Real-time Robust Control Using Digital Signal Processor

T. Slavov, L. Mollov, J. Kralev, P. Petkov

Abstract—**A laboratory system for real-time robust control of multivariable analogue 4th order two input/two output plant is presented. The system consists of Spectrum Digital eZdspTMF28335 development kit with built in Texas Instruments TMS320F28335 Digital Signal Processor (DSP), digital to analogue signal converter, voltage divider, and a test analogue plant. A 16th order discrete-time μ-controller with sampling frequency of 100 *Hz* is implemented by using a DSP. The μ-controller designed ensures robust performance of the closed-loop system in presence of parametric uncertainty in two plant parameters. Frequency domain analysis of the discrete-time closed-loop system is performed. The implementation of high-order robust control law is facilitated by the usage of technology for automatic code generation. An appropriate software in MATLAB®/Simulink® environment is developed which is embedded in DSP by using the Simulink Coder®. Experimental and simulation results are presented which confirm that the control system achieves the prescribed performance.**

Index Terms— **digital signal processor, real-time control, robust control, μ-controller**

I. INTRODUCTION

The Robust Control Theory now is a mature discipline that involves powerful methods for analysis and design of control systems in presence of signal and parameter uncertainties. The most frequently used techniques for robust control design are the H_∞ optimization and the μ-synthesis [1]. The H_∞ optimization is usually preferred in robust design because it produces controller of smaller order which facilitates its implementation. The common disadvantage of all H_∞ optimization methods is that they are suitable for plants with unstructured uncertainties but cannot ensure robust stability and performance in the general case of unstructured and structured (parametric) uncertainties. In contrast, the μ-synthesis which aims at minimization of the structured singular value [2] may ensure robust stability and robust performance in the presence of exogenous disturbances, noises and different type of uncertainties. The high order of the controller obtained is usually pointed out as a disadvantage of μ-synthesis. However, with the appearing of powerful processors in the recent years this peculiarity of the μ-synthesis does not pose a significant difficulty.

In contrast with the theoretical achievements, the practical implementation of robust control laws is still in its beginning. There is a few real life applications of high order robust control laws reported in the literature concerning mainly the control of flying vehicles (see for instance [3] and [4]). The main obstacle of robust control laws implementation is the difficulties related to the development, testing and verification of the necessary real-time software which is highly dependent on the type of digital controller platform used. These difficulties are reduced significantly using the recent technologies for automatic code generation and embedding implemented in MATLAB®/Simulink® program environment [5]. A real-time implementation of linear quadratic control law by using automatic code generation is reported in [6].

In this paper a laboratory system for real-time robust control of multivariable analogue 4th order two input/two output plant is presented. The system consists of Spectrum Digital eZdspTMF28335 development kit with built in Texas Instruments TMS320F28335 DSP, digital to analogue signal converter, voltage divider, and a test analogue plant. A 16th order discrete-time μ-controller with sampling frequency of 100 *Hz* is implemented by using a Digital Signal Processor. The μ-controller designed ensures robust performance of the closed-loop system in presence of parametric uncertainty in two plant parameters. Frequency domain analysis of the discrete-time closed-loop system is performed. An appropriate software in MATLAB®/Simulink® environment is developed which is embedded in DSP by using the Simulink Coder®. Experimental and simulation results are presented which confirm that the control system achieves the prescribed performance.

II. CONTROLLER μ-SYNTHESIS

The block-diagram of the analog two input/two output plant under consideration is shown in Fig. 1.

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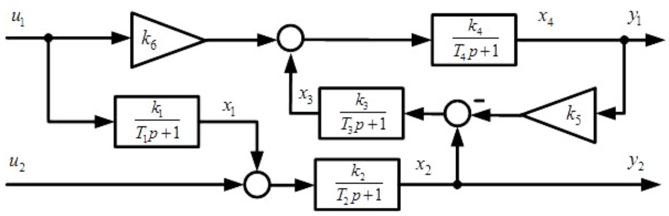

Fig. 1. Block-diagram of the plant

The 4th order plant model is described by the state space equations

$$\left| \begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{array} \right., \qquad (1)$$

where $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$ is the state vector, $u(t) = |u_1(t) \quad u_2(t)|^T$ is the input vector and $y(t) = |y_1(t) \quad y_2(t)|^T$ is the output vector,

$$A = \begin{vmatrix} -\frac{1}{T_1} & 0 & 0 & 0 \\ \frac{k_2}{T_2} & -\frac{1}{T_2} & 0 & 0 \\ 0 & \frac{k_3}{T_3} & -\frac{1}{T_3} & -\frac{k_5 k_3}{T_3} \\ 0 & 0 & \frac{k_4}{T_4} & -\frac{1}{T_4} \end{vmatrix}, B = \begin{vmatrix} \frac{k_1}{T_1} & 0 \\ 0 & \frac{k_2}{T_2} \\ 0 & 0 \\ \frac{k_4 k_6}{T_4} & 0 \end{vmatrix},$$

$$C = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix}, D = 0_{2\times 2},$$

$k_1, k_2, ..., k_6$ are coefficients of proportionality, $T_1, T_2, ..., T_4$ are time constants.

