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ON THE HYDRODYNAMICAL DEVELOPMENT LENGTH FOR PRESSURE-DRIVEN PULSATING LAMINAR FLOW IN A PIPE

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Abstract: In this study, pulsating laminar flow in a pipe is considered numerically. The focus is concentrated on the hydrodynamical development length. At first, steady flow in a pipe is examined and a new correlation for the hydrodynamical development length is developed based on the scale analysis. Then, the pressure-driven pulsating flow is studied. For a constant value of Reynolds number, the effect of the frequency, F and the amplitude, A of the pulsating flow on the development length are determined. It is shown that F for its range of $1 \le F \le 10$ dramatically affects the hydrodynamical development length while this effect becomes negligible either for very high values of F ($F \ge 100$) or for low values of F ($F \le 0.1$). It is also disclosed that the amplitude, A, has a considerable effect on the entrance length for lower values of F ($F \le 0.1$). For some specific cases, results obtained are compared with those available in the open literature and a good agreement is obtained.
Keywords: Pulsating flow, pressure-driven, hydrodynamic development length, frequency, amplitude

BİR BORU İÇERİSİNDE BASINÇ KAYNAKLI ATIMLI AKIŞ İÇİN HİDRODİNAMİK GELİŞME UZUNLUĞU

Özet: Bu çalışmada, bir boru içerisindeki atımlı akış sayısal olarak incelenmiştir. Çalışmanın ilgi odağı, hidrodinamik gelişme uzunluğudur. Öncelikle, bir boru içerisindeki daimi akış incelenmiş ve hidrodinamik gelişme uzunluğu için skala analizine dayalı yeni bir korelasyon geliştirilmiştir. Daha sonra, basınç kaynaklı atımlı akış durumu çalışılmış; Reynolds sayısının sabit bir değeri için, atımlı akış frekans (F) ve genliğinin (A) gelişme uzunluğu üzerindeki etkisi belirlenmiştir. F'in $1 \le F \le 10$ aralığındaki değerleri için hidrodinamik gelişme uzunluğu üzerinde önemli bir etkiye sahipken, F'in çok yüksek ($F \ge 100$) veya düşük ($F \le 0.1$) değerleri için bu etkinin ihmal edilebilir seviyede olduğu görülmüştür. Ayrıca, genliğin de etkisi incelenmiş, F'in düşük değerleri için ($F \le 0.1$), genlik etkisinin önemli olduğu görülmüştür. Bazı özel durumlar için elde edilen sonuçlar literatürde var olanlarla karşılaştırılmış ve iyi bir uyum elde edilmiştir.
Anahtar kelimeler: Atımlı akış, basınç kaynaklı, hidrodinamik gelişme uzunluğu, frekans, genlik

NOMENCLATURE

A	dimensionless amplitude, u_A/u_M
C_1, C_2	coefficients in Eq. [8]
D	diameter [m]
f, F	dimensional [Hz] and dimensionless frequency, $r_0^2 f/\nu$
$F\tau$	dimensionless phase
J_0	zeroth order Bessel function of first kind
L	pipe length [m]
L/D	hydrodynamical development length
n	wave number
$\tilde{P}_{en}^*$	complex form of dimensionless pressure gradient
r, r^*	dimensional [m] and dimensionless radial coordinate, r/r_0
r_0	radius [m]
Re	Reynolds number based on the pipe diameter, $u_M D/\nu$
t, τ	dimensional [s] and dimensionless time (period), $\nu t/r_0^2$
u, U	dimensional [m/s] and dimensionless velocity, u/u_M
u_A	dimensional velocity fluctuation amplitude [m/s]
x, X	dimensional [m] and dimensionless axial coordinate, x/D

Greek symbols

μ	dynamic viscosity [kg/ms]
ν	kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
τ	dimensionless time (period), $\nu t/r_0^2$
ω	angular frequency, $2\pi f$

Subscript

c	center
A	amplitude
M	cycle averaged mean

Superscript

~ Complex conjugate
* dimensionless quantity

INTRODUCTION

Pulsatile flow is a specific unsteady flow in which the resulting flow is composed of a mean component and a periodically varying time-dependent component. Research interest on this flow has been increasing due to its presence in some industrial applications and in biological flows such as blood flow.

