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The Effect Of Peer Collaboration On Children's Arithmetic And Self-Regulated Learning Skills

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Abstract – The present study examines the effect of peer collaboration, teaching children arithmetic in the beginning of 7th grade, age 13 years. Peer collaboration groups are compared to two different structured teaching methods, traditional and independent teaching. Progress made by these students are related to measures of their arithmetic ability, calculation and quantitative concept, as well as their self-regulated learning skills in mathematics, characterised as internal and instrumental motivation, self-concept and anxiety. The results will be discussed with reference to Piaget´s theory of the relation between social interaction and cognitive development. This study has a split-plot factorial design with time as within-subject and type of intervention as a between-subject factor. Students' progress in quantitative concepts is significantly better if teachers teach traditionally or with peer collaboration. The results show that there are no significant differences between teaching methods when assessing arithmetic in total and calculation. Peer collaboration is more effective than traditional and independent work for students' internal motivation. Traditional work and peer collaboration are more effective than independent work for students' self-concept.

Keywords: Peer collaboration, mathematics, arithmetic skills, self-regulated learning skills.

Introduction

The field of mathematics is extremely complex, including areas as arithmetic, and geometry with each of this areas consisting of several subdomains and encompassing many cognitive processes (Kilpatrick et. al., 2001). For elementary schoolchildren achievement test assess a wide range of arithmetic skills such as number sense, procedural knowledge, using problem solving strategies. Although mathematical test often asses diverse mathematical

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skills, this information often is summarized into a total score of mathematical knowledge. Even though relatively little is known about the phenotypic relationships among mathematical skills, less is known about different teaching strategies impact on different mathematical skills. Often a global assessment of mathematics is used by researcher when they report impacts of teaching methods (Reynolds & Muijs, 1999). One hypothesis in this study is that different activities draw attention to different cognitive processes and therefore are more effective according to different mathematical competencies (cf Boaler, 1999).

The activities in the classroom are important because they constitute the knowledge that is produced (Cobb, 1998). Aitkin and Zukovsky (1994) stresses that there is some evidence that different teaching styles have different impacts on student achievement and that the choice of teaching approaches (Wentzel, 2002) can make an important difference in a student's learning. The synthesis of a review of Teddlie and Reynolds (2000) gives evidence for positive relationships between achievement and varied classroom settings. Case (1996) argues that a variation of teaching methods is important because different teaching methods draw attention to different competencies in mathematics (e.g. Boaler, 2002). Thus, the mode of teaching method in mathematics seems to be important for students' performances. In the present study, the effectiveness of peer collaboration are compared to two different structured methods of teaching children arithmetic in the beginning of 7th grade, age 13 years. Progress in mathematics was measured by arithmetic ability, calculation and quantitative concept. Measures of self-regulated learning skills in mathematics were also included. Self-regulated learning is considered in PISA to involve motivation to learn and ability to select appropriate goals and strategies for learning, and that these factors have a positive relationship with students' performance, and this assumption is based on empirical evidence. In this study they are operationalized as internal and instrumental motivation, self-concept and anxiety (OECD, 2004).

Theoretical Perspective

Research examining the relation between social interaction and cognitive development has usually been based on theories of either Piaget or Vygotsky (Tudge, 1993). Piaget (1959) held that children's cognitive development depends on active interaction with the environment. Piaget believed that all children try to strike a balance between assimilation and accommodation, which is achieved through a mechanism Piaget called equilibration. As children progress through the stages of cognitive development, it is important to maintain a balance between applying previous knowledge (assimilation) and changing behaviour to account for new knowledge (accommodation). Equilibration helps explain how children are able to move from one stage of thought into the next. Piaget recommended that peer interaction promoted cognitive conflict by exposing discrepancies between the peer´ own and others knowledge, resulting in disequilibration. As higher level of understanding emerged, through discussion, among individuals of equal status equilibration was restored and cognitive change occurred. Studies grounded in a Piagetian constructivist framework have supported this view that working with peer leads to greater cognitive benefit than working alone (Druyan, 2001; Golbeck & Sinagra, 2000).

Although Piaget (1932) argued that language did not create the structure of thinking, he conceded that language facilitated its emergence. In addition, he accepted that social interaction was an important component of cognitive development. Talking to others provokes some of cognitive disconfirmation, triggering a search for equilibration. If children have the chance to discover the view of others, then, arguably, it is the active interaction with dissimilar perspective that is the critical factor. Theoretically, then the cognitive value of a peer collaboration for learning appears to be linked in to at least two factors, a) the interaction needs to be with one with different knowledge base, to ensure inconsistency between the children's knowledge, b) the child must be an active participant.

Learning Outcomes

The mathematics curriculum during elementary school in Sweden has many components, but there is a strong emphasis on concepts of numbers and operations with numbers. From an international perspective, mathematics knowledge is defined as something more complex than concept of numbers and operations with numbers. Kilpatrick et al. (2001) argue for five strands which together build students' mathematical proficiency. The five strands provide a framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency. In their report they discuss,

1. *Conceptual understanding* is about comprehension of mathematical concepts, operations, and relationships. Students with conceptual understanding know more than isolated facts and methods. Items measuring conceptual understanding are for instance: "Your number is 123.45. Change the hundreds and the tenths. What is your new number?
2. *Procedural fluency* refers to skills in carrying out procedures flexibly, accurately, efficiently, and appropriately. Students need to be efficient in performing basic

computations with whole numbers (e.g., 6+7, 17–9, 8×4) without always having to refer to tables or other aids.

