Isı Bilimi ve Tekniği Dergisi, 28, 2, 17-24, 2008
J. of Thermal Science and Technology

ISSN 1300-3615

FORCED CONVECTION FLOW OF VISCOUS DISSIPATIVE POWER-LAW FLUIDS IN A PLANE DUCT
Part 2. Thermally Developing Flow

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(Geliş Tarihi: 01. 02. 2008, Kabul Tarihi: 12. 05. 2008)

Abstract: In this part of the study, we investigate the problem of hydrodynamically developed but thermally developing laminar forced convection of a power-law fluid in a plane duct when the viscous dissipation is included. The axial heat conduction in the fluid is neglected. Two different thermal boundary conditions are considered: the constant heat flux (H1 boundary condition) and the constant wall temperature (T boundary condition). The combined and interactive influences of the power-law index and the Brinkman number on the distributions for the developing temperature profile and local Nusselt number are determined in the entrance region for the wall heating and cooling cases at T and H1 boundary conditions.
Keywords: Non-Newtonian fluid, Power-law fluid, Thermally developing, Viscous dissipation, Plane duct, Constant heat flux, Constant wall temperature.

BİR DÜZLEMSEL KANAL İÇERİSİNDEKİ VİSKOZ YAYILIMLI POWER-LAW AKIŞKANLARIN ZORLANMIŞ TAŞINIM AKIŞI
Kısım 2. Termal Gelişen Akış

Özet: Çalışmanın bu bölümünde, düzlemsel bir kanalda, hidrodinamik olarak tam gelişmiş termal olarak gelişmekte olan *power-law akışkanın* laminer zorlanmış taşınımı, viskoz yayılım etkileri dahil edilerek incelenmiştir. Akışkan içerisindeki eksenel iletim ihmal edilmiştir. Sabit ısı akısı (H1 sınır koşulu) ve sabit yüzey sıcaklığı(T sınır koşulu) olmak üzere iki farklı termal sınır koşulu ele alınmıştır. Power-law indeksi ve Brinkman sayısının, giriş bölgesinde, gelişmekte olan sıcaklık dağılımı ve yerel Nusselt sayısı üzerindeki etkisi, T ve H1 sınır koşullarında, sıcak ve soğuk cidar durumları için belirlenmiştir.
Anahtar kelimeler: Newtonyen olmayan akışkan, Power-law akışkan, Termal gelişen, Viskoz yayılım, Düzlemsel kanal, Sabit ısı akısı, Sabit yüzey sıcaklığı.

NOMENCLATURE

Br	Brinkman number, Eq. (6)
Br_q	modified Brinkman number, Eq. (16)
c_p	specific heat at constant pressure
k	thermal conductivity [W/mK]
n	power-law index
Nu	Nusselt number
q_w	wall heat flux [W/m^2]
Re	Reynolds number, Eq. (5)
Pr	Prandtl number, Eq. (5)
T	temperature [K]
u	velocity [m/s]
U	dimensionless velocity
W	width of the duct (=2w) (m)
Y	dimensionless vertical coordinate
z	axial direction [m]
Z	dimensionless axial coordinate, Eq. (5)

Greek symbols

α	thermal diffusivity [m^2/s]
η	consistency factor employed in equation (1)
ρ	density [kg/m^3]
υ	kinematic viscosity [m^2/s]
θ	dimensionless temperature, Eq. (5,14)

Subscripts

e	fluids entering
m	mean
w	wall

INTRODUCTION

The flow and heat transfer of non-Newtonian fluids through ducts have wide potential applications in many engineering areas including the chemical, petroleum, polymer, food processing, pharmaceutical and biochemical and biomedical engineering. The most non-Newtonian fluids of practical interest are highly viscous and, therefore, are often processed in the laminar flow regime. Readers are referred to see the excellent reviews by Irvine and Karni (1987) and Hartnett and Choi (1998).

In the first part of this study (Aydın and Avcı, 2008), it was shown that the viscous dissipation had a considerable effect on the hydrodynamically and thermally fully developed flow and heat transfer of a power-law fluid in a plane duct for the constant wall temperature (T type) and the constant heat flux (H1 type) thermal boundary conditions at the wall. Now, we will focus our interest on the hydrodynamically developed, but thermally developing laminar flow problem (the so-called Graetz problem) for the same geometry, fluid type and thermal conditions.