When fluid enters a tube or a duct, a hydrodynamic boundary layer begins to develop on the wall of the tube or the duct. As a fluid flows downstream, the boundary layer on one wall begins to intercept the boundary layer from the opposite wall. At this axial location, a fully developed, i.e. non-varying, velocity profile is reached far downstream from the entrance. Beyond this location, the flow is called the hydrodynamically fully developed flow and, the region is called as the hydrodynamically fully developed flow region. The region up to this location is called as the hydrodynamic entrance region or the hydrodynamic development region. For a steady flow, many studies have appeared in the existing literature studying the length of the hydrodynamic entrance region, i.e. the hydrodynamic development length. Durst et al. (2005) reviewed the existing literature on the entrance length. They evaluated available correlations for the entrance length in terms of the Reynolds number and finally they proposed a new correlation. For time-dependent flows, there is a scarcity of data available on the entrance length. For an unsteady flow, Atabek and Chang (1961) analytically estimated the length of the hydrodynamic entrance region. He and Ku (1994) numerically studied the entrance length and they found that this length decreased with an increase in Womersly number. Krijger et al. (1991) numerically studied pulsating entry flow in a plane channel. In a very recent study, Ray et al. (2012) studied development length as functions of the mean Reynolds number, the amplitude of mass flow rate pulsation and the pulsation frequency in the moderate and high Reynolds number regimes.

In the present computational investigation, it is aimed at studying the hydrodynamic entrance length for a pulsating flow. The effects of the frequency and the amplitude of the pulsation on hydrodynamic development length are determined.

ANALYSIS

Mathematical Formulation

The axially symmetric flow in a pipe is considered. The flow is assumed to be unsteady, 2-D, incompressible and laminar with constant thermophsical properties. The problem geometry is shown in Fig. 1.

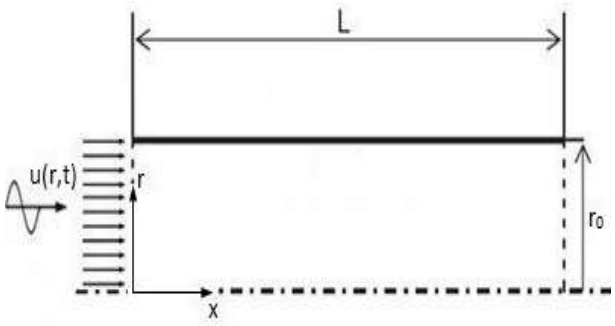

Figure 1. The geometry of the problem.

The partial differential equations governing the momentum transfer are the mass and momentum conservation equations. Considering the above assumptions, the most general form of the governing equations is given as follows:

$$\frac{\partial \rho \phi_k}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i \phi_k - \Gamma_k \frac{\partial \phi_k}{\partial x_i} \right) = S_{\phi_k} \quad k = 1,...,N \qquad (1)$$

Here, ϕ_k represents the general parameter, while Γ_k is the diffusion coefficient and S_{ϕ_k} is the source term. Table 1 lists ϕ_k, Γ_k and S_{ϕ_k} values for the conservations of mass and u-momentum equations.

Table 1. Diffusion coefficient and source terms

Equation	ϕ_k	Γ_k	S_{ϕ_k}
Continuity equation	1	-	-
u- momentum equation	u	μ	$-\frac{\partial p}{\partial x}$

Boundary conditions are given as follows:
At the pipe inlet,

$$u(r,t) = u_0(r,t), \; v = 0 \;\; at \;\; x = 0 \qquad (1)$$

where, $u_0(r,t)$, is constant for a uniform inlet flow, i.e.