3. *Strategic competence* is the ability to formulate, represent, and solve mathematical problems. Kilpatrick et al. (2001, p. 126) give the following example of item testing strategic competence: "A cycle shop has a total of 36 bicycles and tricycles in stock. Collectively there are 80 wheels. How many bikes and how many tricycles are there?"
4. *Adaptive reasoning* refers to the capacity for logical thought, reflection, explanation, and justification. Kilpatrick et al. (2001) gives the following example where students can use their adaptive reasoning. "Through a carefully constructed sequence of activities about adding and removing marbles from a bag containing many marbles, second graders can reason that 5+(–6)=–1. In the context of cutting short bows from a 12-meter package of ribbon and using physical models to calculate that 12 divided by 1/3 is 36, fifth graders can reason that 12 divided by 2/3 cannot be 72 because that would mean getting *more* bows from a package when the individual bow is *larger*, which does not make sense" (p.130).
5. "*Productive disposition* is the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick et al., 2001, p.5). Items measuring productive disposition are for instance: "How confident are you in the following situations? When you count 8-1=___+3 (completely confident, confident, fairly confident, not at al confident)."

The present study focuses on the effect of peer collaboration compared to traditional and independent teaching of students' arithmetic proficiency, conceptual understanding (*quantitative concepts*) and procedural fluency (*calculation*).

The present study investigates different teaching methods influence on aspects of self-regulated learning skills. In this study, tests previously used in PISA were employed. In PISA, self-regulated learning skills include the motivation to learn and the ability to select appropriate goals and strategies for learning. The factors investigated in PISA were categorized as students' interest in and enjoyment of mathematics, instrumental motivation in mathematics, self-concept in mathematics, and anxiety in mathematics, all of which show correlations to skills in mathematics (OECD, 2004). Students' interest in and enjoyment of mathematics and students' instrumental motivation in mathematics are aspects of motivation (OECD, 2004). The former factors are related to internal characteristics while the latter is related to external rewards. Students' self-concepts in mathematics define students' beliefs about their mathematical competencies. These beliefs have influence on the goals students set for themselves and on their choices of learning strategies. Anxiety in mathematics is a complex phenomenon that manifests as panic, fear of failure, and mental disorganisation when solving math problems (Foire, 1999).

Teaching Methods And Learning Outcomes

There are very few previous studies focusing on how different teaching methods affect students' calculation and quantitative concepts as well as self-regulated learning skills, but there are several studies that focus on closely related areas (Reynolds & Muijs, 1999; U.S. Department of Education, 2008).

For learning in general, Granström (2006) shows that different teaching approaches in classrooms affect students' benefits from the lessons. Settings where students are allowed and encouraged to cooperate with classmates and teachers give the students better opportunities to understand and succeed. Similarly, Oppendekker and Van Damme (2006) stress that good teaching involves communication and building relationships with students. Boaler (1999, 2002) reports that practices such as working through textbook exercises or discussing and using mathematical ideas were important vehicles for the development of flexible mathematical knowledge. One outcome of Boaler's research was that students who had worked in textbooks performed well in similar textbook situations. However, these students found it difficult to use math in open, applied or discussion-based situations. The students who had learned math through group-based projects were more able to apply their knowledge in a range of situations. Boaler's studies give evidence for the theory that context constructs the knowledge that is produced.

In a review of successful teaching of mathematics, Reynolds and Muijs (1999) discuss American as well as British research. A result of their review is that effective teaching is signified by a high number of opportunities to learn. Opportunity to learn is related to factors such as length of school day and year and the amount of hours of mathematics. It is also related to the quality of classroom management, especially time-on-task. According to research in the area, achievement is reinforced when teachers create classrooms that include (a) substantial emphasis on academic instruction and students' engagement in academic tasks (Brophy & Good, 1986; Griffin & Barnes, 1986; Lampert, 1988; Cooney, 1994), (b) whole-class instruction (Reynolds & Muijs, 1999), (c) effective question-answer and seatwork practices (Brophy, 1986; Brophy & Good, 1986; Borich, 1996), (d) minimal disruptive behaviour (Evertsson et al., 1980; Brophy & Good, 1986; Lampert, 1988; Secada, 1992), (e) high teacher expectations (Borich, 1996; Clarke, 1997), and (f) substantial feedback to students (Brophy, 1986; Brophy & Good, 1986; Borich, 1996). Many aspect of successful teaching are found in a traditional classroom (lecturing and drill) with one big exception: in successful teaching, teachers are actively asking a lot of questions and students are involved in a class discussion. With the addition of active discussion, students are kept involved in the lesson and the teacher has a chance to continually monitor students' understanding of the concept taught.

On the other hand, negative relationships have been found between teachers who spend a high proportion of time communicating with individuals and students' achievement (Mortimer et al. 1988; OfSTED, 1996). Students' math performances were low when they practiced too much repetitive number work individually (OfSTED, 1996).

Another teaching method discussed in the literature is peer collaboration work. The advantage of peer collaboration lies in the scaffolding process whereby students help each other advance. Giving and receiving help and explanations may widen their thinking skills, and verbalising can help students structure their thoughts (Leikin & Zaslavsky, 1997). This exchange may encourage students to engage in higher-order thinking (Becker & Selter, 1996). Students who work in small groups are developing an understanding of themselves as well as others and learning that others have both strengths and weaknesses. Programmes that have attempted peer collaboration as a teaching method report good results, such as improved conceptual understanding and higher scores on problem-solving tasks (Goods & Gailbraith, 1996; Leikin & Zaslavsky, 1997). Research shows also that children working collaboratively achieve a combined higher performance output than children working individually (Samaha & De Lisi, 2000). However peer collaboration is not always associated with cognitive development (Doise & Mugny, 1984; Levin & Druyan, 1993, Tudge & Winterhoff, 1993). It is suggested that the peer collaborations impact depend on a set of factors as age (Hogan & Tudge, 1999), comparative ability level of partners (Garton & Pratt, 2001), motivation (Gabriele & Montecinos, 2001), confidence (Tudge et al, 1996), gender (Strough, Berg & Meegan, 2001), and the task (Phelps & Damon, 1989). Several researchers (e.g. Rogoff, 1990; Samhan & De Lisi, 2000; Webb & Favier,1999) argue that a key element of effective peer collaboration is the active exchange of ideas though verbal communication.

