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HEAT AND MASS TRANSFER FROM A HORIZONTAL SLENDER CYLINDER WITH A MAGNETIC FIELD EFFECT

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Abstract: The problem of steady laminar magnetohydrodynamic (MHD) forced convective heat and mass transfer about a horizontal slender cylinder is studied numerically. A uniform magnetic field is applied perpendicular to the cylinder. The nonlinear partial differential equations governing the flow are transformed into the similar boundary layer equations, which are then solved numerically using the Keller box method. The transverse curvature parameter, the magnetic parameter, the Prandtl number and the Schmidt number are the main parameters. For various values of these parameters, the local skin friction, heat transfer and mass transfer parameters are obtained. The validity of the methodology is checked by comparing the results with those available in the open literature and a fairly good agreement is observed. Finally, it is determined that the local skin friction coefficient, the local heat transfer coefficient and the local mass transfer coefficient increase with an increase the magnetic parameter Mn and transverse curvature parameter.
Keywords: Horizontal slender cylinder, Heat and mass transfer, Mhd flow.

YATAY İNCE BİR SİLİNDİR ÜZERİNDEN OLAN ISI VE KÜTLE TRANSFERİNE MANYETİK ALANIN ETKİSİ

Özet: Yatay ince bir silindir üzerinden daimi rejimde manyetik alanda, zorlanmış ısı ve kütle transferi problemi nümerik olarak çalışılmıştır. Uniform manyetik alan silindire dik olarak uygulanmıştır. Akışı yöneten non-lineer kısmi diferansiyel denklemler, benzerlik yöntemiyle sınır tabaka denklemlerine dönüştürülmüş ve Keller-box yöntemiyle çözülmüştür. Çaprazlık parametresi, manyetik alan parametresi, Prandtl ve Schmidt sayıları elde edilen denklemlerdeki temel parametrelerdir. Bu parametrelerin farklı değerleri için lokal sürtünme, local ısı ve local kütle transferi parametreleri elde edilmiştir. Metodolojinin doğruluğu literatürdeki mevcut sonuçlarla karşılaştırılmış ve iyi bir uyumun olduğu görülmüştür. Sonuç olarak, manyetik alan parametresi Mn ve çaprazlık parametresinin artmasıyla lokal sürtünme, lokal ısı transferi ve lokal kütle transfer parametrelerinin arttığı tespit edilmiştir.
Anahtar kelimeler: Yatay ince silindir, Isı ve kütle transferi, Mhd akışı.

NOMENCLATURE

B_0	magnetic flux density [Wb/m^2]
c_p	specific heat [kJ/kg K]
D	coefficient of mass diffusivity [m^2/s]
f	dimensionless stream function
Mn	magnetic parameter $\left[=\sigma B_0^2 r_0^2/(2\mu)\right]$
Pr	Prandtl number $\left[=\mu c_p/k\right]$
Sc	Schmidt number $\left[=v/D\right]$
T	temperature [K]
u, v	velocities in x and r directions, respectively [m/s]
x,r	coordinates in axial and radial directions, respectively [m]

Greek symbols

η	similarity variable $\left[=\left\{\left(r^2-r_0^2\right)/4r_0\right\}\left(u_\infty/vx\right)^{1/2}\right]$
ξ	transverse curvature $\left[=\left(4/r_0\right)\left(vx/u_\infty\right)^{1/2}\right]$
σ	electrical conductivity of the fluid
ρ	fluid density [kg/m3]
μ	dynamic viscosity [kg/m s]
υ	kinematic viscosity [m^2/s]
θ	dimensionless temperature
ϕ	dimensionless concentration

Subscripts

w	wall
∞	free stream

INTRODUCTION

Flow over cylinders is considered to be two-dimensional if the body radius is very large compared to the boundary layer thickness. For a thin or slender cylinder, the radius of the cylinder may be of the same order as the boundary layer thickness. Therefore, the flow may be considered as axisymmetric instead of two-dimensional. In this case, the governing equations contain the transverse curvature term which influences both the velocity and temperature fields. The effect of the transverse curvature is important in certain applications such as wire or fiber drawing where accurate prediction of flow and heat transfer is required and thick boundary layer can exist on slender or near slender bodies (Datta et al., 2006).