$$u_0(t) = u_M \qquad (2)$$

while it consists of two components for a pulsating flow:

$$u_0(t) = u_M + u_A Sin(2\pi fT) \qquad (3)$$

At the pipe axis,

$$\frac{\partial u(r,t)}{\partial r} = 0 \;\; at \;\; r = 0 \qquad (4a)$$

At the pipe wall,

$$u(r,t) = 0 \ \ at \ \ r = r_0 \tag{4b}$$

The frequency-time domain of a sinusoid resulting from a pulsating flow is shown in Fig. 2. The sinusoid can be conceived of as existing a distance 1/T out along the frequency axis and running parallel to the time axis. Consequently, when we speak about the behavior of the sinusoid in the time domain, we mean the projection of the curve onto the time plane (Chapra and Canale, 2005). Any point follows a sinusoid along the circle shown in the figure. When the point reaches its initial position, a cycle is completed. Here τ represents the period, a constant, which is the time required for the point to reach its initial position. *F* represents the frequency that is the number of cycles per unit of time. The maximum value of the pulse gives us the amplitude.

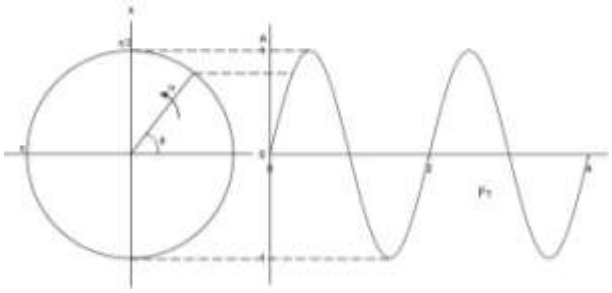

Figure 2. The frequency-time domain of a sinusoid resulting from a pulsating flow.

Numerical Analysis

In the numerical study, the Fluent V6.1.22 CFD package (Fluent 6 User's Guide) based on the finite volume method is used to transform and solve these equations. The discretization scheme used is hybrid for the convective terms in the momentum equations, and the SIMPLE algorithm for pressure-velocity coupling. A user-defined function is introduced to the software for the pulsating inlet velocity profile. The mesh is generated in the Gambit 2.1.6 preprocessor (Gambit 2 User's Guide). Rectangular-type elements are used in the mesh generation. A non-uniform mesh structure is used in radial direction in order to capture very high gradients occurring near the wall while a uniform mesh structure is preferred in the radial direction. For any set of values of the governing parameters studied, mesh structure used is refined until the solution becomes grid-independent. The convergence factor was 10^{-6} for each equation. For many test cases considered, a mesh size of 320x64 nodes in the axial and radial directions is shown to be adequate since further refinement in mesh structure represents negligible changes in regarding parameters.

Scale Analysis

A scale analysis is performed in order to predict the form of the entrance length correlation based on the driving mechanisms of the momentum transfer (Bejan, 1984; Arpacı and Larsen, 1984; Arpacı, 1997).

Scaling the mass conservation equation, we obtain

$$\frac{U_\infty}{X} \sim \frac{U_\infty}{D} \tag{5}$$

which presents a balance between velocity gradients.

Similarly, the u-momentum equation can be scaled as in the following:

$$U_\infty \frac{U_\infty}{X}, U_\infty \frac{U_\infty}{D^2} \sim \nu \frac{U_\infty}{D^2} \tag{6}$$

which presents a balance between inertia and viscous forces. From the above balances, we obtain

$$\mathrm{Re} = \frac{U_\infty D}{\nu} \tag{7}$$

which implies the following correlation for the hydrodynamic entrance length:

$$\frac{L}{D} = \frac{\mathrm{C_1 Re}}{1 + \mathrm{C_2 Re}} \tag{8}$$

RESULTS AND DISCUSSIONS

Here we consider a pipe with a length of 2 m and a diameter of 14 mm. On the choice of these values, an attention is paid to set L/D above the entrance length. The value of the Reynolds number is kept as constant 1000. The working fluid is air.
In this section, results are given for the steady flow and the pulsating flow in the following, respectively:

