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AN IMPLICIT SPECTRAL METHOD FOR THE NUMERICAL SOLUTION OF UNSTEADY FLOWS WITH AN APPLICATION TO ROTATING DISK FLOW AND HEAT TRANSFER

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Abstract: An implicit numerical integration scheme is proposed in the present paper for the solution of the transient flows. The numerical method is based on the spectral Chebyshev collocation technique in the direction normal to the disk and forward marching in time. Besides being free of the numerical oscillations caused by the discontinuities between the initial and boundary conditions inherent to the classical finite difference techniques, the devised technique benefits from the advantages of being robust, unconditionally stable, highly accurate and straightforward to implement. Unlike to the finite difference methods, it is also compact in the sense that it resolves the flow field without necessitating further transformations. The numerical algorithm developed here is applied to the classical time-dependent von Karman swirling flow due to a porous rotating disk impulsively set into motion which progresses into the well-known steady state after a long time. The energy equation is also treated by the method and the physical parameters of paramount interest as such the radial and tangential skin-friction coefficients, the torque and the rate of heat transfer from the disk surface are numerically calculated that are shown to approach their steady state counterparts.
Keywords: Chebyshev collocation, Implicit method, Unsteady flow, Rotating disk, Shear stresses, Heat transfer.

ZAMANA BAGLI AKIŞKANLARIN SAYISAL ÇÖZÜMLEMELERİ İÇİN BİR KAPALI SPEKTRAL YÖNTEM VE DÖNEN DİSK AKIŞI ISI TRANSFERİ PROBLEMİNE UYGULAMASI

Özet: Bu makalede zamana göre geçişken akışkanların çözümü için bir kapalı sayısal integrasyon yöntemi önerilmiştir. Nümerik metod diske dik dogrultuda Chebyshev dügümleme ve zamanda ise ileri adım tekniklerine dayanmaktadır. Klasik sonlu fark yöntemlerine haiz başlangıç ve sınır koşullarındaki süreksizliklerinden kaynaklanan nümerik salınımlardan uzak olmanın yanında, planlanan metod oldukça dayanıklı, şartsız kararlı, yüksek dogruluga sahip ve uygulaması çok kolay olması gibi avantajlardan yararlanmaktadır. Sonlu fark metodlarının aksine, şimdiki metod farklı dönüşümlere gerek kalmaksızın akış alanını kolayca çözümler. Burada geliştirilen nümerik algoritma klasik zamana baglı ve aniden harekete geçirilen, lakin uzun bir süreç sonunda çok iyi bilinen dönen diskten kaynaklanan von Karman dönen akış problemine uygulanmıştır. Enerji denklemi de bu metodla çözülerek radyal ve tegetsel yüzey sürtünme katsayıları, tork ve diskten ısı transfer oranı gibi çok mühim fiziksel parametreler nümerik olarak elde edilmiş ve bunların zamandan bagımsız degerlerine yakınsadıgı gösterilmiştir.
Anahtar Kelimeler: Chebyshev dügümleme, Kapalı metod, Zamana baglı akış, Dönen disk, Yüzey gerilmeleri, Isı transferi.

NOMENCLATURE

Roman symbols

C_P	specific heat
(F,G,H)	self-similar radial, azimuthal and normal velocities
(L,U_c,r_e^*)	characteristic scales [m,m/s,m]
P	self-similar pressure
Pr	Prandtl number
q	heat flux
R	Reynolds number [Uc.L/μ]
r	radial direction in cylindrical polar coordinates
s	suction or injection parameter
T	temperature [K]
(u,v,w)	velocity components in radial, azimuthal and normal directions [m/s]
t	time [s]
z	normal direction in cylindrical polar coordinates [m]

Greek symbols

η	a scaled boundary layer coordinate
K_∞	thermal conductivity at the free stream [W/(m.K)]
μ_∞	dynamic viscosity at the free stream [N.s/m^2]
δ	displacement thickness
ρ	density of the fluid [kg/m^3]

θ	azimuthal direction in cylindrical polar coordinates
τ_r	radial shear stress on the wall
τ_θ	azimuthal shear stress on the wall
υ	kinematic viscosity [m^2/s]
Ω	angular velocity of the disk [m/s]

INTRODUCTION

A number of numerical solution methods has been developed recently to tackle the unsteady flow motion relevant to fluid dynamics phenomena. Besides its reasonable accuracy, convergence, consistency and stability properties, numerical scheme is expected to be computationally efficient and user friendly in terms of its programming and ease of implementation. A new algorithm encompassing all of the aforementioned properties has been developed in the present study based on the spectral Chebyshev collocation technique. Its advantages over the classical finite difference methods have been highlighted with a direct application of the method to the numerical solution of the three-dimensional unsteady porous rotating disk von Karman fluid flow problem.