The effect of transverse curvature has been investigated by several researchers for forced, free and mixed convective flows over a cylinder. Chen and Mucoglu (1975) analyzed the buoyancy and transverse curvature effects on forced convection of Newtonian fluid flow along an isothermal vertical cylinder using the local non-similarity method. The same problem for a uniform surface heat flux case was studied by Mucoglu and Chen (1976). Takhar et al. (2000) studied the combined effect of free and forced convection flows over a vertical slender cylinder. El-Amin (2003) studied the effects of both first- and second-order resistance, Joule heating and viscous dissipation on forced convection flow from a horizontal circular cylinder embedded in porous medium under the action of a transverse magnetic field. Datta et al. (2006) obtained the non-similar solution of a steady laminar forced convection boundary layer flow over a horizontal slender cylinder including the effect of non-uniform slot injection (suction). Roy et al. (2007) developed general analysis for the influence of non-uniform double slot injection (suction) on the steady non-similar incompressible laminar boundary layer flow over a slender cylinder. Also, they investigated the effects of transverse curvature on velocity and temperature profiles. Singh et al. (2008) studied unsteady mixed convection flow over a rotating vertical slender cylinder under the combined effects of buoyancy force and thermal diffusion with injection/suction where the slender cylinder was inline with the flow.

In the present paper, the effect of transverse curvature on MHD forced heat and mass convective flow over a horizontal slender cylinder with uniform surface temperature and concentration is analyzed. The boundary layer equations governing the flow are reduced to local non-similarity equations which are solved using the implicit finite difference method (Keller box). Numerical results for the velocity, temperature and concentration profiles as well as local skin friction, local heat transfer and local mass transfer parameters are presented.

ANALYSIS

Consider the steady, incompressible, laminar, two-dimensional, boundary layer flow over a horizontal slender cylinder of length L and outer radius r_o ($L >> r_o$). The physical model and coordinate system are shown in Fig. 1. The temperature, velocity and concentration at a distance remote from the cylinder are given by T_∞, u_∞ and C_∞, respectively and the body has a uniform temperature T_w and uniform concentration C_w. A uniform magnetic field is assumed to apply in the r-direction causing a resistance on flow force in the x-direction. It is assumed that the induced magnetic field, the external or imposed electric field and the electric field due to the polarization of charges (i.e. Hall effect) is negligible. The plate is considered to be electrically non-conducting.

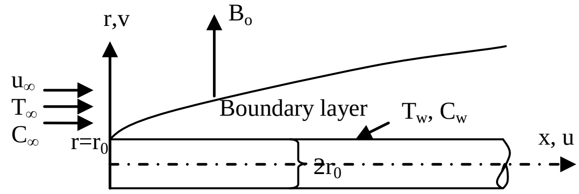


Figure 1. The schematic of the problem.

Under foregoing assumptions and taking into account the Boussinesq approximation and the boundary layer approximation, the system of continuity, momentum, energy and concentration equations can be written:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(r\upsilon)}{\partial r}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial r}=\left(\frac{\nu}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)-\frac{\sigma B_0^2}{\rho}\left(u-u_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial r}=\left(\frac{\nu}{\Pr}\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+\upsilon\frac{\partial C}{\partial r}=\left(D\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) \tag{4}$$

Here u and υ are the velocity components in the x and r direction, respectively, T is the temperature of the fluid, C_p is the heat capacity at constant pressure, ν is the kinematic viscosity, ρ is the fluid density, D is the mass diffusivity and B_0 is the magnetic flux density.

The appropriate boundary conditions are as follows:

$$r=r_0;\quad u=0,\quad \upsilon=0,\quad T=T_w,\quad C=C_w$$
$$r\to\infty;\quad u\to u_\infty,\ \upsilon=0,\ T\to T_\infty,\ C\to C_\infty \tag{5}$$