Steady flow

In order to verify the numerical analysis followed, dimensionless velocity profile in the hydrodynamically fully developed region is obtained and shown in Fig. 3, which agrees fairly very well with the analytical solution. Figure 4 shows the dimensionless developing velocity profiles at different axial locations. The velocity profile at X=24 coincides with that at the outlet the velocity profile is fully developed. The variation of the hydrodynamic entrance length with the Reynolds number is shown in Fig. 5. As expected, this length increases with an increase in Re. Some other results existing in the literature are also included in the figure.
When the results obtained are fitted to the Eq. (5) derived by the scale analysis, the unknown coefficients C_1 and C_2 are obtained and the equation takes the following form:

$$\frac{L}{D} = \frac{0.055703\,\mathrm{Re}}{1 + 0.0000013\,\mathrm{Re}} \tag{9}$$

which is very elegant and in harmony with some data (Sparrow et al, 1964; Hornbeck, 1964; Vrentas et al, 1966) available in the literature. Since its form is constituted from the physics of the problem following a scale analysis, we hope it to be a reliable correlation in future studies and textbooks.

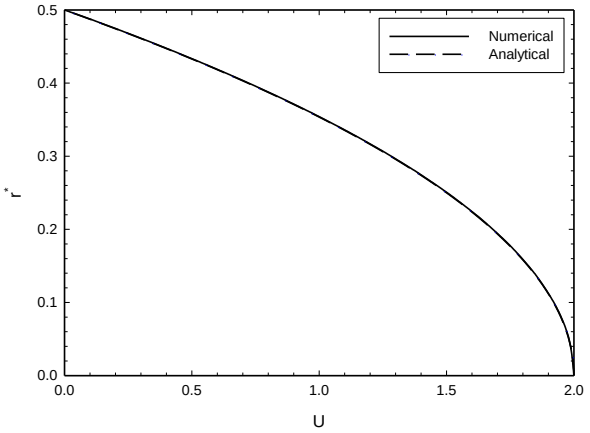

Figure 3. The dimensionless fully developed velocity profile.

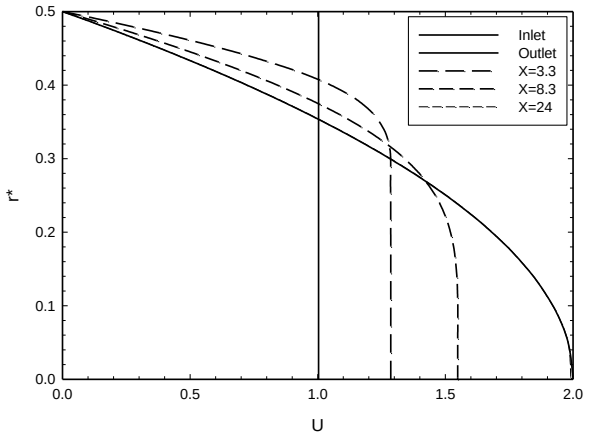

Figure 4. The dimensionless developing velocity profiles at different axial locations.

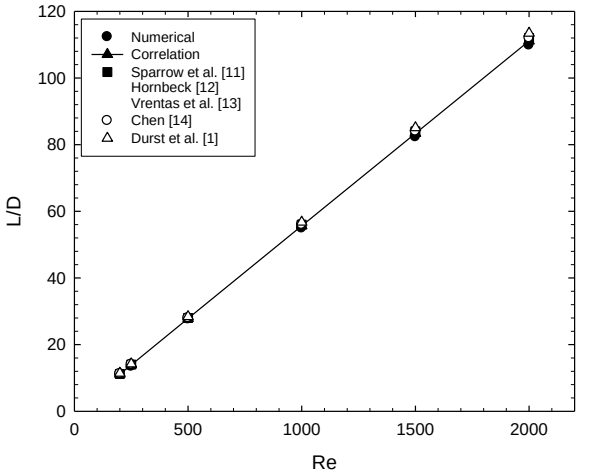

Figure 5. The variation of the hydrodynamic entrance length with the Reynolds number.

Pulsating flow

At first, the numerical methodology followed is validated by comparing the results obtained with those available in the literature. For α=12.5 (Womersly number), A=1 and Re=200, Fig. 6 shows dimensionless hydrodynamically fully developed velocity profile at *wt*=90 and 270 (phase angle) which agrees fairly very well with the analytical solution given as follows (Uchida, 1956):

$$U^* = \frac{u}{u_M} = 2\left(1-r^{*2}\right) - \sum_{n=1}^{\infty} \frac{4\tilde{P}_{en}^* i}{\pi n F}\left[1-\frac{J_0\left(\sqrt{2\pi n F}\, i^{3/2} r^*\right)}{J_0\left(\sqrt{2\pi n F}\, i^{3/2}\right)}\right] e^{2\pi n F \tau i} \quad (10)$$

As seen from Fig. 6, there also exists an excellent harmony among our numerical results and numerical results by He and Ku (1994).

