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ANALYSIS OF INWARD MELTING OF SPHERES SUBJECT TO CONVECTION AND RADIATION

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Abstract: In the present study, inward spherical melting of a solid subject to convection and radiation initially at the fusion temperature has been investigated. The governing equations for liquid phase and the interface have been expressed in dimensionless form and then, computational domain has been fixed using the well-known Landau transformation. The dimensionless governing equations showed that the velocity of the interface depends on Biot number, Stefan number, conduction-radiation parameter and dimensionless temperatures. The dimensionless liquid phase and interface equations have been solved numerically using a finite difference method. Employing the developed numerical model, the effects of the problem parameters on melting process have been investigated and results have been presented graphically.
Keywords: Phase change, Melting, Finite difference method.

TAŞINIM VE IŞINIMA MARUZ KALAN BİR KÜRENİN İÇE DOĞRU ERİMESİNİN ANALİZİ

Özet: Bu çalışmada taşınım ve ışınıma maruz kalan bir kürenin erimesi problemi incelenmiştir. Sıvı faza ait ısı iletim denklemi ile ara yüzey enerji dengesi denklemleri boyutsuz olarak ifade edildikten sonra, hareketli ara yüzey nedeniyle değişken olan problem bölgesi Landau dönüşümü kullanılarak sabitlenmiştir. Boyutsuz denklemler ve sınır koşulları; ara yüzey hızının Biot ve Stefan sayıları, iletim-ışınım parametresi ve boyutsuz sıcaklıklara bağlı olduğunu göstermiştir. Bu yeni sabitlenmiş koordinatlarda ifade edilen problem sonlu farklar yöntemi ile çözülmüştür. Elde edilen sayısal model kullanılarak, erime işlemi üzerine problem parametrelerinin etkisi araştırılmış ve sonuçlar grafiklerle ifade edilmiştir.
Anahtar Kelimeler: Faz değişimi, Erime, Sonlu farklar yöntemi.

NOMENCLATURE

Bi	Biot number
C_P	specific heat [kJ/(kgK)]
D	the difference between dimensionless melting environment temperature
F	radiation shape factor
Fo	Fourier number
h	heat transfer coefficient [W/(m2K)]
k	thermal conductivity [W/(mK)]
L	latent heat of fusion [kJ/kg]
N_c	conduction-to-radiation parameter
r	radial coordinate [m]
R	dimensionless radial coordinate
r_0	sphere radius [m]
s	position of solid-liquid interface [m]
S	dimensionless position of solid-liquid interface
Ste	Stefan number
t	time [s]
T	temperature [K]
U	dimensionless temperature
σ	Stefan-Boltzmann coefficient, [W/(m^{2}K^4)]
α	thermal diffusivity [m^2/s]
β	dimensionless fixed coordinate
ε	emissivity
ρ	density [kg/m^3]
τ	dimensionless time
m	melting
w	surroundings
∞	ambient fluid

INTRODUCTION

Heat transfer problems including solidification or melting processes are called heat transfer problems with phase change. The problems involving unknown boundaries are inherently non-linear even for linear differential equations. The location of interface is a part of the problem in addition to the temperature distribution in the solid and liquid phases. If the temperature gradient appears in only one phase then the problem is called one-phase otherwise two-phase (Özışık, 1980). The phase change problems can be encountered in many applications such as ice production, freezing of foods, casting, and latent heat thermal energy storage. Furthermore, one can face to phase change problems in designing of buildings and pipe lines in cold climates (Lunardini, 1981).

There are few exact solutions about phase change problems for only some idealized situations subject to simple boundary and initial conditions. For the situations for which the exact solutions are not available; approximate, semi-analytical and numerical methods have been used to solve the phase change problems. An extensive review of these methods can be found in (Crank, 1984). Phase change problems with first and third kind of boundary conditions in the spherical domain have been numerically investigated in the literature (Riley and Smith, 1974; Cho and Sunderland, 1970; Pedroso and Domoto, 1973a; Pedroso and Domoto, 1973b; Huang and Shih, 1975; Ismail and Henriquez, 2000; Bilir and İlken, 2005). Goodling and Khader (1974) numerically investigated the freezing of a sphere with convection and radiation at the surface of the sphere. The numerical results were too limited to reproduce, but, if $\sigma\varepsilon T_f^{\,3} r_0 / k < (1/3)Bi^2$, the solidification time with radiation will exceed 90% of the time only convection. Hill and Kucera (1983) presented a series solution method for the problem of freezing a saturated liquid inside a sphere with the effect of radiation at the container surface. Biot's variational method to obtain an approximate analytical solution for the phase change of a finite medium whose one surface is subject to radiative and convective cooling is used by Yeh and Chuang (1979). Also, Yan and Huang (1979) used the perturbation solution for phase change problem in a finite region whose one surface is subject to convective and radiative boundary condition, while the other is insulated. The above approximate solutions are not valid for the ranges of the problem parameters. Therefore it is need a numerical solution which is valid for wide ranges of problem parameters.