A tremendous range of fluid flow phenomena is described by the governing time-dependent equations of motion (Schlichting, 1979; Ramos, 2007; Laizeta and Lamballaisa, 2009). The time evolution of the physical phenomenon needs to be conceived by solving numerically the fluid equations since these equations have no analytical closed-form solutions in most cases. The governing equations generally consist of partial differential equations of mass, momentum, angular momentum and energy conservation depending on the property of the phenomenon.

Several numerical formulations based on different discretization methods, such as Runge-Kutta, finite differences, finite element, and spectral methods, have been proposed to compute the fluid dynamics problems (Dehghan and Taleei, 2010; Arefmanesh and Alavi, 2008; Calgaro et al., 2006; Canuto et al., 1988; Wu et al., 2009). The most common approach for approximating the derivatives is the finite difference methods. Different types and orders of finite difference methods are available as cited in the book Book (1981). Applying conventional first-order finite difference methods like the first-order upwind results in monotonic and stable solutions, but they are also strongly dissipative causing the solution of the strongly convective partial differential equations to become smeared out and often grossly inaccurate. On the other hand higher-order difference methods, e.g. central, Lax-Wendroff, QUICK, etc. are less dissipative but are prone to numerical instabilities, which introduce oscillations across regions of large gradients of the variables (Wang and Hutter, 2001; John and Knobloch, 2007). Crank-Nicolson method is a favorably popular method for solving parabolic equations because it is unconditionally stable and second order accurate (Wade et al., 2007; Jeong and Kim, 2010). One drawback of it is that it responds severely to jump discontinuities in the initial conditions or to the differences between the initial and boundary conditions with oscillations which are weakly damped and therefore may persist for a long time. A selection of methods were later presented to reduce the amplitude of these oscillations (Britz et al., 2003; Huang and Abduwali, 2010).

The same non-physical numerical oscillations were encountered while solving the unsteady rotating disk fluid flow problems by Attia (1998) and Hossain et al. (2001) using a finite difference numerical integration procedure in conjunction with the implicit Crank-Nicolson solver. It appears that the difficulty is inherent to the other unsteady flow problems in fluid mechanics (Ekaterinaris, 2005; Appadu et al., 2008). A fast solution for this numerical problem is generally achieved by using a proper coordinate transformation as suggested by Ames (1977). However, besides the equations to be solved getting complicated, this even does not remedy the problem completely, since the physical domain is infinite, imposing the asymptotic conditions at a finite distance greatly affects the accuracy of the numerical solution, as pointed out in the research of Attia (1998). Therefore, the existing numerical procedures in the literature for the unsteady calculations do the computations in the transformed region up to a predetermined finite time and switches back to the physical domain for the calculation of the rest of the solution in the time domain.

The objective of the present work is to develop a numerical scheme for the computation of transient flows in fluid mechanics. A straightforward approach is the prime target of the study which easily overcomes the aforementioned difficulties and particularly avoids the unwanted numerical oscillations due to the differences between the initial and boundary conditions. To serve to this purpose, Chebyshev polynomials are employed to approximate derivatives in the direction normal to the body surface. Having linearized the nonlinear terms in the governing equations via the usual Newton linearization, the spectral collocation implemented in this way is then furnished with an implicit time differencing for the unsteady terms in the governing equations. The developed compact numerical method is later applied to the von Karman swirling flow equations governing the motion of the unsteady incompressible flow over a porous rotating disk. Numerical oscillations and diminishing of the infinite boundary for large times inherent to the finite difference techniques are no longer present in the method devised. Moreover, the method, being implicit and hence unconditionally stable, produces the steady state solutions using large time steps, i.e., large courant numbers are allowed, with small dissipative and dispersion errors. Finally, the time evolution of some parameters of physical importance has been obtained using the current method.

The following procedure is adopted in the rest of the paper. The implicit spectral numerical scheme is presented in section 2. Application of the method is implemented in section 3 to the special case of unsteady von Karman porous rotating disk flow equations. Section 4 contains results and discussions of the numerical presentations including those of physically important parameters. Finally, conclusions are drawn in section 5.

