



|          |                   |
|----------|-------------------|
| $\mu$    | dynamic viscosity |
| $\rho$   | density           |
| $\psi$   | stream function   |
| $\omega$ | vorticity         |

#### *Subscripts*

|     |                 |
|-----|-----------------|
| a   | average         |
| C   | cold            |
| eff | effective       |
| f   | fluid           |
| H   | hot             |
| o   | reference value |
| p   | partition       |
| s   | solid           |

#### *Superscripts*

|   |                      |
|---|----------------------|
| * | dimensional variable |
|---|----------------------|

## INTRODUCTION

Buoyancy-driven fluid flow and heat transfer in enclosures have received considerable attention due to numerous potential engineering applications such as energy transfer in buildings, cooling of electronic devices and solar collectors. Conventional heat transfer fluids have been used as working fluids in most of previous studies that were performed in parallel with related applications (Frederic and Berbakow, 2002; Rahman and Sharif, 2003; Kahveci, 2007a-c; Kahveci and Öztuna, 2008 and 2009). However, conventional heat transfer fluids have lower thermal conductivity coefficients, which is the basic limitation of the heat transfer performance of the system considered. Recent studies have shown that this limitation, which is due to the lower thermal conductivity coefficient of conventional heat transfer fluids, can be overcome with the use of nanofluids. The term nanofluid is used to describe a solid and liquid mixture that consists of a base liquid and nanoparticles of less than 100 nm in size. Studies show that an anomalous increase is seen in thermal conductivity with the usage of nanoparticles (Eastman et al., 2001; Choi et al., 2001; Xuan and Li, 2000). Keblinski et al. (Keblinski et al., 2002) examined four possible mechanisms that could lead to this unexpected behavior. These mechanisms are: Brownian motion of nanoparticles, molecular level layering of the liquid at the liquid-particle interface, the nature of heat transfer in the nanoparticles, and nanoparticle clustering. Keblinski et al. (Keblinski et al., 2002) have also shown that Brownian motion has no significant effect on conduction while the liquid layering around nanoparticles has a fast.

There are many studies in the literature investigating the effects of nanoparticle usage on convection heat transfer. Xuan and Li (Xuan and Li, 2000) observed that oxide nanoparticles enhance heat transfer reasonably well, but metal nanoparticles have no significant effect. Eastman et al. (Eastman et al., 1998) showed that the convection heat transfer rate increases by more than

15% in water with a volume fraction of CuO of less than 1%. Wen and Ding (Wen and Ding, 2004) examined laminar flow at the entrance region of a tube through which nanofluid-containing alumina-water flowed with a 1.6% solid volume fraction; the local heat transfer coefficient at the entrance region was 41% higher than that of the base fluid with the same flow rate. Maiga et al. (Maiga et al., 2004) studied laminar and turbulent flow of nanofluids in a uniformly heated tube using approximated correlations and found that heat transfer enhancement due to nanoparticle usage becomes more important for the turbulent flow regime with increasing Reynolds numbers. Akbarinia and Behzadmehr (Akbarinia and Behzadmehr, 2007) studied laminar mixed convection of an  $\text{Al}_2\text{O}_3$ -based nanofluid in a horizontal curved tube and found that for a given nanoparticle concentration, increasing the buoyancy forces causes a reduction in skin friction. Izadi et al. (Izadi et al., 2009) performed a numerical investigation on the laminar forced convection of an  $\text{Al}_2\text{O}_3$  and water nanofluid in an annulus and observed that the axial velocity profile does not significantly change with nanoparticle volume fraction; the temperature profiles, however, are affected by the nanoparticle concentration. Khanafer et al. (Khanafer et al., 2003) studied the natural convection of copper-based nanofluids in a differentially heated square cavity. Their results show that the presence of nanoparticles enhances heat transfer by approximately 25% for  $\text{Gr}=10^4$  and  $\text{Gr}=10^5$  at a volume fraction of  $\phi=0.2$ . Hwang et al. (Hwang et al., 2007) studied the buoyancy-driven heat transfer of water-based  $\text{Al}_2\text{O}_3$  nanofluids in a rectangular cavity and found that the ratio of the heat transfer coefficient of nanofluids to that of the base fluid decreases with increasing nanoparticle size. Jou and Tzeng (Jou and Tzeng, 2006) studied convective heat transfer of a copper-based nanofluid in a two-dimensional enclosure and observed that the average Nusselt number at the hot wall exhibits an increasing trend with a decrease in the aspect ratio. Oztop and Abu-Nada (Oztop and Abu-Nada, 2008) studied the natural convection of nanofluids in a partially heated enclosure and found that the increase in heat transfer is more pronounced at low aspect ratios than at high aspect ratios. Aminossadati and Ghasemi (Aminossadati and Ghasemi, 2009) investigated the natural convection cooling of a heat source embedded on the bottom wall of an enclosure filled with nanofluids and found that the type of nanoparticle and the length and location of the heat source significantly affects the maximum temperature of the heat source. Heat transfer augmentation in a two-sided, lid-driven, differentially heated square cavity utilizing nanofluids was studied numerically by Tiwari and Das (Tiwari and Das, 2007). They observed that when both vertical walls move upward in the same direction, heat transfer is reduced. They also found that when vertical walls move in the opposite directions, heat transfer is considerably enhanced for a forced convection-dominated regime. The buoyancy-driven heat transfer of water-based

nanofluids in a differentially heated tilted enclosure was investigated by Kahveci (Kahveci, 2010). He found that the average heat transfer rate increases up to 44% for  $Ra=10^4$ , up to 53% for  $Ra=10^5$ , and up to 54% for  $Ra=10^6$  in the special case of  $\theta=90^\circ$ , which also produces the lowest heat transfer rates, is not taken into consideration. He also found that the maximum heat transfer takes place at  $\theta=45^\circ$  for  $Ra=10^4$  and at  $\theta=30^\circ$  for  $Ra=10^5$  and  $10^6$ .

Convective heat transfer of nanofluids in a rectangular enclosure has been studied extensively in the literature. But the problem of convective heat transfer of nanofluids in a partitioned enclosure has not been analyzed yet. In this study, natural convection heat transfer of water-based nanofluids in a partially divided enclosure is investigated numerically and the effects of partition position and length, nanoparticle volume fraction and Rayleigh number on heat transfer are revealed.

### Polynomial-Based Differential Quadrature Method

In the PDQ method (Shu, 1992; Shu, 2000; Shu and Richards, 1992; Belman et al., 1972), first and second order derivatives of  $f(x)$  at a point  $x_i$  are assumed to be approximated by

$$f_x(x_i) = \sum_{j=0}^n a_{ij} f(x_j) \quad , \text{ for } i=0,1,2,\dots,n, \quad (1)$$

$$f_{xx}(x_i) = \sum_{j=0}^n b_{ij} f(x_j) \quad , \text{ for } i=0,1,2,\dots,n, \quad (2)$$

where  $n$  is the number of the grid points,  $a_{ij}$  and  $b_{ij}$  are the first and second order weighting coefficients respectively. Shu and Richards (Shu and Richards, 1992) derived the following explicit formulations to compute the weighting coefficients.

$$a_{ij} = \frac{M^{(1)}(x_i)}{(x_i - x_j)M^{(1)}(x_j)} \quad j \neq i, \quad a_{ii} = - \sum_{k=1, k \neq i}^n a_{ik} \quad (3)$$

$$b_{ij} = 2a_{ij} \left( a_{ii} - \frac{1}{x_i - x_j} \right) \quad j \neq i, \quad b_{ii} = - \sum_{k=1, k \neq i}^n b_{ik} \quad (4)$$

$$M^{(1)}(x_i) = \prod_{k=1, k \neq i}^N (x_i - x_k) \quad , \quad (5)$$

There is a recurrence relationship to compute weighting coefficients of the higher order derivatives (Belman et al., 1972). When the coordinates of grid points are known, the weighting coefficients for the discretization of derivatives can be easily calculated from equations (3-5).

### ANALYSIS

The schematic of the system with coordinates is shown in Fig. 1. The square enclosure contains a vertical partition of height  $h$ .