However, in the existing convective heat transfer literature on the non-Newtonian fluids, the effect of the viscous dissipation has been generally disregarded. There are only a few studies to be cited. Lawal and Mujumdar (1984,1992) studied viscous dissipation effect on heat transfer for power law fluids in arbitrary cross-sectional ducts. Flores et al. (1991) studied the Graetz problem for the case of a power-law fluid either in the cylindrical geometry or the plane geometry by considering a constant temperature at the duct wall in the presence of viscous dissipation. Dang (1983) studied the effect of viscous dissipation in the thermal entrance region in a pipe using the uniform wall temperature. Barletta (1997) studied the asymptotic behavior of the temperature field for the laminar and hydrodynamically developed forced convection of a power-law fluid which flows in a circular duct taking the viscous dissipation into account. The asymptotic Nusselt number and the asymptotic temperature distribution were evaluated analytically in the cases of either uniform wall temperature or convection with an external isothermal fluid. Wei and Luo (2003) investigated a Graetz-Nusselt type problem of incompressible non-Newtonian fluids with temperature dependent power-law viscous dissipation by using a Galerkin method with linear axisymmetric triangular finite elements.

In a recent study, Aydın and Avcı (2006) studied the thermally developing laminar forced convection flow of a Newtonian-fluid in plane duct considering the effect of the viscous dissipation.

This chapter is followed by the Analysis, Results and Discussion and Conclusions chapters.

ANALYSIS

The flow is considered to be hydrodynamically fully developed but thermally developing. This problem is traditionally termed as the "Graetz" problem. Steady, laminar flow having constant properties (i.e. The thermal conductivity and the thermal diffusivity of the fluid are considered to be independent of temperature) is considered. The axial heat conduction in the fluid and in the wall is assumed to be negligible. The shear stress and strain relationship for the power-law type Ostwald-de Waele fluid is given as

$$\tau_{yz} = \eta \left| \frac{du}{dy} \right|^{n-1} \frac{du}{dy} \tag{1}$$

where n represents the power-law index and the case of n<1, n=1 and n>1 correspond pseudoplastic, Newtonian and dilatant behaviors. The velocity profile for fully developed plane duct flow is given as follows:

$$\frac{u}{u_m} = \left(\frac{2n+1}{n+1} \right) \left[1 - \left(\frac{y}{w} \right)^{(n+1)/n} \right] \tag{2}$$

Since a fully developed velocity profile is assumed for thermally developing flow, the energy equation including viscous dissipation effect can be represented by

$$\frac{\partial^2 T}{\partial y^2} = \frac{u}{\alpha} \frac{\partial T}{\partial z} - \frac{1}{k} \tau_{yz} \frac{du}{dy} \tag{3}$$

where ρ, c_p, k and μ are density, specific heat, thermal conductivity and viscosity. The second term in the right hand side is the viscous dissipation term.

Due to axisymmetry at the center, the thermal boundary condition at $y=0$ can be written as:

$$\left. \frac{\partial T}{\partial y} \right|_{y=0} = 0 \tag{4}$$

Two kinds of thermal boundary condition at wall are considered in this study, namely: constant wall heat flux (H1 type) and constant wall temperature (T type). They are treated separately in the following:

Constant Wall Temperature (T Type)

Introducing the following dimensionless variables

$$U = \frac{u}{u_m}, \quad \theta = \frac{(T_w - T)}{(T_w - T_e)}, \quad Y = \frac{y}{W},$$

$$Z = \frac{z/W}{Re\,Pr}, \quad \Pr = \frac{\eta c_p}{k} \left(\frac{u_m}{W} \right)^{n-1}, \quad \mathrm{Re} = \frac{\rho u_m^{\,2-n} W^n}{\eta} \tag{5}$$

where u_m is the mean velocity in the plane duct and Re is the Reynolds number based on this mean velocity and the width of the duct, W, which is equal to $2w$. The dimensionless variable Z is termed the Graetz number. Then, Eq. 3 becomes

$$U\frac{\partial\theta}{\partial Z}=\frac{\partial^2\theta}{\partial Y^2}-Br\left(\frac{2n+1}{n}\right)^{n+1}2^{(n+1)^2/n}\,Y^{(n+1)/n} \quad (6)$$

where Br is the Brinkman number, which is defined as:

$$Br=\frac{\eta\,u_m^{n+1}}{k\,W^{n-1}\left(T_w-T_e\right)} \quad (7)$$

For this T-type thermal boundary condition, the wall temperature of the wall is kept isothermal in the entrance region, which is mathematically shown as:

$$\text{For } z>0: \quad T=T_w \quad \text{at} \quad y=w \quad (8)$$

In dimensionless form, the thermal boundary conditions that will be applied in the solution of the energy equations are given as:

$$Y=0: \qquad \frac{\partial\theta}{\partial Y}=0$$
$$Y=0.5: \qquad \theta=0 \quad (9)$$