The aim of this work is to solve inward spherical melting problem of a sphere which is initially at the fusion temperature by using a finite difference method. For this purpose, applying the front-fixing transformation which is first proposed by Landau (1950) for heat diffusion equation, the problem has been expressed in a fixed coordinate system. Since the problem has singularity at the initial time, this singularity has been eliminated by the use of the starting small-time solution. In the numerical procedure, the initial layer of the melted mass is calculated using a perturbation solution for Ste<1 given by Bulunti (2003), and the model due to Shih and Tsay (1971) for Ste>1. The results from the numerical model have been presented graphically.

PROBLEM DESCRIPTION

A homogenous sphere of radius r_0 is situated at its fusion temperature, T_m, initially as shown in Figure 1. At time t=0, convective and radiative heating is applied at the sphere surface, $r=r_0$ and melting starts. If the physical properties are independent of temperature and no density change occurs during melting process and no natural convection within the liquid region, the energy equation in the liquid phase is given as;

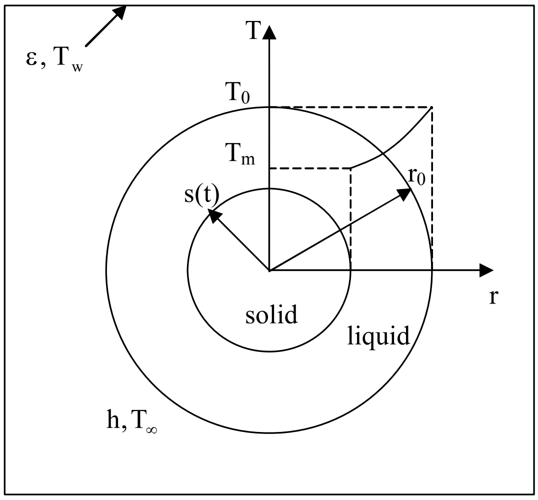

Figure 1. Geometry and coordinates for one-dimensional inward melting problem.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad r_0 < r < s(t)\,,\; t>0 \tag{1}$$

where s(t) is the location of the interface. The boundary and interface conditions of the problem are given by

$$k\frac{\partial T}{\partial r}+h(T-T_\infty)+\sigma\varepsilon(T^4-T_w^{\,4})=0 \quad r=r_0\,, t>0 \tag{2}$$

$$T(s(t),t)=T_m \tag{3}$$

$$-k\frac{\partial T}{\partial r}=\rho L\frac{ds(t)}{dt} \quad r=s(t)\,, t>0 \tag{4}$$

T_∞ and T_m in the above equations are the environment temperature and the fusion temperature, respectively, and L is the latent heat of melting. For simplicity in above equations, both the surroundings temperature T_w and the ambient fluid temperature T_∞ are assumed to be equal in the present analysis. Altough Eq. (1) is transient, due to the lack of any liquid region at initial time of melting the equation has no an initial condition (Özışık, 1980). Therefore, Eq. (1) is singular at this time. This singularity can be eliminated by the use of a starting small-time solution which will be seen in the following "Numerical Analysis" section.