In order to nondimensionalize the governing equations; the following dimensionless variables are used:

$$x = \frac{x^*}{W}, \quad y = \frac{y^*}{W}, \quad x_p = \frac{x_p^*}{W}, \quad L = \frac{H}{W}, \quad u = \frac{u^*}{\alpha_f / W},$$

$$v = \frac{v^*}{\alpha_f / W}, \quad p = \frac{W^2}{\rho_f \alpha_f^2} p^*, \quad T = \frac{T^* - T_C}{T_H - T_C} \quad (6)$$

where  $W$  is the width of the enclosure,  $u^*$  and  $v^*$  are the dimensional velocity components,  $p^*$  is the dimensional pressure,  $T^*$  is the dimensional temperature,  $\rho_f$  is the fluid density, and  $\alpha$  is the thermal diffusivity of the fluid.

For computational purposes, the domain considered in this work is divided into four regions as shown in Figure 1. The flow is assumed to be Newtonian, two-dimensional, steady and incompressible. The dimensionless governing equations can be written as follows in the vorticity-stream function formulation, after invoking the Boussinesq approximation and neglecting viscous dissipation and thermal radiation:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \quad (7)$$

$$u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \frac{\nu_{eff}}{\nu_f} Pr \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + \frac{(\rho \beta_T)_{eff}}{\rho_{eff} \beta_{T,f}} Ra Pr \frac{\partial T}{\partial x} \quad (8)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha_{eff}}{\alpha_f} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \quad (9)$$

Here the Prandtl and Rayleigh numbers are defined as:

$$Pr = \frac{\nu_f}{\alpha_f}, \quad Ra = \frac{g \beta_{T,f} W^3 \Delta T^*}{\nu_f \alpha_f} \quad (10)$$

where  $\nu$  is the kinematic viscosity of the fluid,  $g$  is the gravitational acceleration and  $\beta_T$  is the coefficient of thermal expansion.  $\Delta T^*$  is the temperature difference between the isothermal walls of the enclosure.

The dimensionless stream function and vorticity are defined as follows:

$$u = \frac{\partial \psi}{\partial y}, \quad v = - \frac{\partial \psi}{\partial x}, \quad \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (11)$$

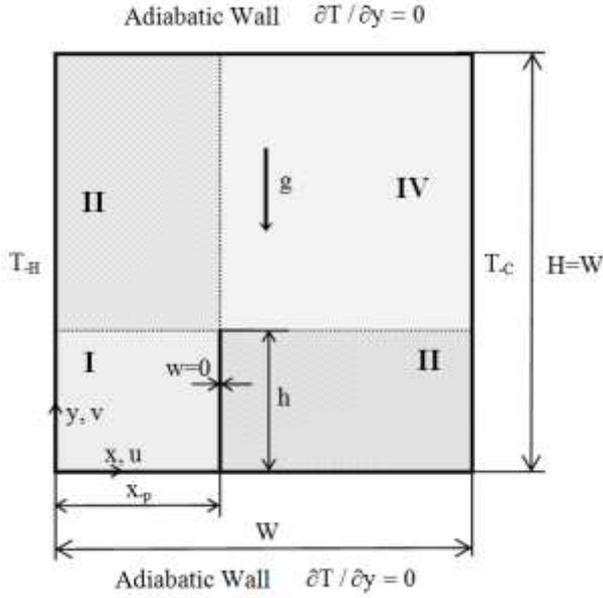


Figure 1. Geometry and coordinate system.

For the effective thermal conductivity, which is the most important thermophysical property, the following approach proposed by Yu and Choi (Yu and Choi, 2003) is used.

$$k_{eff} / k_f = \frac{k_s + 2k_f + 2(k_s - k_f)(1 + \beta)^3 \phi}{k_s + 2k_f - (k_s - k_f)(1 + \beta)^3 \phi} \quad (12)$$

where  $\beta$  is the ratio of the nanolayer thickness to the original particle radius. Yu and Choi (Yu and Choi, 2003) compared their results for  $\beta=0.1$  with existing experimental results from previous studies and obtained reasonably good agreement.

Nanofluid viscosity is generally estimated using existing relations for a two-phased mixture. The following Brinkman model (Brinkman, 1952) was used as the relation for the effective viscosity in this study.

$$\mu_{eff} = \mu_f / (1 - \phi)^{2.5} \quad (13)$$

Xuan and Li (Xuan and Li, 1999) experimentally measured the effective viscosity of a transformer oil–water nanofluid and a water–copper nanofluid within a temperature range of 20–50°C. Their experimental results showed reasonably good agreement with Brinkman’s theory.

Other effective properties that are present in the governing equations can be defined by the following relations.

$$(\rho C_p)_{eff} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \quad (14)$$

$$(\rho \beta_T)_{eff} = (1 - \phi)(\rho \beta_T)_f + \phi(\rho \beta_T)_s \quad (15)$$

Where

$$\rho_{eff} = (1 - \phi)\rho_f + \phi\rho_s \quad (16)$$

The appropriate boundary conditions for the governing equations are as follows.

$$\psi(x, 0) = 0, \quad \left. \frac{\partial T}{\partial y} \right|_{x, 0} = 0, \quad \psi(x, 1) = 0, \quad \left. \frac{\partial T}{\partial y} \right|_{x, 1} = 0 \quad (17)$$

$$\psi(0, y) = 0, \quad T(0, y) = 1, \quad \psi(1, y) = 0,$$

$$T(1, y) = 0 \quad (18)$$

$$\psi(x_p, y) = 0, \quad T(x_p^-, y) = T(x_p^+, y),$$

$$\left. \frac{\partial T}{\partial x} \right|_{x_p^-, y} = \left. \frac{\partial T}{\partial x} \right|_{x_p^+, y} \quad 0 \leq y \leq 1 \quad (19)$$

There is no physical boundary condition for the vorticity. However, an expression can be derived from a Taylor series expansion of the stream function equation as  $\omega_{wall} = -\partial^2 \psi / \partial \eta^2$ . Here,  $\eta$  is the outward direction normal to the surface. The equation above was therefore used as the boundary condition for the vorticity on the partition surfaces as well as for the enclosure surfaces in the present study.

The variation of the Nusselt number can be expressed as follows:

$$Nu = - \frac{k_{eff}}{k_f} \left. \frac{\partial T}{\partial \eta} \right|_{\eta=0} \quad (20)$$

## RESULTS AND DISCUSSION

The PDQ method [25-28], with the following non-uniform Chebyshev-Gauss-Lobatto grid point distribution, was used to transform the governing equations into a set of algebraic equations.

$$x_i = \frac{1}{2} \left[ 1 - \cos\left(\frac{i}{n_x} \pi\right) \right], \quad i = 0, 1, 2, \dots, n_x$$

$$y_j = \frac{1}{2} \left[ 1 - \cos\left(\frac{j}{n_y} \pi\right) \right], \quad j = 0, 1, 2, \dots, n_y \quad (21)$$

The points in this grid system are more closely spaced in regions near the walls, where large velocity and temperature gradients are expected to develop.

The solutions of the governing equations are obtained by the successive over-relaxation (SOR) iteration method for Rayleigh numbers from  $10^4$  to  $10^6$ , for partition locations ranging from 0.2 to 0.8, for partition lengths up to 1, and for solid volume fractions ranging from 0% to 10%. Three different nanoparticles (Cu,

CuO, and Al<sub>2</sub>O<sub>3</sub>) were considered, and the ratio of the nanolayer thickness to the original particle radius was taken as a fixed value of 0.1. Water was used as the base fluid. The thermophysical properties of the fluid and solid phases are shown in Table 1. The convergence

criteria were chosen as  $|R|_{\max} \leq 10^{-5}$ , where  $|R|_{\max}$  is the maximum absolute residual value for the vorticity, stream function and temperature equations.