THE NUMERICAL METHOD

Consider the system of partial differential equations

$$\frac{\partial u}{\partial t} + N(u) = 0 \tag{1}$$

valid inside a domain D, accompanied with the following initial and boundary conditions

$$\begin{aligned} &u(t=0,z) = u_0(z), \qquad z \in D, \\ &u(t,z) = a(t), \qquad (t,z) \in \left(R^+ x \partial D\right), \end{aligned} \tag{2}$$

where u=(u(t,z),v(t,z),w(t,z)) with Z being a normal coordinate in the direction perpendicular to the motion and N in equation (1) is a nonlinear partial differential operator similar to the Navier-Stokes operator arising in many applications of science and engineering.

There are a number of numerical procedures to discretize system (1-2). The most frequently used are the classical explicit or implicit finite difference techniques. But no matter the type of the differencing, the resulting numerical algorithm gives rise to numerical oscillations due to the reason that the initial data and boundary conditions in (2) may possibly constitute a discontinuity to be exemplified later in section 4. This fact was utterly expressed in the numerical studies of the references cited here, in which the numerical oscillations were often reported during the numerical simulation of the unsteady rotating disk flows (Attia, 1998; Hossain, et al. 2001). The cousins of numerical methods based on the finite difference approximations of derivatives as presented in the book by Book (1981) are also susceptible to the same difficulty. A solution for this numerical problem is generally accomplished by means of a proper coordinate transformation, such as

$$\eta = \frac{z}{2\sqrt{t}}$$

as also suggested by Ames(1977). However, the resulting equations need to be solved in the infinite domain

$$0 \le z \le \infty, \qquad t \ge 0.$$

During the numerical computations Z is fixed at a finite distance, but due to the suggested coordinate transformation, this finite domain is diminished with the progression of time and greatly affects the accuracy of the numerical solution of the problem at hand. To cope with these deficiencies, and obtain the unsteady solution at one go, we propose here to use spectral Chebyshev method (Canuto, et al. 1988; Khater and Temsah, 2008). In compliance with this purpose, the infinity physical domain of computation is mapped first onto the interval $\eta \in [-1,1]$ with a suitable transformation η=f(z). Next, the nonlinear operator in (1) is linearized with the usual Newton linearization technique such that a single flow velocity u is written as

$$u = u^{(n)} + \delta u, \tag{3}$$

where $u^{(n)}$ is to denote the value of $\boldsymbol{u}$ at the iteration number n and δu is a small correction term. As a result, the nonlinear operator in (1) will be substituted by its linearized counterpart. A forward time differencing for the derivative of u is appropriate at this stage in the form

$$\frac{\partial u}{\partial t} = \frac{u_{j+1} - u_j}{\Delta t}.$$

Taking into account the advantage of implicit schemes, the linearized terms are also imposed at the time t+Δt. A Chebyshev collocation based on the well-known Chebyshev polynomials is later employed in the wall normal direction η in such a way that the quantities are collocated at the Gauss points

$$\cos\left(\frac{k\pi}{N}\right), \qquad k = 0,1,\ldots,N,$$

where N denotes the number of collocation points used. The spectral Chebyshev collocation method enables one also to represent a derivative of a quantity in terms of the values of that quantity in the whole domain of interest (Canuto et al. 1988). In view of the above remarks, system (1) can be cast into a matrix form

$$A\delta U_{j+1} = B, \tag{4}$$

in which A and B consist of the known values at the nth iteration, and $\boldsymbol{\delta U_{j+1}}$ shows the corrections at the instant of computation for (u,v,w). The matrix system (4) needs to be modified due to the boundary constraints (2). With proper initial approximations $u^{(n)}$ to the variables (which can be assigned from the previously converged solutions) at each time step, the matrix system (4) is eventually solved with an LU matrix factorization technique. The convergence criterion is to force the correction terms δU in (4) to lie within a preassigned small tolerance.

As compared with the finite difference methods, the method devised here is robust, unconditionally stable and easy to programme due to its compact matrix form in equation (4). As will be demonstrated later, the scheme introduced here converges quickly to the steady state solution. The computational efficiency and better accuracy of the spectral Chebyshev method over the finite difference methods are underlined in the textbook by Canuto et al. (1988). More details of the integration scheme without the time derivatives (steady state) can be found in Turkyilmazoglu (1998). We should emphasize that the initial guesses mentioned above are taken as zero initially, which were found to be perfectly capable of generating the results of this study for the entire family of parameters considered.