Figures 7 and 8 illustrate the hydrodynamic length, L/D and the centerline velocity, U_c with Fτ for various values of F at Re=1000 and A=0.1, 0.5 and 0.95. As seen from Fig. 7, either for very high values of F (100 and 1000) or for very low values of F (0.1 and 0.01), the effect of F on L/D is negligible. This figure also shows that the entrance length is also affected strongly by the phase angle. With an increase in the frequency, the value of the phase angle at which the maximum value of the entrance length is attained shifts to higher values.

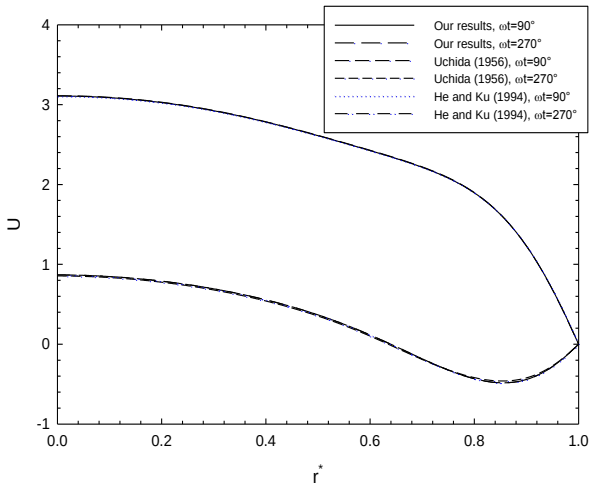

Figure 6. The dimensionless velocity profile at wt=90° and 270°.

For example, the maximum length is reached at the phase angle of 90^o for F=0.01 while it is at 180^o for F=10. Therefore, deciding the length according to a specific angle can be misleading. It is suggested here that the entrance length should be defined as the maximum length obtained by considering all the values of the phase angle. For a more detailed view on variation of the flow field with the regarding