**Table 1.** Thermophysical properties of base fluid and nanoparticles.

| Property                                 | Water | Cu     | CuO   | Al <sub>2</sub> O <sub>3</sub> |
|--|-------|--------|-------|--------------------------------|
| $\rho$ (kg/m <sup>3</sup> )              | 997.1 | 8933   | 6500  | 3970                           |
| $C_p$ (J/kgK)                            | 4179  | 385    | 535.6 | 765                            |
| $k$ (W/mK)                               | 0.613 | 400    | 20    | 40                             |
| $\alpha \times 10^7$ (m <sup>2</sup> /s) | 1.47  | 1163.1 | 57.45 | 131.7                          |
| $\beta_T \times 10^6$ (1/K)              | 210   | 51     | 51    | 24                             |

**Table 2.** Mesh point numbers for each region taken into consideration.

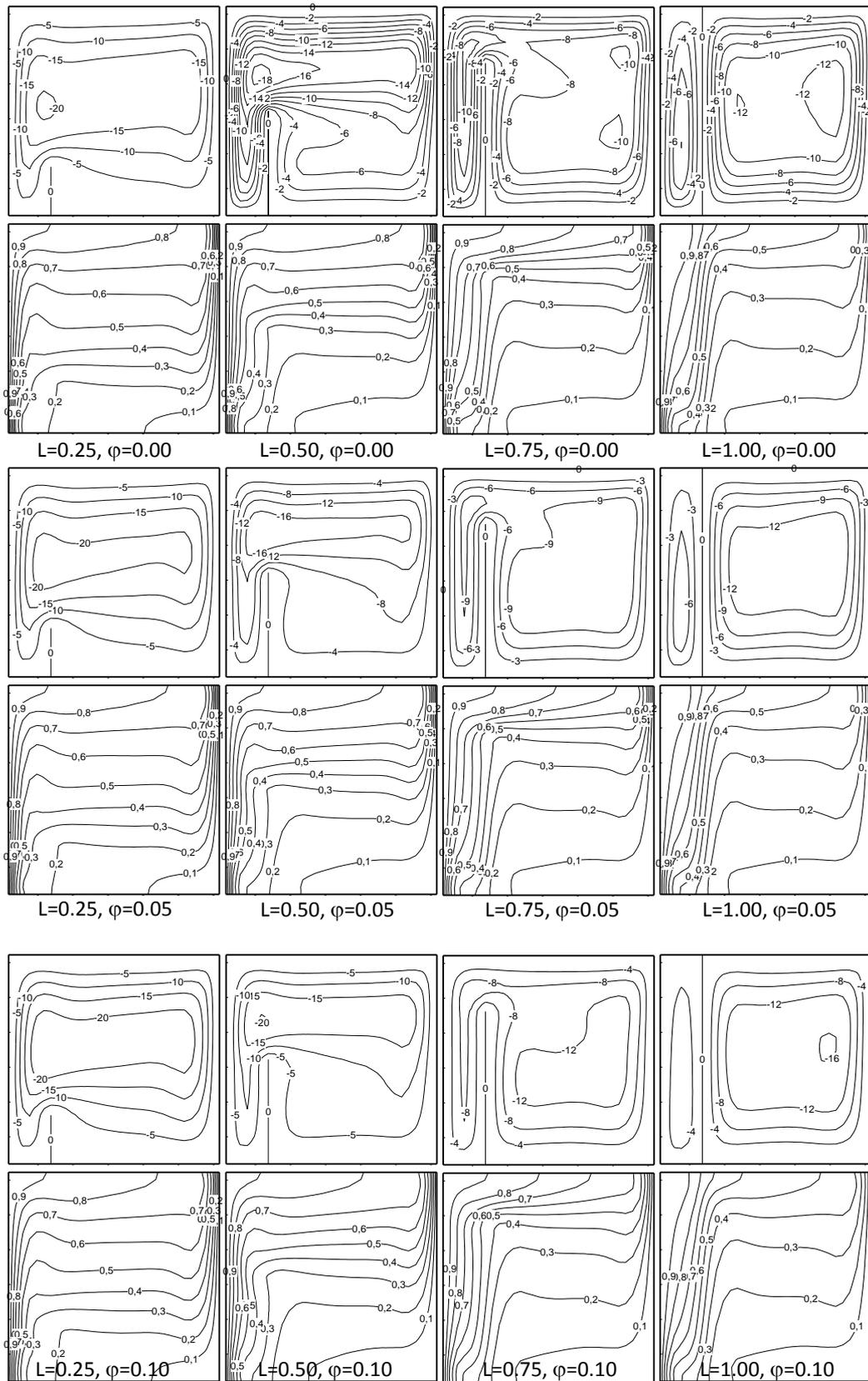
| $x_p$ | L    |       | I  | II | III | IV |
|-------|------|-------|----|----|-----|----|
| 0.2   | 0.25 | $N_x$ | 11 | 11 | 20  | 20 |
|       |      | $N_y$ | 11 | 20 | 11  | 20 |
|       | 0.50 | $N_y$ | 15 | 16 | 15  | 16 |
|       | 0.75 | $N_y$ | 20 | 11 | 20  | 11 |
|       | 1.00 | $N_y$ | 31 | -  | 31  | -  |
| 0.4   |      | $N_x$ | 14 | 17 | 14  | 17 |
| 0.6   |      | $N_x$ | 17 | 14 | 17  | 14 |
| 0.8   |      | $N_x$ | 20 | 11 | 20  | 11 |

**Table 3.** Grid dependency (Ra=10<sup>6</sup>,  $\phi$ =0.10, CuO).

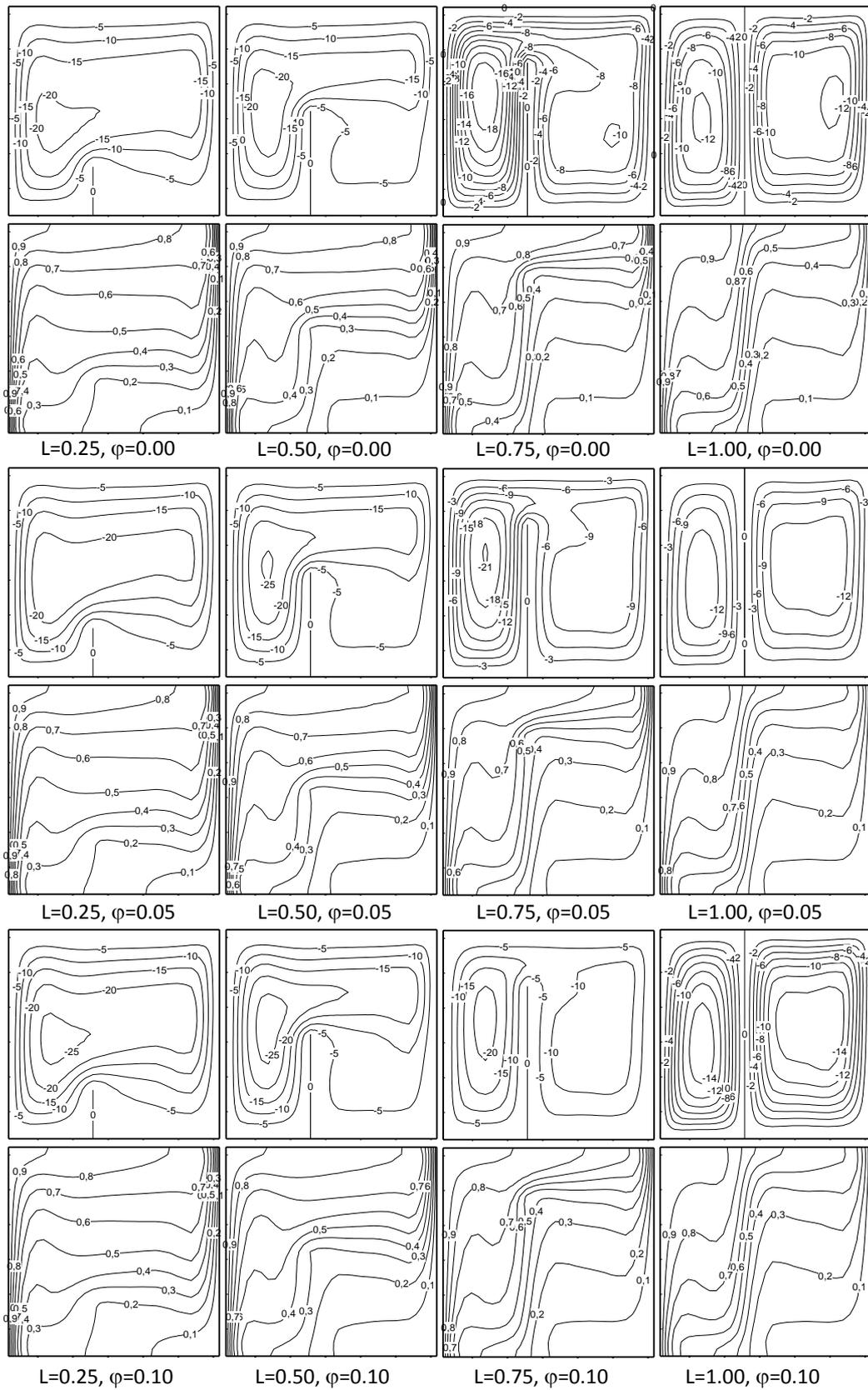
| $x_p$ | L    |                 | 19x19 | 23x23 | 27x27 | 31x31 |
|-------|------|-----------------|-------|-------|-------|-------|
| 0.2   | 0.25 | $Nu_a$          | 10.47 | 10.50 | 10.47 | 10.46 |
|       |      | $ \psi _{\max}$ | 24.3  | 24.5  | 24.3  | 24.3  |
|       | 0.75 | $Nu_a$          | 6.43  | 6.53  | 6.44  | 6.43  |
|       |      | $ \psi _{\max}$ | 14.0  | 13.5  | 13.6  | 13.6  |
| 0.4   | 0.25 | $Nu_a$          | 10.67 | 10.71 | 10.72 | 10.71 |
|       |      | $ \psi _{\max}$ | 26.4  | 26.6  | 26.5  | 26.5  |
|       | 0.75 | $Nu_a$          | 6.93  | 6.87  | 6.81  | 6.80  |
|       |      | $ \psi _{\max}$ | 22.5  | 22.7  | 22.3  | 22.3  |