Different teaching methods also seem to influence students' self-regulated learning skills (interest, view of the subject's importance, self-perception, and attribution) (Boaler, 2002). Students who were expected to cram for examinations describe their attitudes in passive and negative terms. Those who were invited to contribute with ideas and methods describe their attitude in active and positive terms that were inconsistent with the identities they had previously developed in mathematics (Boaler, 2002). A negative attitude towards mathematics can be influenced, for instance, by too much individual practice (Tobias, 1987) as well as by teachers who expose students' inabilities. Students who do well in school (Chapman & Tunmer, 1997) demonstrate appropriate task-focused behaviour (Onatsu-Arvillomi & Nurmi, 2000), and they have positive learning strategies. If the students are reluctant in learning situations and avoid challenges, they normally show low achievement (Midgley & Urdan, 1995; Zuckerman, Kieffer, & Knee, 1998).

The Aim of the Study

The aim of this study was to investigate which teaching approach, peer collaboration, traditional or independent teaching is most effective for developing students' mathematical proficiency in areas such as arithmetic ability, calculation and quantitative concept, as well as students' self-regulated learning skills in mathematics, internal and instrumental motivation, self-concept and anxiety?

Method

Participants

A total of 119 students attending six different classes in mathematics were included in the study. They were all 13 years old, and there were 59 female students and 60 male students. They all attended one school, and this school mainly recruits students from a part of Sweden with average socio-economic status. Their performances on standardised national tests in language and mathematics were representative for Swedish 7th grade students according to the National Agency of Education. Prior to starting grade 7, groups of students were attending different schools in the same town area. They were all mixed in six different classes at the beginning of grade 7. This means that a majority of these students were attending a math class where most classmates were unfamiliar. Thus, there were six groups of mathematic students attending a new school at grade 7 in the mid-August 2006, and these classes were randomly assigned to three different teaching methods. Age in month, gender, and previous performance on national test in language and mathematics were similar across classes for each teaching method. For the first 10-week period of 7th grade, teachers focus on arithmetic in their math classes in Sweden. These 10 weeks of teaching began in mid-August and finished in the beginning of November. Every week, the students had three math lessons, each 50 minutes long. Pre-testing was performed during the first two weeks in school. This testing was performed by the class teacher. Ten weeks later, the class teacher also performed the post-test.

Measures of Math Skills

The tests employed in this study were developed by six teachers in mathematics and three teacher educators. The teacher educators are all involved in textbook writing in mathematics and also are part of the committee working on the national mathematics tests in Sweden. Two tests covering arithmetic ability, calculation, and quantitative concepts were developed. One version was used at pre-testing and the second version was used after the intervention. Both parallel tests measuring each aspect of mathematic skills were performed by students (n=40; 13 years old) prior to the study to make sure that task difficulty was comparable. The first and the second version of each test were performed using a three weeks interval. The means and standard deviations for each test measuring aspects of arithmetic ability were almost identical (see Table 1). In addition, the correlations between each version of the tests measuring calculation, and quantitative concepts were 0.96, and 0.94, respectively. Thus, the reliability between the tests is relatively high.

Table 1 Means And Standard Deviation on Mathematics Tests, Version 1 And 2

	Version 1		**Version 2**	
	Mean	*SD*	*Mean*	*SD*
Calculation	10.15	3.14	10.50	2.69
Quantitative concept	12.40	2.17	12.55	2.05
Arithmetic in total	22.55	5.04	23.05	4.46

The calculation test measures the ability to perform mathematical computations. The items require the person to perform addition, subtraction, multiplication, division. The calculations items also involve decimals, fractions and whole numbers. Because the calculations are presented in a traditional format, the person is not required to make any decisions about what operations to use or what data to include. The test of quantitative concepts measures knowledge of mathematical concepts. The items require knowledge of shapes, sequences, series of numbers, and the ability figure out the pattern and provide the missing numbers.

Cronbach´s α estimates of reliability based on internal consistency for each measure of mathematical skills are based on item-level analyses in all groups. Conceptual understanding was measured by 15 item (α=.91). Calculation was measured by 15 items (α=.93).

The Self-Regulated Learning Skills Questionnaire

Self-regulated learning skills were assessed with a questionnaire originally designed and used in PISA (OECD, 2004). However, in this study a ten-point scale was employed (don't agree = 1 to totally agree = 10) instead of a six-point scale used in the PISA study. The first four statements in the questionnaire were related to *internal motivation*, that is, (a) I enjoy reading about mathematics, (b) I look forward to my mathematics lessons, (c) I do mathematics because I enjoy it, and (d) I am interested in the things I learn in mathematics. Another four statements in the questionnaire were employed to measure *instrumental motivation*. These statements were as follows: (a) Making an effort in mathematics is worth it because it will help me in the work that I want to do later, (b) Learning mathematics is important because it will help me with the subject that I will study further on in school, (c) Mathematics is an important subject for me because I need it for what I want to study later on, and (d) I will learn many things in mathematics that will help me get a job. *Self-concept* was measured by 5 different statements: (a) I am good at mathematics; (b) I get good grades; (c) I learn mathematics quickly; (d) I have always believed that mathematics is one of my best subject; and (e) In my mathematics class, I understand even the most difficult work. Finally, the last five statements focused on *anxiety* about mathematics: (a) I often worry that it will be difficult for me in mathematics classes, (b) I get very tense when I have to do mathematics homework, (c) I get nervous doing mathematics problems, (d) I feel helpless when doing mathematics problem, and (e) I worry that I will get poor grades in mathematics.