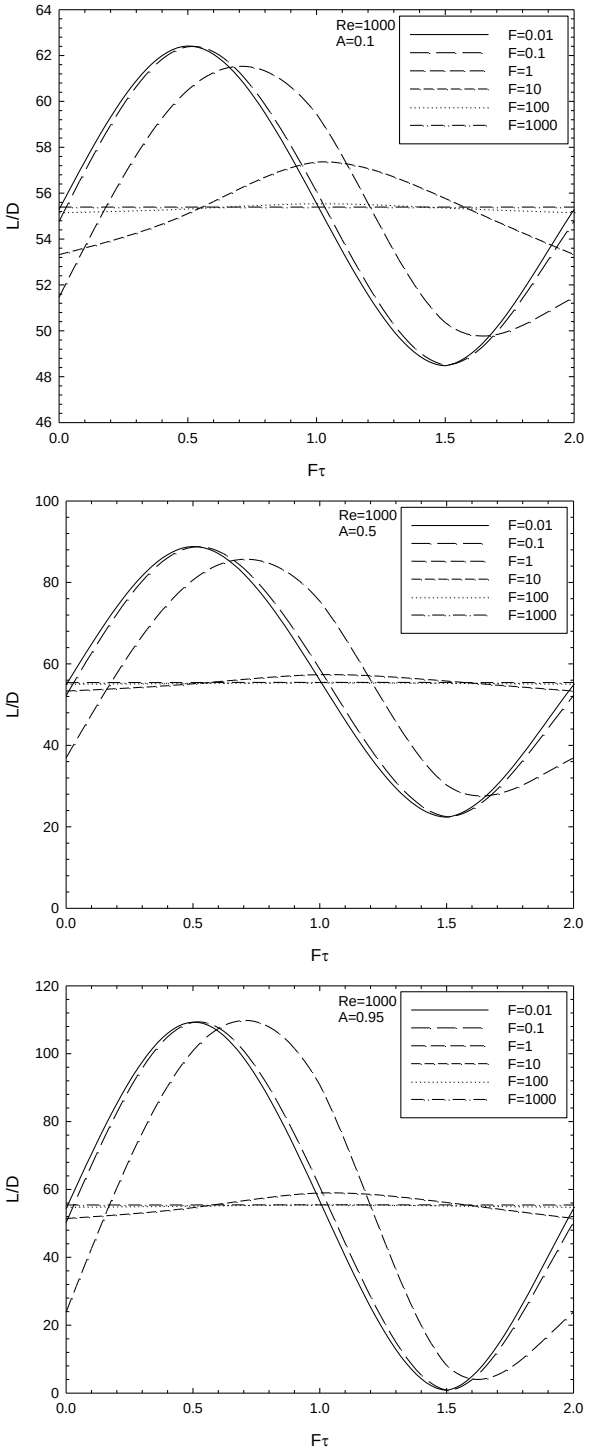

Figure 7. The hydrodynamic length, L/D with Fτ for various values of F at Re=1000 and A=0.1, 0.5 and 0.95.

parameters, the variation of the centerline velocity is also shown (see Fig. 8). Figure 9 illustrates the variation of the maximum entrance length with F for various values of A. For all the values of A considered, either for very high values of F (F≥100) or for very low values of F (F ≤0.1), the influence of F on the entrance length is negligible. With an increase of F beyond 0.1 decreases the entrance length. The most dramatic decrease in the entrance length is observed in the range of 1≤F≤10. The effect of the amplitude, A on the entrance length can also be observed from this figure. A has a considerable effect on the entrance length for lower values of F (F≤0.1). With an increase in F beyond F=0.1, the effect of A decreases and nearly diminishes for F≥100. For the phase angle of Fτ=0 and 0.5, the dimensionless velocity profiles at different axial locations at A=0.1, F=0.1 and Re=1000 are shown in Figs. 10 and 11, respectively. These figures well illustrate velocity fields at some specific axial stations, clearly describe their hydrodynamical development behavior and, in follows, the development length.