**Table 4.** Validation of the numerical code.

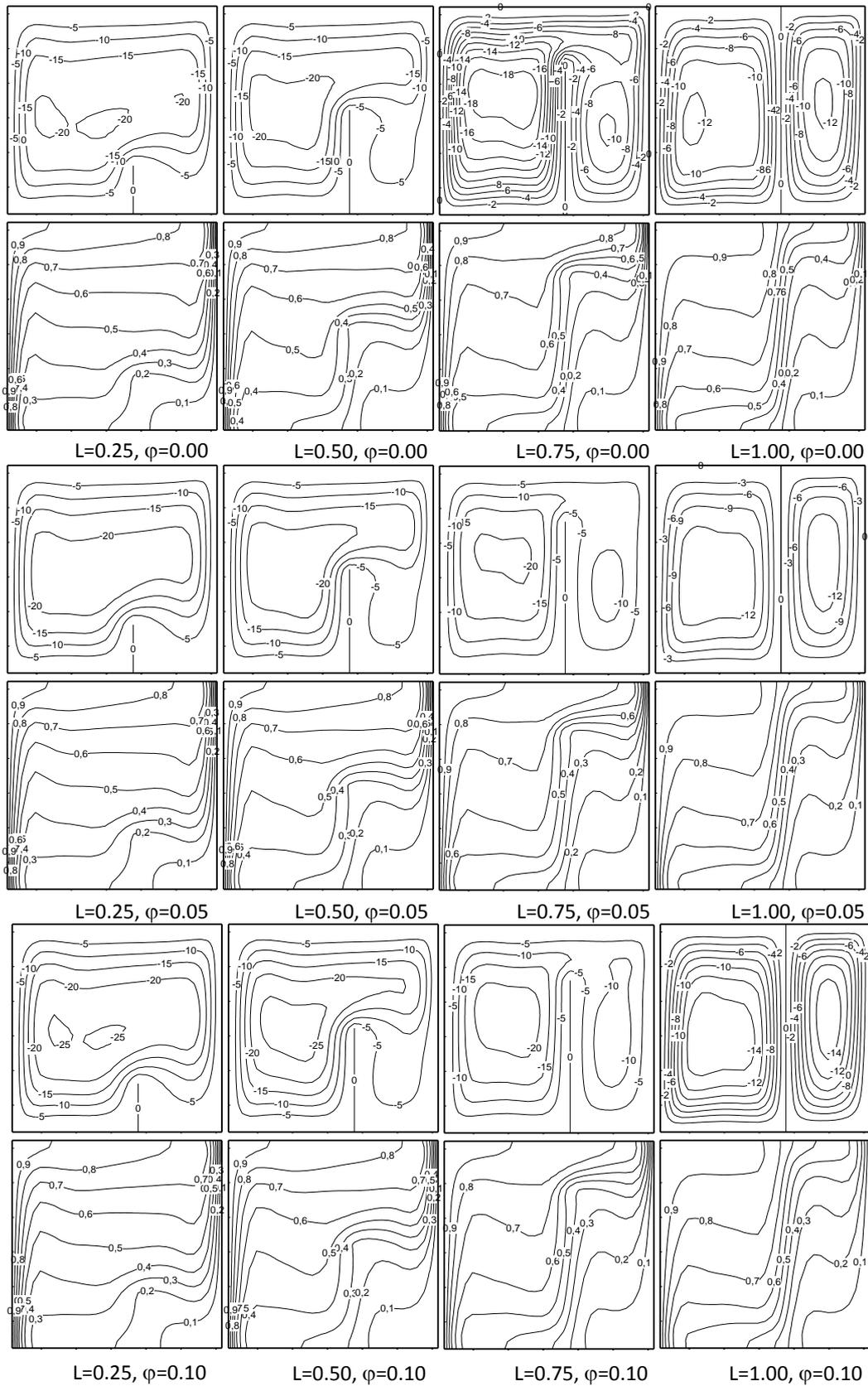
|                 | Ra=10 <sup>4</sup> |         | Ra=10 <sup>5</sup> |         | Ra=10 <sup>6</sup> |         |
|-----------------|--------------------|---------|--------------------|---------|--------------------|---------|
|                 | de Vahl Davis      | Present | de Vahl Davis      | Present | de Vahl Davis      | Present |
| $ \psi _{\max}$ | -                  | 5.07    | 9.61               | 9.60    | 16.75              | 16.72   |
| $Nu_a$          | 2.24               | 2.24    | 4.52               | 4.52    | 8.80               | 8.82    |
| $Nu_{\max}$     | 3.53               | 3.53    | 7.72               | 7.70    | 17.93              | 17.56   |
| $Nu_{\min}$     | 0.59               | 0.59    | 0.73               | 0.73    | 0.99               | 0.98    |