The local Nusselt number is obtained from

$$Nu_W=-\left.\frac{\partial\theta}{\partial Y}\right|_{Y=0.5} \quad (10)$$

It should be noted the Nusselt number being used here is based on the difference between the wall and the inlet temperatures, i.e., on $T_w - T_e$, and not on the difference between the wall and the mean temperatures. Now the mean temperature, i.e. the bulk mean temperature is given by Oosthuizen and Naylor (1999).

$$T_m=\frac{\int\rho\,u\,T\,dA}{\int\rho\,u\,dA} \quad (11)$$

Rewriting this equation in terms of the dimensionless variables:

$$\frac{T_w-T_m}{T_w-T_e}=\frac{\int_0^{0.5}U\,\theta\,dY}{\int_0^{0.5}U\,dY} \quad (12)$$

The Nusselt number based on the difference between the wall and the mean temperature is then given by:

$$Nu_{Wm}=Nu_W\frac{T_w-T_e}{T_w-T_m}=Nu_W\frac{\int_0^{0.5}U\,\theta\,dY}{\int_0^{0.5}U\,dY} \quad (13)$$

Constant Heat Flux (H1 Type)

Now, we will consider the constant heat flux case, $q_w=c$. In this case, the following dimensionless temperature is used:

$$\theta=\frac{\left(T-T_e\right)}{\left(q_wW/k\right)} \quad (14)$$

With this definition, the energy equation can be written in dimensionless form as:

$$U\frac{\partial\theta}{\partial Z}=\frac{\partial^2\theta}{\partial Y^2}+Br_q\left(\frac{2n+1}{n}\right)^{n+1}2^{\frac{(n+1)^2}{n}}\,Y^{(n+1)/n} \quad (15)$$

where Br_q is the modified Brinkman number, which is given as:

$$Br_q=\frac{\eta u_m^{\,n+1}}{q_wW^n} \quad (16)$$

The entrance condition at the beginning of the thermally developing region in this case is defined:

$$Z=0: \quad \theta=0 \quad (17)$$

and the wall thermal boundary condition is

$$k\left.\frac{\partial T}{\partial y}\right|_{y=w}=q_w \quad (18)$$

where q_w is positive when its direction is to the fluid (wall heating), otherwise it is negative (wall cooling).
In dimensionless form, it can be written as:

$$\left.\frac{\partial\theta}{\partial Y}\right|_{Y=1}=1 \quad (19)$$

For this case, the Nusselt number is given:

$$Nu_W=\frac{1}{\theta_w} \quad (20)$$

and

$$Nu_{Wm}=\frac{1}{\theta_w-\theta_m} \quad (21)$$

For each case, the energy equation has been solved numerically using the difference method. The details of the solution procedure can be found in Oosthuizen and Naylor (1999).