The model parameters are estimated by identification procedure using the MATLAB® function *lsqnonlin* based on the nonlinear least square method. The performance criteria minimized is

$$I_e = \int_0^t (y_p(t) - y(t))^T (y_p(t) - y(t))dt, \qquad (2)$$

where $y_p(t) = [y_{p_1}(t) \quad y_{p_2}(t)]^T$ is the analog plant output. The identification input signal is

$$u_{ident}(t) = [1(t) \quad 1(t)]^T$$

where $1(t)$ is a step function. The signal $y_p(t)$ is obtained by real time measuring of the two analogue plant outputs with sample time 100 ms. After minimization of (2) taking $t = 10s$ we obtain

$$I_e = 6.36 \times 10^{-4}.$$

The analog plant output signals and model output signals are shown in Fig. 2 and Fig.3. As can be seen the model outputs coincide very well with the measured plant output signals.

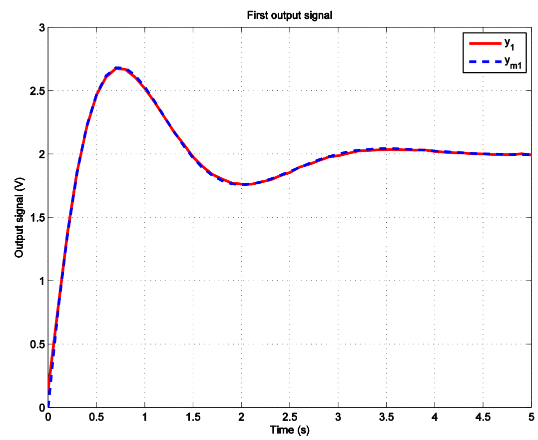

Fig. 2. First output signal

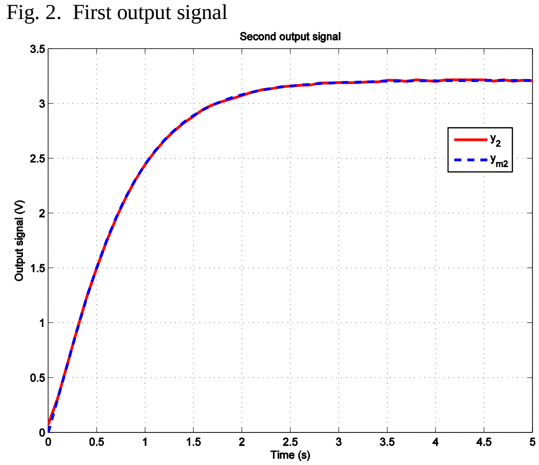

Fig. 3. Second output signal

The nominal model parameters determined are presented in Table 1.

TABLE I
NOMINAL PANT PARAMETERS

Parameter	Value	Parameter	Value
k_1	1.0463	T_1	0.3308
k_2	1.5689	T_2	0.5291
k_3	0.4213	T_3	1.1142
k_4	3.0000	T_4	0.7063
k_5	3.0558		
k_6	1.8970		

Further on it is assumed that the coefficient k_1 is a subject of 25% change around the nominal value and the parameter T_4 undergoes 40% change around its nominal value. In this way the plant involves structured (parametric) uncertainty which is set by using the function *ureal* from Robust Control Toolbox® [7]. In order to ensure robust stability and robust performance of the closed-loop system in the presence of such uncertainty and to ensure acceptable suppression of the disturbances and noises, it is appropriate to design a μ-controller.

The block-diagram of the closed-loop system which includes the performance and control weighting functions is shown in Fig. 4. The transfer function matrix of the uncertain plant is denoted by G and the controller transfer function matrix -by K. The system has a reference vector ref and an output disturbance vector $dist$.