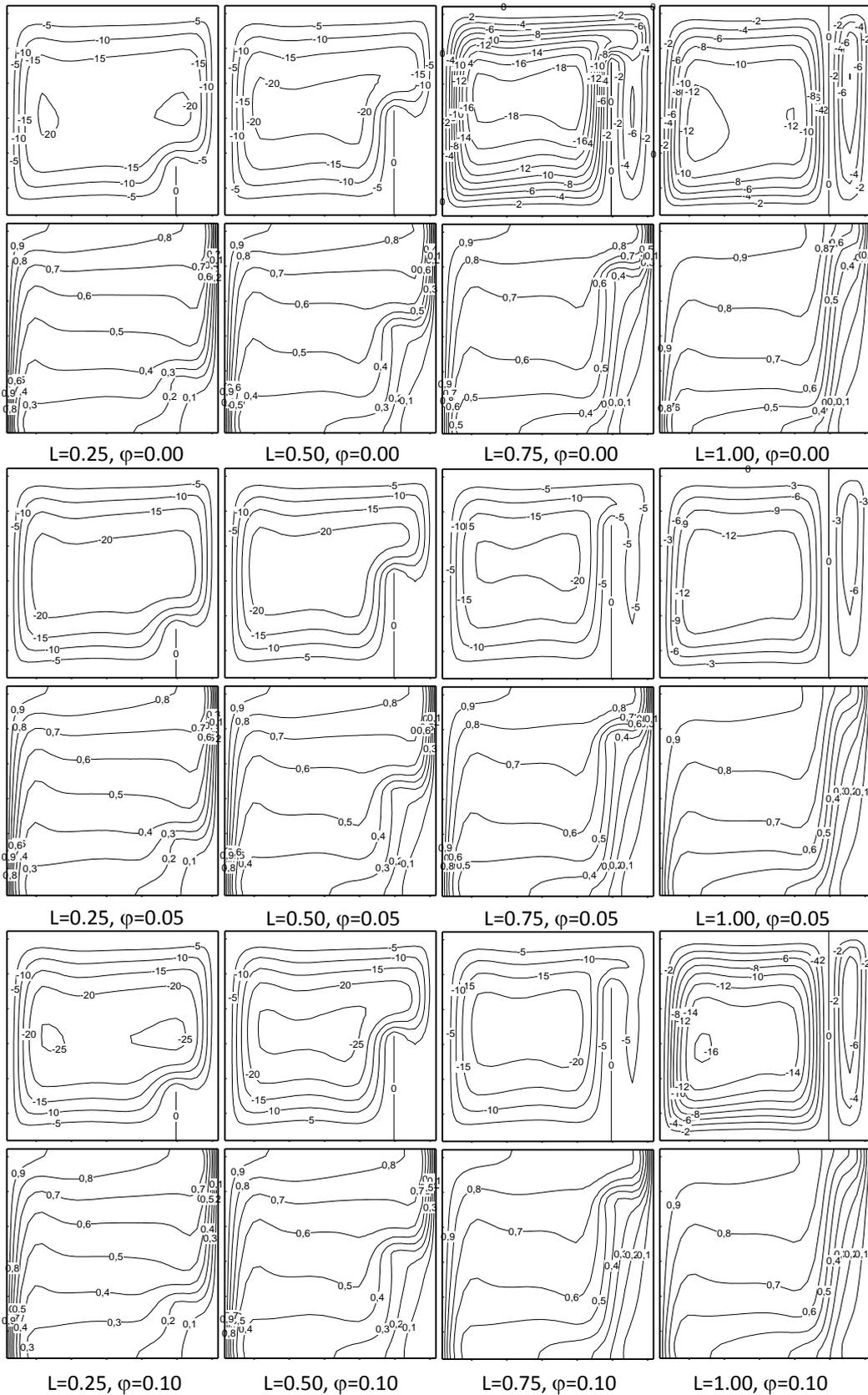
**Figure 2.** Streamlines and isotherms of copper based nanofluid for  $x_p=0.2$  and  $Ra=10^6$ .



**Figure 3.** Streamlines and isotherms of copper based nanofluid for  $x_p=0.4$  and  $Ra=10^6$ .



**Figure 4.** Streamlines and isotherms of copper based nanofluid for  $x_p=0.6$  and  $Ra=10^6$ .



**Figure 5.** Streamlines and isotherms of copper based nanofluid for  $x_p=0.8$  and  $Ra=10^6$ .

A series of grid systems of up to 31\*31 points were performed to obtain a grid-independent mesh size, suggesting that when the mesh size is greater than 26x26,  $Nu_a$  on the hot wall and  $|\psi|_{\max}$  remain almost the same. Some results are shown in Table 2. Therefore a grid system of 31\*31 was used in this study. The numbers of the mesh points used for the regions in the considered geometry are also listed in Table 3.

To validate the numerical code, the solution for the non-partitioned square enclosure with differentially heated sidewalls and adiabatic top and bottom walls were also computed and compared with the benchmark results of de Vahl Davis [32]. The results given in Table 4 showed that there was excellent agreement between the results of the numerical code used in the present study and the benchmark results of De Vahl Davis (De Vahl Davis, 1983).