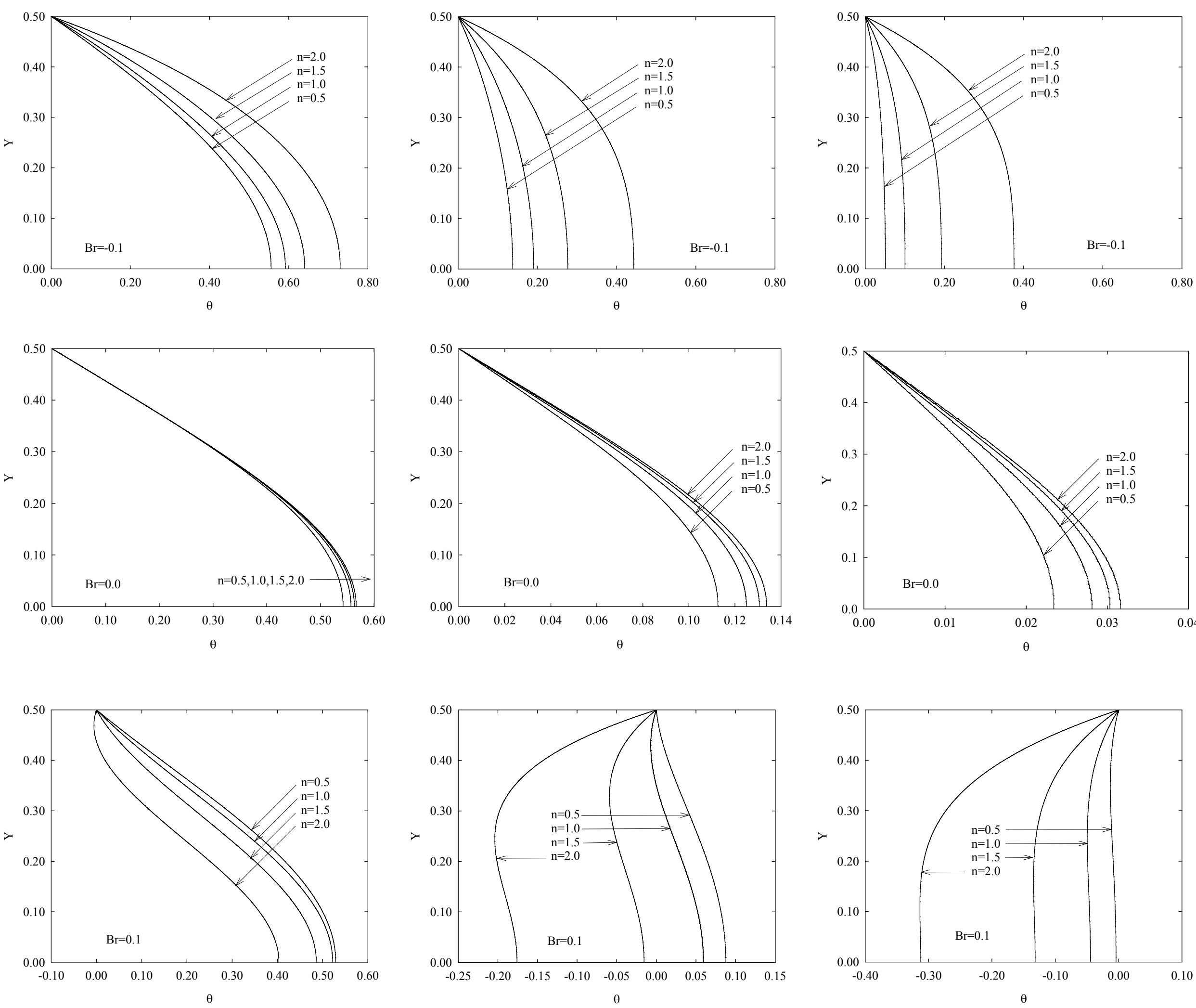
RESULTS AND DISCUSSION

In order to understand heat transfer processes taking place, temperature profiles at certain locations are required. For constant wall temperature at the wall, Fig. 1 depicts the variations in the temperature distributions with the power-law index at different axial locations for Br=-0.1, 0 and 0.1.

Four different values of the power-law index are considered: n=0.5, 1, 1.5 and 2. Remember n=1 represents the Newtonian behavior while n<1 and n>1 representing the pseudoplastic and dilatant behaviors, respectively. As seen, in all the cases considered, with an increase at the Brinkman number, dimensionless temperature gradient decreases. The change of Br significantly influences the temperature profile due to the irreversible energy conversion originating from viscous dissipation. Br=0 represents the case without the viscous dissipation effect. Br=0.1 corresponds to the hot wall or the wall heating (heat is being supplied across the walls into the fluid) case ($T_w>T_c$), while the case of Br=-0.1 representing the cold wall or the wall cooling case ($T_w<T_c$). Viscous dissipation affects the temperature profile by playing a role like an energy source. It increases the temperature of the bulk fluid. Its effect becomes the most significant near the wall due to the highest shear stress occurring there. As seen from the figure, for the wall heating and cooling cases (Br=0.1 and -0.1), the influence of the power-law index on the temperature profile becomes more profound than that for Br=0.