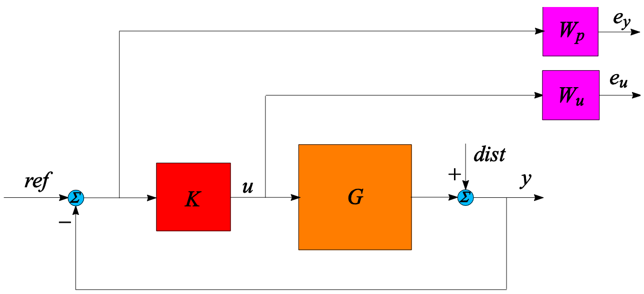

Fig. 4. Closed-loop system with performance requirements

The closed-loop system is described by the equations

$$\begin{aligned} y &= T_o r + S_o d \\ u &= S_i K (r-d) \end{aligned}, \qquad (3)$$

where the matrix $S_i = (I+KG)^{-1}$ is the input sensitivity transfer function matrix, $S_o = (I+GK)^{-1}$ is the output sensitivity transfer function matrix and $T_o = (I+GK)^{-1}GK$ is the output complementary sensitivity function. (Here and further on the reference vector is denoted for brevity by r and the disturbance vector - by d).

The weighted closed-loop system outputs e_y and e_u satisfy the equation

$$\begin{bmatrix} e_y \\ e_u \end{bmatrix} = \begin{bmatrix} W_p S_o & -W_p S_o \\ W_u S_i K & -W_u S_i K \end{bmatrix} \begin{bmatrix} r \\ d \end{bmatrix}. \qquad (4)$$

The performance criterion requires the transfer function matrix from the exogenous input signals r and d to the output signals e_y and e_u to be small in the sense of $\|\cdot\|_\infty$, for all possible uncertain plant models G. The transfer function matrices W_p and W_u are used to reflect the relative importance of the different frequency ranges for which the performance requirements should be fulfilled.

The μ-synthesis is done for the performance weighting function

$$W_p = \begin{bmatrix} 0.92\dfrac{0.15s+1}{0.24s+10^{-4}} & 0 \\ 0 & 0.92\dfrac{0.18s+1}{0.30s+10^{-4}} \end{bmatrix}$$

and for the control weighting function

$$W_u = \begin{bmatrix} 0.02\dfrac{0.005s+1}{0.001s+1} & 0 \\ 0 & 0.02\dfrac{0.005s+1}{0.001s+1} \end{bmatrix}.$$

The performance weighting functions are chosen as low pass filters to suppress the system error $y-r$, and the control weighting functions are chosen as high pass filters with appropriate bandwidth in order to impose constraints on the high frequency spectrum of the control actions [7].

In order to determine a discrete-time μ controller the extended open-loop system including the plant and weighting functions is discretized with sampling period of 0.01 s corresponding to sampling frequency of 100 Hz. The μ-synthesis is performed by using the Robust Control Toolbox® function *dksyn* [8]. Four iterations are performed that decrease the maximum value of μ to 1.000. The final controller obtained is of 16th order.

The singular value plot of the discrete-time controller is shown in Fig.5. The controller has a high low-frequency gains which allows to attenuate efficiently the low-frequency disturbances and to ensure accurate reference tracking.

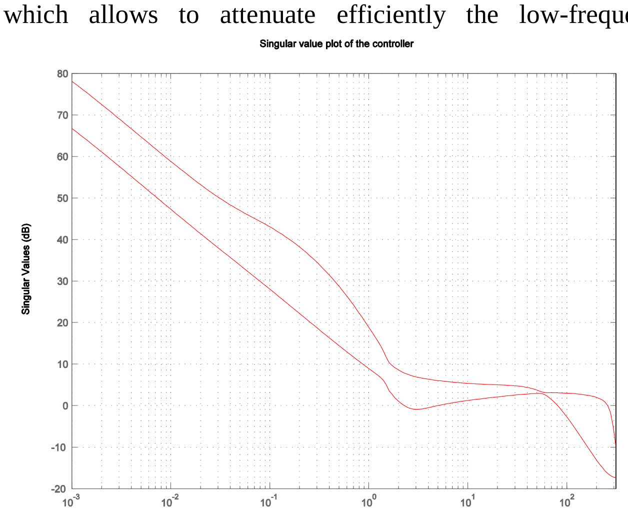

Fig. 5. Controller singular values

The frequency-response plot of the structured singular value for the case of robust stability, obtained by using the function *robuststab* is shown in Fig. 6. The closed-loop system can tolerate up to 201% of the modeled uncertainty which means that the stability is preserved for uncertainty that may be up to two times larger than the assumed one.