To seek a solution, the following dimensionless variables are introduced (Datta et al., 2006 and Roy et al., 2007):

$$\xi=\left(\frac{4}{r_0}\right)\left(\frac{\nu x}{u_\infty}\right)^{1/2},\ \eta=\left[\frac{r^2-r_0^2}{4r_0}\right]\left(\frac{u_\infty}{\nu x}\right)^{1/2},\qquad \theta=\frac{T-T_\infty}{T_w-T_\infty},$$
$$\psi(x,r)=r_0\left(\nu u_\infty x\right)^{1/2}f(\xi,\eta),\qquad \frac{r^2}{r_0^2}=\left[1+\xi\eta\right],$$
$$\phi=\frac{C-C_\infty}{C_w-C_\infty} \tag{6}$$

where $\psi(x,y)$ is the free stream function that satisfies Eq. (1) with $u=(1/r)(\partial\psi/\partial r)$ and $\upsilon=-(1/r)(\partial\psi/\partial x)$

In terms of these new variables, the velocity components can be expressed as

$$u=\frac{1}{2}u_{\infty}f',\ \upsilon=\frac{r_0}{2r}\left(\frac{\nu u_{\infty}}{x}\right)^{1/2}\left[\eta f'-f-\xi\frac{\partial f}{\partial\xi}\right] \tag{7}$$

The transformed momentum, energy and concentration equations together with the boundary conditions, Eqs. (2), (3), (4) and (5), can be written as

$$\begin{aligned}\left(1+\xi\eta\right)f'''+\xi f''+ff''-Mn\xi^2\left[\frac{1}{2}f'-1\right]\\=\xi\left[f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right]\end{aligned} \tag{8}$$

$$\frac{1}{\Pr}\left(1+\xi\eta\right)\theta''+\frac{\xi}{\Pr}\theta'+f\theta'=\xi\left[f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right] \tag{9}$$

$$\frac{1}{Sc}\left(1+\xi\eta\right)\phi''+\frac{\xi}{Sc}\phi'+f\phi'=\xi\left[f'\frac{\partial\phi}{\partial\xi}-\phi'\frac{\partial f}{\partial\xi}\right] \tag{10}$$

with the boundary conditions;

$$\left.\begin{aligned}&\eta=0;\ f+\xi\frac{\partial f}{\partial\xi}=0,\ f'=0,\ \theta=1,\ \phi=1\\&\eta\to\infty;\qquad\qquad f'=2,\ \theta=0,\ \phi=0\end{aligned}\right\} \tag{11}$$

The corresponding dimensionless groups that appeared in the governing equations are defined as

$$\Pr=\frac{\mu c_p}{k}=\frac{\nu}{\alpha},\ Sc=\frac{\nu}{D},\ \ Mn=\frac{\sigma B_0^2 r_0^2}{2\mu} \tag{12}$$

where Pr is the Prandtl number, Sc is the Schmidt number and Mn is the magnetic parameter.

NUMERICAL SOLUTION

The system of transformed equations under the boundary conditions, Eqs. (8)–(11), have been solved numerically using the Keller box scheme, which is proved to be an efficient and accurate finite-difference scheme (Cebeci and Bradshaw, 1977). Readers are referred to Cebeci and Bradshaw (1977) for the details of the numerical methods. This is a very popular implicit scheme, which demonstrates the ability to solve systems of differential equations of any order as well as featuring second-order accuracy (which can be realized with arbitrary non-uniform spacing), allowing very rapid x or ξ variations (Takhar and Beg, 1997; Aydin and Kaya, 2009).

A set of non-linear finite-difference algebraic equations derived are then solved by using the Newton quazi-linearization method. The same methodology followed by Takhar and Beg (1997) is followed. Therefore, for the finite-difference forms of the equations, the reader is referred to Takhar and Beg (1997) for the brevity of the article.

In the calculations, a uniform grid of the step size 0.01 in the η-direction and a non-uniform grid in the ξ-direction with a starting step size 0.001 and an increase of 0.1 times the previous step size were found to be satisfactory in obtaining sufficient accuracy within a tolerance better than 10^{-6} in nearly all cases. The value of η_{∞} = 16 is shown to satisfy the velocity to reach the relevant stream velocity.

In order to verify the accuracy of the present method, the results were compared with those of Chen and Mucoglu (1975), Takhar et al. (2000) and Chang (2006). The comparison is found to be in good agreement, as shown in Table 1.