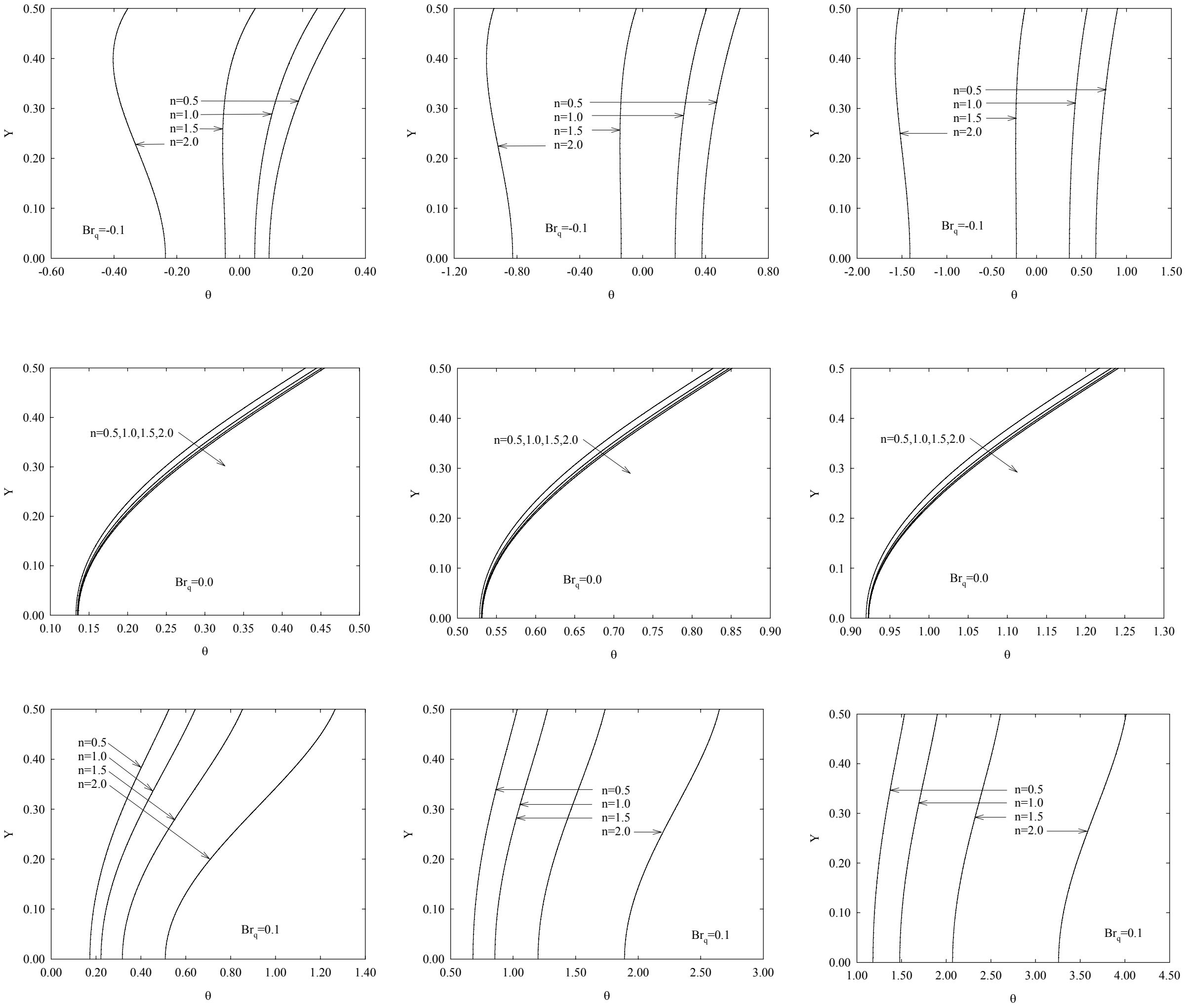
Figure 2 shows the variations in the temperature distributions with the power-law index at different axial locations for Br_q=1, 0 and -1 for the constant heat flux thermal boundary condition at the wall (H1 type). The behaviors seen can be explained similarly to those in Fig. 1.

For the T-type thermal boundary condition at the wall, the behavior of Nu_{Wm} in the downstream for different values of the power-law index is shown in Fig. 3 for different values of Br.



Z=0.1 Z=0.3 Z=0.5

Figure 1. Dimensionless temperature distributions in terms of the power-law index for different Br at the constant wall temperature case.



Z=0.1 Z=0.3 Z=0.5

Figure 2. Dimensionless temperature distributions in terms of the power-law index for different Br_q at the constant wall heat flux case.