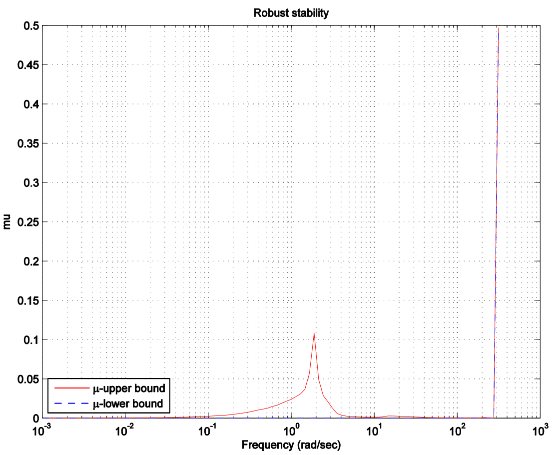

Fig. 6. Robust stability for μ -controller

The frequency response of the structured singular value for the case of robust performance, obtained by using the function *robustperf* is shown in Fig.7. The closed-loop system achieves a robust performance margin of 1.011 which means that the system will satisfy the performance requirements for all possible values of the uncertain parameters. The singular values of the complementary sensitivity function T_o plotted in Fig.8 show that the closed-loop system will track accurately references with frequencies up to 5 rad/s.

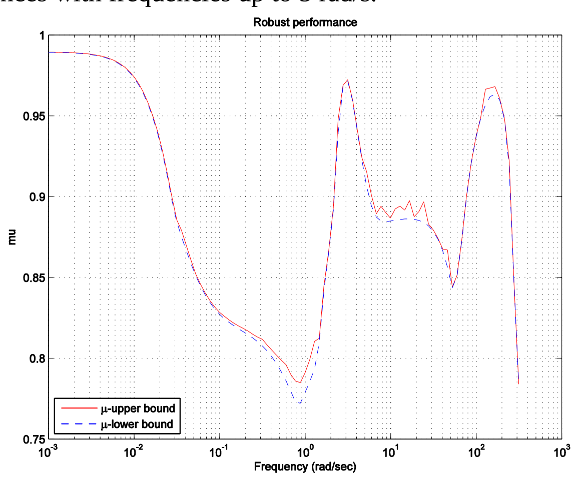

Fig. 7. Robust performance for μ -controller

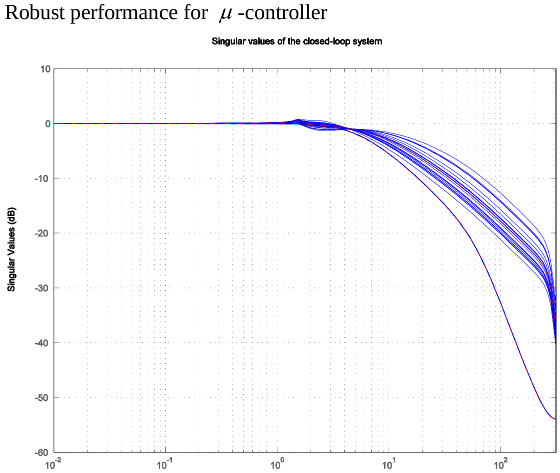

Fig. 8. Singular values of the complementary sensitivity function

In Fig.9 we show the singular value plot of the output sensitivity function S_o compared with the singular value plot of the inverse performance weighting function $W_p{}^{-1}$. The plot confirms that the system attenuates more than 10^3 times low frequency output disturbances. The singular value plot of the transfer function matrix $S_i K = K S_o$ which, according to (3) determines the control action u , is shown in Fig.10. It is seen that the high frequency contents of the control actions is attenuated in accordance with the requirements imposed by the inverse control weighting function $W_u{}^{-1}$.