RESULTS AND DISCUSSION

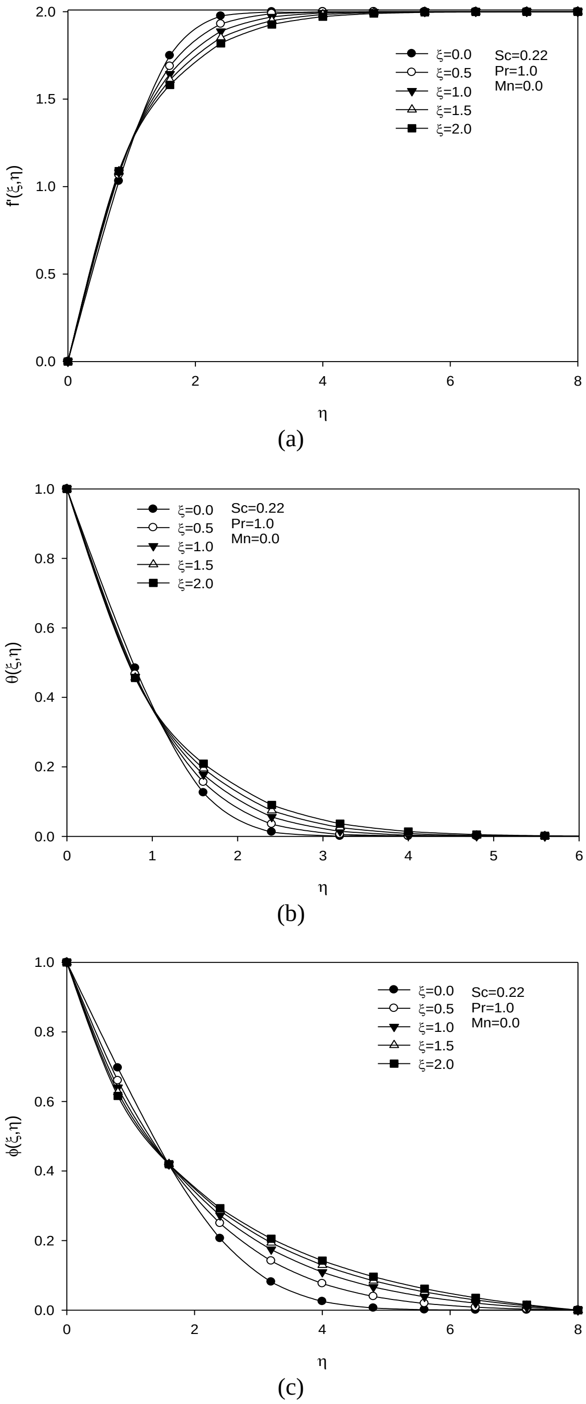
In this study, the effects of transverse curvature on MHD forced heat and mass convection are examined. The following ranges of the main parameters are considered: Pr = 1.0, 7.0 and 10, Sc=0.22, 0.6 and 0.94 and magnetic interaction parameter Mn= 0.0, 0.5, 1.0, 1.5 and 2.0. The combined effects of ξ, Mn, Sc and Pr on the momentum, heat and mass transfer are analyzed.

The effect of surface curvature parameter ξ (or the axial distance) on the velocity (a), temperature (b) and concentration (c) profiles for Mn=0.0, Pr = 0.7 and Sc=0.22 are given in Fig. 2. The velocity, thermal and concentration boundary layers increase due to the surface curvature parameter ξ. Similar trend has been observed by Chen and Mucoglu (1975) and Takhar et al. (2000).

Due to the increase in surface curvature parameter ξ, the steepness in velocity, temperature and concentration profiles near the wall increases. The physical reason is that the increase in ξ acts as a favourable pressure gradient, which enhances the steepness in velocity, temperature and concentration profiles near the wall resulting in higher skin friction, heat transfer and mass transfer rate at wall (Datta et al., 2006).

Table 1. Comparision of the local skin friction and local heat transfer parameters with Pr=0.7, Mn=0.0.