As seen, for the case without the viscous dissipation effect, Nu_{Wm} immediately settles to its fully developed values obtained in the Part 1 [3], which validates the numerical method used here. At Br=0, the case without viscous dissipation, Nu decreases with increasing n for n>1 (the dilatant behavior), while an enhancement in heat transfer is observed (i.e. Nu increases) with decreasing n for n<1 (the pseudoplastic behavior). For the wall heating case, at Br=0.01, we observe some singularities. A decrease at n shifts the singularity point ahead in the downstream. For Br=0.1, these singular points are attained earlier. These singularities can be explained in terms the competition between the heat flux supplied by the wall and the viscous dissipation generated heat. As seen initially, Nu_{Wm} decreases up to the singular point as a result of decreasing temperature difference between the wall temperature and the bulk fluid temperature resulting from the viscous dissipation heat. For the cold or cooling wall situation, the viscous dissipation leads to higher temperature differences between the wall and the bulk fluid with the increasing Br. Interestingly, in the downstream, initially, we obtain superior Nu values for the n>1-fluids than n<1, as opposed the common behavior. Further downstream, we finally reach the common behavior, an enhancement in the heat transfer for the n<1-fluids with the decreases for the n>1-fluids. Interestingly, it should be noted that in the presence of the viscous dissipation (Br≠0), the steady-state value of the Nusselt number does not change for any values of the Brinkman number at a constant value of the power-law index. For n=0.5, 1, 1.5 and 2, Nu receives the following corresponding values, respectively: 10.7800, 8.7413, 8.0543 and 7.7107 for Br≠0 while they are 3.9697, 3.7706, 3.6857 and 3.6391 for Br=0.

Figure 4 illustrates the behavior of Nu_{Wm} in the downstream for different values of the power-law index at different values of Br_q for the constant heat flux case at the wall. For Br_q=0, Nu immediately receives its steady state value for each n given in Part 1 [3], see Fig. 4a. Including the viscous dissipation decreases Nu for each n as result of decreasing temperature differences between the wall and the bulk fluid (Fig. 4b,c). And, an