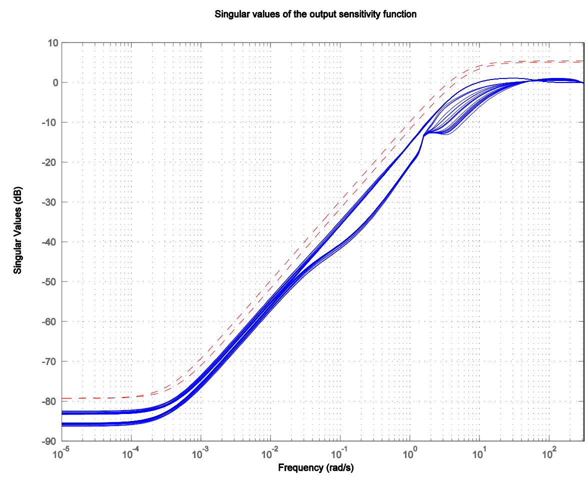

Fig. 9. Singular values of the output sensitivity function

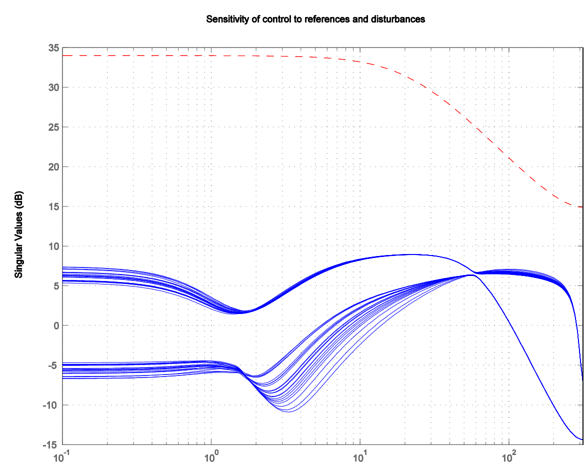

Fig. 10. Singular values of disturbance-to-control transfer function matrix

III. IMPLEMENTATION OF THE μ - CONTROLLER

The experimental setup of the analog plant control system is shown in Fig. 11. The two input/two output plant (1) is modeled on an analog modeling board, consisting of a bus on which functional blocks are operating independently. The bus is powered by a DC power supply. The linear range of input-output signals is ±10 V. The main linear blocks are adder, differentiator, inverter, integrator, aperiodic unit and gain unit.

The analog system makes it easy to model plants with a time constant up to 1 s and gain factor up to 10.

The block-diagram of developed control system with DSP is shown in Fig. 12.

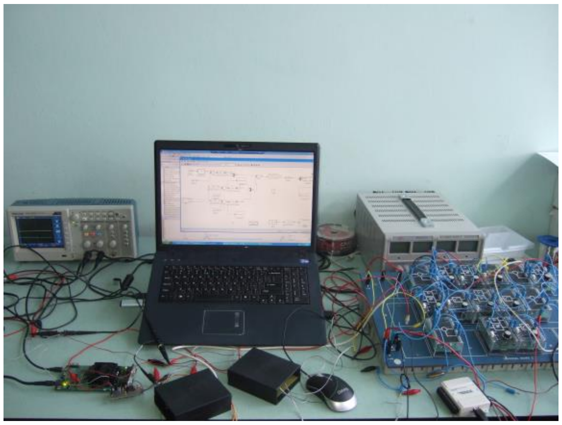

Fig. 11. Experimental setup

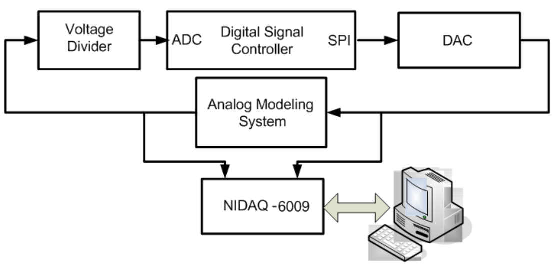

Fig. 12. Block-diagram of the control system with DSP

The block ``Digital Signal Controller" (DSC) is the Spectrum Digital eZdspTMF28335 development board with an integrated Digital Signal Processor Texas Instruments TMS320F28335 [9]. This controller works at 150 MHz and may perform single precision (32-bit) computations by using

The process of generation and implementation of the execution code in the Simulink® model is illustrated in Fig.13. This process relies heavily on the use of Simulink Coder® and Embedded Coder®.

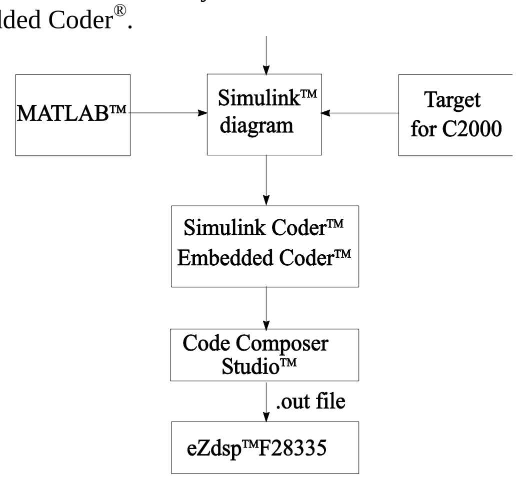

Fig. 13. Generation and embedding of the control code;

FPU (Floating-Point Unit). It has 68K bytes on-chip RAM, 256K bytes off-chip SRAM memory, 512K bytes on-chip Flash memory and on-chip 12 bit Analog to Digital (A/D) converter with 16 input channels.