Using the following dimensionless parameters

$$R=\frac{r}{r_0} \qquad S=\frac{s(t)}{r_0} \qquad \tau=\frac{kt(T_\infty-T_m)}{\rho L r_0^{\,2}} \tag{5a}$$

$$Bi=\frac{hr_0}{k} \quad Ste=\frac{C_P(T_\infty-T_m)}{L} \quad U=\frac{T}{T_\infty-T_m} \tag{5b}$$

$$Fo=\frac{\tau}{Ste} \qquad Nc=\frac{k}{\sigma(T_w-T_m)^3 r_0} \tag{5c}$$

the formulation of the problem reduces to

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial U}{\partial R}\right)=\mathrm{Ste}\frac{\partial U}{\partial \tau} \tag{6}$$

$$-\frac{\partial U}{\partial R}=\mathrm{Bi}(U-U_\infty)+\frac{1}{\mathrm{Nc}}\left(U^4-U_\infty{}^4\right) \tag{7}$$

$$U(S,R=S)=U_m \tag{8}$$

$$\frac{dS}{d\tau}=-\frac{\partial U}{\partial R}\Big|_{R=S} \tag{9}$$

The Landau transformation is used to convert the moving boundary problem into one of fixed domain (Landau, 1950). Thus, movement of the interface is inserted into the liquid phase equation and interface velocity equation. The new spatial variable in the liquid is

$$\beta=\frac{R-S}{1-S} \tag{10}$$

Using the new spatial variable, the governing equations and their initial and boundary conditions can be written as follows:

$$\left[\frac{1}{\mathrm{Ste}(1-S)^2}\right]\frac{\partial^2 U}{\partial \beta^2}+\left[\frac{2}{\beta(1-S)+1}\frac{1}{\mathrm{Ste}(1-S)}+\frac{\beta}{1-S}\frac{dS}{d\tau}\right]\frac{\partial U}{\partial \beta}=\frac{\partial U}{\partial \tau} \tag{11}$$

$$-\frac{1}{1-S}\frac{\partial U}{\partial \beta}=\mathrm{Bi}(U-U_\infty)+\frac{1}{\mathrm{Nc}}\left(U^4-U_\infty{}^4\right),\ \beta=1 \tag{12}$$

$$U=U_m\ \ \beta=0 \tag{13}$$

$$-\frac{1}{1-S}\frac{\partial U}{\partial \beta}=\frac{dS}{d\tau}\ \ \beta=0 \tag{14}$$

NUMERICAL SOLUTION

The numerical solution is realized using the finite difference approximation and the front-fixing approach.

Equation (11) is rewritten as follows.

$$\frac{\partial U}{\partial \tau}=f(S)\frac{\partial^2 U}{\partial \beta^2}+g(S,\beta)\frac{\partial U}{\partial \beta} \tag{15}$$

To obtain the finite difference form of Equation (15), we may use the Cranck-Nicolson method which is an implicit scheme (Pepper and Baker, 1993). The method averages the new and old values in time. Subscript, i, may be used to designate the location of discrete nodal points. The integer n is introduced for discretizing in time as

$$\tau=n\,\Delta\tau \tag{16}$$

The finite difference form of Equation (15) is expressed as follows.

$$\begin{aligned}\frac{U_i{}^{n+1}-U_i{}^{n}}{\Delta\tau}=&\frac{f(S^{n+1})}{2}\frac{U_{i-1}{}^{n+1}-2U_i{}^{n+1}+U_{i+1}{}^{n+1}}{\delta^2}\\&+\frac{f(S^{n})}{2}\frac{U_{i-1}{}^{n}-2U_i{}^{n}+U_{i+1}{}^{n}}{\delta^2}\\&+\frac{g(S^{n+1},i\delta)}{2}\frac{U_{i+1}{}^{n+1}-U_{i-1}{}^{n+1}}{2\delta}\\&+\frac{g(S^{n},i\delta)}{2}\frac{U_{i+1}{}^{n}-U_{i-1}{}^{n}}{2\delta}\end{aligned} \tag{17}$$

There are three unknowns as in the implicit method, i.e., $U_{i-1}^{n+1}, U_i^{n+1}, U_{i+1}^{n+1}$. In this case, we have averaged the unknown values with the previously calculated values at the same nodal locations. The averaging yields a solution that is now $O(\Delta\tau^2)$ in time and is unconditionally stable (Pepper and Baker, 1993).