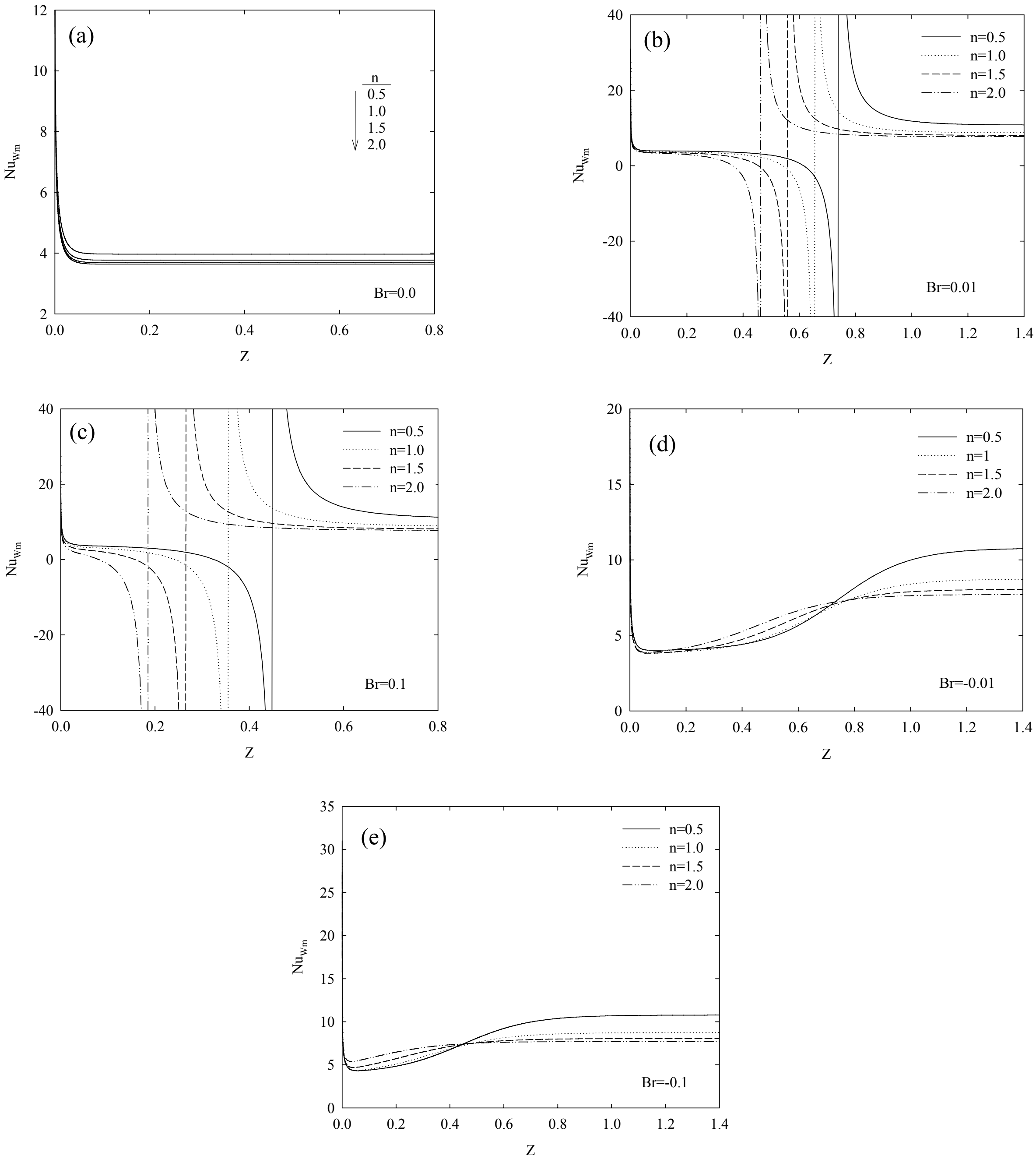


Figure 3. Variation of Nu_{Wm} with n for different Br at the constant wall temperature case.

enhancement in the heat transfer is obtained for the pseudoplastic fluids (n<1), while the opposite is true for the dilatant fluids (n>1). For the cooling wall case, we can easily observe increases at Nu values resulting from decreased temperature differences due to viscous heating inside the fluid. At Br_q=-0.1, interestingly, the pseuplastic fluid (n=0.5) gives lower Nu than the Newtonian fluid (n=1), as opposed to the common behavior (see Fig. 4e). For the dilatant fluids, at n=2, just right after the entrance, a singularity at Nu is observed (Fig. 4e).

Finally, it should be noted that comparisons are possible for steady-state values of the pseudoplastic and dilatant fluids in the presence of the viscous dissipation since we use a different Brinkman number definition, which is based on the duct entrance temperature, than that in Part 1.

CONCLUSION

We have studied hydrodynamically developed, but thermally developing forced convection flow of power-law fluids in a plane duct (the Graetz problem) by taking the effect of viscous dissipation into account. The axial conduction in the fluid is neglected. Two types of wall thermal boundary condition have been considered, namely: constant heat flux (H1-type) and constant wall temperature (T-type). Either wall heating or wall cooling case is examined.