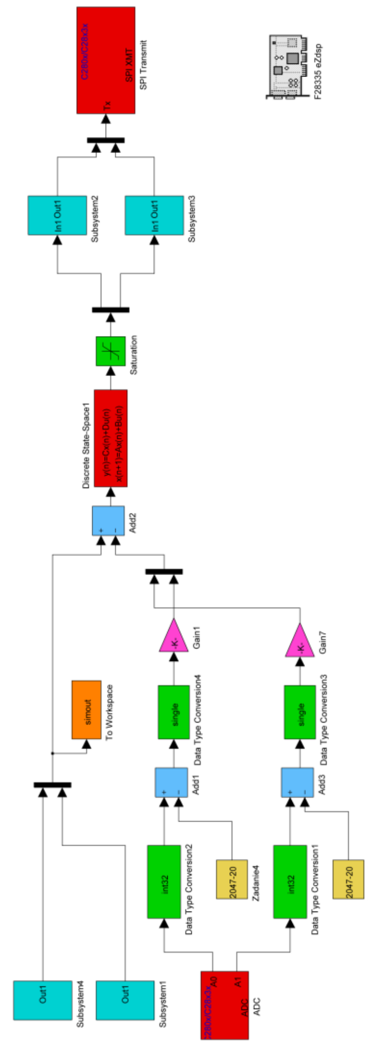

Fig. 14. Block-diagram of the Simulink® model

The block "Voltage Divider" is a specially designed dual channel board, which converts linearly two input analog signals with range -5 to 5 V into two analog output signals with range 0 to 3 V.

The block "DAC" is a digital to analogue converter DAC8734EVM produced by Texas Instruments [10]. The DAC is 16-bit, quad-channel and can be configured to outputs ±10V, ±5V, 0V to 20V or 0V to 10V. It features a standard high-speed serial peripheral interface (SPI) that operates at clock rates of up to 50MHz to communicate with a DSP.

The block ``NIDAQ-6009" is a specialized module for data acquisition NIDAQ-6009 of National Instruments [11]. A specially developed software provides the user interface, connection and exchange input/output data with a standard PC in real time.

The control algorithm is embedded and runs with frequency 100 Hz on the DSC. The software development environment includes MATLAB® v.7.11.0.584(R2010b), \Simulink® v.7.6, Simulink Coder® v.7.6 [5(5)], Embedded Coder® v. 5.6 [12], Microsoft Visual C++ v.8.0 and CodeComposer® (CCS) v.3.3. A technology for automatic generation and embedding the code using the Simulink Coder® is implemented. The main advantages of this technology are the relatively easy implementation of complex control algorithms and the short time to translate the control algorithm from the working simulation environment to the real working environment with physical plants, reducing the overall time for application testing and verification of the developed algorithm.

The block-diagram of the Simulink® model used to generate the execution code is shown in Fig. 14. Some blocks are taken from the specialized library for Texas Instruments C2000 of Simulink®. The experimental results are saved on the PC and are visualized in MATLAB®.

IV. Experimental Results

The performance of the developed robust control system with μ -regulator implemented using digital signal controller is confirmed by experimental investigations. Several experiments are done with the nominal plant and for different combinations of the uncertain plant parameters.

A. *Experiment with the nominal plant*

In case of nominal plant parameters, an experiment is done in which the reference varies every 10 s at three levels: -1V, 0V and 1V. The output and control signals (here and further on all signals are measured in volts) from simulation and experiments are shown in Fig. 15-18 (the experimental results are shown by continuous lines). It is seen from the Fig.15 nd Fig.16 that the experimental signals coincide well with the output signals obtained by simulation.

The Figures displaying control signals show the presence of some deviations from simulation results whose magnitudes may reach values of up to 0.2V. According to the frequency responses shown in Fig. 10, small disturbances and errors may be amplified in the control signals which explain the presence of relatively large noises in these signals.