APPLICATION TO THE ROTATING DISK FLOW

The interest here is with the three-dimensional, unsteady flow of an incompressible, viscous fluid over an infinite disk rotating with a constant angular velocity Ω about its axis of rotation z. A uniform suction or blowing is also applied thorough the surface in the direction normal to the disk. The flow description and geometrical coordinates are depicted in Figure 1.

The governing equations of motion are non-dimensionalized with respect to a length scale $L=r_e^*$, velocity scale $U_c = L\Omega$, time scale L/U_c and pressure scale ρU_c^2, where ρ is the fluid density. Such a dimensionless analysis leads to a global Reynolds number $\mathrm{Re} = \frac{U_c L}{\nu} = R^2$, where the non-dimensional cylindrical polar coordinates R is the Reynolds number based on the displacement thickness $\delta = \left(\frac{\nu}{\Omega}\right)^{\frac{1}{2}}$. Thus, relative to

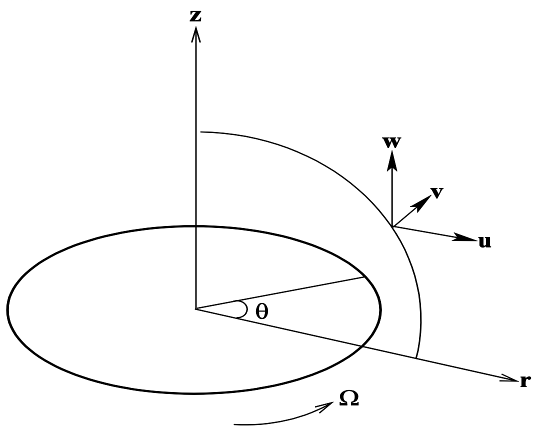

Figure 1. Configuration of the flow and geometrical coordinates.

(r,θ,z), the full time-dependent, Navier-Stokes and energy equations governing the viscous fluid flow are given by

$$\nabla \cdot u = 0, \tag{5}$$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla p + \frac{1}{R^2}\nabla^2 u, \tag{6}$$

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \frac{1}{\Pr R^2}\nabla^2 T. \tag{7}$$

The present analysis assumes that the fluid lies in the $z \geq 0$ semi-infinite space. In the above equations (5-7) ∇^2 is the usual Laplacian operator in cylindrical coordinates. The components of the flow velocity u are (u,v,w), the pressure is P and T is the fluid temperature such that the surface of the rotating disk is maintained at a uniform temperature T_w . Far away from the wall, the free stream is kept at a constant temperature T_∞. As for the velocities, no-slip condition is imposed at the wall together with the vanishing radial and azimuthal velocities far above the disk.

The dimensionless mean flow velocities, pressure and temperature distributions are given by von Karman's exact self-similar solution of the Navier-Stokes equations for the steady laminar flow. Because the boundary layer thickness is of order of magnitude R^{-1}, the steady incompressible boundary layer flow over a rotating disk evolves along a boundary layer coordinate of order unity, defined by Z=Rz. Consequently, the mean flow quantities take the form

$$(u,v,w) = \left(rF(t,Z), rG(t,Z), \frac{1}{R}H(t,Z)\right),$$

$$(p,T) = \left(\frac{1}{R^2}P(t,Z), T_\infty + (T_w - T_\infty)\theta(t,Z)\right), \tag{8}$$

where the similarity functions F, G, H and θ satisfy the following ordinary differential equations

$$\frac{\partial F}{\partial t} + F^2 - G^2 + F'H - F'' = 0,$$

$$\frac{\partial G}{\partial t} + 2FG + G'H - G'' = 0,$$

$$\frac{\partial P}{\partial t} + P' + H'H - H'' = 0, \tag{9}$$

$$2F + H' = 0,$$

$$\frac{\partial \theta}{\partial t} - \frac{1}{\Pr}\theta'' + H\theta' = 0.$$

Here, a prime denotes derivative with respect to Z, $\Pr = \frac{\mu_\infty C_p}{K_\infty}$ is the Prandtl number, C_P is the specific heat at constant pressure and the boundary conditions appropriate to the flow geometry for all time t are given as

$$F = G - 1 = H - s = \theta - 1 = 0, \quad at \quad Z = 0, \tag{10}$$

$$F = G = \theta = 0, \quad as \quad Z \to \infty. \tag{11}$$

System is also supplemented with the subsequent initial conditions valid for all Z

$$F = G = H = \theta = 0, \quad at \quad t = 0. \tag{12}$$

It should be noticed that, as stated before, a discontinuity is present between initial values and boundary conditions in (10-12).