For the interior points, Equation (17) can be rewritten as follows:

$$X_iU_{i-1}^{n+1}+Y_iU_i^{n+1}+Z_iU_{i+1}^{n+1}=T_i \tag{18}$$

where

$$X_i=-\Delta\tau\, f(S^{n+1})+\frac{\delta}{2}\Delta\tau\, g(S^{n+1},i\delta) \tag{19a}$$

$$Y_i=2\delta^2+2\Delta\tau\, f(S^{n+1}) \tag{19b}$$

$$Z_i=-\Delta\tau\, f(S^{n+1})-\frac{\delta}{2}\Delta\tau\, g(S^{n+1},i\delta) \tag{19c}$$

$$T_i=A_iU_{i-1}^{n}+B_iU_i^{n}+C_iU_{i+1}^{n} \tag{19d}$$

The coefficients in Equation (19d) are defined as follows.

$$A_i=\Delta\tau\, f(S^{n})-\frac{\delta}{2}\Delta\tau\, g(S^{n},i\delta) \tag{20a}$$

$$B_i=2\delta^2-2\Delta\tau\, f(S^{n}) \tag{20b}$$

$$C_i=\Delta\tau\, f(S^{n})+\frac{\delta}{2}\Delta\tau\, g(S^{n},i\delta) \tag{20c}$$

In order to complete the finite difference formulation, Equation (17) is written for end points and the boundary conditions given in Equations (12) and (13) are used. Thus, a nonlinear algebraic equation system is obtained as the following matrix form.

$$[K]^{n+1}\{U\}^{n+1} = \{M\}^{n,n+1} \tag{21}$$

where $\{U\}^{n+1}$ and $\{M\}^{n,n+1}$ are the N-dimensional column vectors, and $[K]^{n+1}$ is a NxN dimensional tridiagonal matrix.

The initial layer of the melted mass is calculated using the perturbation series solution given by Bulunti (2003) for Ste<1 and the semi-analytical model due to Shih and Tsay (1971) for Ste>1. The solution of the system of nonlinear equations given Equation (21) can be provided according to following algorithm.

1) Calculate the initial temperature distribution and interface location from two previous studies in the literature (Bulunti, 2003; Shih and Tsay, 1971).
2) Perform the simple predictions using linear interpolation

$$S^{n+1} = 2S^n - 1 \tag{22a}$$

$$U_N^{n+1} = (U_N^n + U_m)/2 \tag{22b}$$

3) In order to obtain $\{U\}^{n+1}$, solve Equation (21) using tridiagonal matrix algorithm.
4) To calculate the interface location at new time, perform the integration of the interface equation given Equation (14) on the interval (τ_n, τ_{n+1})

$$S^{n+1} = 1 - \left[1 + 2\left.\frac{\partial U}{\partial \beta}\right|_{\beta=0} + (S^n)^2 - 2(S^n)\right]^{1/2} \tag{23}$$

5) If $\left|1 - \frac{U_N^{n+1}}{U_N^n}\right| < Tol_{value}$ and $\left|1 - \frac{S^{n+1}}{S^n}\right| < Tol_{value}$ then

let $S^n = S^{n+1}$ and $U_N^n = U_N^{n+1}$ go to (2) otherwise go to (3).

The iterative process is repeated until convergence, i.e., when the maximum norm of the relative difference between two successive iterates is within a tolerance of 10^{-5}. Numerical solution is carried out by taking different values of $\Delta\tau$ for convergence of solution.

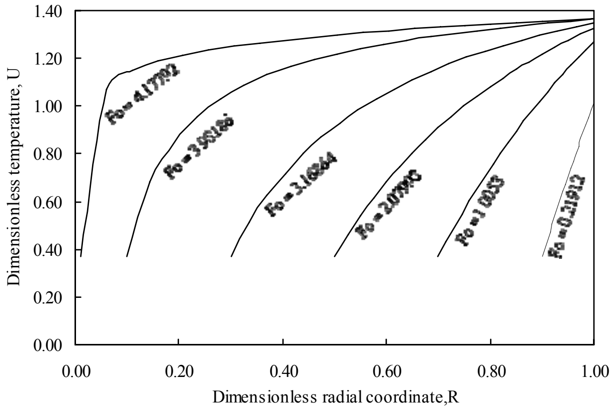

Figure 2. Temperature distributions in the liquid phase (Ste=0.05, Bi=1.0, U_∞=1.37, U_m=0.37, N_c=0.45).

RESULTS AND DISCUSSION

Figure 2 shows the temperature distribution within the liquid phase for different Fourier number and constant values of problem parameters. From the figure, it can be seen that the surface temperature of the sphere increases with Fourier number and approaches the dimensionless ambient fluid temperatures, i.e. U_∞.