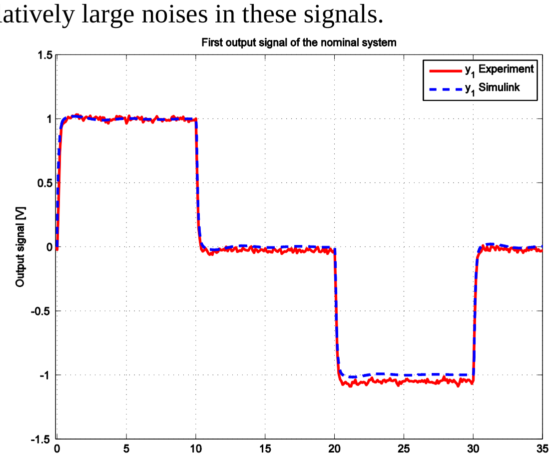

Fig.15. First output signal

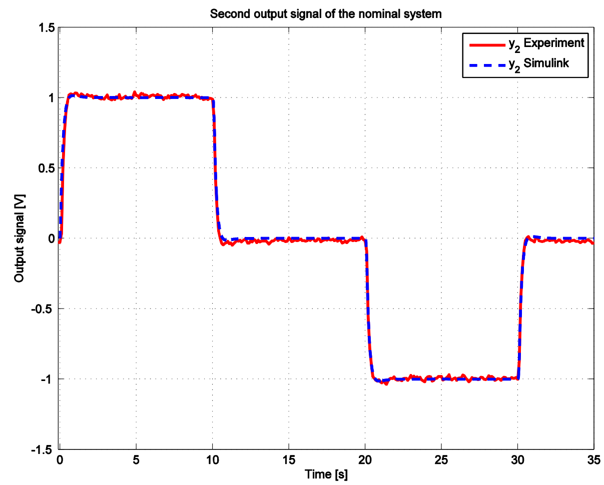

Fig.16. Second output signal

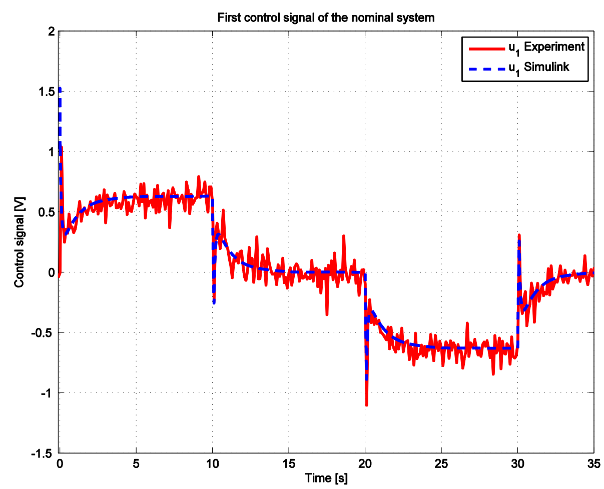

Fig.17. First control signal

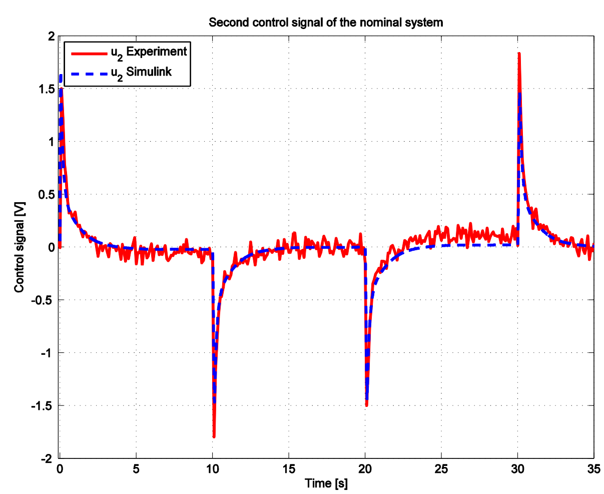

Fig.18. Second control signal

B. *Experiment with different plant parameters*

Five experiments for different values of the uncertain parameters are performed in order to access the influence of the uncertainties on the closed-loop system behavior. The set of uncertain parameters contain combinations of the minimum and maximum values of the uncertain parameters as shown in

Table II.

TABLE II
PARAMETERS VARIATION

Parameter set	k_1	T_4
1	0.75	0.35
2	0.75	1.05
3	1.25	0.35
4	1.25	1.05
nominal parameters	1.0463	0.7063

The set of nominal parameters is also included in order to estimate the maximum possible deviations of the output and control signals from their nominal values.

In Fig.19 and Fig.20 we show the output signals of the system for all five parameter sets. It is seen that the deviations of the outputs from their nominal values are relatively small which a result of the robust performance achieved is. The two control signals shown in Fig.21 and Fig.22 have more significant deviations from their nominal values which is again explained by the large gains associated with the transfer matrix S_iK .