The numerical scheme described in section 2 was made use for the resolution of the velocity and temperature fields from the system of equations (9-12), after mapping the physical domain Z onto $\eta \in [-1,1]$ via the linear transformation $\eta = -1 + \frac{2Z}{Z_{max}}$. Sufficient number of Gauss collocation points were taken together with the proper choice of a large distance Z_{max} above the surface of the disk in order to make sure that the solutions obtained are independent of the parameters involved.

Upon solution of the mean flow quantities from the system (9-12), the skin friction coefficients, the torque and the rate of heat transfer to the surface, which are of principal physical interest, can also be calculated. The action of the viscosity in the fluid adjacent to the disk sets up a tangential shear stress, which opposes the rotation of the disk. As a consequence, it is necessary to provide a torque at the shaft to maintain a steady rotation. To find the tangential shear stress τ_θ and radial shear stress τ_r, we apply the Newtonian formulae

$$\tau_\theta = [v_z + \frac{1}{r} w_\theta](z=0) = \Omega R G'(0)$$
$$\tau_r = [u_z + w_r](z=0) = \Omega R F'(0).$$

The rate of heat transfer from the disk surface to the fluid is computed by the application of Fourier's law as given below

$$q = -T_z(z=0) = -R\theta'(0)$$

from which the normalized Nusselt number can be obtained. Therefore, in what follows we numerically compute $F'(0)$, $G'(0)$ and $\theta'(0)$ to understand the underlying physics of the problem.

RESULTS AND DISCUSSION

In order for testing the efficiency and accuracy of the numerical scheme developed in section 2, we apply it to solve the unsteady flow of incompressible viscous and laminar flow of von Karman equations of motion given in section 3. Numerical simulations are carried out for the motion of a fluid having Prandtl number $\Pr = 1$ for an ideal flow and $\Pr = 0.72$ for air. Time progression of the unsteady velocity profiles as well as radial and tangential shear stresses, vertical suction velocity and the rate of heat transfer at the disk surface are presented against the time.

Starting from the zero initial state, the flow over a disk evolves impulsively by a sudden action of rotation of the disk and as time passes the flow settles down to a steady state. This action of the fluid flow is shown in