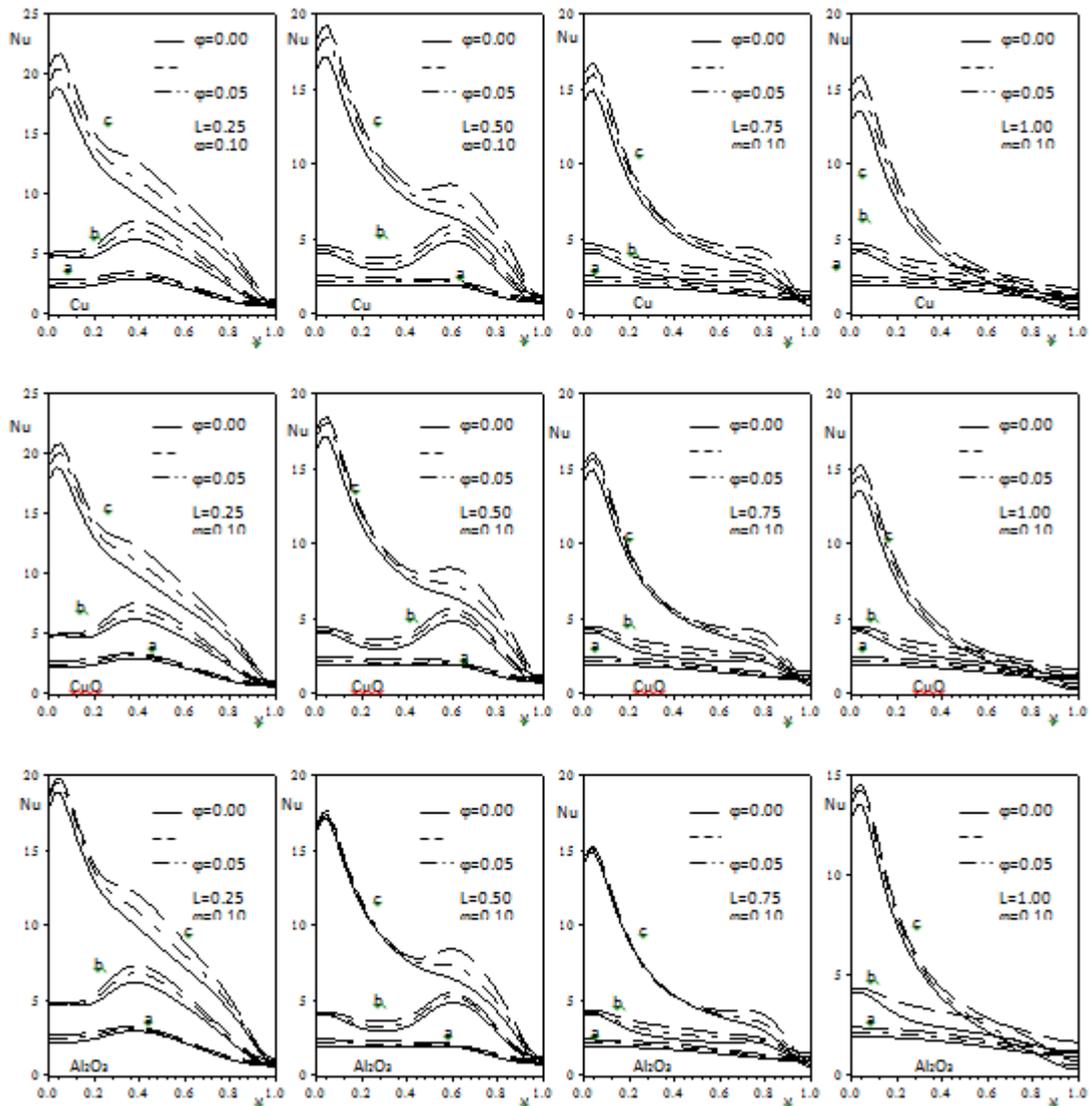


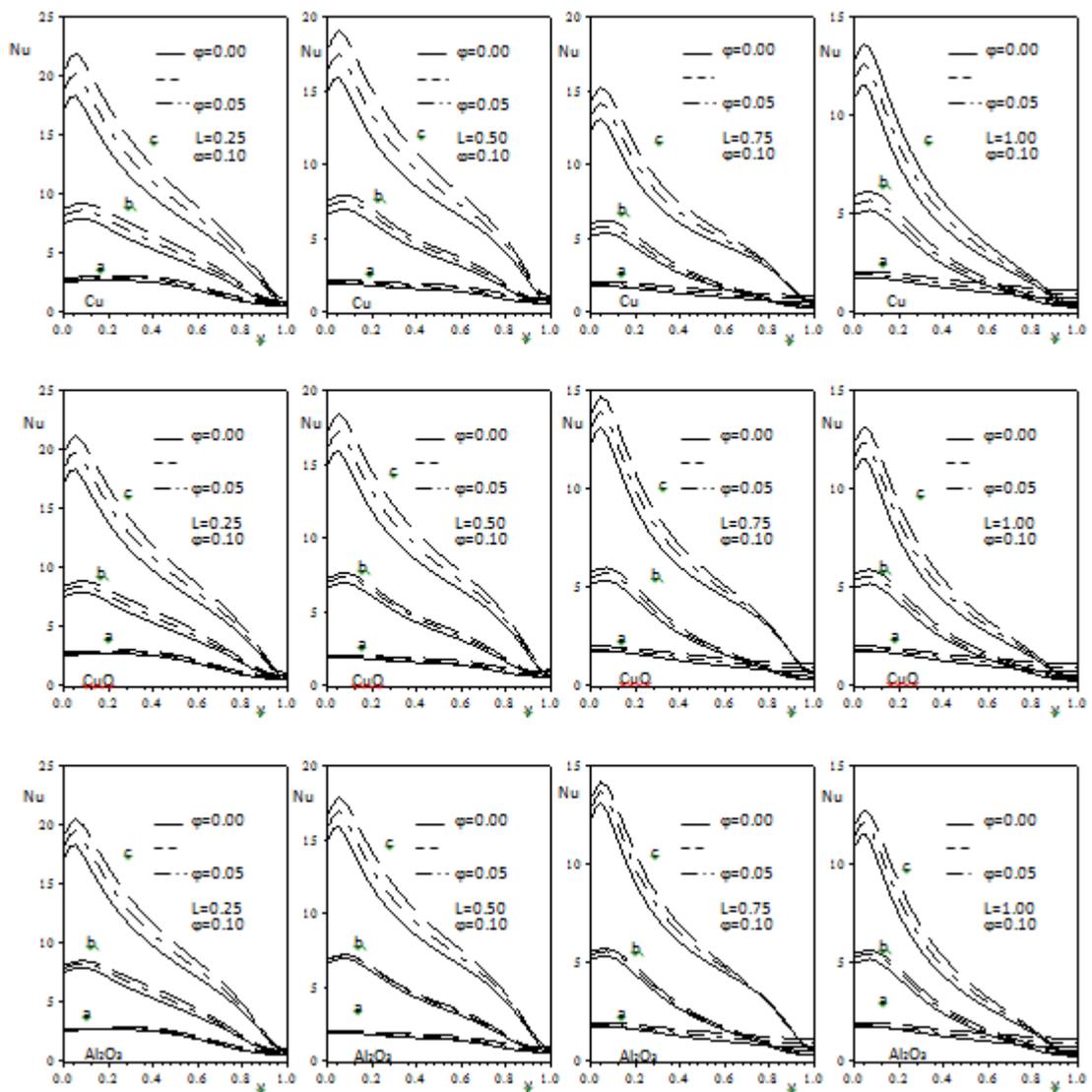
Figure 6. Local Nusselt number for  $x_p=0.2$  and a)  $Ra=10^4$ , b)  $10^5$ , c)  $Ra=10^6$ .

The streamlines and isotherms for various partition location values are shown in Figures 2-5 for CuO nanoparticle case. It can be seen that as the partition height increases, the circulation strength becomes weak as a result of increase in blocking effect of the partition on flow in the enclosure. Flow is single-cellular for low partition heights. Single cellular flow breaks down as partition height increases and a double cellular flow forms the flow field. When the partition divides the enclosure into two separate regions ( $l=1.0$ ), single cellular flows constitutes the flow fields in both regions

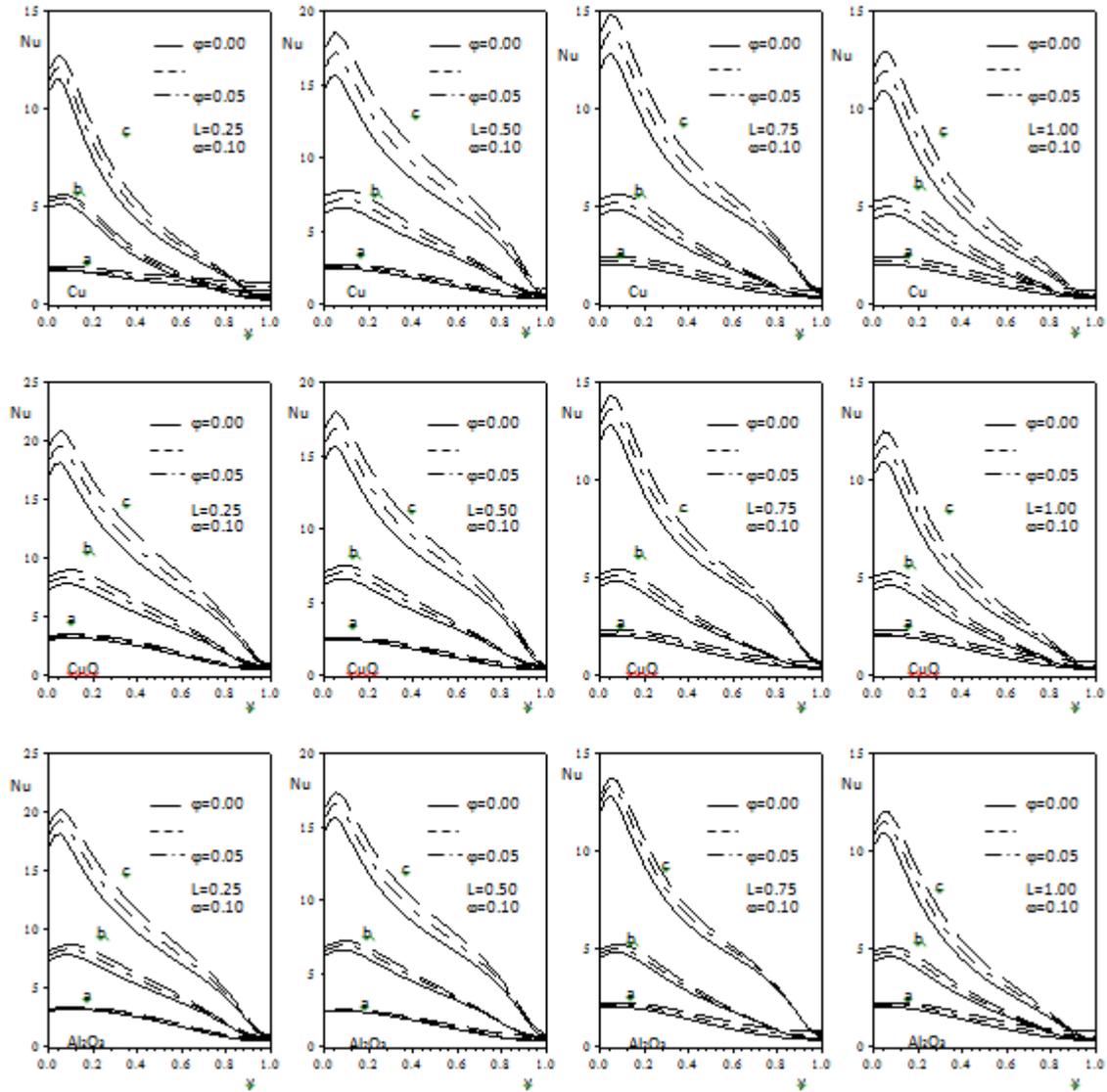
of the enclosure. Flows in these regions form as a consequence of the thermal interaction between these two regions. As expected, circulation strength in the smaller region is weaker for completely divided enclosure case because of higher viscous effects in this region. As can be seen from the figures that the circulation strength increases with increasing solid volume fraction as a result of higher energy transfer from the hot wall to the nanofluid with higher thermal conductivity. Furthermore, the isotherms become gradually parallel to the isothermal walls.