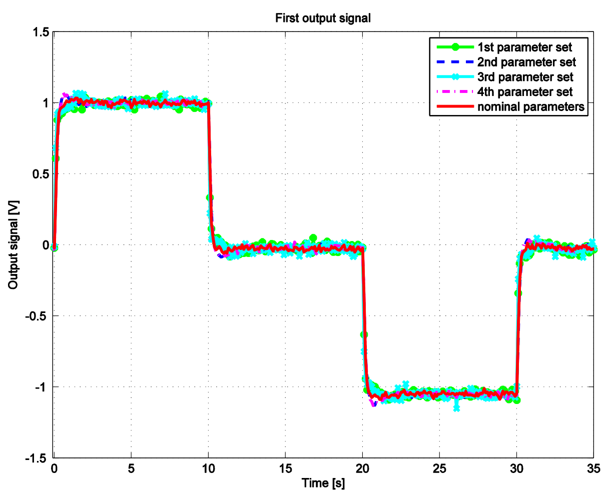

Fig.19. First output signal

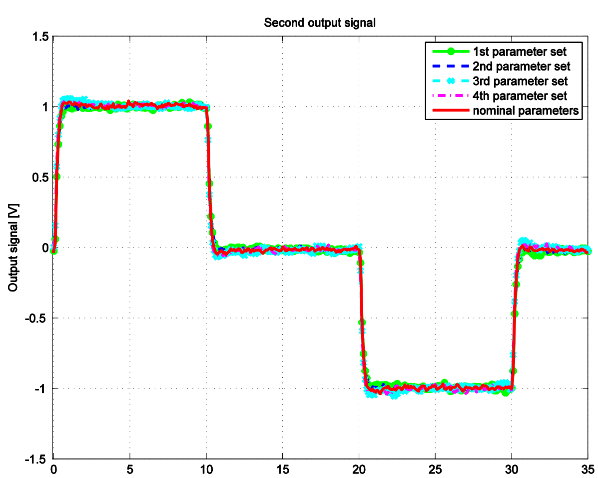

Fig.20. Second output signal

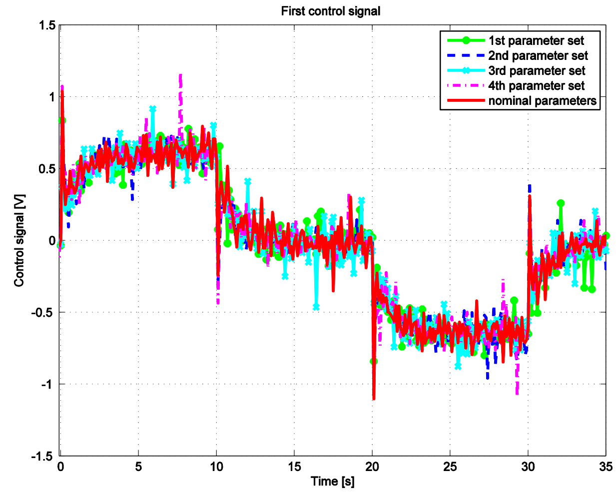

Fig.21. First control signal

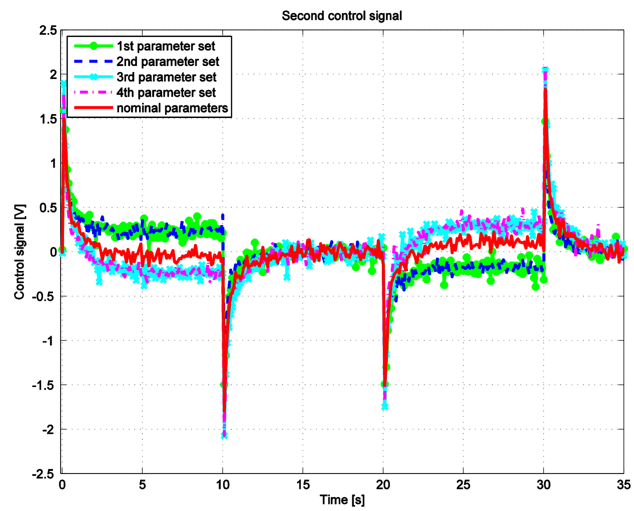

Fig.22. Second control signal

V. CONCLUSION

The paper presents a system for robust real-time control of fourth order plant using Digital System Processor. A specialized software for automatic generation of the control code is developed that facilitates the implementation of high order control algorithms for multivariable plants. The main conclusion is that the existing technologies allow implementing easily a 16th order μ-controller that ensures robust stability and robust performance of the closed-loop system. The experimental results confirm the performance of the closed-loop system whose parameters vary in prescribed boundaries. The control code may be easily modified for control of other plants.

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BIOGRAPHIES

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Petko Petkov received M.S. and Ph.D. degrees in control engineering from the Technical University of Sofia in 1971 and 1979. Since 1995, he has been a Professor of Control Theory in the Department of Systems and Control at the Technical University of Sofia. He is coauthoring *Computational Methods for Linear Control Systems* (Prentice Hall, Hemel Hempstead, U.K., 1991), *Perturbation Theory for Matrix Equations* (North-Holland, Amsterdam, 2003), and *Robust Control Design with MATLAB* (Springer, London, 2005, 2013).