ξ	Chen and Mucoglu (1975)		Takhar et al. (2000)		Chang (2006)		Present results	
	$f''(\xi,0)$	$-\theta(\xi,0)$	$f''(\xi,0)$	$-\theta(\xi,0)$	$f''(\xi,0)$	$-\theta(\xi,0)$	$f''(\xi,0)$	$-\theta(\xi,0)$
0.0	1.3282	0.5854	1.3281	0.5854	1.3280	0.5852	1.3201	0.5846
1.0	1.9172	0.8669	1.9167	0.8666	1.9133	0.8658	1.8934	0.8599
2.0	2.3981	1.0986	2.3975	1.0963	2.3900	1.0940	2.3822	1.0918
3.0	2.8270	1.3021	-	-	2.8159	1.2982	2.8098	1.2902
4.0	3.2235	1.4921	-	-	3.2187	1.4925	3.2102	1.4898



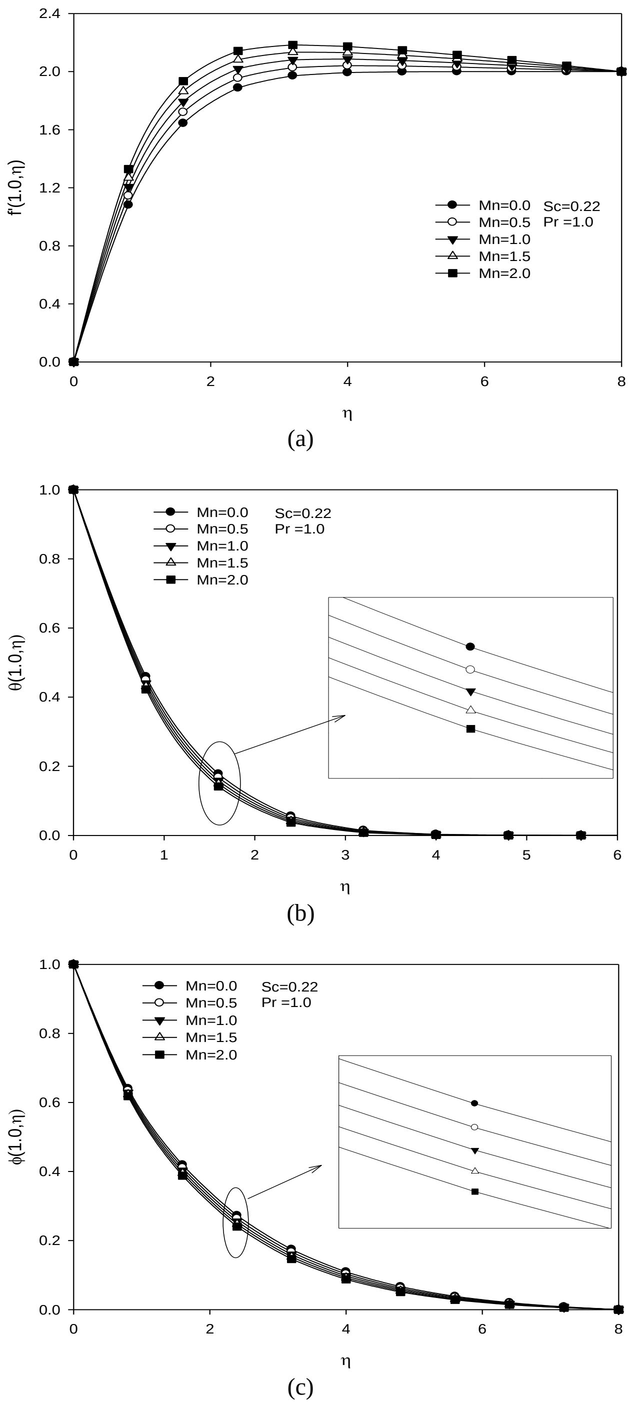


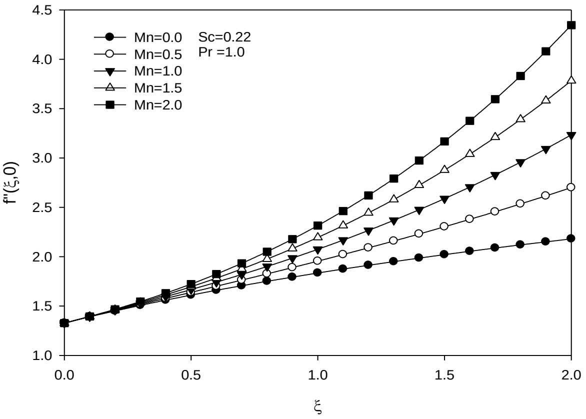
Figure 2. Dimensionless velocity (a), temperature (b) and concentration (b) profiles for different surface curvature parameter ξ at Mn=0.0, Pr=1.0 and Sc=0.22.