Teaching Methods

In this study, three different teaching methods (peer collaboration, traditional and independent work) were compared. Two classes were called *peer collaboration groups*. They were introduced to different ideas and problems that could be investigated and solved using a range of mathematical methods. Students worked in groups of four, and they discussed and negotiated arithmetic issues with each other and with the teacher, both in groups and in whole-class discussions. They also solved problems in textbooks for one third of the lessons.

Two classes were taught in a *traditional way*. This means that the teacher explained methods and procedures from the chalk board at the start of the lessons, and the students then practiced with textbook questions. This could be defined as a teacher-directed instruction where the teacher primarily communicates the mathematics to the student directly (U.S. Department of Education, 2008).

In the last two classes *working independently*, students worked individually on problems from a textbook without a teacher's introduction to the lesson; teachers just helped students who asked for it. In these classes there was no whole-class interaction and very little interaction between students.

One important factor in the teaching process is the teacher (Ma, 1999). To avoid a teacher's effect on the learning outcome, teams were constructed. Three teachers became responsible for three classes, peer collaboration, traditional and independent work. Every lesson was planned by the whole group plus the researcher. Then the teachers circulated in the three classes. They could, for instance, start their teaching in a traditional class, conduct the next lesson in an independent group, and finish with the third in a peer collaboration group during one week. This schedule was repeated, so students met three different teachers each week.

When researchers and teachers planned the lessons, we discussed what mathematical knowledge the textbook taught. When we had agreed on the amount of mathematical knowledge for the independent group, we constructed lessons for the traditional group and the peer collaboration group. By working with the content like this, we hoped that the level of mathematical proficiency would be as equal as possible in different groups.

Design

The design was a split-plot factorial design with group (i.e., peer collaboration, traditional, and independent work) as a between-subject factor and time (i.e., before and after a 10 week intervention) as a within-subject factor. There were a total of seven dependent variables in the study. There were three measures of mathematic abilities, that is, a total score of mathematic ability, calculation, and quantitative concepts. Measures related to self-regulated learning skills such as internal and instrumental motivation, self-concept, and anxiety were also used as dependent variables. Data was analyzed as repeated measures ANOVA with between-subjects factors. To assess the effect of peer collaboration compared to three different teaching methods in mathematics, a total of seven analyses of variance (ANOVA) with group as a between-subject factor and time as a within-subject factor were performed.

Results

The aim of this study was to investigate the effect of peer collaboration compared to two different, ordinary teaching methods in Sweden. The primary data, therefore, come from changes in mathematics ability, including cognitive as well as affective aspects, between pre- and post-tests.

Means and standard deviations for arithmetic in total, calculation, quantitative concept, internal motivation, instrumental motivation, self-concept and anxiety are shown in Table 2.

Table 2 Means and Standard Deviation for the Pre- and Post-Intervention Levels of Arithmetic in Total, Calculation, Quantitative Concept, Internal Motivation, External Motivation, Self-Concept and Anxiety

	Group		
	Peer collaboration (39)	*Traditional (39)*	*Independent work (41)*
Arithmetic in total [a]			
t1	17.90 (6.46)	20.31 (6.38)	17.78 (7.42)
t2	21.10 (6.02)	23.56 (5.83)	20.12 (6.67)
Calculation [b]			
t1	8.38 (3.30)	9.82 (3.62)	9.22 (4.11)
t2	9.54 (3.32)	11.15 (3.41)	11.49 (3.35)
Quantitative concept [b]			
t1	9.52 (3.59)	10.49 (3.16)	8.56 (3.71)
t2	11.56 (2.98)	12.41 (2.74)	8.63 (3.63)
Internal motivation [c]			
t1	5.15 (2.38)	6.38 (2.38)	5.34 (2.17)
t2	6.27 (2.46)	6.44 (2.31)	5.34 (2.04)
Instrumental motivation [c]			
t1	7.53 (2.05)	8.16 (1.59)	7.29 (2.00)
t2	8.47 (1.35)	8.64 (1.43)	7.85 (1.91)
Self-concept [c]			
t1	5.33 (2.27)	5.87 (2.41)	5.41 (2.06)
t2	6.31 (1.95)	6.39 (2.19)	5.45 (2.01)
Anxiety [c]			
t1	4.67 (2.08)	3.90 (1.97)	4.10 (2.30)
t2	4.15 (2.33)	3.65 (2.13)	3.92 (2.05)

[a] maximum scores = 30; [b] maximum scores = 15; [c] maximum scores = 10

The main issue of interest is the extent to which the three groups, peer collaboration, traditional, independent work, have made differential progress on these performance measures. Analyses to address this question need to take into account the fact that the groups are not perfectly matched for their mathematical ability. In a field test like, this it was not possible to match the groups exactly. However, there were no significant differences found between teaching groups on any dependent measures before intervention (all p's>0.05).

The Effect of Teaching Methods on Arithmetic Skills

An ANOVA with total scores of arithmetic skills as dependent measures revealed a significant main effect for time, $F(1,116)=70.1$, $p<.001$, suggesting that skills in arithmetic were improved across teaching groups. There were no main effect of group, $F(1,116)=2.59$, $p>.05$, nor was there an interaction between group and time, $F(2,116)=0.73$, $p>.05$. These findings suggest that there was no general effect of the variation of teaching methods, and that improvement in arithmetic in total was similar across teaching methods.