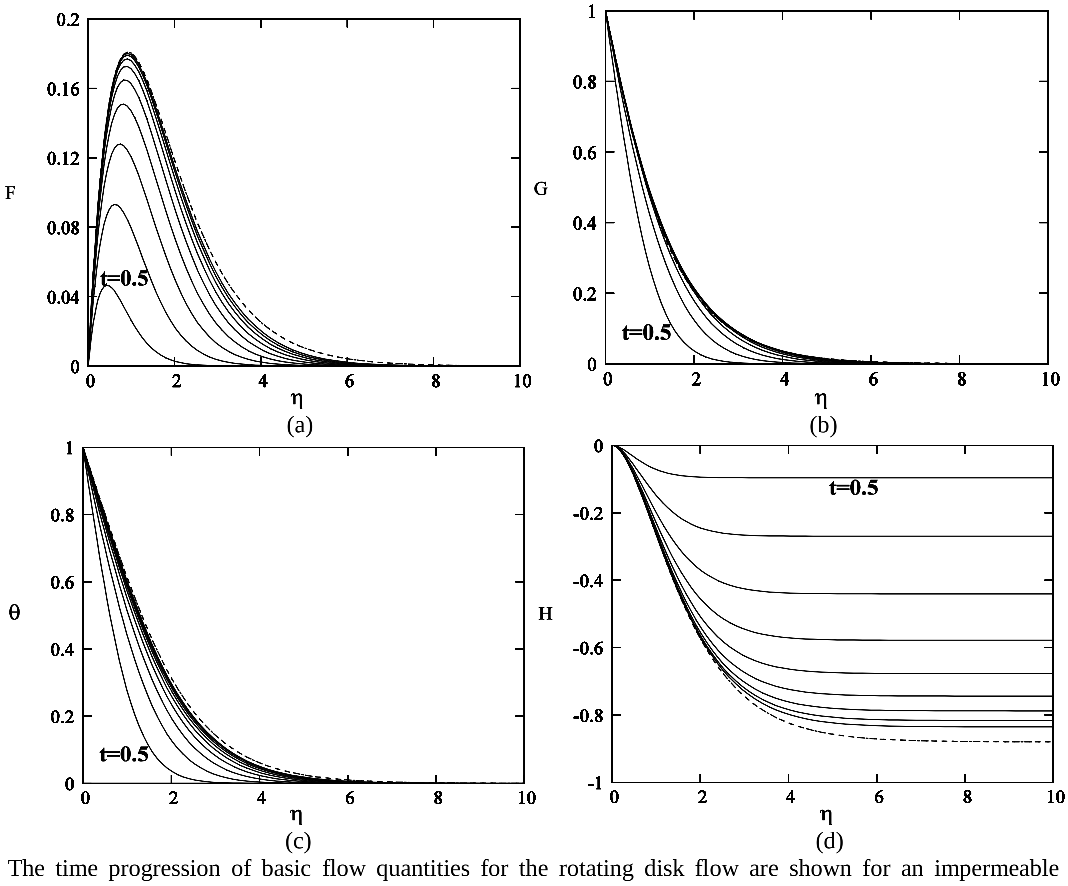

Figure 2. The time progression of basic flow quantities for the rotating disk flow are shown for an impermeable wall case respectively in (a) the radial velocity profiles, (b) the circumferential velocity profiles, (c) the wall normal velocity profiles and (d) the temperature profiles. The snapshots are given at 0.5 increments in time. The dot-dashed curves correspond to the large time limit as well as the steady solution..

Figures 2(a-d) for a nonporous disk. Figures are displayed for the time development of the flow quantities by a time step $\Delta t = 0.05$, but taken at a snapshot of $t = 0.5$. The sufficiently large time solution as well as the steady solution are shown by the dot-dashed curves. Figures show how the impulsive motion ends up with a steady state which were calculated by ignoring the time derivative terms in equations (9-12). The success of the devised numerical method in capturing the steady state solution by assigning larger time steps is also possible (though not demonstrated here) due to its unconditional stability. It is further noticeable from the figure that the circumferential velocity attains its steady state quickest as compared to the other physical variables. Figures show how the impulsive motion ends up with a steady state which were calculated by ignoring the time derivative terms in equations (9-12). The success of the devised numerical method in capturing the steady state solution by assigning larger time steps is also possible (though not demonstrated here) due to its unconditional stability. It is further noticeable from the figure that the circumferential velocity attains its steady state quickest as compared to the other physical variables.