Figure 3. Dimensionless velocity (a), temperature (b) and concentration (b) profiles for different Mn at Pr=1.0, Sc=0.22 and ξ=1.0.

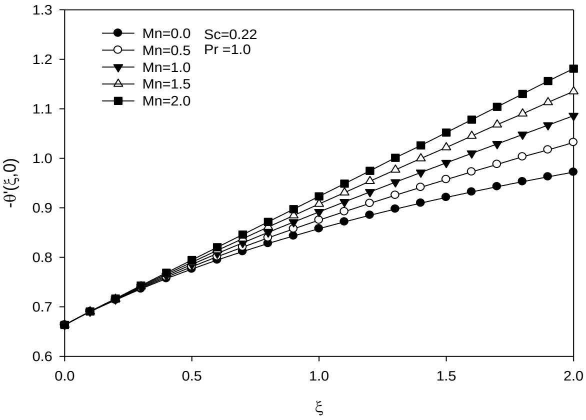
Because of the similarity variable η is multiplied by ξ [see Eqs. (8), (9) and (10)], the momentum and thermal boundary layer thickness (about $\eta<1.0$) and concentration boundary layer thickness (about $\eta<1.5$) decreases with the surface curvature parameter ξ.

Figure 3 shows the dimensionless velocity (a), temperature (b) and concentration (c) profiles inside the boundary layer for different values of the magnetic parameter Mn. The increasing of the magnetic parameter Mn increases velocity, temperature and concentration gradients at the wall. These increased gradients results in increases in the local skin friction and the local heat and mass transfer parameters with the magnetic parameter Mn (Figure 4).

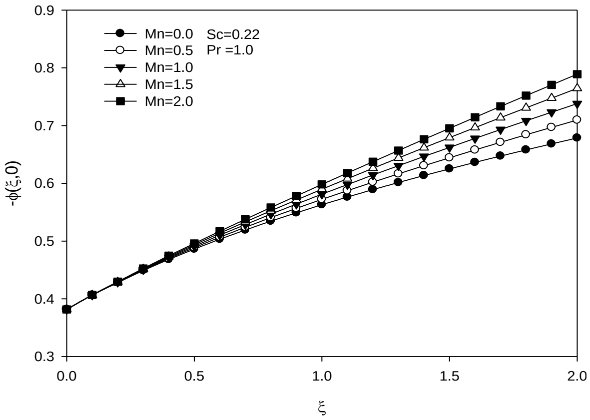
Since the flow problem is uncoupled from the thermal and concentration problems, changes in the values of Pr and Sc will not affect the fluid velocity. For this reason, velocity profiles for this case are not shown. Increasing the Prandtl number tends to reduce the thermal boundary layer thickness along the slender cylinder and



(a)

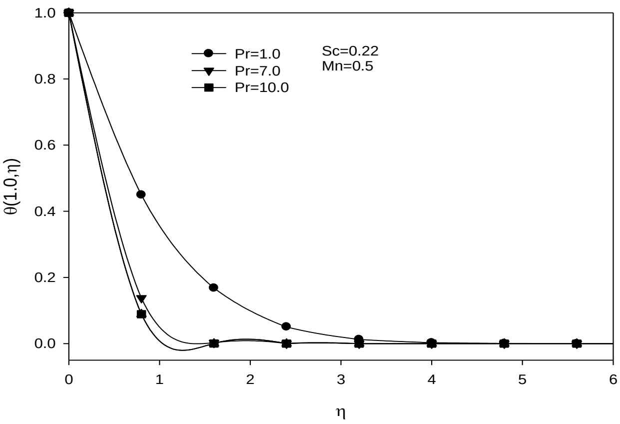


(b)