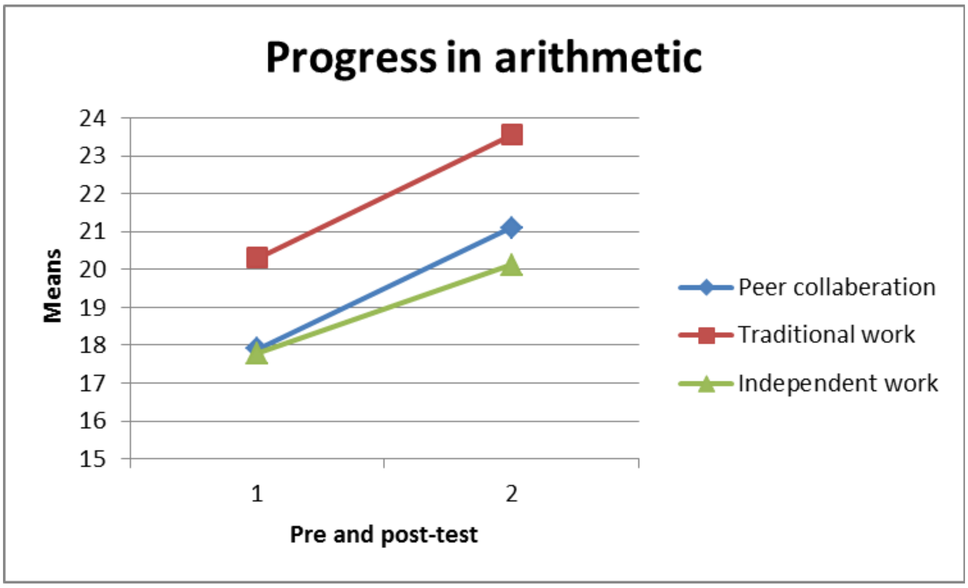

Figure 1 Progress in Arithmetic

Two additional ANOVA's were performed with calculation and quantitative concepts, separately. For calculation as well as for quantitative concepts, there was a main effect of time, $F(1,116)=151.0$, $p<.001$, $F(1,116)=37.5$, $p<.001$, signifying that both calculation ability and quantitative concepts were enhanced across teaching groups. There were also a main effect of group for quantitative concepts, $F(1,116)=8.84$, $p<.001$, which indicate that at least one group perform better according to quantitative concepts. There were no main effects of group, $F(1,116)=2.51$, $p>.05$, when measuring calculation. Finally, there was no interaction between group and time for calculation: $F(2,116)=2.32$, $p>.05$, but there was for quantitative concepts: $F(2,116)=8.56$, $p<.001$.

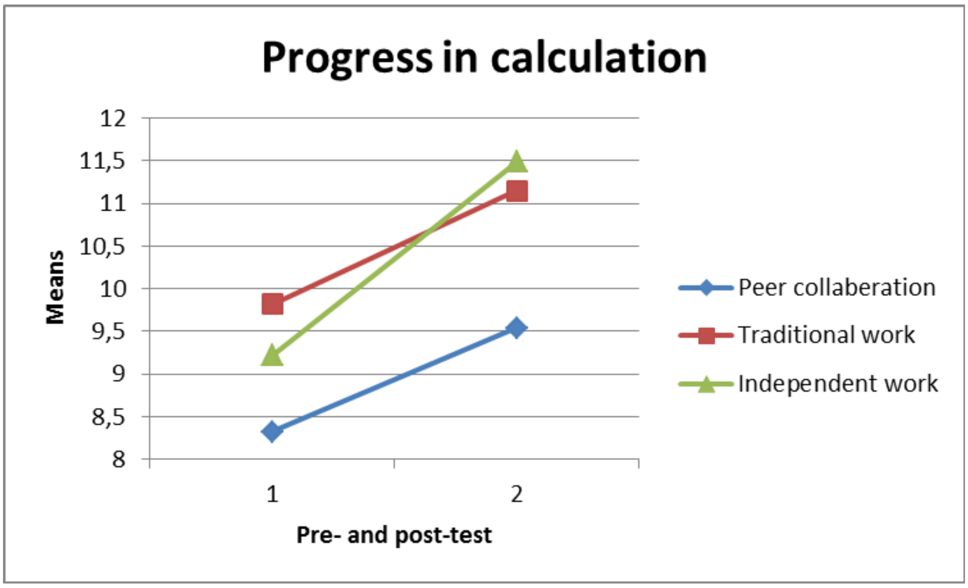

Figure 2 Progress in Calculation

Interaction is explained by differences between the traditional and the independent group as well as the peer collaboration and the independent group. The figure (Fig.3) clearly illustrates a greater progress on quantitative concepts in the traditional and the peer-collaboration groups than in the independent group.