Figure 3 shows that the variation of interface location with Fourier number for different Biot and Stefan numbers. It can be seen that Stefan number plays more important role on interface velocity than Biot number. During the melting process, the heat transfer mechanisms undergone in the phase change material are controlled by two different heat transfer rates. One is the absorbed latent heat during melting, and the other is the heat transfer inside the phase change material. When the melting front moves inward the absorbed latent heat decreases due to the decreased melting mass for unit movement of the interface and the heat transfer rate is reduced due to the increased conduction thermal resistance of the melting phase change material. The curves in Fig. 3 are nearly linear except for their ends. As can be seen from Figure 3, the first and second effects are dominant at the beginning and at the end of melting process, respectively.

Figure 4 presents the dimensionless interface location as a function of Fourier number for four different conduction-to-radiation parameters. The results from the figure show that the total melting time strongly depends on the conduction-to-radiation parameter.

Figure 5 shows the variation of complete melting time with Stefan number for different values of Biot numbers. For the same values of Biot numbers, complete melting time decrease for the different values of Stefan numbers. Consequently, the decrease of Stefan number leads to increasing the time for complete melting.

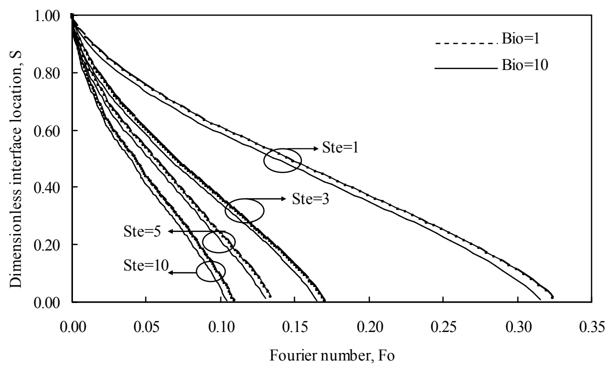

Figure 3. The variation of interface location with Fourier number for different Biot and Stefan numbers (U_∞=1.37, U_m=0.37, N_c=0.45).

CONCLUSIONS

Inward phase change of a spherical body subject to radiation and convection at the surface is considered.

The initial temperature of the body is assumed constant at the fusion temperature and the boundary surface temperature is assumed to change simultaneously. The governing equations for liquid phase and the interface have been expressed in dimensionless form and then, computational domain has been fixed using the well-known Landau transformation. The dimensionless liquid-phase and interface equations have been solved numerically using a finite difference method. In the present study, it has been observed that the velocity of the interface depends on Biot number, Stefan number, conduction-radiation parameter and dimensionless temperatures. Employing the present numerical model, the effects of the problem parameters on melting process are investigated and results are presented graphically.

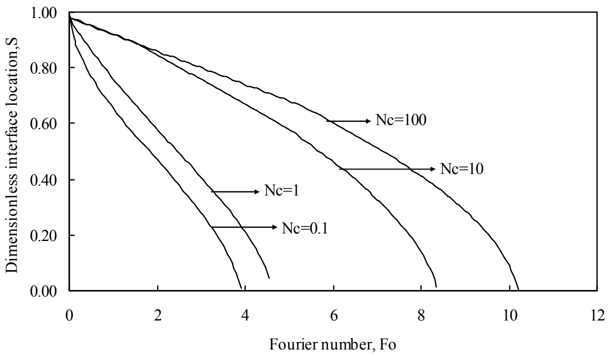

Figure 4. The variation of interface location with Fourier number for different conduction-to-radiation parameter (Ste=0.05, Bi=1.0, U∞=1.37, Um=0.37)

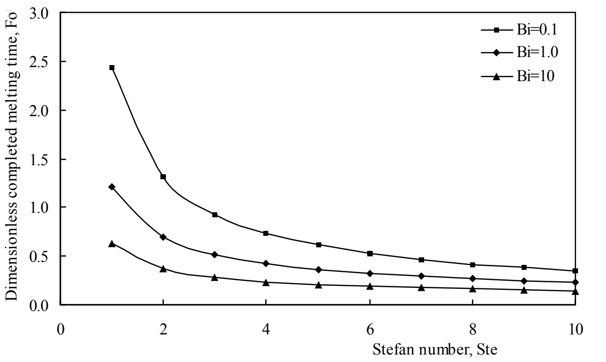

Figure 5. Completed melting time as a function of Stefan number for three different Biot numbers (U_∞=4.44, U_m=3.44, N_c=387).