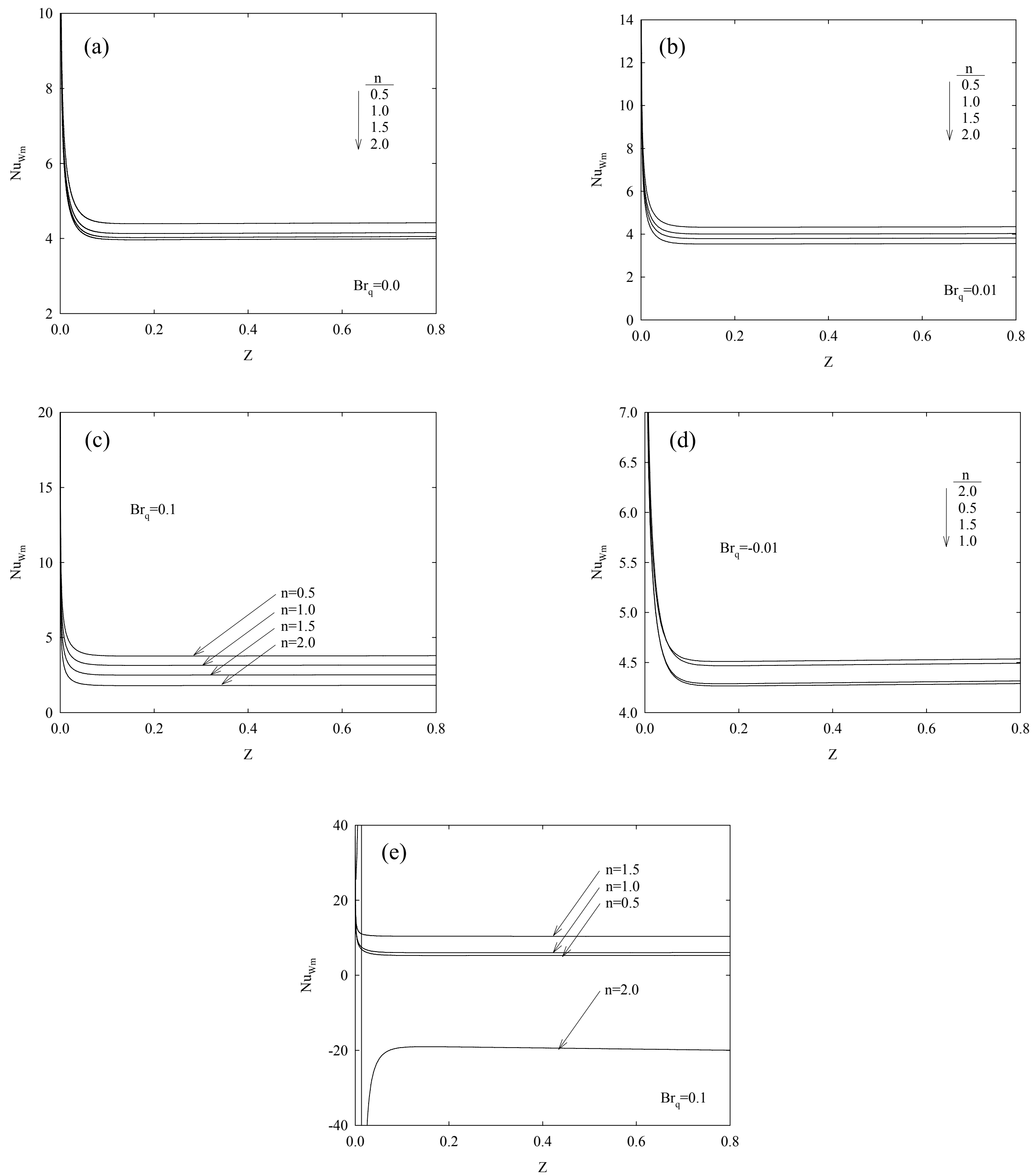


Figure 4. Variation of Nu_{Wm} with n for different Br_q at the constant wall heat flux case.

REFERENCES

Aydın, O. and Avcı, M., Forced convection flow of viscous dissipative power-law fluids in a plane duct Part 1. Hydrodynamically and Thermally developed flow submitted.

Aydın, O. and Avcı, M., Viscous dissipation effects on the heat transfer in a Poiseuille flow, *Applied Energy* 83, 495-512, 2006.

Barletta, A., Fully developed laminar forced convection in circular ducts for power-law fluids with viscous dissipation, *Int. J. Heat Mass Transfer* 40, 15-26, 1997.

Dang, V. D., Heat transfer of a power-law fluid at low Peclet number flow, *J. Heat Transfer* 105 542-549, 1983.

Flores, A. F., Gottifredi, J. C., Morales V. and Quiroga, O. D., Heat transfer to power-law fluids flowing in pipes and flat ducts with viscous heat generation, *Chemical Eng. Sci.* 46, 1385-1392, 1991.

Hartnett, J. P. and Cho, Y. I., Non-Newtonian Fluids, In *Handbook of Heat Transfer* (edited by W.M. Rohsenow, J.P. Hartnett, Y.I. Cho), McGraw Hill, New York, 1998, Chap. 10.

Irvine, T. F. and Kami, J., Non-Newtonian Fluid Flow and Heat Transfer, In *Handbook of Single-Phase Convective Heat Transfer* (Edited by S. Kakaç, R.K. Shah, W. Aung), Wiley, New York, 1987, Chap. 20.

Lawal, A. and Mujumdar, A. S., Forced-convection-heat-transfer to a power-law fluid in arbitrary cross-section ducts, *Can. J. Chemical Eng.* 62, 326-333, 1984.

Lawal, A. and Mujumdar, A.S., The effects of viscous dissipation on heat transfer to power law fluids in arbitrary cross-sectional ducts, *Warme Stoffübertragung* 27, 437-446, 1992.

Oosthuizen, P. H. and Naylor, D., *Introduction to Convective Heat Transfer Analysis*, McGraw-Hill, New York, 1999, Chap. 4.

Wei, D. and Luo, H., Finite element solutions of heat transfer in molten polymer flow in tubes with viscous dissipation, *Int. J. Heat Mass Transfer* 46, 3097-3108, 2003.