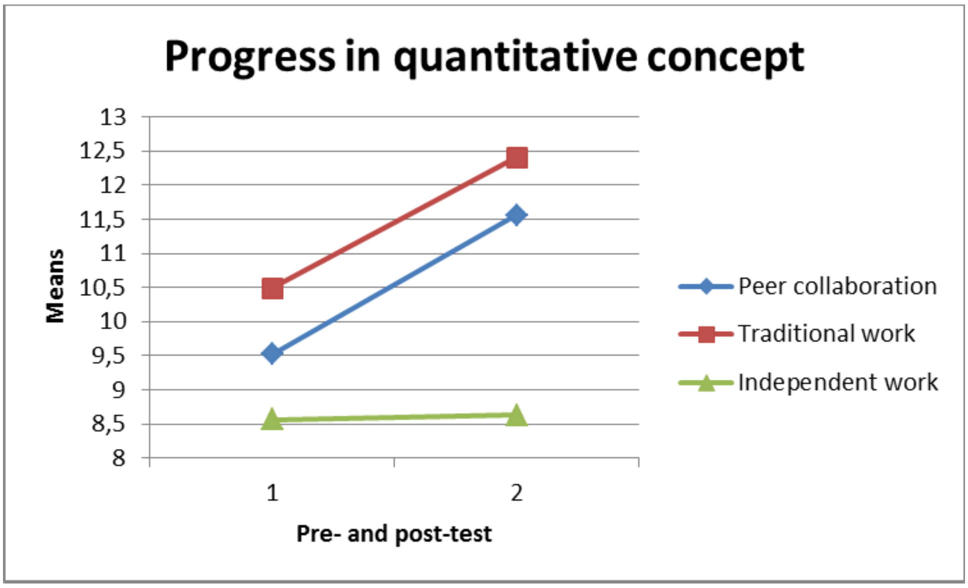

Figure 3 Progress in Quantitative Concept

Summary: There are no significant interaction effects between group and time according to arithmetic in total and calculation. Looking at students' progress in quantitative concept, it is possible to explain differences in progress with the teaching method. Peer Collaboration as well as traditional work seems to have more positive effects on students' development of conceptual knowledge (quantitative concept) than independent work does.

The Effect of Teaching Methods on Self-Regulated Learning Skills

Four ANOVA's with internal motivation, instrumental motivation, self-concept and anxiety as dependent measures, revealed significant main effects for time on each dependent measure. The main effects were internal motivation, $F(1,116)=9.67$, $p<.001$; instrumental motivation, $F(1,116)=30.44$, $p<.001$; self-concept, $F(1,116)=14.77$, $p<.001$; and anxiety, $F(1,116)=4.45$, $p<.05$. All results indicated that these four aspects of self-regulated learning skills improved across groups. For the first three aspects, students scored higher on post-test than pre-test. For anxiety, students scored lower on post-test. It indicates that students internal motivation, instrumental motivation and self-concept were higher and that the students felt less anxious towards mathematics after ten weeks of intervention. The results also show that there were no main effects of group; internal motivation, $F(1,116)=2.47$, $p>.05$; instrumental motivation, $F(1,116)=2.59$, $p>.05$; self-concept, $F(1,116)=1.21$, $p>.05$; or anxiety, $F(1,116)=1.03$, $p>.05$.

Significant interactions between time and group were found in two measures; internal motivation, $F(2,116)= 8.00$, $p<.001$; and self concept, $F(2,116)=4.16$, $p<.05$. Peer collaboration seems to be the most effective teaching method in order to develop students' interest and enjoyment in mathematics.

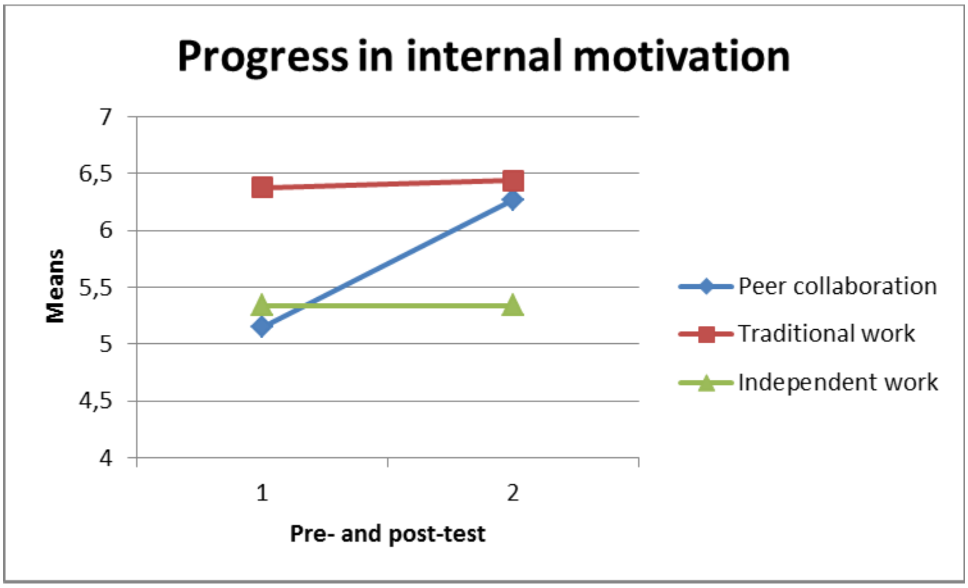

Figure 4 Progress in Internal Motivation

It looks as if students improve their internal motivation more when they learn with peers discussing mathematical issues rather than from teaching at the chalkboard or from individual practicing. Figure 5 clearly illustrates that independent work is not as effective for students' progress in self-concept as peer collaboration and traditional work are.