REFERENCES

Bilir, L., and İlken, Z., Total Solidification Time of a Liquid Phase Change Material Enclosed In Cylindrical/Spherical Containers, *Applied Thermal Engineering,* 25, 1488-1502, 2005.

Bulunti, B., *Küresel Koordinatlarda İçe Doğru Erime Probleminin Analizi*, Yüksek Lisans Tezi, Atatürk Üniversitesi Fen Bilimleri Enstitüsü, Erzurum, 2003.

Caldwell, J. and Kwan, Y.Y., On the Perturbation Method for the Stefan Problem with Time-Dependent Boundary Conditions, *Int. Journal of Heat and Mass Transfer,* 46, 1497-1501, 2003.

Cho, S.H., and Sunderland, J.E., Phase Change of Spherical Bodies, *International Journal of Heat and Mass Transfer,* 13, 1231-1233, 1970.

Chuang, Y.K., and Ehrich, O., On the Integral Technique for Spherical Growth Problems, *International Journal of Heat and Mass Transfer,* 17, 945-953, 1973.

Chung, B.T.F., and Yeh, L.T., Solidification and Melting of Materials Subject to Convection and Radiation, *Journal of Spacecraft,* 12, 329-333, 1975.

Crank, J., *Free and Moving Boundary Problems*, Clarendon Press, 1984.

Dewynne, J.N., and Hill, J.M., On an Integral Formulation for Moving Boundary Problems, *Quart. Appl. Math,* 42, 443-455, 1984.

Fomin, S.A. and Saitoh, T.S., Melting of Unfixed Material in Spherical Capsule with Non-isothermal Wall, *International Journal of Heat and Mass Transfer* 42, 4197-4205, 1999.

Goodling, J.S. and Khader, M.S., Inward Solidification with Radiation-Convection Boundary Condition, *Journal of Heat Transfer,* 96, 114-115, 1974.

Hill, J.M., and Kucera, A., Freezing a Saturated Liquid Inside a Sphere, *International Journal of Heat and Mass Transfer,* 26, 1631-1637, 1983.

Huang, C.L., and Shih, Y.P., A perturbation Method for spherical and cylindrical solidification, *Chem. Eng. Sc.*, 30, 897-906, 1975.

Ismail, K.A.R., and Henriquez, J.R., Solidification of PCM Inside a Spherical Capsule, *Energy Conversion and Management,* 41**,** 173-187, 2000.

Landau, H.G., Heat conduction in a melting solid, *Quart. Appl. Math*, 8**,** 81-94, 1950

Lunardini, V.J., *Heat Transfer in Cold Climates*, Van Nostrand Reinhold Company, 1979.

Moore, F.E., and Bayazitoglu Y., Melting within a Spherical Enclosure, *Journal of Heat Transfer,* 104, 19-23, 1982.

Özışık, M.N., *Heat Conduction,* Wiley & Sons, 1980.

Pedroso, R.I., and Domoto, G.A., Inward Spherical Solidification Solution by the Method of Strained coordinates, *International Journal of Heat and Mass Transfer,* 16, 1037-1043, 1973a.

Pedroso, R.I., and Domoto, G.A., Perturbation Solutions for Spherical Solidification of Saturated Liquids, *Journal of Heat Transfer,* 95, 42-46, 1973b.

Pepper, D.W., and Baker, A.J., Finite Differences versus Finite Elements: *In Handbook of Numerical Heat Transfer*, W.J. Minkowycz et al. (eds.), Wiley & Sons, 1993.

Riley, D.S., and Smith, F.T., The Inward Solidification of Spheres and Circular Cylinders, *International Journal of Heat and Mass Transfer,* 17, 1507-1516, 1974.

Tarzia, D.A., Turner, C.V., Estimation of the Occurrence of the Phase-Change Process in Spherical Coordinates, *Int. Comm. Heat Mass Transfer,* 26, 559-568, 1999.

Yan, M.M. and Huang, P.N.S., Perturbation Solutions to Phase Change Problem Subject to Convection and Radiation, *Journal of Heat Transfer,* 101, 96-100, 1979.

Yeh, L.T. and Chung, B.T.F., A Variational Analysis of Freezing or Melting in a Finite Medium Subject to Radiation and Convection, *Journal of Heat Transfer,* 101, 592-597, 1979.