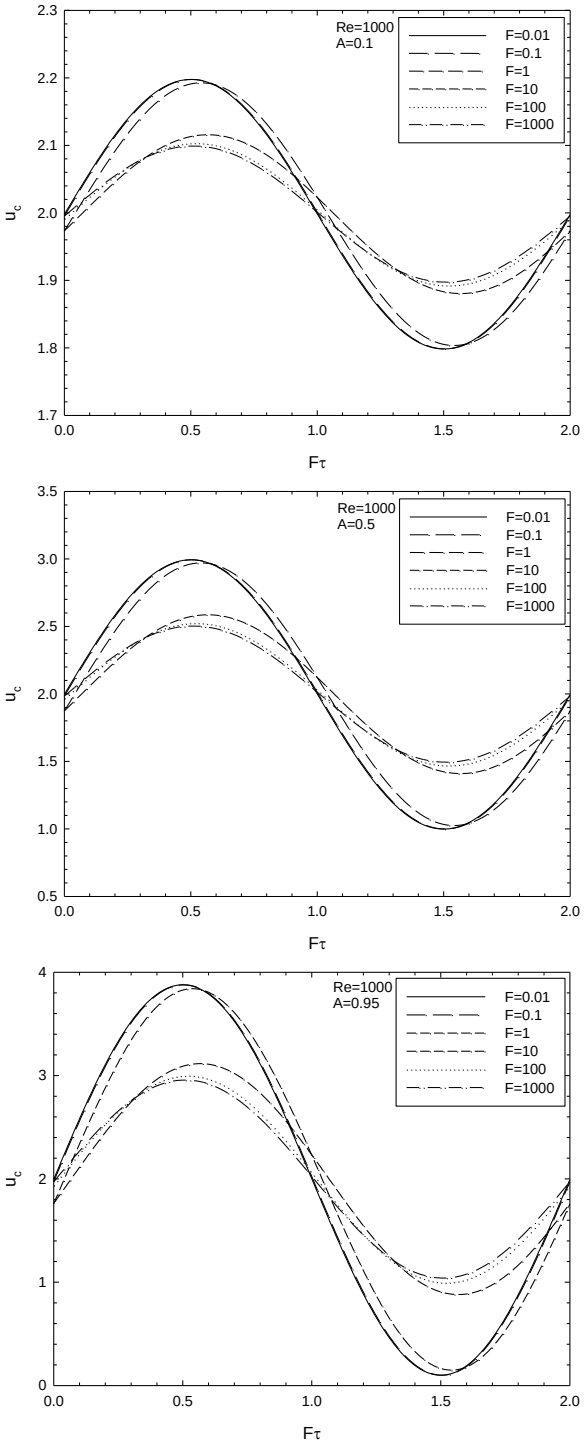

Figure 8. The centerline velocity, U_c with Fτ for various values of F at Re=1000 and A=0.1, 0.5 and 0.95.

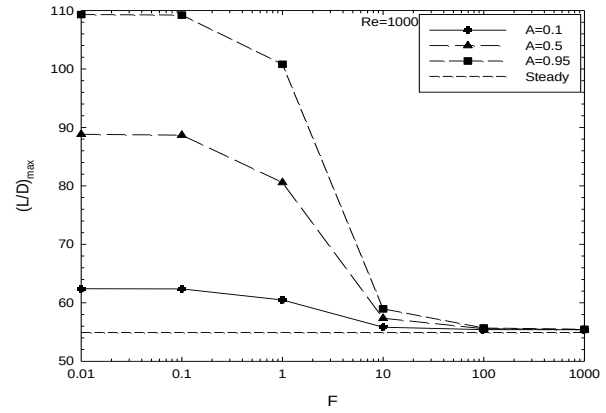

Figure 9. The variation of the maximum entrance length with F for various values of A.

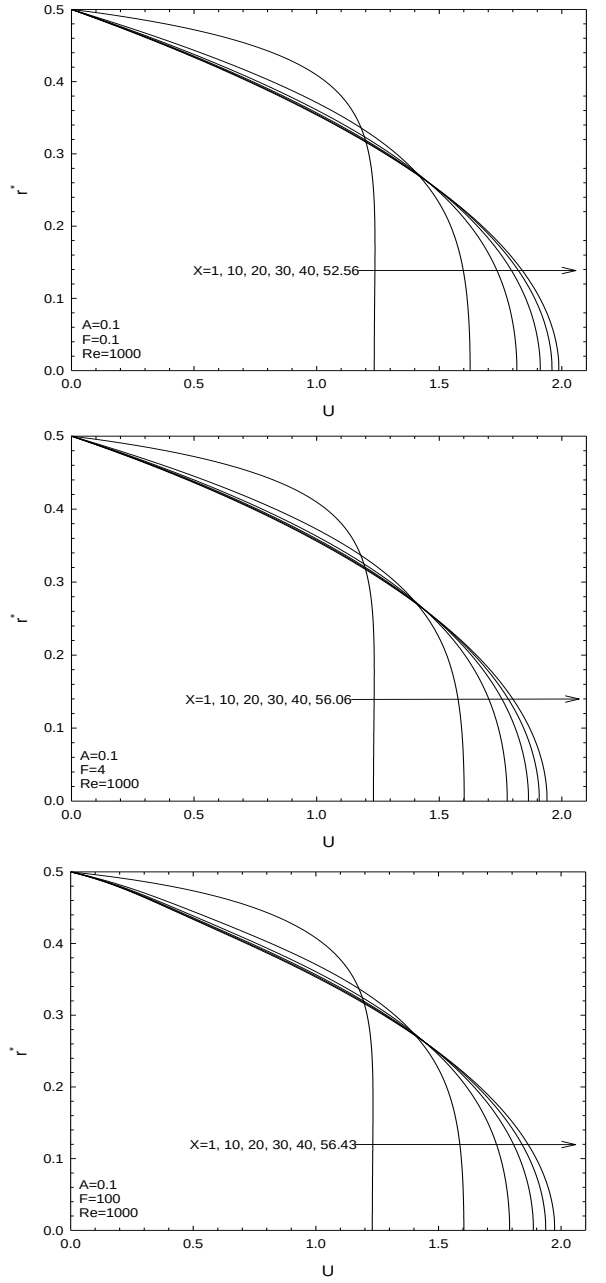

Figure 10. The dimensionless velocity profiles at different axial locations at A=0.1, F=0.1 and Re=1000 for Fτ=0.

