems would change following the visualisation of the relative positions of the planes assisted by the software Cabri 3D.

References

- Accassina, G.& Rogaro, E. (2006). Using cabri 3D diagrams for teaching geometry. *The International Journal for Technology in Math Education*, 13 (1), 11-22.
- Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, 52, 215-241.
- Baki, A. (1996). Matematik öğretiminde bilgisayar her şey midir? *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 12: 135-143.
- Baki, A. Kösa, T.& Karakuş, F. (2008). Uzay geometri öğretiminde 3D dinamik geometri yazılımı kullanımı: Öğretmen Görüşleri. *8th International Education Technology Conference, Anadolu University*, 6-8 May, Eskişehir.
- Baykul, Y. (2002). İlköğretimde matematik öğretimi (6.-8. sınıflar için) Ankara: PegemA Yayıncılık.
- Christou, C., Jones, K., Pitta-Pantazi, D., Pittalis, M., Mousoulides, N., Matos, J.F., Sendova, E., Zachariades, T., & Boytchev, P. (2007). Developing student spatial ability with 3D software applications. Paper presented at the *5th Congress of the European Society for Research in Mathematics Education (CERME)*, Larnaca, Cyprus.
- Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420-464). New York: Macmillan.
- Delice, A., Ertekin, E., Yazici, E. & Aydın, E. (2009). Preservice primary teachers' three dimensional thinking skills. *Procedia-Social and Behavioral Sciences*, 1 (1), 2666-2672.
- Gilbert, J. K. (2005). Visualization: A Metacognitive skill in science and science education. In John K. Gilbert (Ed.). *Visualization in Science Education*, Springer.
- Güven, B. & Karataş, İ. (2002). Dinamik geometri yazılımı cabri ile keşfederek öğrenme: Öğrenci görüşleri, *Turkish Online Journal of Educational Technology*, 2 (2), 67-78.
- Güven, B. & Kösa, T. (2008). The effect of geometry software on student mathematics teachers' spatial visualization skills. *The Turkish Online Journal of Educational Technology*, 7 (4), 100-107.
- Hollebrands, K.F. (2003). High school students understanding of geometric transformations in the context of a technological environments. *Journal of Mathematical Behavior*, 22, 55-72.

- Kösa, T., Karakuş, F. & Çakıroğlu, Ü. (2008). Uzay geometri öğretimi için üç boyutlu dinamik geometri yazılımı kullanarak çalışma yapraklarının geliştirilmesi. International Educational Technology Conference (IETC), Anadolu University, 6-8 May 2008, Eskişehir.
- Kösa, T. & Karakuş, F. (2010). Using dynamic geometry software Cabri 3D for teaching analytic geometry. *Procedia Social and Behavioral Sciences*, 2, 1385-1389.
- Köse, N. Y. & Özdaş, A. (2009). İlköğretim 5. sınıf öğrencileri geometrik şekillerdeki simetri doğrularını cabri geometri yazılımı yardımıyla nasıl belirliyorlar? *İlköğretim Online*, 8 (1), 159-175. [Online]: <http://ilkogretim-online.org.tr>
- Laborde, J. M.; Bainville, E. (2004). *Cabri Géomètre 3D* (Version 1.0). Grenoble: Cabrilog (www.cabri.com).
- Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.) *Problems of representation in the teaching and learning of mathematics*. Hillsdale, NJ: Erlbaum.
- McClintock, E., Jiang, Z. & July, R. (2002). Students' development of three-dimensional visualization in the geometer's sketchpad environment. *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the psychology of mathematics education*.
- Presmeg, N. C. (1997). Generalization using imagery in mathematics. In L. D. English (Ed.), *Mathematical reasoning: Analogies, metaphors and images* (pp. 299-312). Mahwah, NJ: Erlbaum.
- Presmeg, N. (2006). Research on visualization in learning and teaching math. In A. Gutierrez, & P. Boero (Eds.). *Handbook of Research on the Psychology of Math. Education: Past, present and future*, 205-235, Sense Publishers.
- Presmeg, N. (2008). Spatial abilities research as a foundation for visualization in teaching and learning mathematics. In P. Clarkson & N. Presmeg (eds.), *Critical Issues in Mathematics Education*, Springer, New York.
- Schumann, H. (2003). Computer aided treatment of 3d- problems in analytic geometry. *ZDM*, 35 (1), 7-13.
- Sezginman, İ. & Abacı, M. (1985). *Uzay Analitik Geometri: Teori ve problemleri*. Yıldız Üniversitesi Yayınları, İstanbul.
- Thalman, D. (1983). Computer-aided teaching of three-dimensional geometry. *International Journal of Mathematical Education in Science and Technology*, 14 (2), 129-135.
- Tutak, T. & Birgin, O. (2008). Dinamik geometri yazılımı ile geometri öğretiminin öğrencilerin Van Heile geometrik anlama düzeylerine etkisi. International Educational Technology Conference (IETC), Anadolu University, 6-8 May 2008, Eskişehir.
- Ünal, H., Jakubowski, E. & Corey, D. (2009). Differences in learning geometry among high and low spatial ability pre-service mathematics teachers. *International Journal of Mathematical Education in Science and Technology*, 40 (8), 997-1012.
- Young, A. E. (1909). On the teaching of analytic geometry. *The American Mathematical Monthly*, 16 (12), 205-212.