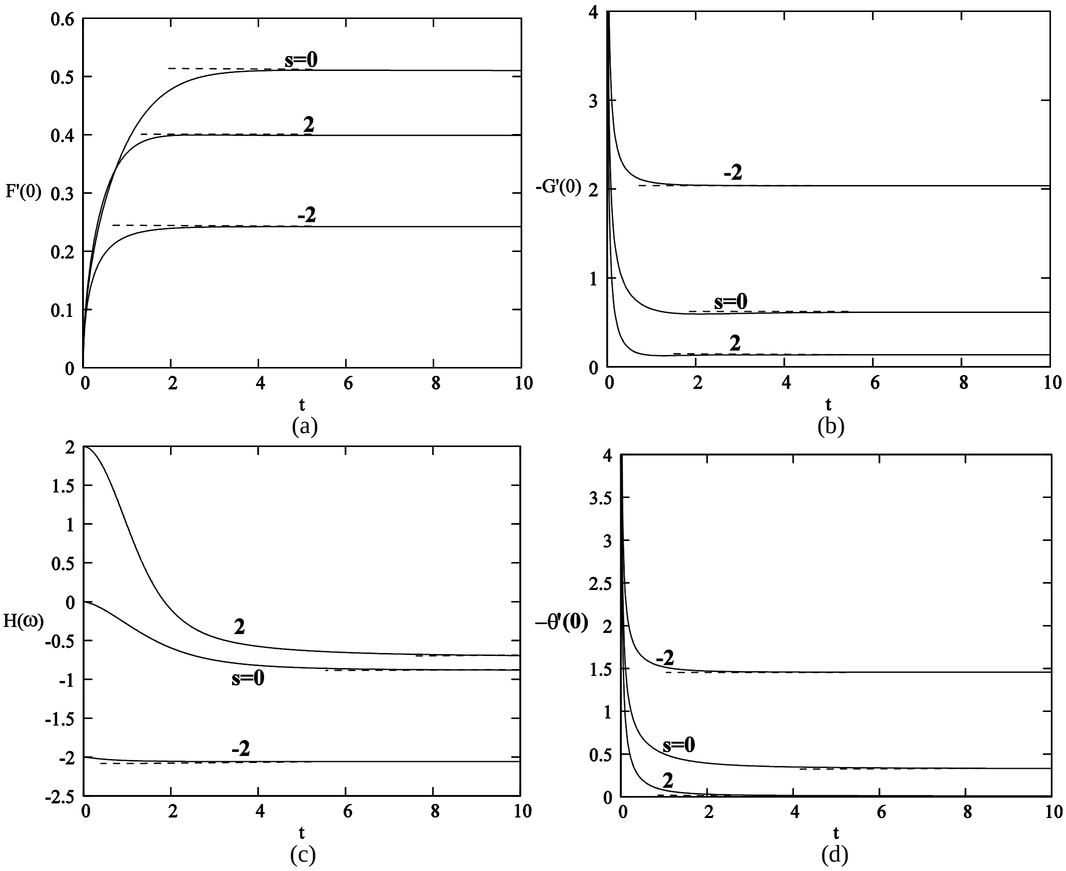

Figure 3. The time progression of physically significant parameters are shown respectively in (a) $F'(0)$, (b) $-G'(0)$, (c) $H(\infty)$ and (d) $-\theta'(0)$. A dashed line corresponds to the steady state value.

Table 1. Values of $F'(0)$, $-G'(0)$, $-H(\infty)$ and $-\theta'(0)$ corresponding to the large time limit steady state zero suction case. First row is from the present computation and second row is from Jasmine and Gajjar (2005).

$F'(0)$	$-G'(0)$	$-H(\infty)$	$-\theta'(0)$
0.510232	0.615922	0.884473	0.328574
0.510232	0.615922	0.884473	0.328573

We finally demonstrate in Figures 3(a-d) the time development of $F'(0)$, $-G'(0)$, $H(\infty)$ and $-\theta'(0)$ which are closely related to the radial skin friction, azimuthal skin friction (also torque), axial velocity at infinity and the local rate of heat transfer for $\Pr = 0.72$ and all computed with a step size of $\Delta t = 0.01$. The steady state values are also shown by the broken lines in the figure and Table 1.

It can again be seen that the method successfully generates the steady state values of the physically important parameters, the fastest for the case of tangential skin frictions consistent with Figure 4.

CONCLUSIONS

A new numerical integration scheme has been proposed in this paper to compute the time-dependent boundary layer flow equations. The method is based on the spectral Chebyshev collocation discretization along the coordinate normal to the fluid flow motion and Euler implicit forward time discretization in time. For the specific example, the three-dimensional unsteady boundary layer flow due to a porous rotating disk has been considered and the mean velocity and temperature fields approaching their steady states have been successfully computed with the method.

The devised spectral method in combined with the implicit Euler time differencing has been found to be robust, unconditionally stable, highly accurate and easy to implement. One of the most important advantage of the presented technique is also its capability to deal with the discontinuities occurring due to the different initial and boundary conditions in an unsteady flow motion. It is a known fact that a finite difference method involves numerical oscillations at such cases and thus requires extreme care, directing the researcher to couple the algorithm with the new transformations and solve more equations than actually needed. However, no such numerical drawbacks which deteriorate the numerical solution, encountered by the proposed compact spectral method.

The developed method has been readily applied to the incompressible, viscous, laminar and time-dependent three-dimensional swirling fluid flow over a rotating disk subject to a wall suction or injection. Starting from zero initial solutions and advancing in time, the method successfully generates the velocity and temperature distributions which evolve into their steady state counterparts after a sufficient time past. The torque, shear stresses, axial suction velocity and heat transfer rate, which are of fundamental importance in view of physics, have also been calculated.

Although the classical von Karman swirling flow of fluid mechanics has been accounted for the application of the method here, the proposed method seems to be highly promising for the evaluation of unsteady flow phenomena in other branches of science and engineering.

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