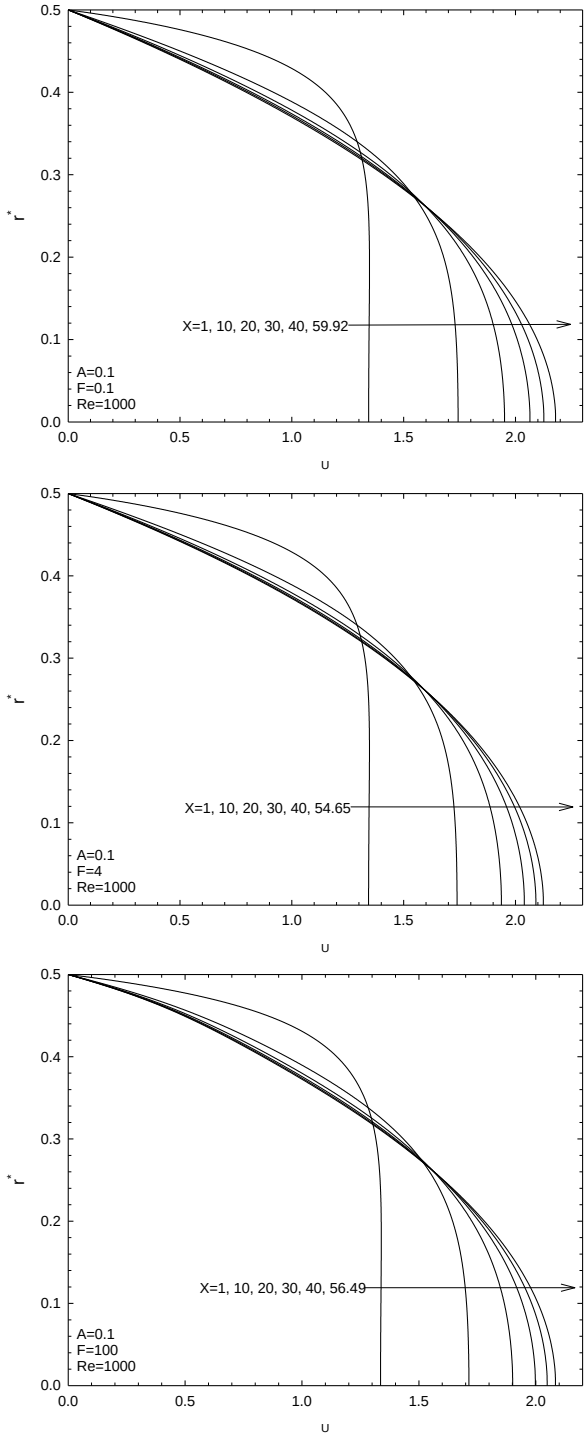

Figure 11. The dimensionless velocity profiles at different axial locations at A=0.1, F=0.1 and Re=1000 for Fτ=0.5.

CONCLUSIONS

The study has been focused on the hydrodynamical development length (i.e. the entrance length). At first, a new and useful correlation developed based on the scale analysis has been proposed for the hydrodynamical development length for the steady flow in pipes. The pulsating flow has been studied for a broad range of the frequency and the amplitude of the pulsating flow is studied at a constant value of Reynolds number. It is disclosed that the development length has been affected

dramatically by F in the range of $1 \leq F \leq 10$ while this effect becomes negligible either for very high values of F ($F \geq 100$) or for very low values of F ($F \leq 0.1$). It is also disclosed that the amplitude, A, has a considerable effect on the entrance length for lower values of F ($F \leq 0.1$). It is obtained that with an increase in F beyond F=0.1, the effect of A decreases and nearly diminishes for $F \geq 100$.

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