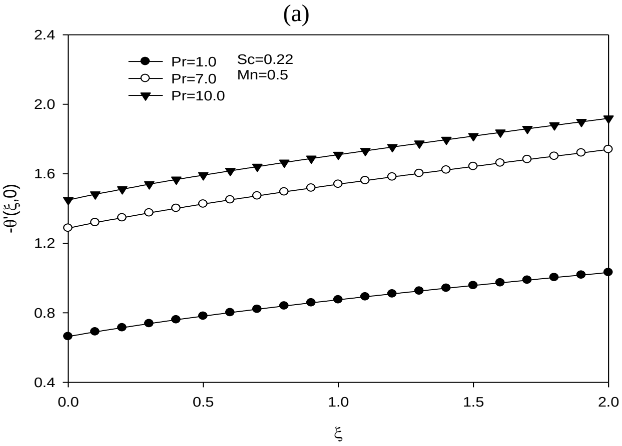


(c)

Figure 4. Effect of Mn on the local skin friction (a), local heat transfer (b) and local mass transfer (c) parameters against ξ at Pr=1.0 and Sc=0.22

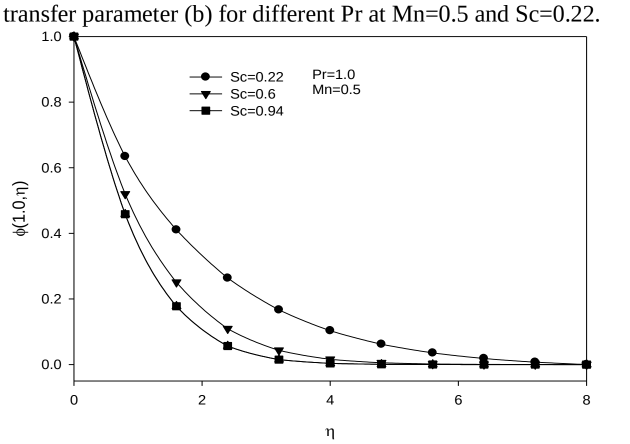


(a)

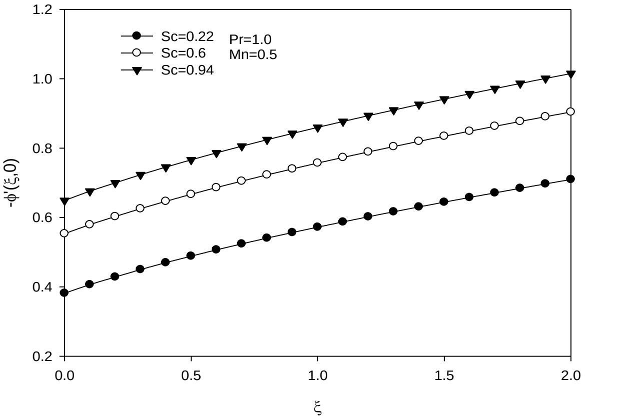


(b)

Figure 5. Dimensionless temperature (a) and local heat transfer parameter (b) for different Pr at Mn=0.5 and Sc=0.22.



(a)



(b)

Figure 6. Dimensionless temperature (a) and local mass transfer parameter (b) for different Sc at Mn=0.5 and Pr=1.0.

the wall temperature gradient increases (Fig.5 (a)). Also, increasing the Prandtl number increases the local heat transfer parameter (Fig.5 (b)).

The effect of the Schmidt number Sc (the values of Sc are chosen so that they represent the diffusing chemical species of most common interest in air like H_2, H_2O, NH_3 and CO_2 where the values of Sc are 0.22, 0.6, 0.78 and 0.94, respectively (Eldabe and Ouaf, 2006)) on the concentration distribution is shown in Fig. 6(a). From this figure it is clear that concentration boundary layer thickness decreases as Sc increases. As indicated in Fig. 6(b), an increase in the Schmidt number Sc produces a rise in the local mass transfer parameter, as expected.

CONCLUSIONS

In this article, the effects of surface curvature parameter ξ, magnetic parameter, Prandtl number and Schmidt number on a steady MHD forced heat and mass transfer convective flows about a horizontal slender cylinder have been studied. A transformed set of non-similar equations have been solved using the Keller box scheme. From the present numerical investigation, the following conclusions can be drawn:

1. An increase in the surface curvature parameter decreases velocity, thermal and concentration gradient at the wall.
2. An increase in the magnetic parameters increases the local skin friction and local heat and mass transfer parameter.
3. Increasing Pr and Sc increases the local heat and local mass transfer parameters.

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