Variation of the local Nusselt number along the hot wall with the Rayleigh number, nanoparticle solid volume fraction, partition location and partition height can be seen in Figures 6- 8 for various nanofluids. As expected, the local Nusselt number is strongly dependent on the Rayleigh number and increases considerably with increasing Rayleigh numbers. When the partition is close to the hot wall, it obstructs the flow within the region between the hot wall and the partition, and relatively weak flow occurs in this region. Therefore, the local Nusselt number gets relatively lower values in this region. When this region is passed, local Nusselt number begins to take higher values. As the fluid particles moving along the hot wall are heated, the local Nusselt number begins to decrease again. When the

partition is close to the hot wall, the Nusselt number displays two peak points. As apparent in the figures, the local Nusselt number gets lower values as the partition height increases because of its negative effect on the flow. As can be seen in the figures, the local Nusselt number exhibits a similar form for all nanofluids studied. The magnitude of the local Nusselt number differs for different types of nanofluids; it was higher for Cu nanoparticles. This is due to the higher thermal energy transport from the hot wall to the fluid with higher thermal conductivity. The figures also show the expected result that an increase in nanoparticle solid volume fraction leads to an increase in the local Nusselt number.



**Figure 7.** Local Nusselt number for  $x_p=0.4$  and a)  $Ra=10^4$ , b)  $10^5$ , c)  $Ra=10^6$ .



**Figure 8.** Local Nusselt number for  $x_p=0.6$  and a)  $Ra=10^4$ , b)  $10^5$ , c)  $Ra=10^6$ .

The variation of the average Nusselt number with various parameters considered in this study is shown in Tables 4-7. As stated above, circulation weakens with an increase in partition height and therefore, average Nusselt number takes lower values with increasing partition height. When the partition is close to the hot wall, it creates a negative effect on the flow in the smaller left region and prevents an effective heat transfer from the hot wall. When the partition is close to the cold wall, it creates a negative effect on the flow in the smaller right region and prevents an effective heat transfer from the cold wall, for high Rayleigh numbers. Consequently, as can be seen in the tables, the average Nusselt number initially increases and then decreases as

the partition moves away from the hot wall and approaches to the cold wall. For low Rayleigh numbers, the Nusselt number initially decreases and then increases as the partition moves away from the hot wall because the partition has more effect on flow in the enclosure for these weak flow cases. As seen from the tables, the values of Nusselt number from the biggest to lowest are for Cu, CuO and  $Al_2O_3$  nanoparticles. Increase in the average Nusselt number due to nanoparticle usage is up to 35% for  $Ra=10^4$ , up to 32% for  $Ra=10^5$ , and up to 25% for  $Ra=10^6$ . Furthermore, the average Nusselt number increases nearly linearly with increasing solid volume fraction.

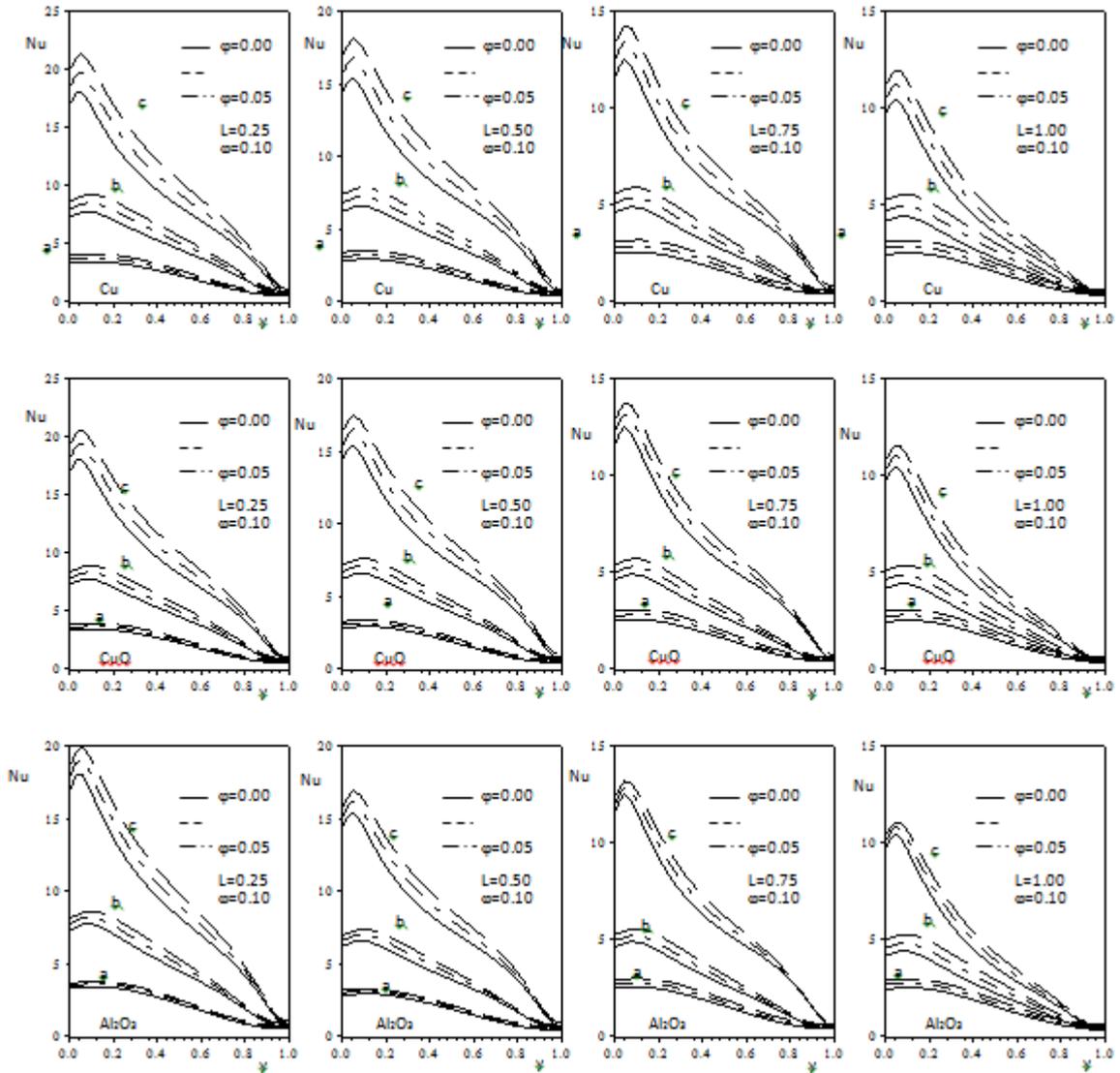


Figure 9. Local Nusselt number for  $x_p=0.8$  and a)  $Ra=10^4$ , b)  $10^5$ , c)  $10^6$ .

Table 4. Average Nusselt number for  $x_p=0.2$ .

| L    | Ra\j            | Cu   |      |       | CuO  |       | Al <sub>2</sub> O <sub>3</sub> |       |
|------|-----------------|------|------|-------|------|-------|--------------------------------|-------|
|      |                 | 0.00 | 0.05 | 0.10  | 0.05 | 0.10  | 0.05                           | 0.10  |
| 0.25 | 10 <sup>4</sup> | 2.01 | 2.24 | 2.47  | 2.20 | 2.38  | 2.17                           | 2.31  |
|      | 10 <sup>5</sup> | 4.16 | 4.62 | 5.08  | 4.53 | 4.90  | 4.47                           | 4.77  |
|      | 10 <sup>6</sup> | 8.84 | 9.86 | 10.85 | 9.67 | 10.46 | 9.54                           | 10.16 |
| 0.50 | 10 <sup>4</sup> | 1.66 | 1.87 | 2.09  | 1.84 | 2.02  | 1.82                           | 1.98  |
|      | 10 <sup>5</sup> | 3.34 | 3.70 | 4.07  | 3.63 | 3.92  | 3.58                           | 3.82  |
|      | 10 <sup>6</sup> | 7.74 | 8.52 | 9.22  | 8.35 | 8.88  | 8.21                           | 8.56  |
| 0.75 | 10 <sup>4</sup> | 1.53 | 1.75 | 1.98  | 1.72 | 1.91  | 1.71                           | 1.883 |
|      | 10 <sup>5</sup> | 2.47 | 2.80 | 3.17  | 2.75 | 3.06  | 2.73                           | 3.02  |
|      | 10 <sup>6</sup> | 5.79 | 6.26 | 6.69  | 6.13 | 6.43  | 6.00                           | 6.15  |
| 1.00 | 10 <sup>4</sup> | 1.53 | 1.75 | 1.98  | 1.72 | 1.91  | 1.70                           | 1.88  |
|      | 10 <sup>5</sup> | 2.35 | 2.70 | 3.09  | 2.65 | 2.99  | 2.64                           | 2.96  |
|      | 10 <sup>6</sup> | 4.34 | 4.84 | 5.34  | 4.74 | 5.14  | 4.68                           | 5.01  |