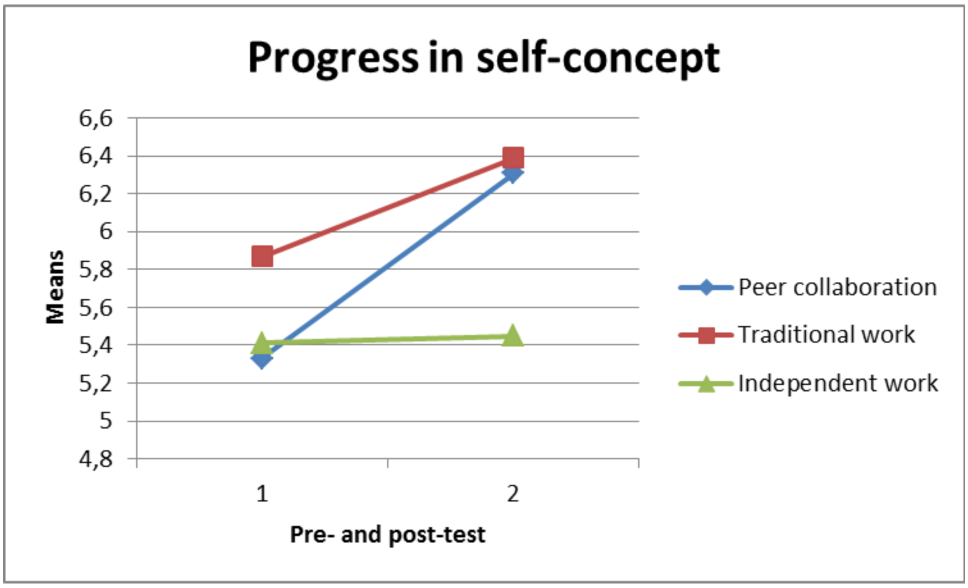

Figure 5 Progress in Self - Concept

Finally, there were no interaction effects on instrumental motivation, $F(2,116)=1.37$, $p>.05$; or anxiety, $F(2,116)=2.58$, $p>.05$.

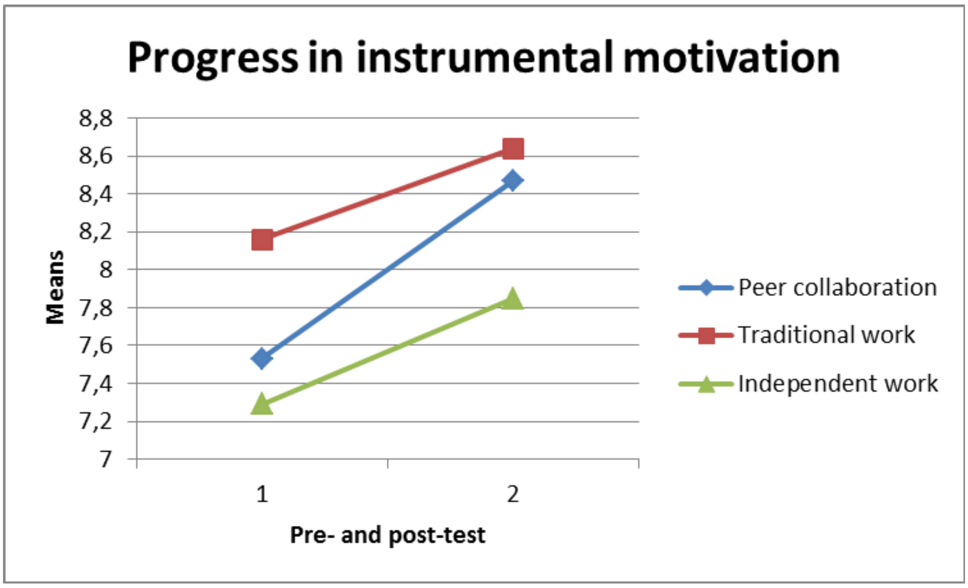

Figure 6 Progress in Instrumental Motivation

Figure 7 shows that students working with peers seem to reduce their anxiety to mathematics more than students taught with traditional and independent method. Thus, in this study there is no statistical significance in such a statement.

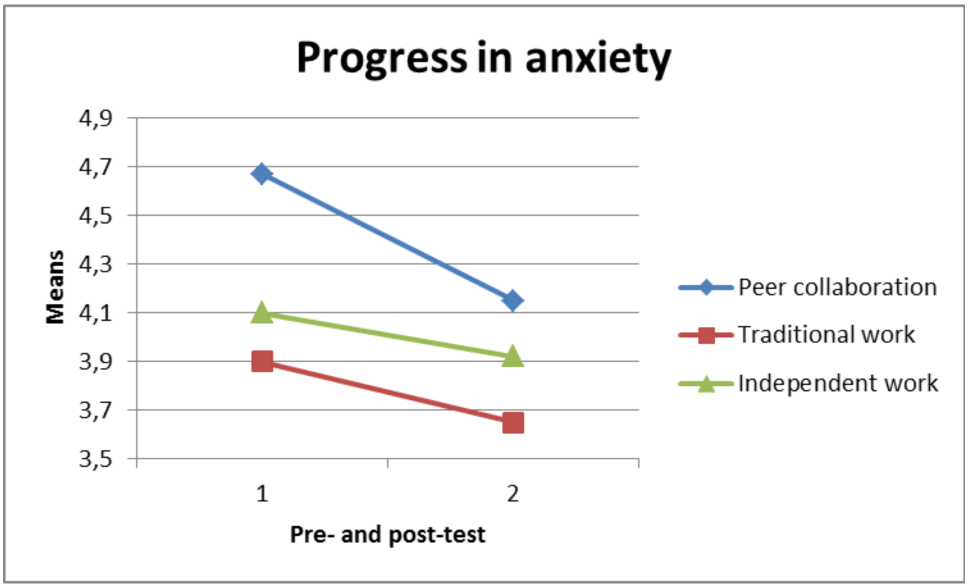

Figure 7 Progress in Anxiety

Summary: To develop aspects of self-regulated learning skills, teachers, according to this study, ought to use traditional work or collaborations with peers. Peer collaboration seems to be more effective in developing students' interest and enjoyment of mathematics than traditional work or independent work is. Traditional work and peer collaboration are more effective than independent work for students' self-concept.

Discussion

This study was designed to investigate the effect of peer collaboration compared to two ordinary, in Sweden, different teaching methods on students' arithmetic skills, and self-regulated learning skills. Before discussing the results, it is important to emphasize that the intervention we conducted did not involve total control over the classroom setting. It is impossible to control everything that could happen in an everyday classroom. Thus, we have tried to minimize the effect of certain variables. The teacher factor and the content teachers should draw attention to in their considerations with students. Teachers circulated from classroom to classroom and researchers and teachers planned and discussed all lessons in order to be as consistent as possible. Another problem with this study is the differences in students' results on the pre-test. The traditional group, performed better on pre-test, but not significant better, than the two others, which was not possible to expect. It is probable more difficult to progress from a higher level than from a lower level.