**Table 5.** Average Nusselt number for  $x_p=0.4$ .

| 1    | Ra\j            | 0.00 | Cu    |       | CuO  |       | Al <sub>2</sub> O <sub>3</sub> |       |
|------|-----------------|------|-------|-------|------|-------|--------------------------------|-------|
|      |                 |      | 0.05  | 0.10  | 0.05 | 0.10  | 0.05                           | 0.10  |
| 0.25 | 10 <sup>4</sup> | 1.94 | 2.13  | 2.31  | 2.08 | 2.22  | 2.05                           | 2.15  |
|      | 10 <sup>5</sup> | 4.49 | 4.98  | 5.45  | 4.88 | 5.25  | 4.81                           | 5.08  |
|      | 10 <sup>6</sup> | 8.90 | 10.00 | 11.12 | 9.81 | 10.71 | 9.69                           | 10.45 |
| 0.50 | 10 <sup>4</sup> | 1.39 | 1.55  | 1.74  | 1.52 | 1.68  | 1.51                           | 1.66  |
|      | 10 <sup>5</sup> | 3.78 | 4.12  | 4.41  | 4.03 | 4.24  | 3.95                           | 4.04  |
|      | 10 <sup>6</sup> | 7.86 | 8.82  | 9.79  | 8.65 | 9.44  | 8.54                           | 9.19  |
| 0.75 | 10 <sup>4</sup> | 1.20 | 1.38  | 1.60  | 1.36 | 1.55  | 1.36                           | 1.55  |
|      | 10 <sup>5</sup> | 2.46 | 2.68  | 2.91  | 2.63 | 2.80  | 2.58                           | 2.70  |
|      | 10 <sup>6</sup> | 5.91 | 6.50  | 7.06  | 6.37 | 6.80  | 6.26                           | 6.56  |
| 1.00 | 10 <sup>4</sup> | 1.19 | 1.38  | 1.60  | 1.36 | 1.55  | 1.35                           | 1.55  |
|      | 10 <sup>5</sup> | 2.27 | 2.53  | 2.79  | 2.48 | 2.69  | 2.45                           | 2.62  |
|      | 10 <sup>6</sup> | 4.39 | 4.93  | 5.48  | 4.84 | 5.29  | 4.78                           | 5.17  |

**Table 6.** Average Nusselt number for  $x_p=0.6$ .

| 1    | Ra\j            | 0.00 | Cu   |       | CuO  |       | Al <sub>2</sub> O <sub>3</sub> |       |
|------|-----------------|------|------|-------|------|-------|--------------------------------|-------|
|      |                 |      | 0.05 | 0.10  | 0.05 | 0.10  | 0.05                           | 0.10  |
| 0.25 | 10 <sup>4</sup> | 2.03 | 2.22 | 2.41  | 2.18 | 2.32  | 2.14                           | 2.24  |
|      | 10 <sup>5</sup> | 4.47 | 4.98 | 5.48  | 4.88 | 5.28  | 4.81                           | 5.12  |
|      | 10 <sup>6</sup> | 8.88 | 9.96 | 11.06 | 9.77 | 10.67 | 9.66                           | 10.40 |
| 0.50 | 10 <sup>4</sup> | 1.50 | 1.64 | 1.81  | 1.61 | 1.74  | 1.58                           | 1.70  |
|      | 10 <sup>5</sup> | 3.77 | 4.15 | 4.51  | 4.07 | 4.34  | 4.00                           | 4.17  |
|      | 10 <sup>6</sup> | 7.82 | 8.77 | 9.71  | 8.60 | 9.36  | 8.49                           | 9.11  |
| 0.75 | 10 <sup>4</sup> | 1.20 | 1.39 | 1.60  | 1.36 | 1.55  | 1.36                           | 1.55  |
|      | 10 <sup>5</sup> | 2.46 | 2.69 | 2.92  | 2.64 | 2.81  | 2.59                           | 2.72  |
|      | 10 <sup>6</sup> | 5.99 | 6.59 | 7.16  | 6.47 | 6.89  | 6.36                           | 6.65  |
| 1.00 | 10 <sup>4</sup> | 1.19 | 1.38 | 1.60  | 1.36 | 1.55  | 1.35                           | 1.55  |
|      | 10 <sup>5</sup> | 2.27 | 2.53 | 2.79  | 2.48 | 2.69  | 2.45                           | 2.62  |
|      | 10 <sup>6</sup> | 4.40 | 4.94 | 5.49  | 4.85 | 5.30  | 4.79                           | 5.17  |

**Table 7.** Average Nusselt number for  $x_p=0.8$ .

| 1    | Ra\j            | 0.00 | Cu   |       | CuO  |       | Al <sub>2</sub> O <sub>3</sub> |       |
|------|-----------------|------|------|-------|------|-------|--------------------------------|-------|
|      |                 |      | 0.05 | 0.10  | 0.05 | 0.10  | 0.05                           | 0.10  |
| 0.25 | 10 <sup>4</sup> | 2.15 | 2.38 | 2.60  | 2.33 | 2.50  | 2.29                           | 2.42  |
|      | 10 <sup>5</sup> | 4.44 | 4.95 | 5.46  | 4.86 | 5.27  | 4.80                           | 5.13  |
|      | 10 <sup>6</sup> | 8.79 | 9.84 | 10.90 | 9.66 | 10.51 | 9.54                           | 10.25 |
| 0.50 | 10 <sup>4</sup> | 1.81 | 2.01 | 2.21  | 1.97 | 2.131 | 1.94                           | 2.07  |
|      | 10 <sup>5</sup> | 3.82 | 4.26 | 4.69  | 4.18 | 4.52  | 4.12                           | 4.39  |
|      | 10 <sup>6</sup> | 7.69 | 8.59 | 9.49  | 8.42 | 9.15  | 8.31                           | 8.90  |
| 0.75 | 10 <sup>4</sup> | 1.54 | 1.76 | 1.99  | 1.72 | 1.92  | 1.71                           | 1.89  |
|      | 10 <sup>5</sup> | 2.69 | 3.01 | 3.35  | 2.95 | 3.23  | 2.92                           | 3.16  |
|      | 10 <sup>6</sup> | 5.94 | 6.55 | 7.14  | 6.42 | 6.87  | 6.31                           | 6.64  |
| 1.00 | 10 <sup>4</sup> | 1.53 | 1.75 | 1.98  | 1.72 | 1.91  | 1.70                           | 1.88  |
|      | 10 <sup>5</sup> | 2.34 | 2.70 | 3.09  | 2.65 | 2.99  | 2.64                           | 2.96  |
|      | 10 <sup>6</sup> | 4.34 | 4.83 | 5.33  | 4.74 | 5.14  | 4.67                           | 5.00  |

## CONCLUSION

The natural convection heat transfer of water-based nanofluids in a differentially heated, partially divided enclosure was studied numerically for a range of partition locations, partition lengths, solid volume fractions and Rayleigh numbers. The results show that heat transfer decreases considerably with increasing partition height. Furthermore, as the distance of the partition from the hot wall increased, an initial decrease and a subsequent increase is observed in the average Nusselt number for low Rayleigh numbers and an initial increase and subsequent decrease is observed for high Rayleigh numbers. Heat transfer is considerably enhanced if the fluid contains nanoparticles. The highest increase in heat transfer occurs when Cu nanoparticles are used, and the lowest increase occurs when Al<sub>2</sub>O<sub>3</sub> nanoparticles are used. Increase in the average Nusselt number due to nanoparticle usage is up to 35% for Ra=10<sup>4</sup>, up to 32% for Ra=10<sup>5</sup> and up to 25% for Ra=10<sup>6</sup>. The results also show that the average Nusselt number increases nearly linearly with solid volume

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