The most notable result is that we have been able to demonstrate selective effects on quantitative concept, internal motivation and self-concept. When these kinds of reports are presented, the interpretation of the results is at least as important as the description. The result will be discussed with respect to Piaget´s theory of the relation between social interaction and cognitive development as well as to earlier research in related areas. The following interpretations seem to be plausible, and they are possible starting points for further studies.

Improvement in Arithmetic

There are researchers arguing that different teaching methods draw attention to different learning outcomes (Cobb, 1998; Case, 1996; Boaler, 2002). For instance, seem peer collaboration have a positive impact on students conceptual understanding as well as on problem-solving tasks (Goods & Gailbraith, 1996; Leikin & Zaslavsky, 1997). In this study it is obvious that different teaching methods have different impacts on different aspects of arithmetic skills. Peer collaboration and traditional work are significantly better for improving students' performances in quantitative concept. The results are consistent with results presented by Goods and Gailbraith (1996), and Leikin and Zaslavsky (1997), with respect to peer collaboration and quantitative concept (conceptual understanding, and to Reynolds and Muijs (1999) with respect to traditional work and learning outcomes in mathematics. One explanation is that students who worked in traditional and peer collaboration classes are exposed to a higher level of reasoning, and that they accept this reasoning as valid (Druyan, 2001; Golbeck & Sinagra, 2000). In traditional work, the teachers provide students with explanations and relevant concepts (Crocker,1986), while students working collaboratively interact with both peers and teachers. The argument can be found in a study made by Oppendekker and Van Damme (2006). In addition, active participation and the communication of thought processes with higher ability people seem to be critical underlying factors when students are developing their conceptual understanding (quantitative concept). The language is a medium for discussing how to proceed and for restructuring ideas of peers' divergent and sophisticated range of strategies (Piaget, 1932). From this perspective, discussion provides students with the opportunity to explore variations between their own and their partners' knowledge and thinking, correct misconceptions and fill gaps in understanding (Piaget, 1959; Granström, 2006). Most importantly, in a collaborative activity, students need to convince themselves and their partners of the correctness of a particular method. In traditional teaching groups, teacher's help students to move forward and equilibration could be restored (Piaget, 1932). In addition, working with peers and learning from teachers who teach from the chalk board and draw attention to important mathematical concepts provide students with a better understanding of quantitative concepts. It seems like learning conditions characterised by communication and active students are positive for students' understanding of conceptual understanding (quantitative concepts) (Piaget, 1932; Boaler, 2002).

The communication processes do not have the same impact on procedural skills (calculation) in this study . Student who worked individually progressed more, but not significantly more, than students who worked with peers or in a traditional environment according to calculation. This is not surprising; they have practiced their procedural skills more than the traditional and problem-solving groups. Time on the specific types of tasks was longer in this group (Brophy & Good, 1986; Griffin & Barnes, 1986; Lampert, 1988;Cooney, 1994).

The results from this study tell us something different from earlier studies. In an overview of research of effective mathematics teaching, Reynolds and Muijs (1999) argue for one specific teaching method, similar to what is called a traditional method in this study, as the most effective method for learning mathematics. This study's contribution is a more specific discussion of how different teaching methods affect different mathematical proficiencies. From a teacher's perspective, when a mathematic is complex (Kilpatrick et. al., 2001), it is essential to know how different methods affect students' learning outcomes.

Improvement in self-regulated learning skills

This study also shows that different teaching methods also seem to affect students' self-regulated learning skills in different ways (Boaler). While earlier studies has shown that internal and instrumental motivation, self-concept and anxiety correlate with students' performance, the results of this study provide us with important knowledge of how to improve these self-regulated learning skills (OECD, 2004).

Prior in this article, it was shown that students who work with peers or in traditional classrooms progress more in the area of quantitative concept. A safe hypothesis is that students who notice that they understand the mathematic will find it more interesting. But this is not completely true. Students who had been taught from the chalk board did not progress in interest as much as students who had worked with peers. In order to progress in this aspect, it seems essential that students have the opportunity to discuss mathematical issues with their peers. Working together with peers appears to help students to develop a greater interest than traditional teaching and independent work do. In earlier research, Tobias (1987), argued that a traditional method with too much individual work could affect students' interest and enjoyment of mathematics in a negative way. The result of this study does not fully support the result Tobias presents. In this study, students who regularly practice alone did not lose interest. The interest and enjoyment in mathematics seem to be almost equal before and after the intervention.

Students' self-concept is affected significantly more if students work traditionally or with problem-solving. One interpretation is that, in these classes, students become aware of their knowledge more than they do when working independently. In both the traditional and peer collaboration groups, they are provided with feedback from teachers and peers. These results are strengthened by Boaler's (1999) study which found that a strong predictor of a positive self-concept in mathematics is a group climate where students interact with each other and feel support from teachers and peers. Students who work independently do not have the same opportunity to get positive feedback on their reasoning as students who interact with teachers and peers.

The educational implications of this study will be obvious. The quantity of teaching received by the students in the study is at a level that makes the application educationally realistic. The magnitude of gains achieved also makes them educationally as well as statistically significant. Theoretically, the findings could be understood with support of a Piagets theoretical framework of relation between social interaction and cognitive development. Teaching methods where students are able to interact with teachers and peers seem to promote cognitive conflicts by exposing discrepancies between peer´ own other knowledge. Discussing mathematical problems and take advantage of higher ability persons restore the equilibration and seem to be positively correlated with cognitive (quantitative concept) as well as affective outcomes (internal motivation and self-concept).

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